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Abstract

This paper uses a relatively new total factor productivity (TFP) index to measure productivity change in the Australian economy. Unlike the TFP indexes used by most statistical agencies, the index used in this paper satisfies a suite of common sense axioms from index number theory. It can also be exhaustively decomposed into measures of technical change, environmental change, and various types of efficiency change. This paper uses least squares methods to estimate these components. The main driver of productivity change in the Australian economy is found to have been scale-mix efficiency change. Scale-mix efficiency is a measure of how well a firm is capturing economies of scale and scope.

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1. Introduction

Total factor productivity (TFP) indexes are measures of output quantity change divided by measures of input quantity change. In the growth accounting literature, it is common to measure the output change component using the change in real gross domestic product (GDP). The input change component is usually measured using a Törnqvist index. For many growth accountants, the main advantage of the Törnqvist index is that it provides for a convenient multiplicative decomposition of total input change into changes in capital, labour and other inputs. The main disadvantage of the index is that it violates a common sense transitivity axiom. Transitivity says that a direct comparison of two observations should yield the same measure of input change as an indirect comparison via a third observation. For example, if all inputs in period 2 were 20% higher than in period 1, and if all inputs in period 3 were 20% higher than in period 2, all direct and indirect comparisons should say that inputs in period 3 were $1.2 \times 1.2 - 1 = 44\%$ higher than in period 1. The fact that the Törnqvist index doesn't satisfy this axiom means it cannot be used to make reliable comparisons involving more than two firms or time periods. This paper measures the input change component of TFP change using the geometric Young (GY) input index defined by O'Donnell (2014, p. 19).[1] The GY input index satisfies a suite of common sense axioms from index number theory, including the transitivity axiom.

Most, if not all, TFP indexes can, by construction, be deterministically decomposed into changes in individual output and input quantities. These accounting-type decompositions are often used by statistical agencies to provide policy-makers with useful information on, for example, the relationship between "capital deepening" and labour productivity. Economic decompositions are also possible. Specifically, most TFP indexes can be decomposed into measures of technical progress (a measure of outward movements in the production frontier due to the discovery of new technologies), environmental change (a measure of movements in the frontier due to changes in characteristics of the production environment), technical efficiency change (a measure of movements towards or away from the frontier) and scale and mix efficiency change (measures of movements around the frontier surface to capture economies of scale and

[1] O'Donnell (2014, p. 19) also defines the GY output and TFP indexes. The GY output index is also defined in O'Donnell (2012*b*, eq. 5). The GY TFP index can be traced back to O'Donnell (2011).

scope) (O'Donnell, 2012*a*, 2014). There are at least two reasons for wanting to identify these types of components. First, all other things being equal, productivity growth that is driven by technical progress, improvements in the production environment, and/or increases in technical efficiency will always be associated with higher net incomes. However, productivity growth that is driven by increases in scale and mix efficiency could easily be associated with lower net incomes. Thus, identifying the components of productivity change is critically important for determining whether productivity growth is associated with higher or lower net incomes (and therefore wealth). Second, different policies will generally have different effects on the different components of productivity change. For example, increases in R&D expenditure will generally have a larger impact on the rate of technical change than on scale efficiency, and policies that change input prices are likely to have a larger effect on input mix efficiency than on technical efficiency. This paper uses econometric methods to decompose TFP change into a combined measure of technical and environmental change, a measure of technical efficiency change, and a measure of scale and mix efficiency change.

The structure of the paper is as follows. Section 2 explains the difference between a technology and a metatechnology. It also lists weak assumptions that are sufficient for a functional representation of a metatechnology to exist. Section 3 explains how to construct a proper TFP index. Examples of proper TFP indexes include the Färe-Primont TFP index defined by O'Donnell (2012*c*) and the Lowe TFP index defined by O'Donnell (2012*d*). Section 4 defines important measures of environmental efficiency, output-oriented technical efficiency, and output-oriented scale-mix efficiency. Changes in these measures are important drivers of changes in TFP. Section 5 explains how any proper TFP index can be exhaustively decomposed into measures of technical change and efficiency change. It also discusses the conditions under which a proper TFP index will collapse to the Solow (1957) residual. Section 6 explains that, if it exists, the output distance metafunction can be written in the form of a stochastic production frontier. Importantly, no assumptions concerning the functional form of the output distance metafunction are required. Section 7 reports estimates of the drivers of TFP change in the Australian economy. The main driver is found to be changes in scale-mix efficiency (i.e., economies of scale and scope). Section 8 concludes.

2. Technologies and Metatechnologies

In this paper, a *technology* is defined as a technique, process or method for transforming inputs into outputs. Examples of technologies include R-DNA technologies for producing human insulin, and the Bessemer process for making steel. In practice, it is convenient to follow O'Donnell (2014) and think of a technology as a book of instructions. Relatedly, the set of all technologies available in period t is referred to as the *period-t metatechnology*. If we think of a technology as a book of instructions, then we should also follow O'Donnell (2014) and think of the period-t metatechnology as a library.

Let $q \in \mathbb{R}_+^{N^*}$, $x \in \mathbb{R}_+^{M^*}$ and $z \in \mathbb{R}_+^{J^*}$ denote vectors of outputs, inputs and characteristics of the production environment respectively. In this paper, characteristics of the production environment are variables that are directly involved in the production process but *never* chosen by firms. The set of output-input combinations that are possible using the period-t metatechnology in an environment characterised by z is formally defined as $T^t(z) \equiv \{(x,q) : x \text{ can produce } q \text{ in period } t \text{ in environment } z\}$. It is common to assume that:

A1: $P^t(x,z) \equiv \{q : (x,q) \in T^t(z)\}$ is bounded for all $x \in \mathbb{R}_+^{M^*}$ (boundedness),

A2: $q \geq 0 \Rightarrow (0,q) \notin T^t(z)$ (weak essentiality),

A3: $(x,q) \in T(z,g)$ and $0 \leq a \leq q \Rightarrow (x,a) \in T(z,g)$ (strong disposability of outputs),

A4: $(x,q) \in T(z,g)$ and $b \geq x \Rightarrow (b,q) \in T(z,g)$ (strong disposability of inputs),

A5: $P^t(x,z)$ is closed for all $x \in \mathbb{R}_+^{M^*}$ (output metasets closed) and

A6: $L^t(q,z) \equiv \{x : (x,q) \in T^t(z)\}$ is closed for all $q \in \mathbb{R}_+^{N^*}$ (input metasets closed).

Assumption A1 (boundedness) says there is a limit to what can be produced using a finite amount of inputs. Assumption A2 (weak essentiality) says that a positive amount of at least one input is needed to produce a positive amount of output. Assumption A3 (strong disposability of outputs) says that the same inputs can be used to produce fewer outputs. Assumption A4 (strong disposability of inputs) says that the same outputs can be produced using more inputs. Finally, assumptions A5 and A6 say that output and input metasets contain all the points on their boundaries.

