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Sustainable Productivity Change in the U.S. Dairy Sector*

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Abstract

Sustainable production refers to the creation of goods and services in ways that minimize the production of bad outputs and the use of natural capital. We explain how to measure productivity change in a way that reflects well on firms that adopt sustainable production practices. We illustrate the properties of our so-called sustainable productivity index using simulated data. We then compute sustainable productivity index numbers for a sample of U.S. dairy producers. Finally, we estimate the extent to which changes in productivity have been driven by technical progress, environmental change and various types of efficiency change.

Keywords: U.S. dairy sector, sustainable productivity index, total factor productivity index, good outputs, bad outputs, stochastic frontier model, productivity change, GHG emissions

JEL Classification: Q12, D22, D24

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1 Introduction

The majority of U.S. dairy operations are family-owned and operated. They contribute to the national economy through employment, producer income, and numerous ecosystem services that help preserve landscapes and open spaces (Johnston 2002). According to the U.S. Department of Agriculture (USDA 2024*b*), the sector produced 226.36 billion pounds of milk in 2023, accounting for $46.13 billion in farm revenue. The sector also plays a significant role in global markets: in 2023 it exported 2.63 million metric tonnes of dairy products, generating $8 billion in export income.

In recent years there has been a structural shift in the U.S. dairy sector, with the majority of milk production becoming concentrated in a few states: California, Wisconsin, Idaho, Texas, New York, Michigan, Minnesota, Pennsylvania, New Mexico, and Washington. In 2023, these states collectively accounted for approximately 74% of national milk production. The sector has also been increasingly characterized by a shift in production towards larger and fewer dairy operations. In 2023, there were 24,082 dairy operations that sold milk off-farm. The 2,013 farms that had herd sizes greater than 1,000 accounted for 66% of total milk sales (USDA 2024*a*).

On the downside, the dairy sector was a large contributor to U.S. greenhouse gas (GHG) emissions. Methane (CH_4) and nitrous oxide (N_2O) emissions from enteric fermentation processes and manure management systems were the main culprit. In 2021, the sector generated CH_4 and N_2O emissions amounting to 90.5 million metric tonnes in carbon dioxide equivalents (CO_2e), the equivalent of 32.5% of the aggregate 278.3 million metric tonnes in CO_2e that was emitted from all forms of livestock production in the United States (USEPA 2023).

The United States has committed to GHG emissions reduction as part of an effort to address climate change. Achieving the goal of emissions reduction involves tracking and measuring GHG emissions across every sector (USEPA 2023). The Inflation Reduction Act of 2022 (U.S. Congress 2022) appropriated funding to establish and advance a research network to collect data on GHG emissions. This included $300 million to support the U.S. Department of Agriculture's measuring, monitoring, reporting and verification of GHG emissions. The overarching objective is to aid producers' efforts at adopting climate-smart practices such as nutrient management plans, cover cropping, tillage practices, and livestock management practices.

A recent study on productivity in the U.S. dairy sector (Njuki 2022) estimated that total factor productivity (TFP) grew by an average of 2.51% per year between 2000 and 2016. Other measures of productivity such as milk yield have also seen growth in recent years. For example, in 2023 average milk production per cow in the U.S. dairy sector stood at 24,117 pounds per head, an increase of 8.39% from 22,249 pounds per head in 2014. The problem with these traditional measures is that they only account for the rate of growth in 'good outputs' relative to the rate of growth in conventional inputs. Because they neglect the environmentally detrimental aspects of dairy production technologies, they very likely overstate the productive performance of farms. To address this concern we need an alternative sustainable productivity measure that can be used to assess the long-term viability of farms. This paper develops such a measure.

The structure of the paper is as follows. Section 2 explains how we combine output and input volume indexes in such a way that the resulting sustainable productivity index (SPI) numbers increase as good outputs increase and bad outputs and inputs fall. Sections 3 and 4 explain how a combination of economic theory and statistical methods (i.e, econo-metrics) can be used to identify the main economic drivers of productivity change: technical progress, environmental change, and various measures of technical, scale and mix efficiency change. Section 4 describes our dataset: we have 1657 observations on milk production, GHG emissions and inputs used on dairy farms in 28 states in the years 2016 and 2021. Section 5 presents and discusses our results. Section 6 summarizes the paper, and Section 7 offers some concluding remarks.

2 Measuring Productivity Change

Productivity indexes are ratio measures of output volume change divided by ratio measures of input volume change (i.e., output volume indexes divided by input volume indexes). Well-known productivity indexes include the multilateral productivity index of Caves, Christensen and Diewert (1982) (CCD) and the Bjurek-Hicks-Moorsteen (BHM) index loosely described by Diewert (1992, p. 240) and explicitly defined by Bjurek (1996, eq. 9). Productivity indicators, on the other hand, are *difference* measures of output volume change minus *difference* measures of input volume change (i.e., measures of change in net output). The best-known productivity indicator is arguably the Malmquist-Luenberger indicator of Chambers et al. (1996). In this paper, we are only concerned with measuring changes in productivity, not net output (i.e., we are only concerned with indexes, not indicators).

Efforts to incorporate bad outputs into productivity indexes can be traced back at least as far as Pittman (1983). The multilateral index proposed by Pittman is effectively a CCD index with bad outputs valued at shadow prices rather than (unobserved) market prices (Pittman 1983, p. 886). One of the problems with this index is that it does not satisfy an identity axiom. In a multilateral context, for example, the identity axiom says that if two firms use exactly the same inputs to produce exactly the same outputs, then they must be regarded as being equally productive.

Other indexes that attempt to incorporate bad outputs include the environmentally-adjusted binary Törnqvist (EABT) index described by Shaik (1998, eq. 5), the enhanced hyperbolic productive efficiency (EHPE) measure of Färe et al. (1989, eq. 19), and the environmentally-sensitive hyperbolic Malmquist (ESHM) index of Ball et al. (2004, eq. 11). One of the problems with these indexes is that, except in restrictive special cases, they do not satisfy a transitivity axiom. In a multilateral context, this axiom says that if we compare the productivity of two firms directly, then we should get the same index numbers as when we compare the productivity of the two firms indirectly via a third firm.

O'Donnell (2022) considers a sustainable productivity index (SPI) that overcomes these problems. Like all productivity indexes, the SPI can be written as an output volume index divided by an input volume index. On the output side, the index that compares the outputs firm i in period t with the outputs of firm k in period s is:

$$QI(g_{ks}, b_{ks}, g_{it}, b_{it}) \equiv \frac{Q(g_{it}, b_{it})}{Q(g_{ks}, b_{ks})} \tag{1}$$

where $g_{it} = (g_{1it}, \ldots, g_{Nit})'$ is a vector of good outputs, $b_{it} = (b_{1it}, \ldots, b_{Kit})'$ is a vector of bad outputs, and $Q(.)$ is any nonnegative, scalar-valued aggregator function that is nondecreasing in the goods and nonincreasing in the bads. In practice, the choice of aggregator function is a matter of taste. O'Donnell (2022) suggests:

$$Q(g_{it}, b_{it}) \propto G(g_{it})^{1-\alpha} B(b_{it})^{-\alpha} \tag{2}$$

where $G(.)$ and $B(.)$ are nonnegative, nondecreasing, linearly-homogeneous, scalar-valued aggregator functions, and $\alpha \in [0, 1]$ is a parameter that measures the extent to which policy-makers want to account for the bads. If $\alpha = 0$, then equation (2) measures the volume of good outputs only. If $\alpha = 0.5$, then it is equivalent to the square root of the measure of economic performance defined by Stetter & Sauer (2022, eq.2). The Stetter & Sauer measure has its antecedents in the measure of eco-efficiency (EE) defined by Schmidheiny & Zorraquin (1996) and subsequently applied by Kuosmanen & Kortelainen (2005), Kortelainen (2008), Picazo-Tadeo et al. (2011), Picazo-Tadeo et al. (2014), and Perez-Urdiales et al. (2016). If $\alpha = 0.5$ and special types of distance functions are used as aggregator functions, then the output index defined by equations (1) and (2) is equivalent to the square root of the environmental performance index defined by Färe et al. (2004, eq. 5). Thus, there is precedent in the productivity literature for setting $\alpha = 0.5$.

