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Productivity Growth and Convergence: Revisiting Kumar and Russell (2002)

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Abstract

In this paper, we first investigate the replicability of the seminal work of Kumar and Russell (*The American Economic Review*, 2002) on productivity growth and convergence. We then compare Kumar and Russell's results with estimates obtained using alternative approaches that allow for statistical noise (e.g., measurement error) in the data. When statistical noise is accounted for, some of the conclusions found in their study are confirmed and some are not. In particular, capital deepening still remains as the key factor driving the transformation of the distribution of labor productivity from a unimodal distribution into a bimodal distribution with a higher mean. However, we cannot confirm that capital deepening contributes to world convergence. We find instead statistical evidence that indicates efficiency change is a key driver of world convergence.

Key words : DEA, Stochastic DEA, Local Likelihood Estimator, Productivity Growth, Convergence

JEL Codes: O47, D24, C14, C44

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1. Introduction

Since the seminal work of Färe *et al.* (1994), and Kumar and Russell (2002), frontier methodology has received considerable attention in the literature on productivity growth and convergence (see Ray and Desli (1997), Henderson and Russell (2005), Mastromarco and Simar (2014), among many others). The approach allows for the decomposition of output growth into: technological change (shift of the frontier), efficiency change (movement towards or away from the frontier) and input change (movement along the frontier). Kumar and Russell (2002) (hereafter KR), as well as many others, used deterministic nonparametric Data Envelopment Analysis (DEA) to construct the frontier. The advantage of DEA is that it does not require *a priori* assumptions about the functional form of the frontier (e.g., Cobb-Douglas, translog function, etc.). However, it assumes no statistical noise (e.g., measurement error) in the estimation of the frontier. This paper aims to account for statistical noise and investigate the robustness of KR's results when such noise is considered.

The most well-known approach dealing with statistical noise is a parametric stochastic frontier model proposed by Aigner *et al.* (1977) and Meeusen and van den Broeck (1977). In their models, an assumption about the parametric specification of the frontier function is required. Kumbhakar *et al.* (2007) relaxed the assumption by using the local likelihood estimator (LLE). Their approach was first applied for continuous explanatory variables; Park *et al.* (2015) later generalized it to encompass categorical variables. Both approaches were originally applied to cross-sectional data. In this paper, we adapt their methods to the context of two-period panel data to measure technological change nonparametrically. We also adapt the stochastic DEA (SDEA) proposed by Simar and Zelenyuk (2011) for comparison purposes.

Section 2 presents the estimation of the worldwide production frontier using nonparametric deterministic (DEA) and nonparametric stochastic methods (SDEA, LLE), and discusses the similarities and differences across the methods. Section 3 discusses the robustness of KR's results across the approaches and Section 4 concludes the paper.

2. Methodology

For each country i, $i = 1, \ldots, n$, we denote a period-t input vector as $X_{it} = (K_{it}, L_{it}) \in \mathbb{R}^2_+$, where K_{it} is physical capital and L_{it} is labor. A single output (gross domestic product–GDP) is denoted as $Y_{it} \in \mathbb{R}_+$. We assume the common technology set, describing a conversion of inputs to GDP in period t for all countries, can be characterized via a set $\mathcal{T}_t = \{(X_{it}, Y_{it}) : X_{it} \in \mathbb{R}^2_+ \text{ can produce } Y_{it} \in \mathbb{R}_+ \text{ in period } t\}$. The boundary of the technology set is referred to as the production frontier.

2.1. Data Envelopment Analysis (DEA)

KR used DEA as a tool to construct the worldwide production frontier and the associated efficiency scores of individual countries (distances from the frontier). In DEA, the technology set, denoted as $\mathcal{T}_t^{DEA}$, is often assumed to meet a number of regularity conditions such as convexity, free disposibility in inputs and outputs, and returns to scale (see Färe and Primont (1995) for further details). Under these assumptions, the technology set can be characterized by the Shephard (1970) output distance function

$$D(X_{it}, Y_{it}) \equiv \inf\{\delta > 0 : (X_{it}, Y_{it}/\delta) \in \mathcal{T}_t^{DEA}\}. \tag{2.1}$$

The distance function can also be viewed as a criterion of efficiency of a country i, denoted as $TE_{it} \equiv D(X_{it}, Y_{it})$. A country is considered as fully efficient when $TE_{it} = 1$, and technically inefficient if $TE_{it} < 1$.