Assumptions A1–A6 are maintained throughout this paper. If A1–A6 are satisfied then equivalent representations of $T^t(z)$ are the output and input distance metafunctions of Shephard (1970):[2]

$$D_O^t(x,q,z) = \inf\{\delta > 0 : (x, q/\delta) \in T^t(z)\} \quad (1)$$

and $$D_I^t(x,q,z) = \sup\{\rho > 0 : (x/\rho, q) \in T^t(z)\}. \quad (2)$$

Technically-feasible input-output combinations are characterised by $D_O^t(x,q,z) \leq 1$ and $D_I^t(x,q,z) \geq 1$. Technically efficient input-output combinations are characterised by $D_O^t(x,q,z) = 1$ or $D_I^t(x,q,z) = 1$.

If A1–A6 are satisfied then the output (resp. input) distance metafunction is nonnegative (NN), nondecreasing (ND) and homogeneous of degree one (HD1) in outputs (resp. inputs). Additional assumptions concerning the production technology place additional structure on the these functions. For example, it is common to assume:[3]

A7: $(x,q) \in T^t(z) \Leftrightarrow (\lambda x, \lambda^r q) \in T^t(z)$ for all $\lambda > 0$ (homogeneity of degree r),

A8: $P^t(x,z) = a(x,z,\bar{s},t)P^{\bar{s}}(x,z)$ for any $\bar{s}$ (implicit Hicks output neutrality),

A9: $P^t(x,z) = f(x,z,\bar{x},\bar{z},t)P^t(\bar{x},\bar{z})$ for any $\bar{x}$, $\bar{z}$ (output homotheticity),

A10: $L^t(q,z) = b(q,z,\bar{s},t)L^{\bar{s}}(q,z)$ for any $\bar{s}$ (implicit Hicks input neutrality) and

A11: $L^t(q,z) = h(q,z,\bar{q},\bar{z},t)L^t(\bar{q},\bar{z})$ for any $\bar{q}$, $\bar{z}$ (input homotheticity)

where $\bar{x}$, $\bar{q}$ and $\bar{z}$ are reference inputs, outputs and environmental variables; $\bar{s}$ is a reference time period; and $a(.)$, $f(.)$, $b(.)$ and $h(.)$ are scalar-valued functions with properties that are consistent with the properties of $T^t(z)$.

Assumption A7 (homogeneity of degree r) says that if inputs are increased by one percent then outputs can be increased by r percent. The metatechnology is said to exhibit decreasing returns to scale (DRS), constant returns to scale (CRS), or increasing returns to scale (IRS) as r is less than, equal to, or greater than one. If the metatechnology is

[2] Shephard (1970, pp. 206, 207) does not explicitly allow for multiple technologies and/or production environments. In such cases, the z and t notation is usually suppressed.

[3] These definitions can be found in O'Donnell (2014). If there is no environmental change then the letter z can be suppressed and A8–A11 collapse to definitions that can be found in Balk (1998, pp. 16-19).

homogeneous of degree r (HDr) then $D_O^t(x,q,z) = D_I^t(x,q,z)^{-r}$ (e.g. O'Donnell, 2012*b*, Proposition 6).

Assumption T8 (implicit Hicks output neutrality) says that the outputs that can be produced using a given input vector in a given environment in a given period are a scalar multiple of the outputs that can be produced using the same input vector in the same environment in any other period. Assumption A9 (output homotheticity) says that the outputs that can be produced in a given period using given inputs in a given environment are a scalar multiple of the outputs that can be produced in the same period using any other inputs in any other environment. If technical change is implicit Hicks output neutral (IHON) and the metatechnology is output homothetic (OH) then the output distance metafunction takes the form $D_O^t(x,q,z) = Q(q)/F^t(x,z)$ where $Q(q) = D_O^{\bar{s}}(\bar{x},q,\bar{z})$ is an aggregate output and $F^t(x,z)$ is a production metafunction (O'Donnell, 2014, Proposition 5).

Assumption A10 (implicit Hicks input neutrality) says that the inputs that can produce a given output vector in a given production environment in a given period are a scalar multiple of the inputs that can produce the same output vector in the same environment in any other period. Assumption A11 (input homotheticity) says that the inputs that can produce a given output vector in a given production environment in a given period are a scalar multiple of the inputs that can produce any other output vector in any other environment in that period. If technical change is implicit Hicks input neutral (IHIN) and the metatechnology is input homothetic (IH) then the input distance metafunction takes the form $D_I^t(x,q,z) = X(x)/H^t(q,z)$ where $X(x) = D_I^{\bar{s}}(x,\bar{q},\bar{z})$ is an aggregate input and $H^t(q,z)$ is an input metafunction (O'Donnell, 2014, Proposition 6).

Finally, if technical change is both IHON and IHIN then and only then it is said to be Hicks neutral (HN) (O'Donnell, 2014). Similarly, if a metatechnology is both OH and IH then and only then it is said to be homothetic (H) (O'Donnell, 2014). If technical change is HN and the metatechnology is H and HDr (i.e., if assumptions A7–A11 are true) then the output distance metafunction takes the form $D_O^t(x,q,z) = Q(q)/[A^t(z)X(x)^r]$ where $A^t(z) \propto 1/D_O^t(\bar{x},\bar{q},z)$ is the Solow (1957) residual (O'Donnell, 2014, Proposition 7). An example of such a metafunction is (the antilogarithm of):

$$\ln D_O^t(x,q,z) = \ln\left(\sum_{n=1}^{N^*} \alpha_n q_n\right) - \sum_{n=1}^{M^*} \beta_m \ln x_m - \sum_{j=1}^{J^*} \rho_j \ln z_j - \gamma_t \qquad (3)$$

where $\alpha_n \geq 0$, $\beta_m \geq 0$ and $\sum_m \beta_m = r$. The inequality restrictions ensure that this particular metafunction is nonincreasing and quasiconvex inputs, and nondecreasing and convex in outputs. If there is only one output involved in the production process then it collapses to a Cobb-Douglas function.

3. Measures of TFP Change

It is convenient at this point to introduce firm and time subscripts into the notation and let $q_{it} = (q_{1it}, \ldots, q_{N^*it})'$ and $x_{it} = (x_{1it}, \ldots, x_{M^*it})'$ denote the outputs and inputs of firm i in period t. In this paper, the TFP of this firm is defined as $TFP_{it} \equiv Q(q_{it})/X(x_{it})$ where $Q(.)$ and $X(.)$ are NN, ND and HD1 scalar aggregator functions. This definition can be found in O'Donnell (2012*a*).[4] The associated index that compares the TFP of firm i in period t with the TFP of firm k in period s is

$$TFPI_{ksit} \equiv \frac{TFP_{it}}{TFP_{ks}} = \frac{Q(q_{it})/X(x_{it})}{Q(q_{ks})/X(x_{ks})}. \tag{4}$$