An alternative to setting $\alpha = 0.5$ is to follow Cobourn et al. (2024) and set α to the value that minimizes the variance of the logarithm of the output index defined by equations (1) and (2). This involves rewriting equations (1) and (2) in the form of the following regression model:

$$\ln GI(g_{ks}, g_{it}) = \delta \ln BI(b_{ks}, b_{it}) + e_{it} \tag{3}$$

where $GI(g_{ks}, g_{it}) \equiv G(g_{it})/G(g_{ks})$ is a good output index, $BI(b_{ks}, b_{it}) \equiv B(b_{it})/B(b_{ks})$ is a bad output index, $\delta \equiv \alpha/(1-\alpha) \geq 0$ is a parameter to be estimated, and $e_{it} \propto \ln QI(g_{ks}, b_{ks}, g_{it}, b_{it})$ is an unobserved variable that in any other context would be interpreted as statistical noise. In this paper we estimate $\delta \geq 0$ under the assumption that e_{it} is an independent normal random variable with a mean of zero and a variance of σ_e^2. We then recover α using the fact that $\alpha = \delta/(1+\delta)$.

On the input side, the index that compares the inputs of firm i in period t with the inputs of firm k in period s is:

$$XI(x_{ks}, x_{it}) \equiv \frac{X(x_{it})}{X(x_{ks})} \tag{4}$$

where $x_{it} = (x_{1it}, \ldots, x_{Mit})'$ is a vector of inputs and $X(.)$ is any nonnegative, nondecreasing, linearly-homogeneous, scalar-valued aggregator function. The SPI that compares the sustainable productivity of firm i in period t with the sustainable productivity of firm k in period s can then be written as:

$$SPI(.) \equiv \frac{QI(g_{ks}, b_{ks}, g_{it}, b_{it})}{XI(x_{ks}, x_{it})} = \frac{GI(g_{ks}, g_{it})^{1-\alpha} BI(b_{ks}, b_{it})^{-\alpha}}{XI(x_{ks}, x_{it})}. \quad (5)$$

This index satisfies the identity, weak monotonicity, commensurability, time-space reversal and transitivity axioms listed in Appendix 1. Indexes that satisfy these axioms can be used to make sensible comparisons of productivity across both time and space. Indexes that fail one or more of these axioms yield nonsensical numbers and are of no practical use.

To make some of these ideas more concrete, consider the simulated good output volumes (in the columns labeled g_1 and g_2), bad output volumes (b_1 and b_2), input volumes (x_1 and x_2), good output prices (p_1 and p_2), bad output shadow prices (s_1 and s_2) and input prices (w_1 and w_2) reported in Table 1. The bad output shadow prices were calculated at points on the boundary of an estimated output set; the estimated output set was the set defined by Kuosmanen (2005, eq. 4), and the frontier points were identified by scaling down the bad output vectors as far as possible while holding the good output and input vectors fixed.

Five sets of associated productivity index numbers are reported in Table 2: the CCDP index is the CCD-type index of Pittman (1983); the EABT index is the environmentally-adjusted binary Törnqvist index described by Shaik (1998, eq. 5); the EHPE index is the enhanced hyperbolic productive efficiency measure of Färe et al. (1989, eq. 19); the SPIA index is the sustainable productivity index defined by equation (5), constructed using additive volume indexes (with average prices as weights) and $\alpha = 0.5$; and the SPIB index is an alternative sustainable productivity index constructed using benefit-of-the doubt (BOD) volume indexes and, again, $\alpha = 0.5$. For technical details concerning additive and BOD volume indexes, see O'Donnell (2018, ch. 3). Observe that the CCDP index number for firm A differs from the number for firm D, even though both firms use the same inputs to produce the same outputs. This is nonsense. Similarly, the EABT index number for firm C differs from the number for firm E, even though those two firms also use the same inputs to produce the same outputs. The EHPE numbers are of no practical use in this example because there are not enough degrees of freedom to reliably estimate the relevant distance functions; a much larger sample would be needed to illustrate the pathological properties of the EHPE index. Only the SPIA and SPIB indexes yield sensible numbers: both indexes indicate that firms A and D are equally productive (the identity axiom), that firm C is more productive than firm B (the weak monotonicity axiom), and that firms C and E are equally productive (a combination of the identity and transitivity axioms).

Table 1: Simulated Volumes and Prices

Firm	g_1	g_2	b_1	b_2	x_1	x_2	p_1	p_2	s_1	s_2	w_1	w_2
A	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-1.0	0.0	1.0	1.0
B	0.3	0.5	0.5	0.1	1.0	0.6	1.0	8.0	0.0	-10.0	4.0	1.0
C	0.7	0.5	0.2	0.1	0.8	0.4	6.0	1.0	-5.0	0.0	1.0	2.0
D	1.0	1.0	1.0	1.0	1.0	1.0	3.0	1.0	-1.0	0.0	1.0	3.0
E	0.7	0.5	0.2	0.1	0.8	0.4	1.0	3.0	-5.0	0.0	3.0	1.0

Table 2: Selected Productivity Index Numbers

Firm	CCDP	EABT	EHPE	SPIA	SPIB
A	1	1	1	1	1
B	0.835	0.633	1	1.377	1.581
C	5.316	5.519	1	3.146	2.339
D	0.707	1.000	1	1	1
E	4.515	4.057	1	3.146	2.339

3 Explaining Productivity Change

In order to develop evidence-based policies that have the potential to raise levels of productivity, policy-makers must first find an economic explanation for productivity change. O'Donnell (2018) explains how a combination of economic theory and statistical methods (i.e., econo-metrics) can be used for this purpose. This section explains how the approach can be used to identify the four main factors that drive changes in the productivity index defined by (5): technical progress (i.e., the discovery of new techniques for transforming inputs into outputs), environmental change (i.e., changes in variables that are physically involved in the production process but never within the control of managers, such as rainfall), technical efficiency change (i.e., changes in how well managers select and implement production techniques) and economies of scale and substitution (i.e., changes in the scale of operations, the output mix, and the input mix).

Estimating these components involves estimating production frontiers. The most relevant frontiers are the boundaries of period-and-environment-specific sets. A period-and-environment output set, for example, is a set containing all outputs that can be produced using given inputs in a given period in a given production environment. For example, let $z_{it} = (z_{1it}, \ldots, z_{Jit})'$ denote a vector that characterises the production environment of firm i in period t. The set of outputs that can be produced using x_{it} in period t in an environment characterised by z_{it} is:

$$P^t(x_{it}, z_{it}) = \{(g, b) : x_{it} \text{ can produce the good output vector } g \text{ and the bad output vector } b \text{ in period } t \text{ in environment } z_{it}\}. \quad (6)$$

To estimate the boundary of this set, we need to make some assumptions about what can and cannot be produced using the set of production technologies available in period t. In this paper we assume that, with the so-called period-t technology set:

A1: it is possible to produce zero output;

A2: there is a limit to what can be produced using a finite amount of inputs;

A3: a positive amount of at least one input is needed in order to produce a strictly positive amount of any output;

A4: output sets contain all the points on their boundaries;

A5: if particular inputs can be used to produce a given output vector, then they can also be used to produce a scalar contraction of that output vector;

A6: if particular outputs can be produced using given inputs, then they can also be produced using more inputs; and

A7: if given inputs can be used to produce particular outputs, then they can also be used to produce the same bad outputs and fewer good outputs.

Sets can be difficult to work with mathematically. Fortunately, assumptions A2 and A5 mean that $P^t(x_{it}, z_{it})$ can be conveniently represented by a period-and-environment-specific output distance function. Such a function gives the reciprocal of the largest factor by which it is possible to scale up the output vector before reaching the boundary of the output set. In the case of firm i in period t, the value of the output distance function is:

$$D_O^t(x_{it}, g_{it}, b_{it}, z_{it}) = \min\{\rho > 0 : (g_{it}, b_{it})/\rho \in P^t(x_{it}, z_{it})\}. \quad (7)$$

By construction, this function is nonnegative and linearly homogeneous in outputs. Assumptions A6 and A7 mean it is also nonincreasing in inputs and nondecreasing in good outputs. A specific example of an output distance function that has these properties (and is also consistent with assumptions A1 to A5) is:

$$D_O^t(x_{it}, g_{it}, b_{it}, z_{it}) = \left[\exp\left(\alpha_t + \sum_{j=1}^{J} \delta_j z_{jit} + \sum_{j=1}^{J} \delta_{jj} z_{jit}^2 \right) \prod_{m=1}^{M} x_{mit}^{\beta_m} \right]^{-1} \times \left(\frac{\theta B(b_{it})^2}{B(b_{it}) - \gamma G(g_{it})} \right) \tag{8}$$

where $G(.)$ and $B(.)$ are nonnegative, nondecreasing, linearly-homogeneous, scalar-valued aggregator functions, $\alpha_t \geq \alpha_{t-1}$, $(\delta_1, \ldots, \delta_J, \beta_1, \ldots, \beta_M, \theta)' \geq 0$, $(\delta_{11}, \ldots, \delta_{JJ})' \leq 0$ and $0 \leq \gamma < B(b_{it})/G(g_{it})$.

