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FORMULATION OF TECHNICAL, ECONOMIC AND ENVIRONMENTAL EFFICIENCY MEASURES THAT ARE CONSISTENT WITH THE MATERIALS BALANCE CONDITION*

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Abstract

The materials balance condition is a fundamental adding up condition, which essentially says that: "what goes in must come out". In this paper we argue that a number of the recently developed methods of incorporating pollution measures into standard productive efficiency models are inconsistent with this fundamental condition. We propose an alternative method that involves the incorporation of materials balance information into the production model in a similar manner to which price information is normally incorporated. This produces a new environmental efficiency measure that can be decomposed into technical and allocative components, in a similar manner to the conventional cost efficiency decomposition. The approach is illustrated with the case of phosphorus emission on Belgian pig-finishing farms, using data envelopment analysis (DEA). Our results indicate that a substantial proportion of nutrient pollution on these farms can be abated in a cost *reducing* manner.

Keywords: environmental efficiency, materials balance condition, nutrient pollution, pig-finishing farms

1. Introduction

During the past two decades, the environmental side effects of economic activities have entered the core of public and political debate. In order to allow for better monitoring and evaluation of firms and their production processes, researchers have recognised the need to adjust traditional methods of productivity and efficiency analysis in order to integrate environmental concerns into the standard technical and economic efficiency measures.

Several attempts have already been made to integrate technical, economic and environmental performance measures (e.g., see review by Tyteca, 1996 or Scheel, 2001). Generally these environmental performance measures are obtained by making adjustments to standard parametric and non-parametric efficiency analysis techniques.[1] The majority of these studies have approached the problem by incorporating an extra pollution variable into the production model to be estimated, either as another input or as a weak disposable bad output (e.g., Färe *et al..,* 1989; Ball *et al..,* 1994; Piot-Lepetit and Vermersch, 1998; Reinhard, Lovell and Thijssen, 2000; Shaik, Helmers and Langemeier, 2002).

These methods implicitly assume that (for a technically efficient firm) a reduction in pollution can only occur via an increase in one or more traditional inputs and/or a reduction in one or more traditional outputs. That is, pollution reduction is implicitly costly. This assumption discounts the possibility that the firm could alter its input mix to achieve lower pollution, which is a viable option in many industries. These methods also suffer from some

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[1] See Coelli *et al.* (2005) for an introductory treatment of these standard methods.

infeasibility problems when the materials balance condition is recognised, as we explain shortly. In this paper we propose an alternative method that allows one to deal with these shortcomings.

The research that is reported in this paper was actually motivated by a practical problem. That is, how can one conduct an analysis of the efficiency of pig-finishing farms in Belgium that takes into account the problem of nutrient pollution on these farms? Intensive pig farming produces a lot of manure. In regions such as Flanders in Belgium, there are geographically concentrated groups of intensive farms that produce large amounts of manure that is difficult to dispose of in an economic and environmentally appropriate manner. This manure is rich in nutrients, such as phosphorus and nitrogen (in particular phosphorus), which leads to environmental problems, such as eutrophication and acidification.

The standard way of calculating nutrient pollution in intensive livestock production is via the materials balance condition. For example, see the analysis of nitrogen pollution in dairy farms in the Netherlands conducted by Reinhard and Thijssen (2000). The nutrient balance of a farm is calculated as the amount of nutrient that enters the farm in inputs minus the amount that leaves the farm bound up in useful output. Thus it is simply calculated as a linear function of the traditional input and output variables.

In this study, we argue that a number of the commonly used methods of incorporating pollution into efficiency models are inconsistent with the materials balance condition. In particular, those methods that involve the inclusion of a pollution variable as an input variable or (bad) output variable into a production technology. In the following section we review

some of these methods and show that the materials balance condition renders a number of them mathematically infeasible.

Following this, in Section 3 we describe an alternative strategy (building upon the unpublished work of Lauwers, Van Huylenbroeck and Rogiers, 1999) that uses standard optimisation methods to identify the emissions minimising input vector in a standard production model. This allows one to define an environmental efficiency measure that is consistent with the materials balance equation, and that can also be decomposed into technical and allocative components in a similar way to the traditional cost efficiency decomposition method described in Farrell (1957).

In section 4 we explain how data envelopment analysis (DEA) methods can be used to calculate these measures. The methods are then illustrated using data on Belgian pig-finishing farms in Section 5, with a summary and concluding comments provided in Section 6.

2. Existing methods

Various researchers have proposed performance indicators that seek to model the links between environmental pressure and economic or social activities. Tyteca (1996) gives a detailed literature review on the methods that have been used to measure the environmental performance of firms. He discusses various issues relevant to the development of environmental performance indicators, such as aggregation, normalisation, standardisation, relative or absolute measures and accounting issues, and finally, he stresses the potential of methods in the productive efficiency literature to deal with many of these issues.

The early literature linking pollution with productivity and efficiency measures mainly focussed upon the effects of pollution controls upon (macro) economic growth (Christainsen and Haveman, 1981; Gollop and Roberts, 1983; Färe, Grosskopf and Pasurka, 1989). A few micro economic studies were conducted, such as Pittman (1981) and Pashigian (1984). However, these studies also focussed primarily upon the effects of pollution controls upon the production process, in particular the effect upon scale economies.

Pittman (1983), in an analysis of Wisconsin paper mills, was the first to attempt to incorporate environmental pollution into conventional productivity measures. This was done by making adjustments to the Caves, Christensen and Diewert (1982) multilateral productivity index. Unlike traditional inputs and outputs, for which market prices are generally available, proxies were used for the undesirable output (i.e., pollution) prices in these pollution adjusted productivity indices. These proxies were derived from observed values, such as pollution taxes and marketable permits, or from shadow prices obtained from previous studies. This method implicitly assumes that pollution is costly, as discussed in the introduction section.

The first incorporation of environmental variables into firm-level efficiency analyses methods also assumed that pollution reduction would be costly. Färe *et al.*. (1989), utilising the Pittman data, included pollution measures as bad outputs into a production model estimated using hyperbolic data envelopment analysis (DEA) methods. They introduced the notion of using the weak disposability concept to account for the fact that the bad outputs (pollution) cannot be freely disposed. Strong disposability implies that it is free of charge to dispose of unwanted inputs or outputs, weak disposability implies expensive disposal. Färe *et al.*. (1989) suggested that comparing the (hyperbolic) productive efficiency measures of two models, one imposing strong disposability of all outputs on the technology and another preventing the

strong disposability of undesirable outputs, yields information of the extent to which environmental regulations are binding. Moreover, they showed how it is possible to derive a producer-specific measure of potential output loss due to the lack of strong disposability. Färe *et al.*. (1993) repeated this analysis using parametric output distance functions. This was done to allow them to more easily measure the shadow prices of the undesirable outputs.

A number of subsequent applied studies have used similar approaches in other industrial applications (e.g., Färe, Grosskopf and Tyteca, 1996; Coggins and Swinton, 1996; Chung, Färe and Grosskopf, 1997; Färe, Grosskopf and Pasurka, 2001) and also in agricultural applications involving nutrient pollution (e.g., Ball *et al.*., 1994; Piot-Lepetit and Vermersch, 1998; Reinhard and Thijssen, 2000; Shaik, Helmers and Langemeier, 2002; Asmilde and Hougaard, 2004). A selection of these studies are discussed below.