Any NN, ND and HD1 aggregator functions can be used to construct such an index. For example, if assumptions A1–A6 are satisfied, then the output and input distance metafunctions are NN, ND and HD1 in outputs and inputs respectively. Thus, natural aggregator functions are $Q(q) = D_O^{\bar{s}}(\bar{x}, q, \bar{z})$ and $X(x) = D_I^{\bar{s}}(x, \bar{q}, \bar{z})$ (these functions were introduced in Section 2). In this case, the index (4) takes the form:

$$TFPI_{ksit} = \frac{D_O^{\bar{s}}(\bar{x}, q_{it}, \bar{z})}{D_O^{\bar{s}}(\bar{x}, q_{ks}, \bar{z})} \frac{D_I^{\bar{s}}(x_{ks}, \bar{q}, \bar{z})}{D_I^{\bar{s}}(x_{it}, \bar{q}, \bar{z})}. \tag{5}$$

This is the proper TFP index defined by O'Donnell (2014, p. 12). It is proper in the sense that it can be written as a proper output quantity index divided by a proper input

[4]Measuring TFP as the ratio of an aggregate output to an aggregate input is an idea that goes back to Jorgenson and Griliches (1967) and Nadiri (1970). However, the distinguishing feature of the O'Donnell (2012*a*) definition is that the aggregator functions are NN, ND and HD1. Other authors are silent about the properties of their aggregator functions, and many use aggregator functions that are not NN, ND and HD1. In such cases, quantity indexes are not proper in the sense of O'Donnell (2012*b*). For example, Nadiri (1970, p. 1138, eq. (1)(b)) uses an input aggregator function that is not HD1. Consequently, his TFP index does not satisfy a proportionality axiom. The proportionality axiom says, for example, that if all inputs double and all outputs double then the productivity index should take the value one (indicating that average product has not changed).

quantity index. In this paper, a quantity index is said to be proper if and only if it satisfies the identity, transitivity, circularity, homogeneity, proportionality, time-space reversal and weak monotonicity axioms listed in O'Donnell (2012*b*).

If there are no environmental variables involved in the production process then the TFP index (4) collapses to the Färe-Primont TFP index defined by O'Donnell (2012*c*, eq. 11). Two other special cases are of interest. First, if the output distance metafunction is given by (3), then (5) takes the form:

$$TFPI_{ksit} = \left[\frac{\alpha' q_{it}}{\alpha' q_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mks}}{x_{mit}}\right)^{\lambda_m} \tag{6}$$

where $\alpha = (\alpha_1, \ldots, \alpha_{N^*})'$ and $\lambda_m \equiv \beta_m / r$. This form of the index has the convenient property that it does not depend on $\bar{q}$, $\bar{x}$, $\bar{z}$ or $\bar{s}$. Second, if output and input markets are perfectly competitive, firms maximise profits, and the output distance metafunction is given by (3), then (5) takes the form:

$$TFPI_{ksit} = \left[\frac{\bar{p}' q_{it}}{\bar{p}' q_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mks}}{x_{mit}}\right)^{\bar{s}_m} \tag{7}$$

where $\bar{p}$ is a vector of average output prices and $\bar{s}_m$ is the m-th average input cost share.[5] This form of the index is convenient because it can be computed without having to estimate the unknown parameters of the output distance metafunction. Mainly for this reason, this is the measure of TFP change that will be used for the empirical work in Section 7. The output change component in (7) is the Lowe output quantity index defined by Hill (2008, eq. 9). The input change component is the input analogue of the GY output quantity index defined by O'Donnell (2012*b*, eq. 5). Importantly, the empirical work in Section 7 will allow for the possibility that the assumptions underpinning this so-called Lowe-GY TFP index are not true [i.e., allowance will be made for the strong possibility that the output and input markets are not perfectly competitive, firms do not maximise profits, and the output distance metafunction is not given by (3)].

[5] If markets are perfectly competitive, firms maximise profits, and the output distance metafunction is given by (3), then prices and input cost shares will be fixed over both space and time. In this case, the first-order conditions for an interior solution to the profit maximisation problem can be solved for $\bar{p} \propto \alpha$ and $\bar{s}_m = \lambda_m$.

4. Measures of Efficiency

An overall measure of firm performance is the measure of TFP efficiency (TFPE) defined by O'Donnell (2012*d*, p. 880):

$$TFPE_{it} \equiv TFP_{it}/TFP_t^* \tag{8}$$

where TFP_t^* denotes the maximum TFP that is possible in period t. This measure can be written as the product of several other measures of efficiency, including measures of firm efficiency (FE), environmental efficiency (EE), output-oriented technical efficiency (OTE), and output-oriented scale-mix efficiency (OSME).

FE and EE are measures defined by O'Donnell (2014). FE is associated with variables that are, at some point, chosen by firms (i.e., inputs, outputs and technologies). EE is associated with variables that are *never* chosen by firms (i.e., characteristics of the production environment). The FE and EE of firm i in period t are formally defined as:

$$FE_{it} \equiv TFP_{it}/TFP^t(z_{it}) \tag{9}$$

$$\text{and} \qquad EE_{it} \equiv TFP^t(z_{it})/TFP_t^* \tag{10}$$

where $TFP^t(z_{it})$ is the maximum TFP that is possible using the period-t metatechnology in an environment characterised by z_{it}. If there are no environmental variables involved in the production process then $EE_{it} = 1$ and $TFPE_{it} = FE_{it}$ for all i and t.

OTE is a measure of efficiency that can be traced back to Debreu (1951) and Farrell (1957). OSME is a measure of economies of scale and scope defined by O'Donnell (2012*a*). The OTE and OSME of firm i in period t are formally defined as:

$$OTE_{it} \equiv D_O^t(x_{it}, q_{it}, z_{it}) \tag{11}$$

$$\text{and} \qquad OSME_{it} \equiv FE_{it}/OTE_{it}. \tag{12}$$

Equation (12) says that OSME is the component of FE that remains after the OTE component has been removed. Several closely-related measures of efficiency are available. For example, output-oriented scale efficiency (OSE) is a measure of pure economies of scale, and residual mix efficiency (RME) is the component of OSME that remains after the OSE component has been removed (i.e., $RME_{it} = OSME_{it}/OSE_{it}$). Input-oriented measures of technical, scale and mix efficiency are also available. For more details, see O'Donnell (2012*a*).

In this paper, the only measures of efficiency that are of interest are those defined by equations (8) to (12). It is clear from these equations that $TFPE_{it} = FE_{it} \times EE_{it}$ and $FE_{it} = OTE_{it} \times OSME_{it}$. Thus, the TFPE of firm i in period t can be written as

$$TFPE_{it} = EE_{it} \times OTE_{it} \times OSME_{it}. \tag{13}$$

This provides a basis for an exact output-oriented decomposition of TFP change. If there are no environmental variables involved in the production process then $EE_{it} = 1$. In this case, equation (13) can be rewritten in the form of either equation (8) or (9) in O'Donnell (2012*d*, p. 880).