To better understand the relationship between distance functions and output sets, consider the case where $\gamma = \theta = 1$ and the first term on the right-hand side of equation (8) (i.e., the term raised to the power of -1) is equal to one. In this case, the boundary of the output set is given by the curve passing through point B in Figure 1. It is clear from this figure that assumptions A1, A2, A4, A5 and A7 are satisfied. It is also clear that the firm operating at point A could have scaled up its outputs by a factor of 4 before reaching the frontier at point B. The value of the output distance function is therefore $1/4 = 0.25$. In this paper, we use the value of the output distance function as our measure of technical efficiency; in this paper, the firm operating at point A is said to be only 25% technically efficient.

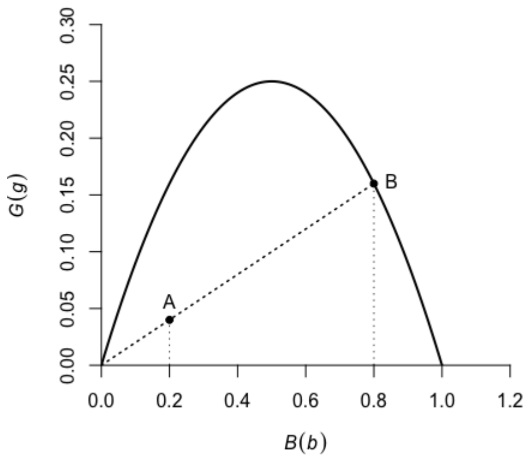

Figure 1: An Output Set

Finally, to see how distance functions can be used to identify the economic drivers of productivity change, it is perhaps easiest to rearrange equation (8) as:

$$\begin{aligned}1 = {} & \exp(\alpha_t) \times \exp\left(\sum_{j=1}^{J} \delta_j z_{jit} + \sum_{j=1}^{J} \delta_{jj} z_{jit}^2\right) \\ & \times \prod_{m=1}^{M} x_{mit}^{\beta_m} \left(\frac{B(b_{it}) - \gamma G(g_{it})}{\theta B(b_{it})^2}\right) \times D_O^t(x_{it}, g_{it}, b_{it}, z_{it}). \end{aligned} \tag{9}$$

A similar equation holds for firm k in period s. Dividing one equation by the other and then multiplying both sides of the result by the SPI defined by (5) yields:

$$
\begin{aligned}
SPI(.) = \; & \left[\frac{\exp(\alpha_t)}{\exp(\alpha_s)}\right] \times \left[\frac{\exp\left(\sum_{j=1}^{J}\delta_j z_{jit} + \sum_{j=1}^{J}\delta_{jj} z_{jit}^2\right)}{\exp\left(\sum_{j=1}^{J}\delta_j z_{jks} + \sum_{j=1}^{J}\delta_{jj} z_{jks}^2\right)}\right] \\
& \times \left[\prod_{m=1}^{M}\left(\frac{x_{mit}}{x_{mks}}\right)^{\beta_m}\frac{X(x_{ks})}{X(x_{it})}\right. \\
& \quad \left. \times \left(\frac{B(b_{it}) - \gamma G(g_{it})}{B(b_{ks}) - \gamma G(g_{ks})}\right)\frac{G(g_{it})^{1-\alpha}}{G(g_{ks})^{1-\alpha}}\left(\frac{B(b_{ks})}{B(b_{it})}\right)^{2+\alpha}\right] \\
& \times \left[\frac{D_O^t(x_{it}, g_{it}, b_{it}, z_{it})}{D_O^s(x_{ks}, g_{ks}, b_{ks}, z_{ks})}\right]. \qquad (10)
\end{aligned}
$$

The first term in square brackets on the right-hand side is an output-oriented technology index (OTI) (a measure of technical progress); the second term is an output-oriented environment index (OEI) (a measure of changes in the production environment); the third term (broken across two lines) is an output-oriented scale-and-mix efficiency index (OSMEI) (a measure of changes in economies of scale and substitution); and the last term is an output-oriented technical efficiency index (OTEI) (a measure of changes in how well technologies are used). As we said earlier, estimating these components involves estimating production frontiers.

4 Estimating Frontiers

It is common to estimate production frontiers using data envelopment analysis (DEA) models. DEA models that explicitly account for the production of bad outputs include the models of Färe et al.(1989, 2004), Kuosmanen (2005), Kortelainen (2008), Picazo-Tadeo et al. (2014) and Beltran-Esteve & Picazo-Tadeo (2017), among others. The main problem with DEA models is that they do not allow for omitted variables and other sources of statistical noise. Another problem is that, because they involve linear programming, they do not lend themselves to decomposing productivity indexes that have been constructed using nonlinear aggregator functions (e.g., the SPI given by equation (5)). For these reasons, we have chosen to estimate the drivers of productivity change using a stochastic frontier model.

The particular stochastic frontier model we use is nonlinear in the parameters. To derive our model, we begin by supposing that the output distance function is given by the theoretically-plausible equation (8). After some simple algebra, this equation can be rewritten as:

$$\ln g_{1it} = \alpha_t + \sum_{j=1}^{J} \delta_j z_{jit} + \sum_{j=1}^{J} \delta_{jj} z_{jit}^2 + \sum_{m=1}^{M} \beta_m \ln x_{mit} \\ - \ln\left(\frac{\theta B(b_{it}^*)^2}{B(b_{it}^*) - \gamma G(g_{it}^*)}\right) - u_{it} \tag{11}$$

where $g_{it}^* \equiv g_{it}/g_{1it}$ and $b_{it}^* \equiv b_{it}/g_{1it}$ are vectors of normalized outputs and $u_{it} \equiv -\ln D_O^t(x_{it}, g_{it}, b_{it}, z_{it})$ is a nonnegative output-oriented technical inefficiency effect. Our stochastic frontier model is obtained by simply adding a random noise component, v_{it}, to this equation. This noise component accounts for omitted variable errors, measurement errors, and the strong possibility that the true output distance function is not, in fact, given by equation (8). One of the implications of adding a noise component to equation (11) is that we must also add a statistical noise index (SNI) to the productivity decomposition given by equation (10).

It is common to estimate stochastic frontier models using the method of maximum likelihood (ML). The main problem with ML estimators is that there are no satisfactory methods for imposing inequality constraints of the type listed below equation (8): binding inequality constraints will lead to ML parameter estimates that lie on the constraint boundary, with standard errors of zero, implying we know the true values with certainty. The most practical way forward in these situations is to use a Bayesian estimator.