Färe, Grosskopf and Tyteca (1996) obtained environmental performance indicators for U.S. fossil fuel-fired electric utilities, using input-orientated DEA methods containing "bad output" pollution variables, in a similar manner to the earlier hyperbolic DEA methods used in Färe *et al.*. (1989). They decomposed overall productive efficiency into input efficiency and environmental efficiency. In fact for each firm, two input-oriented DEA models are run, the first allowing for the conventional proportional contraction of all inputs, the second having an extra constraint, which takes weak disposability of bad outputs into account. The environmental performance indicator was then defined as the ratio of the efficiency score obtained with the first model over the score obtained with the second model. The indicator takes values less than or equal to one, corresponding to environmental inefficiency or efficiency, respectively. Also Tyteca (1997) adapted the original Färe *et al.*. (1989) and Ball *et al.*. (1994) models with the explicit objective to derive environmental efficiency scores, by

measuring the degree to which the pollution variable could be reduced, with the quantities of inputs and outputs held fixed.

In an agricultural example, Reinhard, Lovell and Thijssen (2000) studied the effects of nitrogen pollution on intensive dairy farms in the Netherlands. They utilised DEA models in which a pollution variable is specified as an additional input variable (as opposed to the "bad output" approach discussed above). Their nitrogen pollution variable was calculated using a materials balance equation. They defined three different efficiency models. The first involved the contraction of the pollution input variable, holding the conventional inputs and output constant. The second was a generalisation of the classical Banker, Charnes and Cooper (1984) output-oriented technical efficiency model, which allowed for the radial expansion of the outputs with the inputs (inclusive of the pollution input) held fixed. The third was the input-oriented formulation of the former. These models yielded three types of efficiency scores, (i) an environmental efficiency score; (ii) an output-oriented technical efficiency (*TE*) score; and (iii) an input-oriented *TE* score.

Applicability to the materials balance case

In this section we argue that some of the above methods are likely to suffer from infeasibility problems when the materials balance condition is applicable. First we define some notation. Consider the situation where we have a firm that produces a vector of *m*=1,2,...,*M* outputs, $\mathbf{y} \in \mathbf{R}_+^M$, using a vector of *k*=1,2,...,*K* inputs, $\mathbf{x} \in \mathbf{R}_+^K$. The production activity also produces pollution as a by-product. The amount of pollution is defined by the materials balance equation: $z = \mathbf{a}'\mathbf{x} - \mathbf{b}'\mathbf{y}$, where **a** and **b** are (*K*×1 and *M*×1) vectors of known non-negative constants.

First let us consider the original Färe *et al.* (1989) method. Their hyperbolic efficiency measure involves trying to find the largest scalar, λ, such that the scaled vector ($\lambda\mathbf{y}$, $\mathbf{x}/\lambda$, z/λ) is within the feasible production set. If we apply this scaling to the materials balance equation we obtain:

$$z/\lambda = \mathbf{a}'\mathbf{x}/\lambda - \mathbf{b}'\mathbf{y}\lambda$$

which implies:

$$z - \mathbf{a}'\mathbf{x} = -\mathbf{b}'\mathbf{y}\lambda^2$$

and hence $\lambda=1$ is the only positive value that can satisfy both the hyperbolic efficiency measure and the materials balance equation.[2] Thus an interior point in the production technology (i.e., an inefficient point) is not feasible in this instance. This is not an attractive property. Hence the Färe *et al.* (1989) method is not a viable approach in this materials balance case.

A similar problem will arise with the Färe, Grosskopf and Tyteca (1996) approach. Their input orientated efficiency measure involves trying to find the largest scalar, λ, such that the scaled vector ($\mathbf{y}$, $\mathbf{x}/\lambda$, z) is within the feasible production set. If we apply this scaling to the materials balance equation we obtain:

$$z = \mathbf{a}'\mathbf{x}/\lambda - \mathbf{b}'\mathbf{y}$$

which implies:

$$(z + \mathbf{b}'\mathbf{y})\lambda = \mathbf{a}'\mathbf{x}$$

and thus $\lambda=1$ is again the only value that is mathematically feasible.

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[2] Except for trivial cases where, for example, all values in **a** and **b** are zero.

If we also apply a similar test to the three alternative efficiency measures used in Reinhard, Lovell and Thijssen (2000), namely $(\mathbf{y}, \mathbf{x}, z/\lambda)$, $(\mathbf{y}\lambda, \mathbf{x}, z)$ and $(\mathbf{y}, \mathbf{x}/\lambda, z/\lambda)$, we come to the same conclusion. That is, the only efficiency score that is consistent with the materials balance condition is a value of one, implying that inefficient production is not permitted.

It is true that some of the above studies did not explicitly discuss a materials balance condition in their papers and hence one could argue that this infeasibility problem is not relevant in those cases. However, it is difficult for one to conceptualise a production system in which a materials balance condition (e.g., for nitrogen or for carbon or for sulphur) does not exist.

Some other researchers that are familiar with the typical biological nature of the nutrient emission problem in agriculture have also begun to question the applicability of some of these methods. For example, Piot-Lepetit, Vermersch and Weaver (1997) observed that one should be able to reduce the external impacts of nutrient pollution through reducing persistent inefficient levels of input use. Additionally, Reinhard and Thijssen (2000) discuss the notion of an environmentally optimal allocation of inputs, determined by their nitrogen contents. They used a shadow cost system in which shadow prices can deviate from the market prices. Similar to price distortion factors, they calculated nitrogen distortion factors. That is, the degree to which the input mixes deviate from those that would minimise nutrient pollution. However, they do not attempt to use this model for measuring and decomposing environmental efficiency indices in the manner we outline in the next section.

3.   Proposed efficiency measures

In this section we consider, as above, a firm that produces a vector of $m=1,2,...,M$ outputs, $\mathbf{y} \in \mathbf{R}_+^M$, using a vector of $k=1,2,...,K$ inputs, $\mathbf{x} \in \mathbf{R}_+^K$.   The feasible production set, T, is defined as:

$$T = \left\{ \langle \mathbf{y}, \mathbf{x} \rangle \in \mathbf{R}_+^{M+K} \mid F(\mathbf{x}, \mathbf{y}) \leq 0 \right\}, \tag{1}$$

where F(.) is a continuously differentiable production technology, which is convex and non-increasing in inputs, non-decreasing in outputs, and exhibits strong disposability in inputs and outputs.[3]

We define a surplus variable, $z \in \mathbf{R}_+$, that is calculated using a material balance equation, which is a linear function of the output and input vectors.   That is:

$$z = \mathbf{a}'\mathbf{x} - \mathbf{b}'\mathbf{y}, \tag{2}$$

where $\mathbf{a}$ and $\mathbf{b}$ are ($K\times1$ and $M\times1$) vectors of known non-negative constants.   Note that we have called these z-variables "surplus variables" as opposed to "pollution variables".   This distinction is important, as will become apparent when we discuss abatement strategies later in this section.   Furthermore, note that the possibility of two or more pollution variables is also considered later in this section.