5. Decomposing TFP Change

To decompose the TFP index defined by (4), it is convenient to first rearrange (8) as $TFP_{it} = TFP_t^* \times TFPE_{it}$. Substituting (13) into this result yields $TFP_{it} = TFP_t^* \times EE_{it} \times OTE_{it} \times OSME_{it}$. A similar equation holds for firm k in period s. Thus, the TFP index defined by (4) can be decomposed as:

$$TFPI_{ksit} = \frac{TFP_t^*}{TFP_s^*} \times \frac{EE_{it}}{EE_{ks}} \times \frac{OTE_{it}}{OTE_{ks}} \times \frac{OSME_{it}}{OSME_{ks}}. \tag{14}$$

The first term on the right-hand (RH) side of (14) is a measure of technical change. The remaining terms are obvious measures of efficiency change. If there are no environmental variables involved in the production process then the environmental efficiency change component vanishes and (14) collapses to equation (11) in O'Donnell (2012*d*, p. 881). Again, two other special cases are of interest.

First, if the output distance metafunction is given by (3), then the technical efficiency change component in (14) is

$$\left[\frac{OTE_{it}}{OTE_{ks}}\right] = \left[\frac{\alpha' q_{it}}{\alpha' q_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mks}}{x_{mit}}\right)^{\beta_m} \left[\frac{\exp(\gamma_s)}{\exp(\gamma_t)}\right] \prod_{j=1}^{J^*} \left(\frac{z_{jks}}{z_{jit}}\right)^{\rho_j}. \tag{15}$$

A simple rearrangement of this equation yields:

$$\left[\frac{\alpha' q_{it}}{\alpha' q_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mks}}{x_{mit}}\right)^{\lambda_m} = \left[\frac{\exp(\gamma_t)}{\exp(\gamma_s)}\right] \prod_{j=1}^{J^*} \left(\frac{z_{jit}}{z_{jks}}\right)^{\rho_j} \left[\frac{OTE_{it}}{OTE_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mit}}{x_{mks}}\right)^{\beta_m - \lambda_m} . \quad (16)$$

The left-hand (LH) side of this equation is the measure of TFP change (hereafter ΔTFP) defined by (6). The terms on the RH side are measures of technical change (ΔT), EE change (ΔEE), OTE change (ΔOTE), and OSE change (ΔOSE) respectively. There is no RME change (ΔRME) component in (16) because the output and input aggregator functions are proportional to the output and input distance metafunctions.[6] Equation (16) says that ΔTFP = ΔT × ΔEE × ΔOTE × ΔOSE. If the technology exhibits CRS then $\beta_m = \lambda_m$ and ΔOSE = 1. If the technology exhibits CRS and all firms are technically efficient then ΔOTE = ΔOSE = 1 and ΔTFP = ΔT × ΔEE. In the growth accounting literature, this residual is known as the Solow (1957) residual.[7]

Second, the TFP index that will be used for the empirical work in this paper is the Lowe-GY TFP index defined by (7). If the output distance metafunction is given by (3), then this index can be decomposed as:[8]

$$TFPI_{ksit} = \left[\frac{\exp(\gamma_t)}{\exp(\gamma_s)}\right] \prod_{j=1}^{J^*} \left(\frac{z_{jit}}{z_{jks}}\right)^{\rho_j} \left[\frac{OTE_{it}}{OTE_{ks}}\right] \prod_{m=1}^{M^*} \left(\frac{x_{mit}}{x_{mks}}\right)^{\beta_m - \bar{s}_m} \left[\frac{\bar{p}' q_{it}}{\alpha' q_{it}} \frac{\alpha' q_{ks}}{\bar{p}' q_{ks}}\right] . \quad (17)$$

Again, the first three terms on the RH side are measures of technical change, EE change, and OTE change. Collectively, the remaining terms are a measure of OSME change (ΔOSME). Thus, this equation says that ΔTFP = ΔT × ΔEE × ΔOTE × ΔOSME. If there is only one output then last term in square brackets vanishes and the ΔOSME component can be decomposed as ΔOSME = ΔX1 × ... × ΔXM where the definitions are obvious. If markets are perfectly competitive and firms maximise profits then $\bar{p} \propto \alpha$ and $\bar{s}_m = \lambda_m$.[9] In this case, equation (17) collapses to equation (16). In practice, estimating the components on the RH side of (17) involves estimating the unknown

[6] For details, see O'Donnell (2012*a*, Section 3.7).

[7] Solow (1957, p. 312) uses the term technical change "as a shorthand expression for *any kind of shift* in the production function" (his italics). In this paper, shifts in the production function are due to technical change and environmental change (only).

[8] To see this, simply multiply both sides of (16) by the RH side of (7) and then rearrange the result.

[9] See footnote 5.

parameters of the output distance metafunction.

6. Econometric Estimation

If assumptions A1–A6 are true, then the output distance metafunction can be rewritten in the form:

$$\ln Q_{it} = \gamma_0 + \gamma_1 t + \sum_{j=1}^{J} \rho_j \ln z_{jit} + \sum_{n=1}^{M} \beta_m \ln x_{mit} + v_{it} - u_{it} \tag{18}$$

where $Q_{it} \propto \bar{p}' q_{it}$ is an aggregate output, $u_{it} \equiv -\ln D_O^t(x_{it}, q_{it}, z_{it}) \geq 0$ is an unobserved output-oriented technical inefficiency effect, and v_{it} is an unobserved error accounting for statistical noise. In the present case, the noise component accounts for functional form errors [e.g., the possibility that the output distance metafunction does not have the same form as (3)], measurement errors (e.g., the possibility that Q_{it} is not proportional to $\bar{p}' q_{it}$) and omitted variables (e.g., the possibility that $J < J^*$ or $M < M^*$). If noise is random then (18) is a stochastic production frontier of the type discussed by Meeusen and van den Broeck (1977) and Aigner et al. (1977). If the output distance metafunction is written in the form of (18), then, whether or not noise is random, the Lowe-GY TFP index defined by (7) can be written as:

$$TFPI_{ksit} = \left[\frac{\exp(\gamma_1 t)}{\exp(\gamma_1 s)}\right] \prod_{j=1}^{J} \left(\frac{z_{jit}}{z_{jks}}\right)^{\rho_j} \left[\frac{OTE_{it}}{OTE_{ks}}\right] \prod_{m=1}^{M} \left(\frac{x_{mit}}{x_{mks}}\right)^{\beta_m - \bar{s}_m} \left[\frac{\exp(v_{it})}{\exp(v_{ks})}\right]. \tag{19}$$

This is an empirical version of equation (17). Again, the first three terms on the RH side are measures of technical change, EE change and OTE change. Even though it does not involve outputs, the fourth term is still a measure of OSME change.[10] The last term measures the change in statistical noise (ΔNOISE). Thus, equation (19) says that dTFP = dT $\times$ dEE $\times$ dOTE $\times$ dOSME $\times$ dNOISE. Estimating the first four components involves estimating $\theta \equiv (\gamma_1, \rho_1, \ldots, \rho_J, \beta_1, \ldots, \beta_M)'$ and OTE_{it}. The noise component can then be estimated as a residual. In practice, it is common to assume that:

[10]The term *output-oriented* derives from the fact that OSME is an output-oriented decomposition of the scale-mix effect: OSME = OSE $\times$ RME. If there is only one output, then the RME component represents input mix effects only.