Bayesian estimation involves summarizing everything we know about the unknown parameters in the form of a posterior probability density function (pdf). Bayes's Theorem says the posterior pdf is proportional to the likelihood function (i.e., the function that is maximized in the ML approach) times the prior (i.e., the pdf that summarizes everything we know about the parameters before the data are observed). The exact form of the likelihood function depends on the assumptions we make about the noise and inefficiency effects. We follow standard practice in the Bayesian stochastic frontier literature and assume that the noise effects are independent $N(0, h^{-1})$ random variables and the inefficiency effects are independent exponential random variables with rate parameter λ. Our prior pdf is a combination of proper pdfs on h, λ and each of the unknown parameters in equation (11). To be specific, our joint prior pdf is the product of the following:

$$p(h) = f_G(h|1, \kappa), \tag{12}$$

$$p(\lambda) = f_G(\lambda|1, -\ln\tau), \tag{13}$$

$$p(\alpha_t) \propto f_N(\alpha_t|0, 1/\kappa)I(\alpha_t \geq \alpha_{t-1}) \text{ for all } t, \tag{14}$$

$$p(\delta_j) \propto f_N(\delta_j|0, 1/\kappa)I(\delta_j \geq 0) \text{ for all } j, \tag{15}$$

$$p(\delta_{jj}) \propto f_N(\delta_j|0, 1/\kappa)I(\delta_{jj} \leq 0) \text{ for all } j, \tag{16}$$

$$p(\beta_m) \propto f_N(\beta_m|0, 1/\kappa)I(\beta_m \geq 0) \text{ for all } m, \tag{17}$$

$$p(\theta) \propto f_N(\theta|0, 1/\kappa)I(\theta \geq 0) \text{ and} \tag{18}$$

$$p(\gamma) \propto f_N(\gamma|0, 1/\kappa)I(0 \leq \gamma \leq \min_{i,t}\{B(b_{it})/G(g_{it})\}) \tag{19}$$

where the notation $f_G(h|1, \kappa)$ says h is a gamma random variable with shape parameter 1 and rate parameter κ (equivalently, an exponential random variable with rate parameter κ), $f_N(\alpha_t|0, 1/\kappa)$ says α_t is a normal random variable with mean zero and variance $1/\kappa$, and $I(.)$ is an indicator function that takes the value 1 if the argument is true (and 0 otherwise). In this paper we set $\kappa = 0.0001$ to ensure that, apart from the inequality constraints, the joint prior is noninformative. In the Bayesian stochastic frontier literature it is common to set the hyperparameter τ to the prior median of efficiency, usually something in the range 0.7 to 0.9. In this paper we follow Koop et al. (1994, p. 341) and van den Broeck et al. (1994, p. 293) and set the prior median of efficiency to $\tau = 0.875$.

Bayesian point estimation involves evaluating characteristics of marginal posterior pdfs (e.g., means and variances). In practice, this involves simulating from the joint posterior pdf. The most widely used simulation algorithms yield chains of draws that have the properties of Markov processes. This paper drew Markov chains of length 500,000 using the R package of Plummer (2019). The stationarity (or 'convergence') of the chains was confirmed visually.

5 Data

The data used in this study were derived from the Agricultural Resource Management Survey (ARMS). The ARMS is a primary source of information for the U.S. Department of Agriculture on production practices, resource use, and financial characteristics of U.S. farms. The farm surveys are conducted over three phases. The first phase is a screening phase intended to qualify the enterprise as a farm. A second phase targets specific crops and collects information on cost and production practices at randomly selected field-levels. The third phase collects information on the whole farm operation, including farm household characteristics.

For this study we used input-output data on farm-level conventional (i.e., non-organic) dairy operations from the 2016 and 2021 surveys. The inputs were number of milk cows, acres of land dedicated to dairy production, paid and unpaid labor hours, quantity of purchased feed, the real cost of operating capital, real expenditures on fuel and lubricants, and the real cost of veterinary services. The real cost of capital (an implicit quantity index) was obtained by dividing the cost of capital by the interest rate on non-farm debt. The real costs of fuel, lubricants and veterinary services were obtained by dividing monetary values by the producer price index for farm products published by the National Agricultural Statistics Service (NASS). Production characteristics of the land input were captured using dummy variables for the nine ERS farm resource regions within which farms are characterized by similar physiography (i.e., Heartland, Northern Crescent, Northern Great Plains, Prairie Gateway, Eastern Uplands, Southern Seaboard, Fruitful Rim, Basin and Range, and Mississippi Portal). The good output was milk in cwt, and the bad output was GHG emissions measured in carbon dioxide equivalents (CO_2e). This variable was constructed by aggregating methane and nitrous oxide emissions using conversion factors from the scientific literature: details are provided in Appendix 2.

We augmented the input-output data with annual county-level contemporaneous mean temperature and cumulative precipitation information generated from the Parameter-elevation Regressions on Independent Slopes Model (PRISM) Climate group. The PRISM data are available at the county-level and interpolate maximum and minimum temperatures over 4-by-4 kilometer grid cells across the conterminous United States; the interpolation procedure utilizes information from nearby weather stations, while considering distance, elevation, topographic position and coastal proximity (Daly et al., 2008; Daly et al., 2015).

Table 3 provides descriptive statistics and units of measurement for our input, output and environmental variables. The first ten variables in this table are dummy variables. The mean of the Period variable, for example, indicates that

34.8% of observations were from 2021, while the mean of the Heartland variable indicates that 23.6% of observations were from farms in the Heartland.

Table 3: Descriptive Statistics

	Min	Max	Mean	Median
Period (2021 = 1)	0	1	0.348	0
Heartland	0	1	0.236	0
Northern Crescent	0	1	0.346	0
Northern Great Plains	0	1	0.014	0
Prairie Gateway	0	1	0.038	0
Eastern Uplands	0	1	0.117	0
Southern Seaboard	0	1	0.057	0
Fruitful Rim	0	1	0.143	0
Basin and Range	0	1	0.045	0
Mississippi Portal	0	1	0.003	0
Milk (cwt)	671	4580000	147954	40000
GHG (tonnes CO_2 equiv.)	57	145989	4617	1322
Mean Temperature (deg. F)	11.20	61.50	28.39	26.80
Cum. Precipitation (inches)	2.41	150.96	40.47	39.31
Land (acres)	0.00	3500	86	20
Cows (no.)	8	21000	651	195
Operating capital (index)	4	110059	2497	705
Labour (hours)	1	1083940	23999	7800
Purchased Feed (index)	0.00	30000000	125064	3017
Vet Services (index)	0.00	2618061	84990	23183
Fuel (index)	0.00	6073741	118549	42857

6 Results

We generated our results in five steps. The first step was to choose the aggregator functions in the output and input indexes defined by equations (1), (2) and (4). Since we only have one good output (milk) and one bad output (GHG emissions), we only needed to choose a function to aggregate the inputs. There are many nonnegative, nondecreasing, linearly homogenous aggregator functions that can be used for this purpose. Furthermore, irrespective of the functional form of the aggregator function, there are many measures of relative importance that can be used as weights. In the case of inputs, the most common measures of importance are observed input prices and cost shares. Because we don't have these data, we chose to aggregate inputs using a log-linear aggregator function with average estimated shadow cost shares as weights.

The second step was to estimate the stochastic frontier model and use the estimated parameters to compute estimates of shadow cost shares. Bayesian point estimates of the parameters are reported in the first column of Table 4. The second column reports the standard deviations of the relevant estimated marginal posterior pdfs. The columns labeled 2.5% and 97.5% are percentiles of those estimated pdfs. The priors given by equations (12) to (19) ensure that the parameter estimates and the percentiles are consistent with our prior beliefs: among other things, the estimated coefficient of the Period dummy variable implies the sector experienced technical progress at a plausible average rate of $1.0178^{1/(2021-2016)} - 1 = 0.35\%$ per annum; the estimated coefficients of the temperature variables indicate that, within in the range of temperatures recorded in our dataset, increases in temperature lead to increases in output;[1] the estimated coefficients of the precipitation variables indicate that the output-maximizing cumulative precipitation is 122 inches per year; the estimated coefficients of the log-land-times-regional-dummy variables indicate that land in the Basin and Range (resp. Eastern Uplands) was more (resp. less) productive than land in any other region; the estimated coefficient of log-cows indicates that a 1% increase in the number of cows leads to a 1% increase in output; and the coefficients of the remaining log-inputs indicate that a 1% increase in any other input leads to an output increase of less than 0.012%. The estimated elasticity of scale is 1.015, indicating that production frontiers exhibit slightly increasing returns to scale. The estimated shadow cost shares are obtained by dividing the coefficients of the log-inputs by the estimated elasticity of scale: the estimated shadow cost shares of cows, operating capital and labor are $0.986, 0.011$ and 0.003; the estimated shadow cost shares of the remaining inputs are zero when rounded to three decimal places. These estimates mean that variations in our input index numbers are almost totally driven by variations in numbers of cows.

[1]The estimated output-maximizing mean temperature is 82 degrees Fahrenheit, which is beyond the maximum reported in Table 3.