To provide an example of the above variables, consider the case of the pig finishing industry. We could have $M=1$ output variables (pig meat), $K=5$ input variables (piglets, feed, labour, capital, other) and one surplus variable (phosphorus).   The phosphorus contents of pig meat,

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[3] See Coelli *et al.* (2005) for further discussion of these properties.

piglets and feed are known non-zero constants, while the contents of the other inputs (labour, capital and other) are zero.

Given that a particular amount of output is to be produced, one question that could be of interest is: What combination of inputs would result in the lowest possible quantity of surplus (i.e., pollution), for a specified amount of output? One could argue that this is not a sensible question to ask, given that a manager will also be interested in the relative prices of the inputs as well as their pollution consequences. But from pollution minimising policy point of view it is the solution of interest. If one can find this solution, the next policy step could be to ask: How one can introduce appropriate economic incentives to encourage producers to reach this combination? Therefore, this surplus minimising input combination is of particular interest, and hence we will assume initially that "price is no barrier" and seek to find the surplus minimising input combination.

The answer to this question can be found by using similar techniques to those that are used to solve for cost minimising input quantities in many microeconomics texts, such as Varian (1999). We will first describe the cost minimisation problem and then describe the surplus minimising problem.

Cost minimisation

Given a vector of $k=1,2,...,K$ input prices, $\mathbf{w} \in \mathbf{R}_+^K$, one can define the minimum cost of producing a particular output vector as:

$$C(\mathbf{y}, \mathbf{w}) = \min_{\mathbf{x}} \left\{ \mathbf{w}'\mathbf{x} \mid \langle \mathbf{x}, \mathbf{y} \rangle \in T \right\}. \tag{3}$$

The input vector which corresponds to the point at which cost is minimised is denoted by $\mathbf{x}_c$ and hence the minimum cost equals $\mathbf{w}'\mathbf{x}_c$. In the event that a particular firm does not use the

cost minimising input vector, its observed input vector is denoted as **x** and hence the observed cost equals **w′x**.

In the event that the observed vector is not located on the boundary of the technology set, the firm is said to be "technically inefficient". Using the Farrell (1957) definition of technical efficiency, one can identify the technically efficient input vector, $\mathbf{x}_t$, by proportionally shrinking the observed input vector until it is projected onto the boundary of the technology set. That is, by solving the optimisation problem:

$$TE(\mathbf{y},\mathbf{x}) = \min_{\theta}\left\{\theta \mid \langle\theta\mathbf{x},\mathbf{y}\rangle \in T\right\}. \tag{4}$$

In equation 4 the technical efficiency of the firm is equal to the scalar θ, which takes a value between zero and one. This reflects the degree to which the observed input vector can be proportionally reduced while holding the output vector fixed. For example, a value of θ=0.8, indicates that the same output could be produced with 80 percent of observed inputs, while a value of 1 indicates that the firm is technically efficient (i.e., it is operating on the boundary of the production technology). The technically efficient input vector is calculated as $\mathbf{x}_t=\theta\mathbf{x}$, and the cost of this input vector is $\mathbf{w}'\mathbf{x}_t$.

These various input vectors can be illustrated on a simple diagram for the case where there are only two input variables. This is done in Figure 1, where the boundary of the production technology (for the given level of output) is defined by the isoquant, and all points to the Northeast of this isoquant are feasible production points. The input price information is reflected in the slopes of the iso-cost lines. Note that an iso-cost line in this simple two input case is

$$C = w_1 x_1 + w_2 x_2, \tag{5}$$

which after rearrangement becomes:

$$x_1 = \frac{C}{w_1} - \frac{w_2}{w_1} x_2 .$$

(6)

Thus the iso-cost lines in Figure 1 have intercepts equal to C/w_1 and (identical) slopes equal to the negative of the input price ratio. Clearly the iso-cost line that passes through the observed point (x_1, x_2) has a larger intercept than that line which passes through cost minimising point (x_{1c}, x_{2c}), implying that it must also be associated with a larger amount of cost (C). Similarly, the iso-cost line that passes through the technically efficiency point (x_{1t}, x_{2t}) must have an intercept (and hence cost) that lies between these two levels.
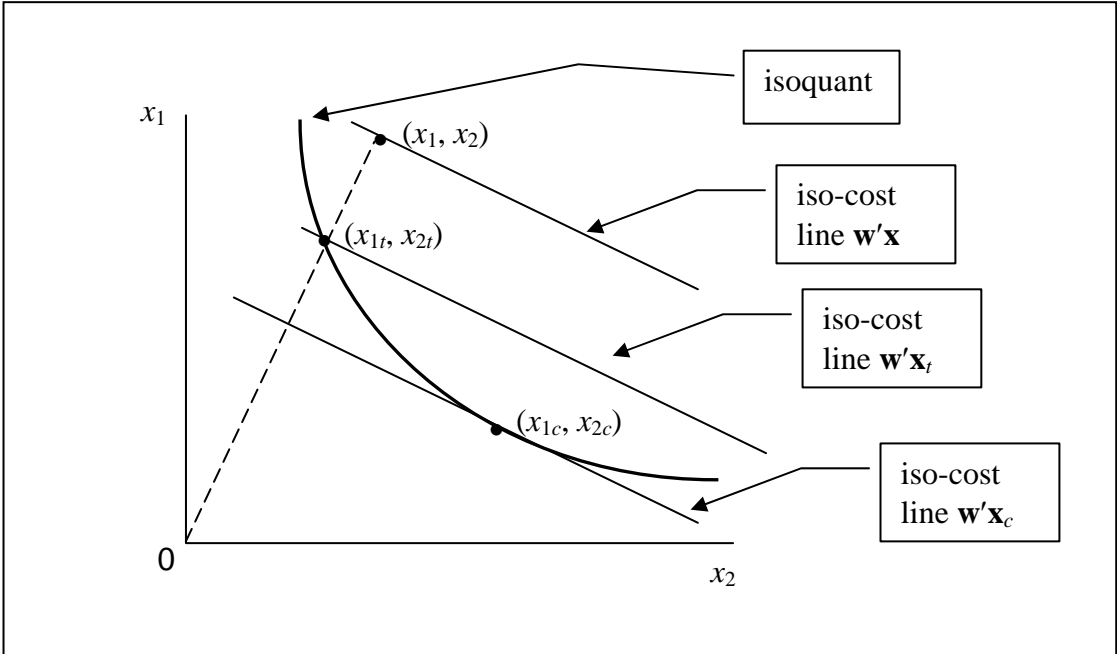

Figure 1: Cost minimisation

On can also define a range of efficiency measures. The cost efficiency (*CE*) of a firm is defined as the ratio of minimum cost over observed cost:

$$CE = \mathbf{w'x}_c / \mathbf{w'x}.$$

(7)

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This will take a value between zero and one, with a value of one indicating full cost efficiency.