Since the true technology set $\mathcal{T}_t^{DEA}$ is not known, individual technical efficiencies must be estimated. The associated efficiency score for a country i, under the constant returns to scale, period-t technology, is obtained via the following optimization problem

$$\widehat{TE}_{it}^{DEA} = \inf_{\delta, \alpha_{1t}, \ldots, \alpha_{nt}} \{\delta > 0 : (X_{it}, Y_{it}/\delta) \in \hat{\mathcal{T}}_t^{DEA}\}, \tag{2.2}$$

where

$$\hat{\mathcal{T}}_t^{DEA} = \Big\{(X, Y) \in \mathbb{R}^2_+ \times \mathbb{R}^1_+ : \sum_{i=1}^{n} \alpha_{it} Y_{it} \geq Y; \sum_{i=1}^{n} \alpha_{it} X_{it} \leq X; \alpha_{it} \geq 0, i = 1, \ldots, n\Big\}, \tag{2.3}$$

where α_{it} are intensity variables.

2.2. Local Likelihood Estimator (LLE)

Our model here is a variant of the stochastic frontier models proposed by Kumbhakar *et al.* (2007) and Park *et al.* (2015). We need to adapt their approaches to measure technological change non-parametrically.

Let y_{it} denote the logarithm of output $(\log(Y_{it}))$ for a country i in period t, and x_{it} denote the logarithm of inputs $(\log(X_{it}))$. Given the sample of 57 countries observed during two periods 1965 and 1990, our localized version of the stochastic frontier model is

$$y_{it} = m(x_{it}, d_t) - u_{it} + v_{it}, \tag{2.4}$$

where $m(.,.)$ is the unknown frontier function. The two-sided error term v_{it} is statistical noise and $v_{it}|x_{it} = x, d_t = d \sim N(0, \sigma_v^2(x, d))$. The one-sided error term $u_{it} \geq 0$ represents technical inefficiency and $u_{it}|x_{it} = x, d_t = d \sim |N(0, \sigma_u^2(x, d))|$. A dummy variable d_t takes the value of 1 if an observation is in the period 1990, and 0 otherwise. The coefficient associated with the dummy variable d_t is a measurement of technological change between the periods 1965 and 1990.

Given the model (2.4), $\theta(x, d) = [m(x, d), \sigma_u^2(x, d), \sigma_v^2(x, d)]'$ are unknown parameters and are required to be estimated. The conditional log-likelihood function of θ is

$$L(\theta) = \sum_{t=1}^{2} \sum_{i=1}^{n} \log\Big[g(y_{it}, \theta(x_{it}, d_t))\Big], \tag{2.5}$$

where $g(.,.)$ is the known conditional probability density function of y_{it} given $x_{it} = x$ and $d_t = d$.

Direct maximization of the log-likelihood function over θ is intractable and suffers from over-fitting in an infinite-dimensional function space (Tibshirani and Hastie (1987)). The idea of LLE is to approximate $\theta(x_{it}, d_t)$ by polynomial approximation (e.g., local linear approximation) in a neighborhood of a fixed interior point (x, d) following the direction of continuous variables x. The neighboring observations around (x, d) are given more weight than those further from (x, d). The weight is chosen by using a kernel approach. For time discrete variables, we use the kernel function $W_\gamma(d_t, d) = \gamma^{\mathbf{I}(d_t \neq d)}$, with a bandwidth $\gamma \in [0, 1]$, and an indicator function $\mathbf{I}(B)$ taking a value of 1 if B holds, and 0 otherwise. The kernel choice has also been employed by Park *et al.* (2015). For a vector of $p \times 1$ of continuous variables x_{it}, we choose the most common kernel in the applied literature, which is the product kernel defined on $\mathbb{R}^p$, $K_H(x_{it}, x) = |H|^{-1} \prod_{s=1}^{p} k\left(\frac{x_{it}^s - x^s}{h^s}\right)$, where $H = (h^1, \ldots, h^p)$, with h^s being a bandwidth of continuous variables x^s ($s = 1, \ldots, p$), $|H|$ being the determinant of H, $|H| = h^1 h^2 \ldots h^p$, and k being a Gaussian density (see Pagan and Ullah (1999, pp.54-55) for other kernel choices).