Table 4: Parameter Estimates

Variable	Est.	SD	2.5%	97.5%
Constant	2.18e+00	1.50e-01	1.94e+00	2.48e+00
Period	1.78e-02	5.71e-03	6.48e-03	2.98e-02
Temperature	5.14e-04	2.30e-04	1.53e-04	1.08e-03
Precipitation	2.45e-05	2.14e-05	5.88e-07	7.60e-05
Temperature Squared	-3.13e-06	2.85e-06	-1.13e-05	-1.31e-07
Precipitation Squared	-1.00e-07	9.23e-08	-3.47e-07	-4.21e-09
ln(Land) × Heartland	1.76e-04	1.65e-04	3.20e-06	6.16e-04
ln(Land) × Northern Crescent	5.99e-04	4.08e-04	2.84e-05	1.54e-03
ln(Land) × Northern Great Plains	2.05e-03	1.08e-03	2.03e-04	4.19e-03
ln(Land) × Prairie Gateway	4.39e-03	1.15e-03	2.18e-03	6.51e-03
ln(Land) × Eastern Uplands	1.04e-04	1.11e-04	2.11e-06	4.03e-04
ln(Land) × Southern Seaboard	1.81e-04	1.74e-04	2.70e-06	6.35e-04
ln(Land) × Fruitful Rim	4.24e-03	5.53e-04	3.17e-03	5.31e-03
ln(Land) × Basin and Range	5.07e-03	1.11e-03	2.84e-03	7.22e-03
ln(Land) × Mississippi Portal	2.58e-03	2.56e-03	6.46e-05	9.40e-03
ln(Cows)	1.00e+00	2.89e-03	9.95e-01	1.01e+00
ln(Operating Capital)	1.16e-02	2.63e-03	6.46e-03	1.66e-02
ln(Labour)	2.65e-03	1.05e-03	4.96e-04	4.67e-03
ln(Purchased Feed)	2.66e-05	2.42e-05	1.16e-06	8.65e-05
ln(Vet Services)	9.44e-05	8.26e-05	3.99e-06	3.12e-04
ln(Fuel)	5.99e-05	5.66e-05	1.74e-06	2.11e-04
θ	1.38e+00	2.12e-01	1.08e+00	1.84e+00
γ	1.00e-03	1.62e-04	6.73e-04	1.31e-03

The third step was to obtain an estimate of α by estimating equation (3). Our ordinary least squares (OLS) estimate of δ was 0.9604. The associated estimate of $\alpha = \delta/(1+\delta)$ was 0.4899. We rounded this value to one decimal place and set α to 0.5; recall that this is also the value that underpins the performance measures of Stetter & Sauer (2022) and Färe et al. (2004).

The fourth step was to compute various output, input and productivity index numbers. These are summarized in Figures 2 to 4 below, and in Table 5 in Appendix 3. The total factor productivity index (TFPI) numbers in Figure 2 were obtained by setting $\alpha = 0$ in equation (5), while the sustainable productivity index (SPI) numbers were obtained by setting $\alpha = 0.5$. The good output index (GI) numbers in Figure 3 measure changes in milk production, while the bad output index (BI) numbers measure changes in GHG emissions. The aggregate output index (QI) numbers in Figure 5 measure changes in the milk-production-to-GHG-emissions ratio, while the input index (XI) numbers essentially measure changes in the number of cows (the input index gives a weight of 0.986 to cows and a weight of only 0.014 to all other inputs combined). The bars in each figure (and the numbers in Table 5) compare a given state in a given year with Arizona in 2016: for example, the last bar in the left-hand panel of Figure 3 (and the last entry in the second-last column of Table 5) indicates that TFP in Wisconsin in 2021 was 9.1% higher than TFP had been in Arizona in 2016. The variations we see in Figures 2 to 4 reflect the fact that there is relatively little variation in milk output and GHG emissions per cow. This means that total milk production and total GHG emissions are highly correlated[2]. Mathematically, the values of GI and BI are very similar, which means the values of QI = $(\text{GI}/\text{BI})^{0.5}$ are reasonably close to one. In turn, this means that values of SPI = QI/XI are mainly driven by changes in XI. That is, sustainable productivity in the dairy sector is mainly driven by changes in the number of cows (i.e., the size of the sector).

[2]It is noteworthy that only 3.47% of sampled farms reported having a system in place for capturing methane that would otherwise be released into the atmosphere, thus reducing overall GHG emissions

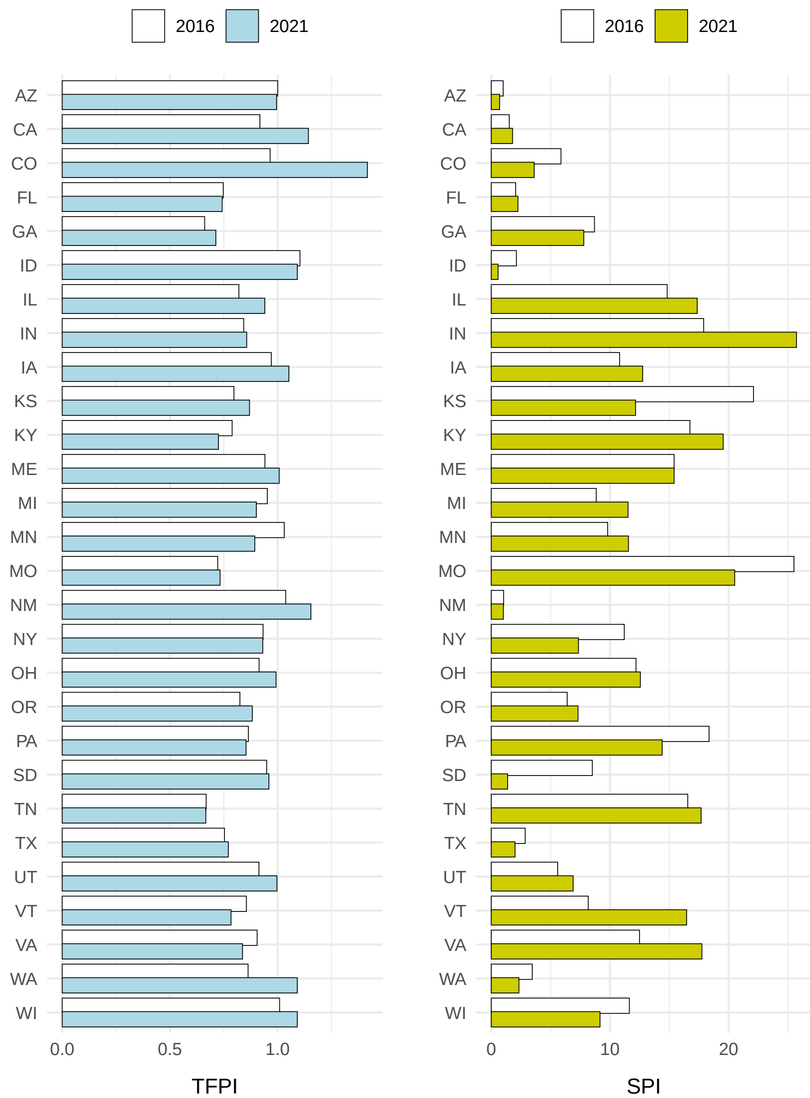

Figure 2: TFPI and SPI Numbers (AZ in 2016 = 1)