One can decompose *CE* into two components: technical efficiency (*TE*) and allocative efficiency (*AE*), where

$$TE = \mathbf{w}'\mathbf{x}_t \,/\, \mathbf{w}'\mathbf{x} = \mathbf{w}'(\theta\mathbf{x}_t) \,/\, \mathbf{w}'\mathbf{x} = \theta. \tag{8}$$

and

$$AE = \mathbf{w}'\mathbf{x}_c \,/\, \mathbf{w}'\mathbf{x}_t. \tag{9}$$

Essentially, *AE* relates to having the correct input mix, given observed input price relativities, while *TE* relates to operation on the boundary of the technology (i.e., the production frontier). All three efficiency measures take a value between zero and one, with a value of one indicating full efficiency. We also note that the three measures are related, such that

$$CE = TE \times AE. \tag{10}$$

Surplus minimisation

One can approach the question of surplus minimisation in an analogous manner. First we note that since the output vector (**y**) is fixed, nutrient surplus ($S = \mathbf{a}'\mathbf{x} - \mathbf{b}'\mathbf{y}$) will be minimised when the aggregate nutrient content of the inputs ($N = \mathbf{a}'\mathbf{x}$) is minimised. Thus, given a vector of $k=1,2,...,K$ nutrient contents, $\mathbf{a} \in \mathbf{R}_+^K$, one can define the minimum nutrients associated with producing a particular output vector as:

$$N(\mathbf{y},\mathbf{a}) = \min_{\mathbf{x}} \left\{ \mathbf{a}'\mathbf{x} \mid \langle \mathbf{x},\mathbf{y} \rangle \in T \right\}. \tag{11}$$

The input vector which involves minimum nutrients (which implies minimum nutrient surplus and hence the best environmental result) is denoted by $\mathbf{x}_e$ and the minimum nutrients equals

$\mathbf{a}'\mathbf{x}_e$. Furthermore, the nutrients at the observed input vector is equal to $\mathbf{a}'\mathbf{x}$, while that at the technically efficient input vector is $\mathbf{a}'\mathbf{x}_t$.

These various alternative input vectors can again be illustrated on a simple diagram for the case where there are only two input variables. This is done in Figure 2, where this time the nutrient content information (per unit of each input) is reflected in the slopes of the iso-nutrient lines. Note that an iso-nutrient line in this simple two input case is

$$N = a_1 x_1 + a_2 x_2, \tag{12}$$

which after rearrangement becomes:

$$x_1 = \frac{N}{a_1} - \frac{a_2}{a_1} x_2. \tag{13}$$

Thus the iso-nutrient lines in Figure 2 have intercepts equal to N/a_1 and (identical) slopes equal to the negative of the nutrient content ratio. Clearly the iso-nutrient line that passes through the observed point (x_1, x_2) has a larger intercept than that line which passes through the nutrient minimising point (x_{1e}, x_{2e}), implying that it must also be associated with a larger amount of nutrient (N). Similarly, the iso-nutrient line that passes through the technically efficiency point (x_{1t}, x_{2t}) must have an intercept (and hence nutrient) that lies between these two levels.

Next we define our new environmental efficiency measures. The environmental efficiency (EE) of a firm equals the ratio of minimum nutrients over observed nutrients:

$$EE = \mathbf{a}'\mathbf{x}_e / \mathbf{a}'\mathbf{x}. \tag{14}$$

This will take a value between zero and one, with a value of one indicating full environmental efficiency efficiency. That is, given the available technology, it is not possible to produce the specified amount of output with a smaller nutrient surplus.
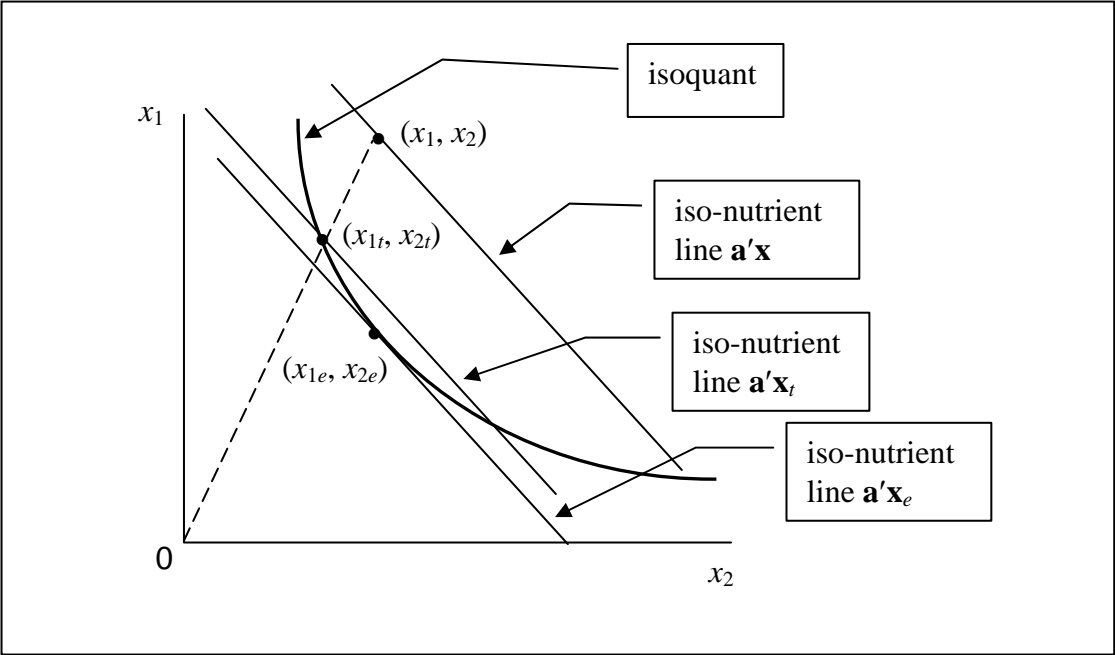

Figure 2: Nutrient minimisation

One can decompose *EE* into two components: that part due to technical efficiency (*TE*) and that part due to environmental allocative efficiency (*EAE*), where

$$TE = \mathbf{a}'\mathbf{x}_t / \mathbf{a}'\mathbf{x} = \mathbf{a}'(\theta\mathbf{x}_t) / \mathbf{a}'\mathbf{x} = \theta, \tag{15}$$

and

$$EAE = \mathbf{a}'\mathbf{x}_e / \mathbf{a}'\mathbf{x}_t, \tag{16}$$

where *EAE* essentially relates to having the correct input mix, given observed nutrient content relativities, while *TE* relates to operation on the boundary of the technology (i.e., the production frontier). All three efficiency measures take a value between zero and one, with a

value of one indicating full efficiency. We also note that the three measures are related, such that