E1: $\varepsilon_{it} \equiv v_{it} - u_{it}$ is a random variable with $E(\varepsilon_{it}) = -\mu \leq 0$ and

E2: $Cov(\varepsilon_{it}, x_{mit}) = Cov(\varepsilon_{it}, z_{jit}) = 0$ for all $m = 1, \ldots, M$ and $j = 1, \ldots, J$.

If these assumptions are true, then the ordinary least squares (OLS) estimator is a consistent estimator of θ. Moreover, the corrected least squares (CLS) estimator of Winston (1957, p. 283) is a consistent estimator of OTE_{it}.[11] Importantly, if the CLS estimator is used to estimate OTE, then the noise component in (19) vanishes (i.e., ΔNOISE $= 1$). Common statistical tests can be used to assess E1 and E2. If they are not true, then the OLS and CLS estimators are inconsistent. However, other consistent estimators are available (e.g., 2SLS, GMM).

7. Productivity Change in the Australian Economy

This section reports estimates the drivers of TFP change in $I = 18$ sectors of the Australian economy for the $T = 38$ years from 1970 to 2007. TFP change is measured using the Lowe-GY TFP index defined by (7).[12] Decomposing this index into measures of technical change and efficiency change involves estimating the unknown parameters in equation (18).

7.1. Data

The dataset comprises observations on one output (Q_{it} = total output), $M = 3$ inputs (x_{1it} = capital, x_{2it} = labour, x_{3it} = materials) and $J = 18$ sectoral dummy variables ($z_{jit} = 1$ if $i = j$, 0 otherwise). The output and input data were drawn directly from the EU-KLEMS database. For details on the construction of this database, see O'Mahony and Timmer (2009). The sectoral dummy variables were included to account for unobserved environmental and technological factors that have different effects on production in different sectors (e.g., rainfall directly affects agricultural production but does not directly affect the production of steel; the technologies used to transport goods and services differ from the technologies used to generate electricity). The time trend in equation (18) accounts for unobserved environmental and technological factors that

[11] The CLS estimator of OTE_{it} is $\exp(\hat{e}_{it} - \hat{\mu})$ where $\hat{e}_{it}$ is the (i,t)-th least squares residual and $\hat{\mu} = \max_{i,t}\{\hat{e}_{it}\}$.

[12] There is only one output in the dataset, so the Lowe output quantity index in (7) could, in fact, be replaced by any other output quantity index.

vary systematically over time (e.g., climate change; steady incremental improvements in technologies). The fact that the dummy variables and the time trend both account for unobserved environmental and technological factors means that, in this paper, it is not possible to separately identify the technical change and EE change components of TFP change. Formal definitions and descriptive statistics for the output and input variables are reported in Table 1. This table also reports descriptive statistics for input costs and input cost shares (the average input cost shares are used to compute the GY input quantity index). A listing of the eighteen sectors is provided in Table 2.

7.2. Parameter Estimates

OLS stepwise regression was used to determine if any sectoral dummy variables should be omitted from the estimating equation (18). The estimates obtained at the end of the stepping sequence are reported in the first column of Table 3. All of the estimated coefficients are correctly signed (i.e., consistent with assumptions A1–A6) and significant at the 5% level of significance. Unfortunately, a Hausman test led to the rejection of assumption E2 at the 5% level of significance. This test was conducted using input prices and lagged output prices as instruments.[13] Rejection of assumption E2 suggests that inputs of capital, labour and/or materials are endogenous, and that the OLS estimator is biased and inconsistent. Consequently, the model was re-estimated using two-stage least squares (2SLS). The 2SLS estimates are reported in the second column of Table 3. Again, input prices and lagged output prices were used as instruments. A test of over-identifying restrictions confirmed that these instruments are valid. All of the 2SLS estimates are statistically significant, but the estimated elasticity of capital is incorrectly signed. Consequently, the model was re-estimated using inequality-restricted two-stage least squares (R2SLS). The inequality restrictions were imposed using the simulation methodology of Geweke (1986).[14] The R2SLS estimates are reported in the third column of Table 3. This estimated model explains more than 96% of the variation in log-output (around its mean). All estimated elasticities are correctly signed and

[13]Output prices were constructed by dividing the value of gross output (variable GO in the EU-KLEMS database) by the gross output quantity index (GO_QI). Similarly, input prices were constructed by dividing capital, labour and intermediate input costs (CAP, LAB and II) by the associated quantity measures.

[14]The Geweke (1986) algorithm is an accept-reject algorithm. The estimates reported in Table 3 were obtained using two million draws.

statistically significant by construction. The R2SLS estimate of the elasticity of scale is slightly higher than the OLS and 2SLS estimates, and significantly different from one. All three estimates plausibly indicate that the metatechnology exhibits decreasing returns to scale. Among other things, this implies that the Solow (1957) residual is an unreliable measure of TFP change. The R2SLS estimates of OTE (obtained using the CLS estimator) are generally similar to the 2SLS estimates (see the last row in Table 3).

7.3. The Drivers of TFP Change

Estimates of TFP change in selected sectors of the Australian economy are reported in Figure 1. This figure reveals that there have been large variations in TFP performance across sectors and over time. Effective public policy-making requires an understanding the sources of these variations. This section uses the R2SLS parameter estimates reported in Table 3 to estimate the the components on the RH side of equation (19).[15] The CLS estimator is used to estimate OTE so there is no residual noise (i.e., ΔNOISE $= 1$).

Figures 2 to 5 report estimates of technical and environmental efficiency change (ΔT $\times$ ΔEE), output-oriented technical efficiency change (ΔOTE), output-oriented scale-mix efficiency change (ΔOSME) and, for completeness, changes in residual noise (ΔNOISE). All the measures reported in these figures are indexes that use agriculture ($i = 1$) in 1970 ($t = 1$) as the reference sector/period. The estimates reported in Figure 2 indicate that technical and environmental factors have only made a small contribution to TFP change over the sample period. This figure also indicates that the hotel sector generally operates in the most favourable production environment and/or uses the most productive technology. The estimates reported in Figure 3 indicate that changes in technical efficiency account for a moderate amount of short-term variation in TFP. Hotels and restaurants were among the most technically efficient firms in the 1970s, and were among the least technically efficient firms from the early 1990s until the end of the sample period. The estimates reported in Figure 4 indicate that scale-mix efficiency has been a strong driver of cross-sectional variations in TFP. Observe that the mining/quarrying sector was one of the most scale-mix efficient sectors from the early 1970s until the early 1990s, and that the electricity/gas/water sector was the most scale-mix efficient sector from the early 1990s until the end of the sample period. Finally, Figure 5 confirms that there is

[15] The OLS and 2SLS estimates yield qualitatively similar results.

no residual noise (by construction).