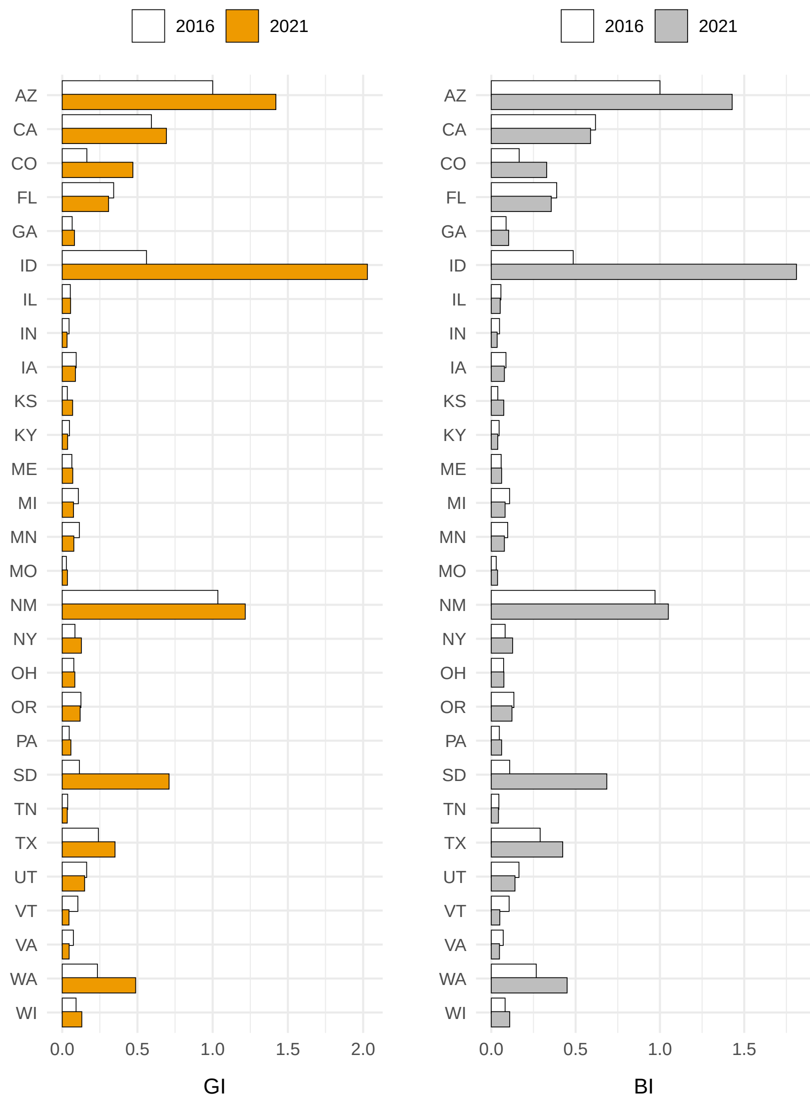

Figure 3: GI and BI Numbers (AZ in 2016 = 1)

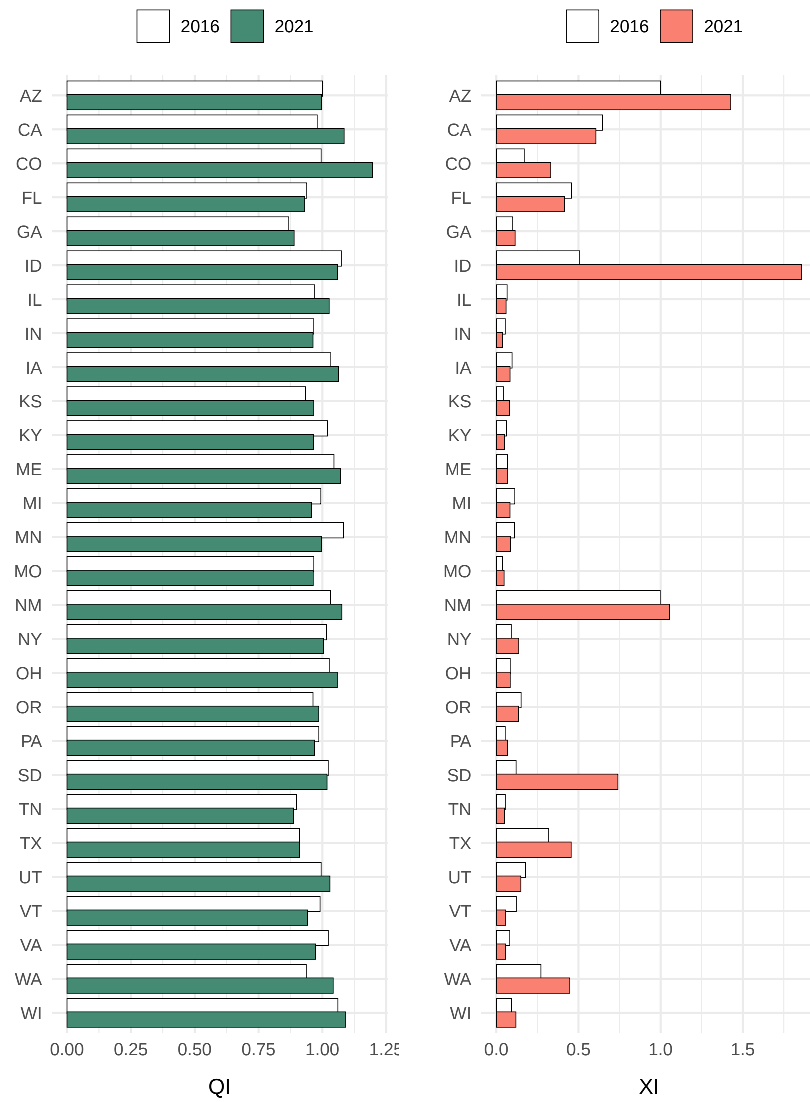

Figure 4: QI and XI Numbers (AZ in 2016 = 1)

The final step was to identify the economic drivers of productivity change. These were estimated by substituting the parameter estimates in Table 4 into the various terms in the SPI decomposition given by equation (10). A full set of results is reported in Table 6 in Appendix 3. In this table the SNI component was computed as a residual: SNI = SPI/(OTI×OEI×OSMEI×OTEI). It is evident that the main economic drivers of productivity change were technical efficiency change (i.e., the OTEI component) and scale-and-mix efficiency change (i.e., the OSMEI component). Results for these two components are summarized in Figure 5. Again, the bars in this figure (and the numbers in Table 6) compare a given state in a given year with Arizona in 2016: for example, the last bar in the left-hand panel of Figure 5 (and the last entry in the second-last column of Table 6) indicates that technical efficiency in Wisconsin in 2021 was only 1.5% lower than technical efficiency had been in Arizona in 2016. By far the most important driver is the scale-and-mix efficiency component. Our estimate of the elasticity of scale indicates that returns to scale are relatively small. Thus, differences in sustainable productivity are mainly driven by changes in the output mix and the input mix.

7 Conclusion

Increased awareness of the environmental impacts of negative externalities in agricultural production has led to a push to embrace sustainable production (USEPA 2023). Sustainable production requires that goods and services are produced in a manner that minimizes the production of bad outputs. Traditional total factor productivity indexes (TFPIs) ignore bad outputs, so they tell policymakers very little about how well firms are meeting their sustainability goals. In this study, we moved beyond traditional TFPIs and developed a sustainable productivity index (SPI) that explicitly accounts for the production of bads. Our SPI was constructed by combining a good output index (GI) and a bad output index (BI) into a total output index (QI), and by then dividing this total output index by a total input index (XI). Our SPI satisfies a set of basic axioms from index theory, namely transitivity, identity, weak monotonicity, proportionality, commensurability, and time-space-reversal. Indexes that satisfy these axioms are consistent with measurement theory, and they can be used to make economically meaningful comparisons of productivity across both time and space.

Our SPI was applied to data on U.S. dairy farms collected as part of the Agricultural Resource Management Surveys of 2016 and 2021. The dairy farms in the sample generated both good and bad outputs: the good output was fluid milk, whereas the bad output was greenhouse gas (GHG) emissions produced from enteric fermentation processes and manure management systems.

We decomposed our SPI numbers into an output-oriented technological index (OTI) that captures technological progress, an output-oriented environment index