$$EE = TE \times EAE. \tag{17}$$

Figures 1 and 2 can be merged to form Figure 3. Using this diagram we can identify two additional quantities of interest. That is, the cost of the nutrient minimising input bundle, $\mathbf{w}'\mathbf{x}_e$, and the nutrients corresponding to the cost minimising input bundle, $\mathbf{a}'\mathbf{x}_c$. Using the first of these measures we can identify the cost associated with moving from the cost minimising point to the nutrient minimising point, $(\mathbf{w}'\mathbf{x}_e - \mathbf{w}'\mathbf{x}_c)$. This could be interpreted as the shadow cost of pollution. However, if alternative cheaper abatement strategies, such as manure transportation, were available, this measure would overstate the shadow cost.
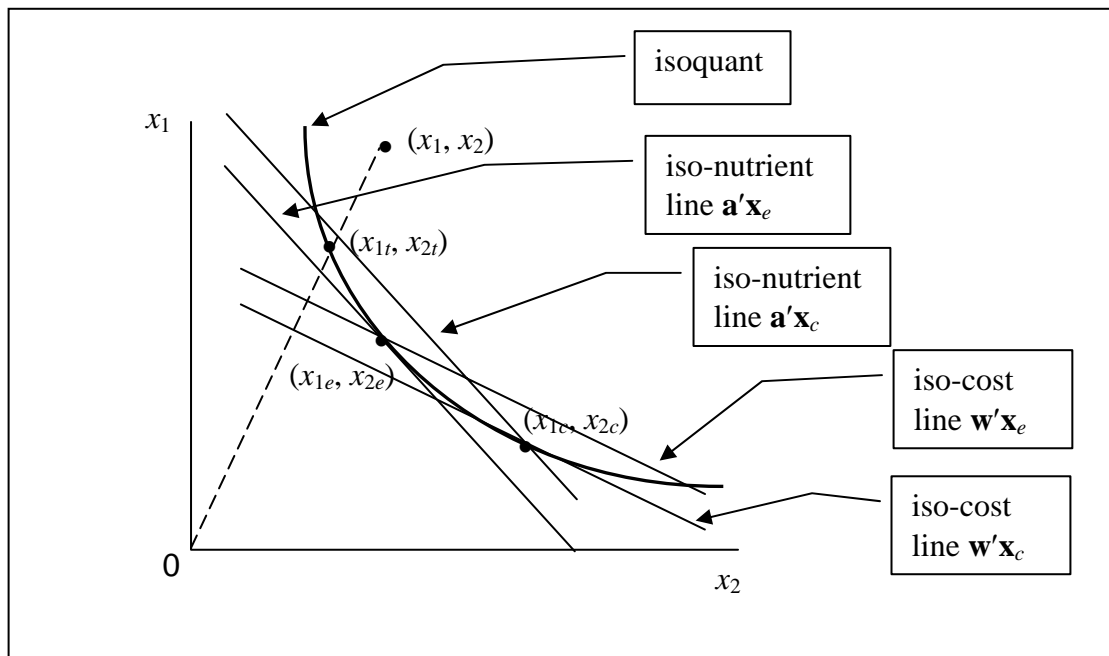

Figure 3: Costs and benefits of nutrient minimisation

The second of these new measures, $\mathbf{a'x}_c$, can be used to identify the pollution consequences associated with movement from the nutrient minimising point to the cost minimising point, $(\mathbf{a'x}_c - \mathbf{a'x}_e)$. This provides an upper bound measure of the surplus reduction that could be achieved if the relative prices of these inputs are adjusted (e.g., via taxation) to encourage nutrient minimisation. Obviously, the two optimal points will coincide when the input price relativities are adjusted so that $\mathbf{w} = \alpha\mathbf{a}$, where α is a positive scalar.

In the example provided in Figure 3, the technically efficiency point is located to the left of both the optimal points. Hence, a movement along the isoquant from the technically efficiency point to the nutrient minimising point results in a reduction in costs in this instance. This need not always be the case. An improvement in *EE* could be associated with either an increase or a decrease in *CE*. If the improvement in *EE* is due to an improvement in *TE* then *CE* will improve. However, if it is due to an improvement in *EAE* then it could result in a rise or fall in *CE*, depending upon whether the movement is towards or away from the cost minimising point. This point is emphasised in Figure 4, where a unit isoquant is drawn with both an iso-nutrient line and an iso-cost line. If a firm is located on the isoquant in sections **I** or **III**, then an improvement in *EAE* will result in an associated increase in *CE* because the movement will also be towards the cost minimising point. However, if the firm is within section **II** any increase in *EAE* will result in a corresponding decrease in *CE* because as the firm moves towards the nutrient minimising point it will be moving away from the cost minimising point.

Before discussing some generalisations of this method, we pause to note that the two main advantages of modelling pollution in this manner are as follows. First, it is mathematically consistent with the materials balance condition. Second, it emphasises the fact that pollution

reduction can in some instances be cost reducing, a point that has been often overlooked in many recent studies of environmental efficiency.
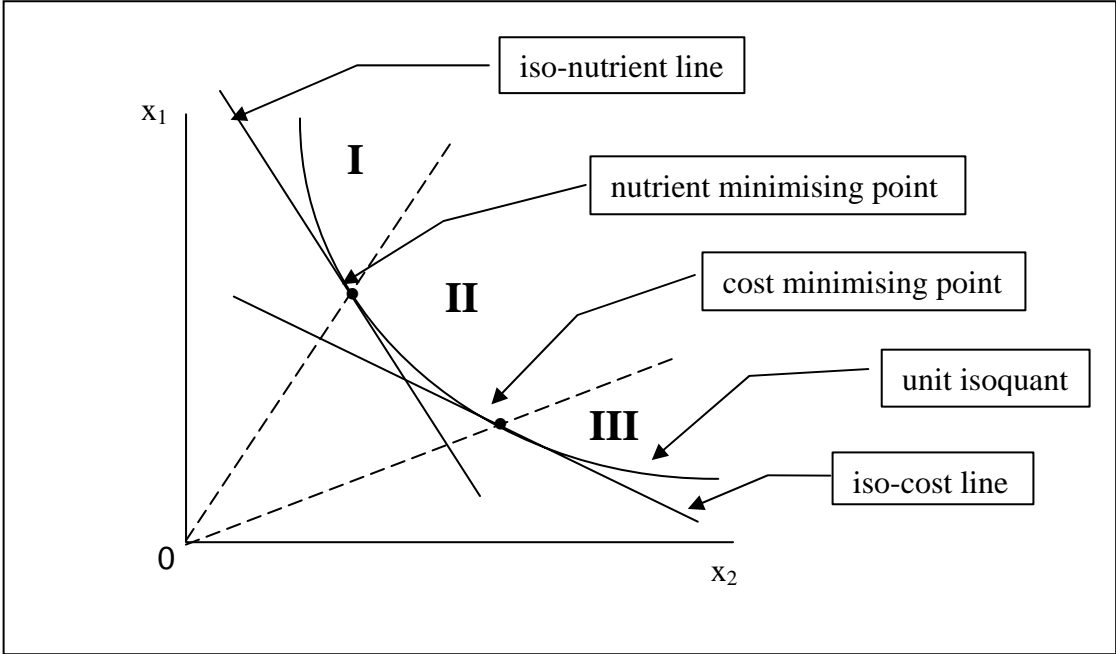