For comparison purposes, Figures 1 to 5 were deliberately drawn using the same vertical scale. A comparison the five figures reveals that the most important source of productivity differences across sectors has been differences in OSME (i.e., economies of scale and scope). For many sectors, changes in OSME have also been the most important driver of productivity change over time. To illustrate, the components of TFP change in the mining/quarrying sector have be consolidated into Figure 6. This figure indicates that changes in mining sector TFP were mainly driven by changes in both OTE and OSME in the early 1970s, by changes in OTE during the late 1970s and early 1980s, and by changes in OSME since the early 2000s. To obtain deeper insights into the drivers of OSME, it is useful to write ΔOSME $= \Delta$K $\times$ ΔL $\times$ ΔM where K, L, and M refer to capital, labour and materials inputs respectively. This type of decomposition was discussed at the end of Section 5. Results for the mining/quarrying sector are presented in Figure 7. This figure indicates that long-term declines in OSME have been associated with the use of capital and labour. Further insights into roles of capital and labour are provided in Figure 8. This figure presents capital, labour and materials input quantity indexes (KI, LI and MI) as well as the capital, labour and materials components of OSME change (ΔK, ΔL and ΔM). This figure reveals that the use of capital and labour inputs is negatively correlated with the capital and labour components of OSME change (correlation coefficients are -0.92 and -0.82 respectively). On the other hand, the use of materials inputs is positively correlated with materials component of OSME change (the correlation coefficient is 0.96). These results indicate that declines in mining sector productivity are mainly due to increases in the use of capital and labour.

8. Conclusion

TFP indexes are important measures of economic performance. Common economic assumptions concerning technologies, markets and behaviour are often used to inform the selection of meaningful index formulas. For example, if technologies are CRS Cobb-Douglas technologies, markets are perfectly competitive, firms maximise profits, and all inputs and outputs involved in the production process are observed and measured without error, then a meaningful measure of productivity change is the Solow (1957) residual. The Solow residual is the bedrock of the growth accounting approach to measuring productivity change.

This paper uses a Lowe-GY TFP index to measure productivity change in eighteen sectors of the Australian economy over the period 1970 to 2007. The Lowe-GY TFP index can be computed without knowing anything about technologies, markets or behaviour. However, we need to know something about technologies in order to decompose it into measures of technical change and efficiency change. This paper uses least squares econometric methods to estimate these components. Importantly, the paper makes no assumptions concerning the functional form of the unknown production frontier. Assumptions that underpin the growth accounting approach to productivity measurement are rejected at usual levels of significance.

The empirical results reveal that changes in scale-mix efficiency are one of the most important drivers of TFP change in the Australian economy. Scale-mix efficiency is a measure of how well a firm is capturing economies of scale and scope. Changes in scale-mix efficiency usually occur when rational optimising firms change the scale of their operations and/or their output and input mixes in response to changes in prices. In the case of the Australian mining industry, for example, significant declines in scale-mix efficiency in the last decade have been associated with increases in the use of capital and labour. In turn, these increases in input use have been associated with improvements in the sectoral terms of trade. For government policy-makers, the appropriate policy response depends on whether increases in firm profits are more or less important than increases in productivity.

Table 1: Variables and Descriptive Statistics

	Variable[†]	Mean	St. Dev.	Min.	Max.
Q	gross output (GO_QI)	85.049	34.209	13.280	211.93
X1	capital services (CAP_QI)	89.875	45.075	14.090	369.76
X2	total hours worked (H_EMP)	680.90	902.86	75.790	5121.3
X3	materials inputs (II_QI)	82.257	40.424	5.7500	233.55
C1	capital cost (CAP)	7347.0	14060	6.1800	0.13E+6
C2	labour cost (LAB)	11594	21238	221.41	0.17E+6
C3	materials cost (II)	17948	24264	327.63	0.18E+6
S1	capital cost share	0.1814	0.1258	0.13E-2	0.5158
S2	labour cost share	0.3024	0.1217	0.1033	0.7088
S3	materials cost share	0.5162	0.1422	0.1787	0.7835

[†] EU-KLEMS variable names in parentheses.

Table 2: List of Sectors

Code	Sector
1	Agriculture, hunting, forestry and fishing
2	Mining and quarrying
3	Food, beverages and tobacco
4	Textiles, leather and footwear
5	Wood and of wood and cork
6	Pulp, paper, printing and publishing
7	Chemical, rubber, plastics and fuel
8	Other non-metallic minerals
9	Basic metals and fabricated metal
10	Machinery n.e.c
11	Electricity, gas and water supply
12	Wholesale and retail trade
13	Hotels and restaurants
14	Transport and storage
15	Post and telecommunications
16	Financial intermediation
17	Real estate, renting and business activities
18	Community, social and personal services

Table 3: Parameter Estimates

Parameter	Variable	OLS	2SLS	R2SLS
γ_1	Time	0.004	0.004	0.002
		(0.001)	(0.003)	(0.256)
ρ_2	Mining	-0.050	-0.040	-0.054
		(0.013)	(0.021)	(0.200)
ρ_4	Textiles	0.046	0.085	0.047
		(0.013)	(0.025)	(0.211)
ρ_5	Wood	0.102	0.057	0.064
		(0.014)	(0.026)	(0.262)
ρ_8	OtherMin	0.111	0.079	0.076
		(0.014)	(0.029)	(0.287)
ρ_{10}	OtherMach	0.060	0.056	0.040
		(0.013)	(0.025)	(0.242)
ρ_{11}	ElecGasWat	-0.058	-0.055	-0.074
		(0.013)	(0.025)	(0.237)
ρ_{13}	Hotels	0.124	0.188	0.163
		(0.012)	(0.035)	(0.343)
ρ_{14}	Transport	-0.055	-0.051	-0.048
		(0.012)	(0.019	(0.187)
ρ_{18}	Community	-0.089	-0.168	-0.107
		(0.015)	(0.037)	(0.286)
β_1	Capital	0.087	-0.116	0.021
		(0.014)	(0.056)	(0.188)
β_2	Labour	0.039	0.057	0.037
		(0.005)	(0.012)	(0.942)
β_3	Materials	0.624	0.817	0.727
		(0.007)	(0.072)	(0.638)
γ_0	Constant	1.008	0.956	0.898
		(0.047)	(0.237)	(0.238)
Elasticity of Scale		0.749	0.758	0.785
Average OTE		0.742	0.693	0.704

Figure 1: TFP Change (ΔTFP)