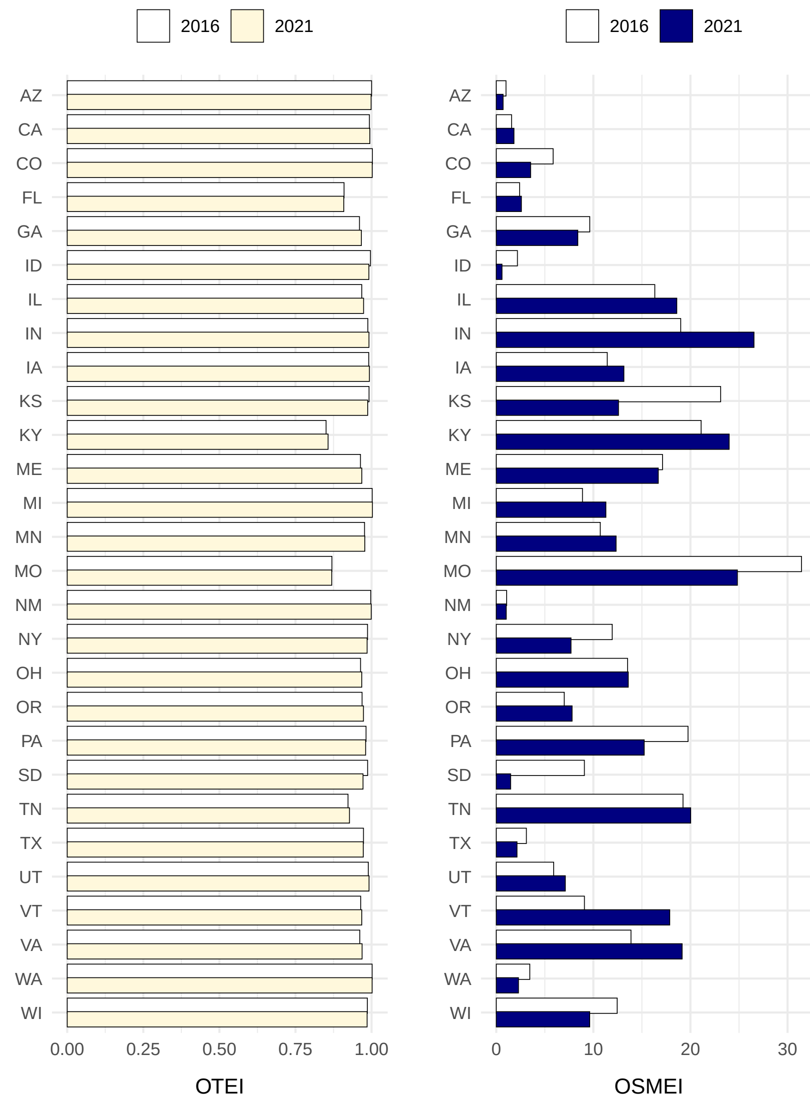

Figure 5: OTEI and OSMEI Numbers (AZ in 2016 = 1)

(OEI) that measures changes in the production environment, an output-oriented scale-and-mix efficiency index (OSMEI) that captures productivity gains due to economies of scale and substitution, and an output-oriented technical efficiency index (OTEI) that measures how well technologies are used. We found that productivity change was primarily driven by technical, scale and mix efficiency change.

These findings have implications for economic policy. If sustainable production is the intended goal, policy could be used to incentivize investments in dairy management practices that result in lower GHG emissions. Moreover, since we found that technical, scale and mix efficiency change are important economic drivers of productivity change, policy interventions could be directed at imparting knowledge on how to select and use the best available technologies, as well as how to achieve the optimal balance of scale and mix in dairy operations, in order to raise productivity in a sustainable way.

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Appendix 1: Axioms

An axiom is a property that is regarded as being self-evidently true. If all outputs and inputs are positive, then the SPI defined by (5) satisfies the following axioms:

A1 Identity:
$SPI(x_{ks}, g_{ks}, b_{ks}, x_{ks}, g_{ks}, b_{ks}) = 1.$

A2 Weak monotonicity:
$g_{rl} \geq g_{it},\ b_{rl} \leq b_{it}$ and $x_{rl} \leq x_{it} \Rightarrow SPI(x_{ks}, g_{ks}, b_{ks}, x_{rl}, g_{rl}, b_{rl}) \geq SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}).$

A3 Commensurability:
$SPI(\delta x_{ks}, \lambda g_{ks}, \theta b_{ks}, \delta x_{it}, \lambda g_{it}, \theta b_{it}) = SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it})$ for all $(\lambda, \delta, \theta)' > 0.$

A4 Time-space reversal:
$SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}) = 1/SPI(x_{it}, g_{it}, b_{it}, x_{ks}, g_{ks}, b_{ks}).$

A5 Transitivity:
$SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}) = SPI(x_{ks}, g_{ks}, b_{ks}, x_{rl}, g_{rl}, b_{rl})$
$\times\ SPI(x_{rl}, g_{rl}, b_{rl}, x_{it}, g_{it}, b_{it}).$

In a time series context, for example, the identity axiom says that if a firm uses exactly the same inputs to produce exactly the same outputs in two different periods, then it must be regarded are being equally productive in the two periods. The weak monotonicity axiom says that productivity increases as good outputs rise and bad outputs and inputs fall. The commensurability axiom says that the index numbers are invariant to changes in units of measurement (e.g., if volumes are measured in kilograms instead of tonnes). The time-space reversal axiom says that the index number that compares productivity in two periods using the first period as the reference period is equal to the reciprocal of the index number obtained when using the other period as the reference period (i.e., when the reference and comparison periods are flipped). The transitivity axiom says that if we compare the productivity of a firm in two periods directly, then we should get the same index numbers as when we make the comparison indirectly via a third period.

Appendix 2: GHG Emissions

The GHG emissions variable was constructed by first estimating methane (CH_4) and nitrous oxide (N_2O) emissions from dairy production, and then converting these emissions into carbon dioxide equivalents (CO_2e) (USEPA, 2009). Methane emissions are generated through the anaerobic decomposition of manure, and as part of the normal digestive process in milk cows, whereas nitrous oxide is produced through the nitrification and denitrification of the nitrogen in dung and urine (USEPA, 2023).

Methane emissions (in metric tonnes per year) from farm i at time t were estimated as:

$$
\begin{aligned}
CH_{4,it} = \Big[& TVS_{it} \times V_{it} \times (1 - SS_{it}) \\
& \times (days/365)_{it} \times MCF_{MMS,it} \times B_0 \qquad (20) \\
& \times 0.662\ kg\ CH_4/m^3 \Big] \qquad (21)
\end{aligned}
$$

TVS_{it} is the total volatile solids excreted by the milk cow; V_{it} represents the fraction of the total manure that is managed in the manure management system; SS_{it} is the proportion of volatile solids removed through solid separation (if no solid separation occurs this term is set to zero); days/365 represents the number of days per year the milk cows were held in the facility; MCF_{MMS} is a CH_4 conversion factor which varies by the type of manure management system; B_0 is the maximum CH_4 producing capacity of the milk cow (the default value for milk cows is 0.24); and 0.662 is the conversion factor of CH_4 from cubic meters (m^3) to kilograms (kg). The total volatile solids (TVS_{it}) excreted by the milk cows in farm i at time t were estimated as:

$$
TVS_{it} = Population_{it} \times 604kg \times VE_{it}/1000 \qquad (22)
$$

where *Population* is the number of milk cows at the dairy operation; 604 kg is the typical animal mass of the milk cow; and VE_{it} is the volatile excretion rate of a typical milk cow per 1,000kg of animal mass (which varies by State).

Nitrous oxide emissions (in metric tonnes per year) in farm i at time t were estimated as:

$$N_2O_{,it} = \Big[N_{ex,it} \times V_{it} \times (1 - SS_{it}) \times EF_{MMS,it} \times (days/365)_{it}\Big] \tag{23}$$

$$\times (44\, N_2O/28\, N_2O) - N \tag{24}$$

where $N_{ex,it}$ is the daily nitrogen excreted by the milk cow; V_{it} is the proportion of manure that is managed in the manure management system; SS_{it} is the proportion of nitrogen removed through solid separation (if no solid separation occurs this value is set to zero); $EF_{MMS,it}$ is the N_2O emissions factor for the manure management system; days/365 represents the number of days per year the milk cows were held in the facility; and $(44N_2O/28N_2O) - N$ is a ratio used to convert the mass of nitrogen within a nitrous oxide molecule $(N_2O - N)$ to the mass of the entire nitrous oxide molecule (N_2O). Further, $N_{ex,it}$ above is estimated as:

$$N_{ex,it} = Population_{it} \; \times \; 604kg \; \times \; N_{it}/1000 \tag{25}$$

where *Population* is the number of milk cows at the dairy operation; 604kg is the typical animal mass of the milk cow; and $N_{it}/1000$ is the total nitrogen excreted per day per 1,000kg of animal mass (which varies by State).