Figure 4: Cost minimisation and nutrient minimisation

Some generalisations

a) Multiple pollutants

The above method can be generalised to the case of two or more pollutants. One could use the method to identify the surplus minimising point for each pollutant individually. For example, one could identify one point for phosphorus and one point for nitrogen in an agricultural application. Alternatively, if one wishes to identify an "aggregate" surplus minimising point, this would require the specification of weights (or relative prices) for the two (or more) types of pollution. For example, in the case where there are two pollutants, two inputs and one output, the two balance equations could be:

$$z_1 = a_{11}x_1 + a_{21}x_2 - b_1 y, \tag{18}$$

and

$$z_2 = a_{12}x_1 + a_{22}x_2 - b_2 y, \tag{19}$$

and if the chosen weights were v_1 and v_2, the *aggregate* balance equation would become:

$$(v_1 z_1 + v_2 z_2) = (v_1 a_{11} + v_2 a_{12})x_1 + (v_1 a_{21} + v_2 a_{22})x_2 - (v_1 b_1 + v_2 b_2)y, \tag{20}$$

or equivalently

$$z^* = a_1^* x_1 + a_2^* x_2 - b^* y, \tag{21}$$

and the method would then proceed normally.[4]

b) Including the social costs of the pollutants

In our earlier discussion we identified two optimal points on the production surface: the cost minimising point and the nutrient minimising point. If the price of pollution was known (e.g., the social cost) then one could use this information to identify a new comprehensive cost (CC) minimising optimal point that takes into account both the private costs of the firm and the social costs of pollution. If the per unit price of pollution, u, is given then the optimisation statements in equations 3 and 11 can to combined to form:

$$CC(\mathbf{y}, \mathbf{w}) = \min_{\mathbf{x}} \left\{ \mathbf{w}'\mathbf{x} + u(\mathbf{a}'\mathbf{x}) \mid \langle \mathbf{x}, \mathbf{y} \rangle \in T \right\}. \tag{22}$$

Given that $\mathbf{w}'\mathbf{x} + u(\mathbf{a}'\mathbf{x}) = (\mathbf{w} + u\mathbf{a})'\mathbf{x}$, it is clear that this is equivalent to a standard cost minimisation problem where the prices of the inputs have been adjusted by a factor equal to their pollution content multiplied by the price of pollution. This framework could be used by policy makers in various ways. For example, to assess the impact of possible pollution taxes upon the levels of pollution in various industries.

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[4] In the eutrophication case, the choice of weights is straightforward: the eutrofying power of phosphorus is ten times more than that of nitrogen.

c) Pollution abatement activities

In the above model, we have assumed that the production process does not involve any form of specific (input consuming) pollution abatement activity, such as the transport of manure to other farms for use as a (appropriately regulated) fertiliser, or the installation of scrubbers in the smoke stacks of electricity generation plants. Pollution abatement of this type generally implies the need for the use of extra inputs, such as extra capital in the scrubbers case and extra labour, fuel and transport equipment in the manure transport case.[5]

In this instance it *could* seem logical for one to include a pollution quantity variable as an explicit production variable (input or bad output) into the production model. However, we would encounter two problems associated with the materials balance condition if this was done. The first is that the materials balance condition will produce an upward biased estimate of the pollution quantity variable unless one can adjust it in some way by subtracting the amount of pollution that is abated by these particular activities. The second problem is that the inclusion of this pollution variable will mean that the problems discussed earlier, regarding the mathematical infeasibility of inefficient data points, will return.

One possible solution to these problems is to include an extra "abatement output" variable, such as "environmentally approved manure" or "scrubbed materials", into the production model, in association with the above materials balance analysis of environmental efficiency. It should be emphasised that this would be a *good output* variable not a *bad output* variable. In the case of nutrient pollution in manure, one can measure this variable directly. In the case

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[5] Note that in the case of manure, the installation of manure treatment equipment is also an option.

of electricity generation, if one has data on pollution that is emitted, one could calculate the amount scrubbed by subtracting the pollution measure from the surplus calculation.

With this type of model formulation one can then accommodate four different pollution reduction strategies:

1. reductions in technical inefficiency;

2. reductions in environmental allocative inefficiency;

3. employment of extra inputs for pollution abatement; and

4. output reduction,

in a comprehensive and mathematically feasible manner. Note that option 1 is cost reducing; option 2 can be either cost reducing or increasing; option 3 will be costly; while option 4 will reduce profits (if the production of the marginal unit of output is profitable).

4. Implementation using data envelopment analysis

The DEA linear programs (LPs) that may be used to calculate the above *TE*, *EAE* and *EE* measures are similar to those defined in Färe, Grosskopf and Lovell (1994) for the cost efficiency case. In this section the notation is defined as follows. There are data on K inputs and M outputs for each of N farms, where for the i-th farm, these data are represented by the column vectors $\mathbf{x}_i$ and $\mathbf{y}_i$, respectively. The $K \times N$ input matrix, $\mathbf{X}$, and the $M \times N$ output matrix, $\mathbf{Y}$, represent the data for all N farms. The linear program (LP) used for calculation of technical efficiency is:

$$\min_{\theta, \lambda} \theta,$$

$$\text{st} \quad -\mathbf{y}_i + \mathbf{Y}\lambda \geq \mathbf{0},$$

$$\theta\mathbf{x}_i - \mathbf{X}\lambda \geq \mathbf{0},$$

$$\lambda \geq \mathbf{0}, \tag{23}$$

where θ is a scalar and $\boldsymbol{\lambda}$ is a N×1 vector of constants. The value of θ obtained is the technical efficiency score for the i-th farm. It will satisfy: $0 \leq \theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient farm. Note that the linear programming problem must be solved *N* times, once for each farm in the sample. A value of θ is then obtained for each farm.

The environmental efficiency and environmental allocative efficiency scores are obtained by solving an additional linear program for each farm in the sample. Namely, the following nutrient minimisation DEA: [6]

$$\min_{\lambda, \mathbf{x}i*} \quad (\mathbf{a}_i'\mathbf{x}_i*),$$

$$\text{st} \quad -\mathbf{y}_i + \mathbf{Y}\boldsymbol{\lambda} \geq \mathbf{0},$$

$$\mathbf{x}_i* - \mathbf{X}\boldsymbol{\lambda} \geq \mathbf{0},$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \tag{24}$$

where $\mathbf{a}_i$ is a K×1 vector of nutrient contents for the *i*-th farm and $\mathbf{x}_i*$ (which is calculated by the LP) is the nutrient minimising vector of input quantities for the *i*-th farm, given the nutrient contents $\mathbf{a}_i$ and the output levels $\mathbf{y}_i$. The environmental efficiency (*EE*) of the *i*-th farm is calculated as

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[6] Note that the LPs in equations 23 and 24 assume CRS. VRS versions are obtained by inserting the convexity constraint (that the elements of $\boldsymbol{\lambda}$ must sum to 1) into each LP.

$$EE = \mathbf{a}_i'\mathbf{x}_i^* / \boldsymbol{a}_i'\mathbf{x}_i. \tag{25}$$

That is, *EE* is the ratio of minimum nutrients to observed nutrients, for the *i*-th farm. The environmental allocative efficiency is then calculated residually as

$$EAE = EE/TE. \tag{26}$$

5. An empirical illustration using Belgian pig-finishing farms

Intensive pig farming is one of the main causes of nutrient surplus problems in intensive livestock regions such as Flanders (Belgium), the Netherlands or Brittany (France). The high density of production in these regions results in a volume of nutrient excretion far higher than is needed for fertilisation. The resulting nutrient surplus leads to environmental problems such as eutrophication and acidification. Phosphorus (P) is the most important nutrient in question. One kg of P has the same eutrophying power as 0.1 kg of nitrogen (N). Expressed in phosphate, the total production from livestock in Flanders (where almost the entire Belgian pig-finishing activity is located) is estimated at 85 million kg P_2O_5 per year (of which more than half comes from pigs), whereas fertilisation limits restrict the environmentally acceptable disposal on agricultural land to about 60 million kg P_2O_5 per year. Strong competition for this disposal room leads to disposal costs of about 3 euro per kg P_2O_5. The costs of manure treatment are about double this amount. Economically feasible solutions for the remaining 25 million kg P_2O_5 per year are still yet to be found.