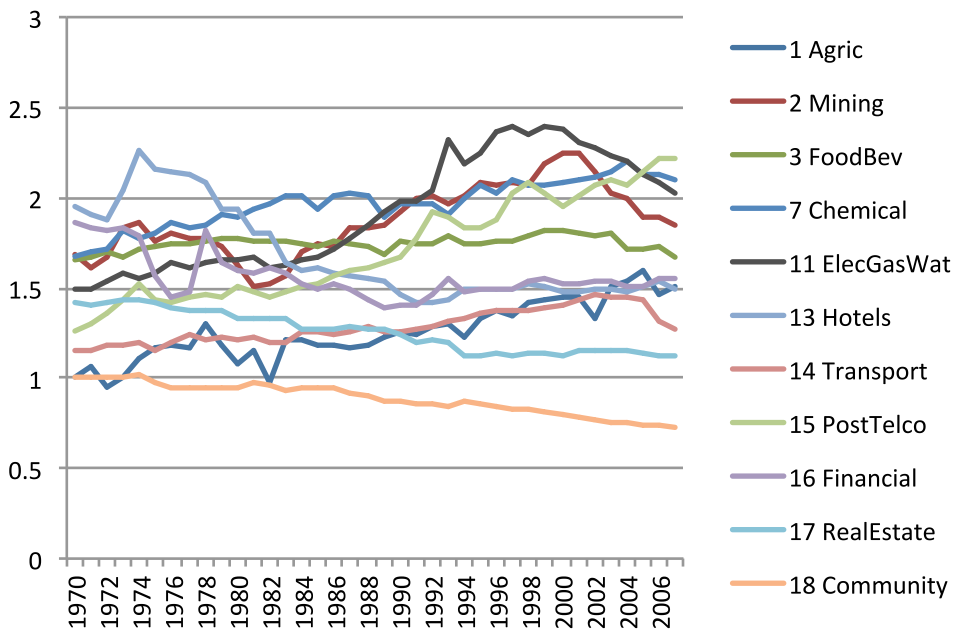


Figure 2: Technical and Environmental Change ($\Delta T \times \Delta EE$)

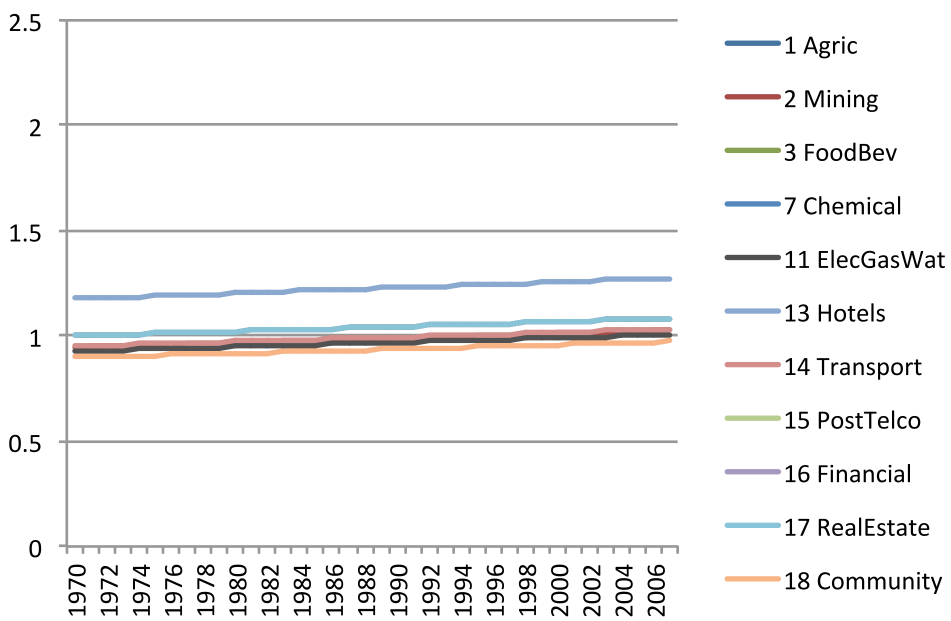


Figure 3: Output-Oriented Technical Efficiency Change (ΔOTE)

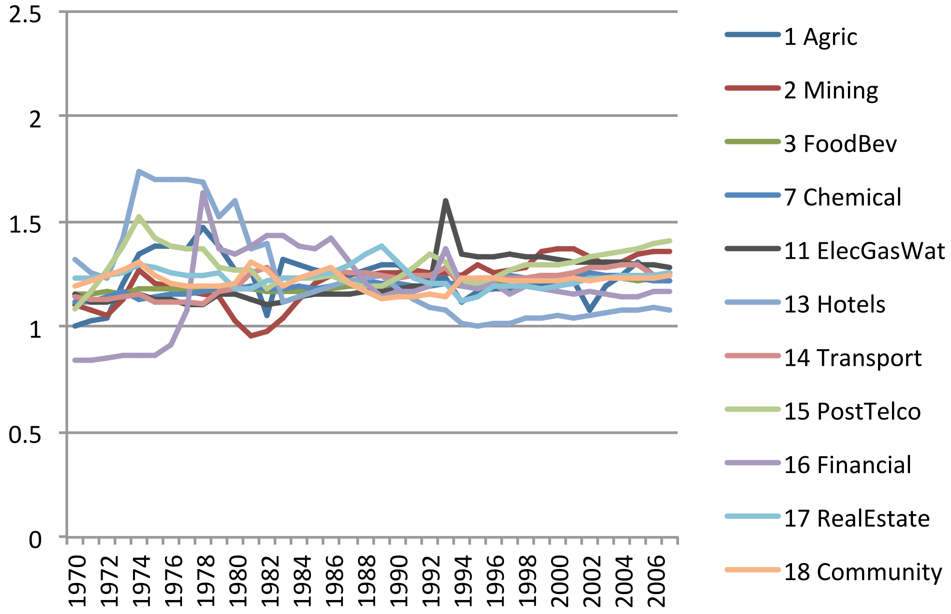


Figure 4: Output-Oriented Scale-Mix Efficiency Change (ΔOSME)