Finally, methane and nitrous oxide emissions were converted to carbon dioxide equivalent (CO_2e) GHG emissions (in metric tonnes per year) using their respective global warming potentials:

$$GHG_{it} = (21 \times CH_{4,it}) + (310 \times N_2O_{it}). \tag{26}$$

Appendix 3: Tabulated Results

Table 5: Measures of Volume Change and Productivity Change

State	Year	GI	BI	QI	XI	TFPI	SPI
AZ	2016	1.000	1.000	1.000	1.000	1.000	1.000
CA	2016	0.593	0.618	0.980	0.646	0.918	1.517
CO	2016	0.164	0.165	0.995	0.170	0.965	5.869
FL	2016	0.342	0.388	0.939	0.457	0.747	2.052
GA	2016	0.066	0.088	0.868	0.100	0.661	8.691
ID	2016	0.560	0.486	1.074	0.508	1.104	2.116
IL	2016	0.054	0.057	0.970	0.065	0.820	14.821
IN	2016	0.045	0.049	0.966	0.054	0.842	17.894
IA	2016	0.093	0.087	1.033	0.096	0.971	10.805
KS	2016	0.034	0.039	0.934	0.042	0.797	22.091
KY	2016	0.048	0.046	1.019	0.061	0.789	16.734
ME	2016	0.064	0.058	1.045	0.068	0.941	15.402
MI	2016	0.107	0.108	0.994	0.112	0.952	8.843
MN	2016	0.114	0.097	1.082	0.110	1.031	9.804
MO	2016	0.027	0.029	0.966	0.038	0.722	25.496
NM	2016	1.035	0.971	1.032	0.997	1.037	1.035
NY	2016	0.085	0.082	1.016	0.091	0.932	11.204
OH	2016	0.077	0.073	1.027	0.084	0.914	12.191
OR	2016	0.124	0.134	0.963	0.151	0.825	6.393
PA	2016	0.046	0.048	0.986	0.054	0.864	18.347
SD	2016	0.114	0.109	1.023	0.120	0.949	8.513
TN	2016	0.036	0.045	0.899	0.054	0.668	16.552
TX	2016	0.240	0.290	0.910	0.319	0.753	2.855
UT	2016	0.162	0.164	0.995	0.178	0.913	5.594
VT	2016	0.104	0.106	0.991	0.121	0.855	8.169
VA	2016	0.074	0.071	1.023	0.082	0.905	12.486
WA	2016	0.234	0.267	0.937	0.271	0.863	3.452
WI	2016	0.092	0.082	1.061	0.091	1.009	11.632
AZ	2021	1.420	1.429	0.997	1.427	0.995	0.699
CA	2021	0.693	0.588	1.085	0.606	1.143	1.790
CO	2021	0.470	0.328	1.196	0.331	1.417	3.607
FL	2021	0.308	0.356	0.931	0.415	0.742	2.244
GA	2021	0.081	0.103	0.889	0.114	0.713	7.793
ID	2021	2.028	1.810	1.059	1.859	1.091	0.570
IL	2021	0.056	0.053	1.026	0.059	0.940	17.350
IN	2021	0.032	0.035	0.963	0.037	0.856	25.711
IA	2021	0.088	0.078	1.063	0.083	1.052	12.755
KS	2021	0.069	0.074	0.966	0.080	0.870	12.154

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Table 5 cont.

State	Year	GI	BI	QI	XI	TFPI	SPI
KY	2021	0.036	0.038	0.965	0.049	0.725	19.538
ME	2021	0.070	0.061	1.070	0.069	1.007	15.400
MI	2021	0.075	0.082	0.957	0.083	0.901	11.517
MN	2021	0.077	0.078	0.996	0.086	0.894	11.557
MO	2021	0.034	0.037	0.964	0.047	0.733	20.508
NM	2021	1.216	1.050	1.076	1.054	1.154	1.021
NY	2021	0.127	0.126	1.004	0.137	0.931	7.343
OH	2021	0.084	0.075	1.058	0.084	0.993	12.559
OR	2021	0.119	0.122	0.986	0.135	0.882	7.303
PA	2021	0.057	0.061	0.970	0.067	0.853	14.394
SD	2021	0.710	0.685	1.018	0.740	0.960	1.376
TN	2021	0.033	0.043	0.887	0.050	0.666	17.677
TX	2021	0.351	0.423	0.910	0.455	0.771	2.000
UT	2021	0.149	0.140	1.029	0.149	0.997	6.894
VT	2021	0.045	0.051	0.942	0.057	0.784	16.455
VA	2021	0.046	0.048	0.973	0.055	0.837	17.742
WA	2021	0.488	0.449	1.042	0.447	1.091	2.330
WI	2021	0.130	0.109	1.091	0.119	1.091	9.157

Table 6: The Economic Drivers of Productivity Change

State	Year	SPI	OTI	OEI	OSMEI	OTEI	SNI
AZ	2016	1.000	1.000	1.000	1.000	1.000	1.000
CA	2016	1.517	1.000	1.000	1.574	0.993	0.970
CO	2016	5.869	1.000	1.000	5.856	1.003	1.000
FL	2016	2.052	1.000	1.000	2.400	0.910	0.940
GA	2016	8.691	1.000	1.000	9.629	0.960	0.940
ID	2016	2.116	1.000	1.000	2.178	0.996	0.975
IL	2016	14.821	1.000	1.000	16.328	0.968	0.938
IN	2016	17.894	1.000	1.000	18.997	0.988	0.954
IA	2016	10.805	1.000	0.999	11.430	0.991	0.955
KS	2016	22.091	1.000	1.000	23.111	0.992	0.964
KY	2016	16.734	1.000	1.000	21.099	0.850	0.933
ME	2016	15.402	1.000	1.000	17.121	0.963	0.934
MI	2016	8.843	1.000	1.000	8.878	1.002	0.995
MN	2016	9.804	1.000	0.999	10.711	0.977	0.938
MO	2016	25.496	1.000	1.000	31.435	0.870	0.933
NM	2016	1.035	1.000	1.000	1.061	0.997	0.978
NY	2016	11.204	1.000	1.000	11.941	0.987	0.951
OH	2016	12.191	1.000	1.000	13.520	0.964	0.936
OR	2016	6.393	1.000	1.000	7.000	0.969	0.943
PA	2016	18.347	1.000	1.000	19.750	0.982	0.946
SD	2016	8.513	1.000	0.999	9.074	0.987	0.951
TN	2016	16.552	1.000	1.000	19.232	0.923	0.933
TX	2016	2.855	1.000	1.000	3.099	0.973	0.946
UT	2016	5.594	1.000	1.000	5.910	0.989	0.957
VT	2016	8.169	1.000	1.000	9.079	0.964	0.934
VA	2016	12.486	1.000	1.000	13.872	0.961	0.937
WA	2016	3.452	1.000	1.000	3.453	1.002	0.998
WI	2016	11.632	1.000	0.999	12.452	0.986	0.948
AZ	2021	0.699	1.018	1.000	0.702	0.998	0.979
CA	2021	1.790	1.018	1.000	1.821	0.994	0.971
CO	2021	3.607	1.018	1.000	3.537	1.002	1.000
FL	2021	2.244	1.018	1.000	2.586	0.909	0.938
GA	2021	7.793	1.018	1.000	8.397	0.966	0.943
ID	2021	0.570	1.018	1.000	0.588	0.991	0.959
IL	2021	17.350	1.018	1.000	18.590	0.974	0.942
IN	2021	25.711	1.018	1.000	26.544	0.991	0.960
IA	2021	12.755	1.018	1.000	13.144	0.993	0.960
KS	2021	12.154	1.018	1.000	12.586	0.987	0.961
KY	2021	19.538	1.018	1.000	23.987	0.857	0.934
ME	2021	15.400	1.018	1.000	16.685	0.968	0.937
MI	2021	11.517	1.018	1.000	11.288	1.003	1.000

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Table 6 cont.

State	Year	SPI	OTI	OEI	OSMEI	OTEI	SNI
MN	2021	11.557	1.018	1.000	12.340	0.978	0.941
MO	2021	20.508	1.018	1.000	24.835	0.869	0.934
NM	2021	1.021	1.018	1.000	1.021	0.999	0.984
NY	2021	7.343	1.018	1.000	7.697	0.985	0.951
OH	2021	12.559	1.018	1.000	13.593	0.968	0.938
OR	2021	7.303	1.018	1.000	7.808	0.973	0.944
PA	2021	14.394	1.018	1.000	15.236	0.980	0.947
SD	2021	1.376	1.018	1.000	1.478	0.972	0.941
TN	2021	17.677	1.018	1.000	20.024	0.927	0.935
TX	2021	2.000	1.018	1.000	2.133	0.973	0.947
UT	2021	6.894	1.018	1.000	7.110	0.992	0.961
VT	2021	16.455	1.018	1.000	17.865	0.968	0.935
VA	2021	17.742	1.018	1.000	19.141	0.969	0.940
WA	2021	2.330	1.018	1.000	2.285	1.002	1.000
WI	2021	9.157	1.018	1.000	9.622	0.985	0.949