Pig finishing is not a complex process. Essentially, it is an activity of feed transformation into meat, which starts from a 10 week-old piglet and ends with a market hog of about 106 kg.

This process usually takes approximately 19 weeks. Hence in one *pig place* (the space needed to house one pig), this process can be repeated approximately 2.7 times a year. Due to idle occupation between two rotations or as result of mortality, a pig place may not be always fully occupied. Therefore, in this industry, performance indicators are usually presented per *average present finisher* (APF) per year. That is, per pig place per year, corrected for the average occupation throughout the year.

The main inputs used in pig finishing, in terms of their contribution to total costs, are piglets and feed. Labour and capital are minor inputs. The nutrients embodied in two of the inputs, piglets and feed, are not entirely recuperated in the marketable output, pig meat (i.e., pork), with the balance being excreted in manure.

The data used for this research consist of a representative cross-section of 183 Belgian pig finishing farms in the accounting year 1996-1997. This data is taken from the Belgian Farm Accountancy Data Network, FADN (the official Belgian network being part of the European FADN, see http://europa.eu.int/comm/agriculture/rica). Some key summary statistics for these pig-finishing farms are given in Table 1. The size of the pig finishing operation is expressed in APF per year. Production is measured in (live weight) kilograms of pig meat that is produced per APF per year. Revenues are obtained by multiplying pig production by the (live weight) pig price. Inputs (feed, piglets, other direct costs, fixed capital costs) are also expressed per APF per year. The costs of piglets and feed dominate the direct costs. Revenues minus direct costs yield a gross margin measure. Subsequent subtraction of the

fixed costs (i.e., capital costs) gives a labour income (because the farmer is the residual claimant). Finally, nutrient surplus information is provided.[7]

Preliminary econometric analysis of the available data indicates that the capital expenses and other expenses (mostly labour) variables are not significant explanators of output. Moreover, this econometric analysis indicates that the technology exhibits constant returns to scale (CRS).[8] Hence in the empirical analysis reported in this paper we have a production model with one output (pig meat), two inputs (piglet meat and feed) and a CRS technology.

Table 1: Sample data summary statistics

Variable	Mean	Stdev.	Min	Max
Size of operations (in APF per year)	644.25	456.77	9.20	2430.10
Pig meat production (in kg per APF per year)	269.41	34.62	160.75	365.35
Pig meat price (in euro per live weight kg)	1.34	0.05	1.18	1.60
Total revenues (in euro per APF per year)	362.05	46.96	216.60	486.12
Piglet input (in kg per APF per live weight year)	58.84	10.93	34.55	89.53
Piglet meat price (in euro per live weight kg)	2.35	0.17	1.79	2.96
Concentrated feed input (in kg per APF per year)	662.20	73.71	467.42	921.74
Concentrated feed price (in euro per kg)	0.20	0.01	0.17	0.26
Total piglet cost (in euro per APF per year)	137.41	22.16	81.22	197.95

[7] The phosphorus contents of pig meat, feed and piglet meat (in kg per kg) are 0.0117, 0.0124 and 0.0117, respectively. These values were obtained from CAE (1998).

[8] We were unable to reject the null hypothesis of CRS at the 10% level of significance. These econometric results are available from the authors on request. Note also that we also repeated our DEA analysis with a variable returns to scale (VRS) technology and found that mean scale efficiency was 0.983 and hence that the empirical results changed very little between CRS and VRS.

Total feed cost (in euro per APF per year)	135.58	17.21	90.00	211.58
Other direct costs (in euro per APF per year)	6.98	4.10	0.98	28.33
Total direct costs (in euro per APF per year)	273.00	33.94	180.64	365.27
Fixed costs (in euro per APF per year)	27.50	12.34	4.27	78.08
Gross margin (in euro per APF per year)	89.05	28.39	7.09	179.26
Labour income (in euro per APF per year)	61.55	30.65	-44.95	174.99
Phosphorus surplus (in kg P_2O_5 per APF per year)	5.75	0.72	4.19	8.32

Our DEA results are summarised in Table 2. The mean technical efficiency (*TE*) score of 0.897 suggests that the average farm should be able to produce their current output with 10.3% fewer inputs. The mean environmental efficiency (*EE*) score of 0.843 indicates that the average farm should be able to produce their current output with an input bundle that contains 15.7% less phosphate. Approximately two thirds of this *EE* is due to technical inefficiency (operating below the production frontier) and one third is due to environmental allocative efficiency (*EAE*) (i.e., using a sub-optimal mix of feed and piglets). The mean *EAE* score was 0.940.

Table 2: DEA results

Efficiency measure	Mean	Stdev.	Min	Max
Technical efficiency (*TE*)	0.897	0.055	0.727	1.000
Environmental allocative efficiency (*EAE*)	0.940	0.046	0.763	1.000
Environmental efficiency (*EE*)	0.843	0.065	0.670	1.000
Allocative efficiency (*AE*)	0.985	0.021	0.877	1.000
Cost efficiency (*CE*)	0.883	0.057	0.722	1.000

Cost efficiency (*CE*) and allocative efficiency (*AE*) scores were also calculated using the Färe, Grosskopf and Lovell (1994) cost-minimising DEA model. Mean cost efficiency (*CE*) is 0.883, which suggests that the average farm could reduce costs by 11.7% and still produce the same output. This cost inefficiency is primarily due to technical inefficiency. The mean *AE* score is quite high, at 0.985. This suggests that most farms are using an input mix that approximates the cost minimising input mix. This high mean *AE* score is unusual but not surprising for this particular industry. It is most likely due to the fact that the technology is

well known, the environment is controlled (mostly under cover), and the advice given by agricultural extension advisers widely known and applied.

What are the implications of these results if we extrapolate them to the industry level? Given that we know that this sample is representative of the population and given that the efficiency levels are essentially uncorrelated with farm size,[9] we make the following observations. First we note that the above *EE* scores are expressed as a percentage of the total phosphorus input on the farm. The impact on phosphorus surplus itself will hence be larger. Indeed, pig finishing in Flanders (based on livestock statistics of 1997 in order to remain coherent with the data in this study) is responsible for a surplus of 24.5 million kg of P_2O_5 per year (5.7 kg per APF per year multiplied by 4.3 million APF). This surplus results from 38.1 million kg of P_2O_5 per year from inputs (feed and piglets) and 13.6 million kg of P_2O_5 per year of output (pig meat).