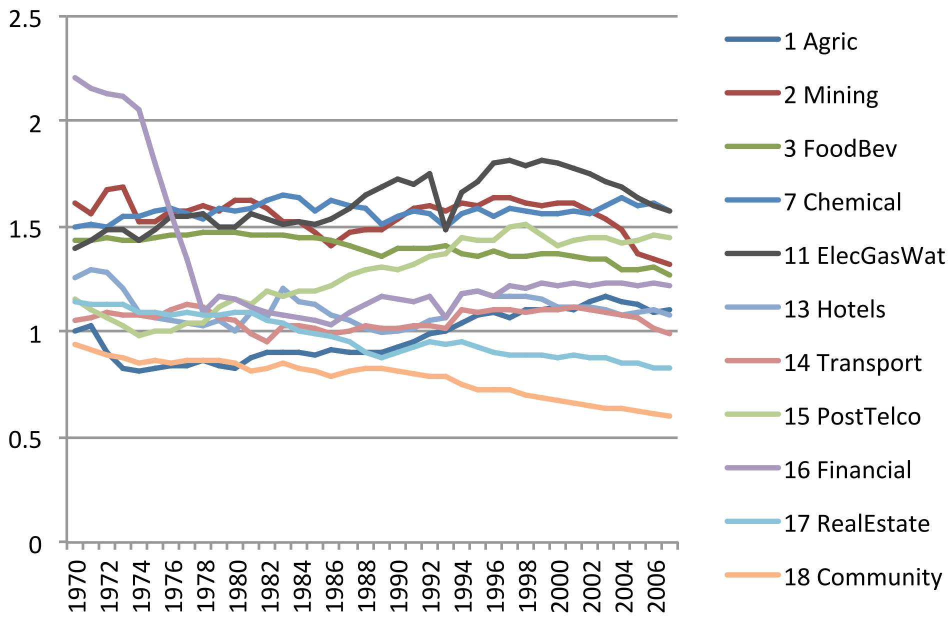


Figure 5: Change in Statistical Noise

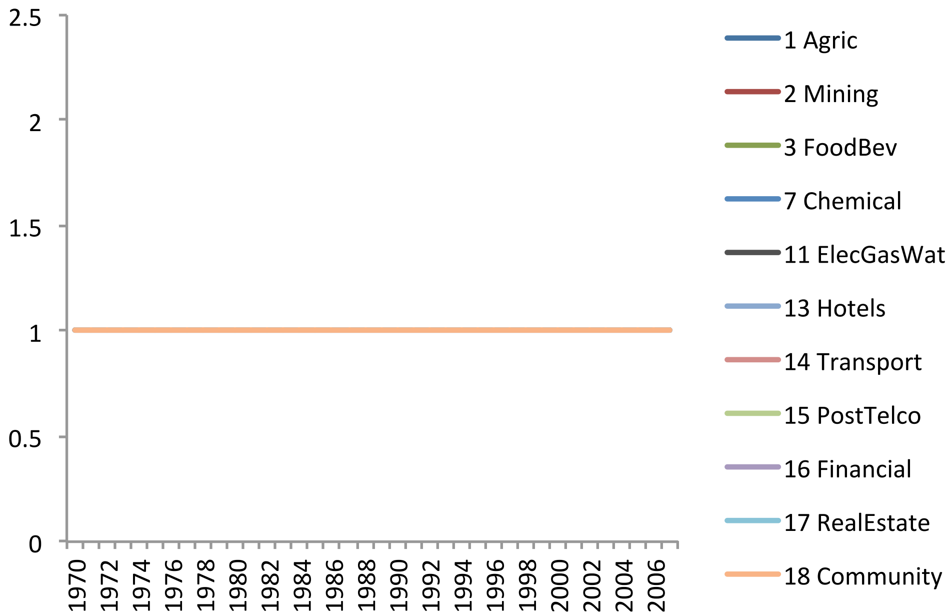


Figure 6: Components of TFP Change: Mining and Quarrying

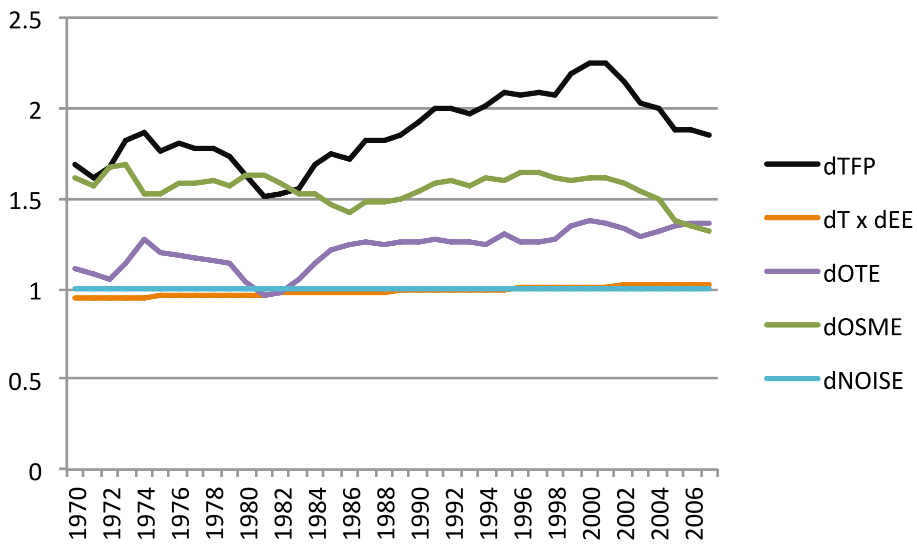


Figure 7: Components of OSME Change: Mining and Quarrying

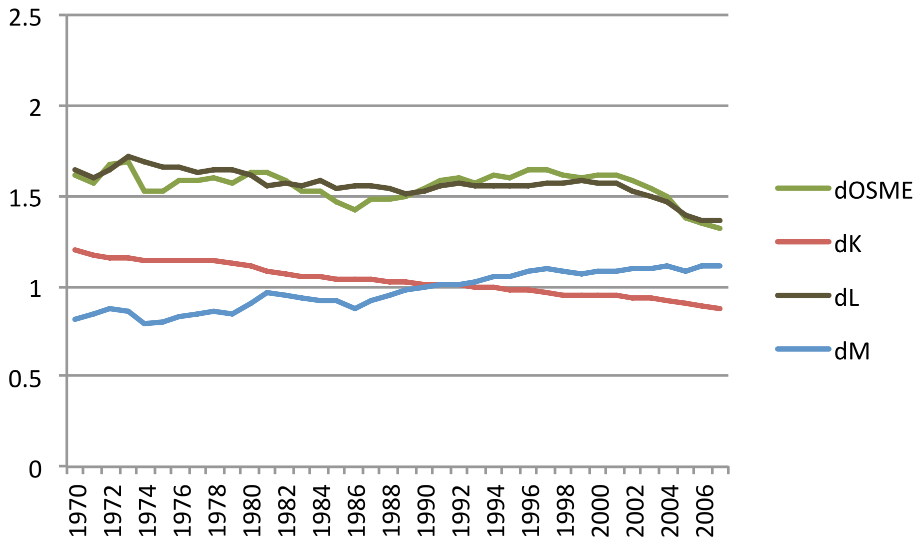
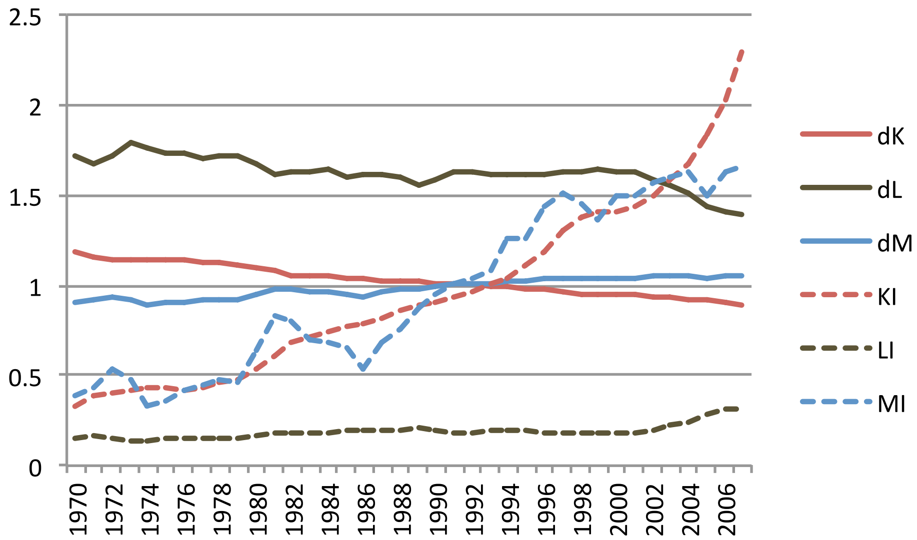


Figure 8: Capital and Labour Change: Mining and Quarrying



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