If the estimate of 15.7% average environmental inefficiency was applicable to the whole pig-finishing industry, it suggests that approximately 5.98 million kg P_2O_5 input could be avoided if all farms were to achieve environmental efficiency. Expressed over the original surplus amount, this nutrient saving (with output fixed) suggests a surplus reduction of approximately one quarter.

This estimated potential for phosphorus surplus reduction in the pig farming analysis is a substantial amount, for which one has no need to find extra and expensive new technologies

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[9] The correlation coefficient between TE and farm size (measured in APF) is 0.02 and is not significantly different from zero at any standard level of significance.

for pollution reduction. However, one must recognise that there is likely to be a cost associated with operation at the emission minimising point. On the one hand, improving *TE* will both reduce pollution and reduce cost, but as noted earlier, improving *EAE* is likely to result in increased cost in some (if not many) cases, as one moves away from the cost-minimising point.

In order to consider this issue further, we have plotted our sample data in Figure 5. Given that we have one output, two inputs and a CRS technology, we can represent our DEA analysis in this two dimensional diagram. The sample data, the DEA frontier (i.e., unit isoquant) and the iso-cost and iso-nutrient lines are presented in Figure 5. Note that the iso-cost line is that which corresponds to the mean prices from Table 1. This is done because the diagram would become too messy if we attempted to plot the iso-cost lines of each farm in the sample (prices vary to some extent from farm to farm).
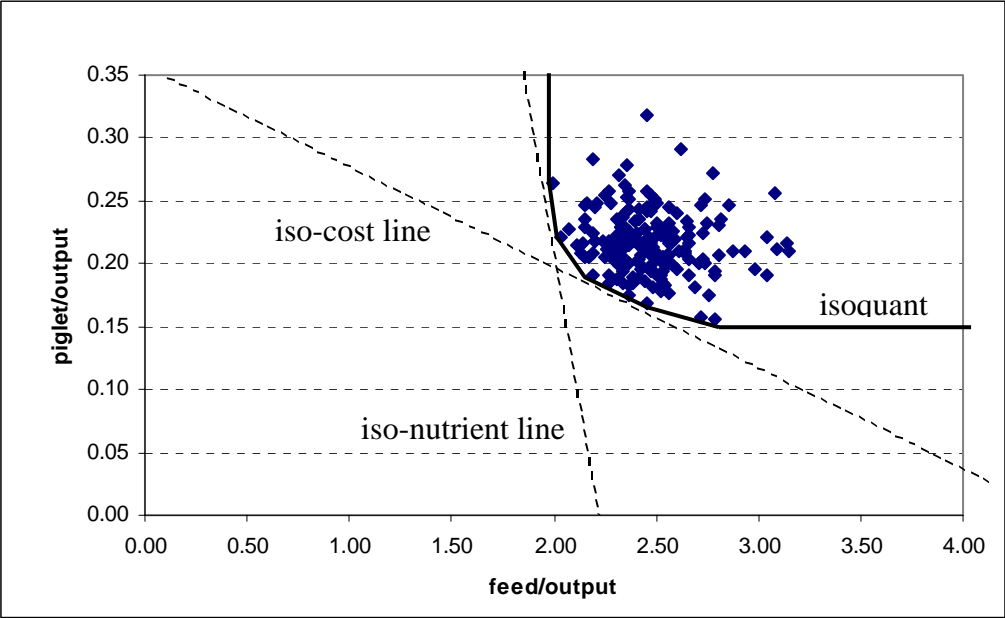

Figure 5: DEA analysis of pig-finishing data

Two optimal points are identified in Figure 5. The nutrient minimising optimal point is 2.038 feed kg per pig meat kg and 0.221 piglet meat kg per pig meat kg. The cost minimising optimal point is 2.187 feed kg per pig meat kg and 0.1903 piglet meat kg per pig meat kg. The costs at these two points (using the mean prices from Table 1) are approximately 0.937 euros and 0.896, respectively. This suggests that movement from the nutrient minimising point to the cost minimising point will reduce unit costs by 4.6%. The nutrients input at these two points are 27.84 grams and 29.34 grams, respectively. This suggests that movement from the cost minimising point to the nutrient minimising point will reduce nutrient inputs by 5.3%.

This information can also be used to construct a shadow cost estimate. The shadow cost of this nutrient surplus reduction is (0.937 - 0.896) / (29.34 - 27.84) = 0.027 Euros per gram or 27 Euros per kg. This is larger than alternative abatement strategies, such as manure treatment, which is approximately six Euros per kg.[10] Hence, in this case the advice to the farmer would most likely be to adjust the input mix so as to reach this cost minimising point, but not attempt to move further to the nutrient minimising point, because the alternative abatement strategies are less costly. However, in other applications these need not be the case.

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[10] This estimate is only provided for illustrative purposes at this stage, given that DEA frontiers can be susceptible to measurement errors in input and output measures. In future work we plan to repeat this analysis using stochastic frontier methods (see Coelli *et al.* 2005) to see how robust these results are to the use of a frontier estimation methodology that explicitly attempts to accommodate data noise.

6. Conclusions

A new method of measuring the environmental efficiency of firms is proposed that involves the identification of nutrient minimising input vectors in the context of a stand production model. The method is applicable when pollution is calculated using a materials balance equation, and can be calculated using traditional data envelopment analysis (DEA) methods. The new measure has the additional advantage that it can also be decomposed into technical and allocative components.

Discussion of previously proposed methods indicates that they have certain shortcomings when the materials balance condition is applicable. In contrast to the conventional environmental efficiency methods, which model pollutants as weakly disposable outputs or as environmentally detrimental inputs and imply a costly disposal or control of these pollutants, the new method allows, up to a certain pollution abatement level, for negative abatement costs. Above that level, additional technologies for pollution reduction or bad output disposal will remain necessary, which may then justify the modelling of this type of pollution abatement activity as an extra output variable.

With regards to the Belgian pig-finishing industry that featured in the empirical illustration, this study suggests that substantial potential exists for nutrient pollution abatement via efficiency improvements, which are cost *enhancing* as opposed to cost reducing. Given that pig production is a conditioned indoor activity, which is highly manageable and does not suffer (like most agricultural activities) from persistent inefficiencies due to quasi-fixed environmental conditions, it can be argued that the degree of inefficiency measured in this study provides a realistic measure of the potential improvement. Furthermore, given the observation that current abatement strategies in Flanders rely almost exclusively on new

technologies or production processes, this knowledge may be of significant benefit to this industry.

Finally, we note that the methods developed in this paper are not only applicable to the case of pig-finishing farms, but are also directly applicable to other agricultural industries, such as dairy farming, egg production, broiler farming and cattle feed lots. Furthermore, these methods could also be adapted to deal with industrial examples, such as coal-fired electricity generation, where the link between thermal efficiency and pollution is well known.[11]

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[11] For example, the Australian Coal Association states that: "Every 1% increase in thermal efficiency results in a 2-3% decrease in greenhouse gas emissions" (http://www.australiancoal.com.au/environmentfuture.htm).

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