The local log-likelihood for the local linear approximation at a fixed interior point (x, d) is then given by

$$\begin{aligned} L_n(\theta^{(0)}, \theta^{(1)}; x, d) &= \sum_{t=1}^{2} \sum_{i=1}^{n} \Big(\log\Big[g(y_{it}, \theta^{(0)}(x, d) + \theta^{(1)}(x, d)(x_{it} - x)) \Big] \\ &\times \quad K_H(x_{it} - x) W_\gamma(d_t, d) \Big). \end{aligned} \tag{2.6}$$

At the point (x, d), the local linear estimator of $\theta(x, d)$, denoted as $\hat{\theta}(x, d) = \hat{\theta}^{(0)}(x, d)$), and its first derivative, denoted as ($\hat{\theta}^{(1)}(x, d)$), are obtained by maximizing (2.6)

$$(\hat{\theta}^{(0)}(x, d), \hat{\theta}^{(1)}(x, d)) = \underset{\theta^{(0)}, \theta^{(1)}}{\operatorname{argmax}} \, L_n(\theta^{(0)}, \theta^{(1)}; x, d). \tag{2.7}$$

The bandwidths γ and H in (2.6) can be estimated via a number methods such as rule-of-thumb, plug-in, and cross-validation. Here we choose the leave-one-out cross-validation estimator, a data-driven approach, to jointly estimate γ and H. The bandwidths are chosen by maximizing

$$MLCV(H, \gamma) = \frac{1}{2n} \sum_{t=1}^{2} \sum_{i=1}^{n} \log[g(y_{it}, \hat{\theta}_{H,\gamma}^{-i}(x_{it}, d_t))], \tag{2.8}$$

where $\hat{\theta}_{H,\gamma}^{-i}(x_{it}, d_t)$ is the 'leave-the ith observation-out' version of LLE discussed above.

The procedure in Battese and Coelli (1988, p.391) is used to obtain the predictor of an individual efficiency score

$$\widehat{TE}_{it}^{LLE} \approx E(\exp(-u_{it}) | \hat{\epsilon}_{it}), \tag{2.9}$$

where $\hat{\epsilon}_{it} = y_{it} - \hat{m}^{(0)}(x_{it}, d_t)$.

2.3. Stochastic DEA (SDEA)

Here we adapt SDEA proposed by Simar and Zelenyuk (2011) to the context of continuous and discrete time variables. The associated individual efficiencies are estimated using a similar procedure as described in DEA (equation (2.2))

$$\widehat{TE}_{it}^{SDEA} = \inf_{\delta,\alpha_{1t},\ldots,\alpha_{nt}} \{\delta > 0 : (X_{it}, Y_{it}/\delta) \in \hat{\mathcal{T}}_t^{SDEA}\}. \tag{2.10}$$

However, in SDEA the construction of the constant returns to scale, period-t technology, $\hat{\mathcal{T}}_t^{SDEA}$, is based on 'filtered' data $\{(X_{it}, \hat{Y}_{it}^*)\}_{i=1}^n$, where $\hat{Y}_{it}^*$ is filtered from noise using LLE, rather than the original data $\{(X_{it}, Y_{it})\}_{i=1}^n$

$$\hat{\mathcal{T}}_t^{SDEA} = \Big\{ \sum_{i=1}^n \alpha_{it}\hat{Y}_{it}^* \geq Y; \sum_{i=1}^n \alpha_{it}X_{it} \leq X, \alpha_{it} \geq 0, i = 1, \ldots, n \Big\}, \tag{2.11}$$

where $\hat{Y}_{it}^* = \exp(\hat{y}_{it}^*)$ and $\hat{y}_{it}^* = \hat{m}^{(0)}(x_{it}, d_t)$, being the optimal output estimated in (2.7). SDEA can be viewed as a combination of DEA and LLE estimations.

2.4. Comparison among DEA, SDEA and LLE

For a single output case, we can present a deterministic and stochastic frontier model as $Y_{it} = f_t(K_{it}, L_{it})e^{-u_{it}}e^{v_{it}}$, where $f_t(K_{it}, L_{it})$ is a production function in period t ($f_t(K_{it}, L_{it}) = m(K_{it}, L_{it}, d_t)$). The key differences across DEA, SDEA and LLE approaches lie in the following aspects: the presence of statistical noise, the regularity conditions and the estimation of the associated individual efficiency scores.

DEA assumes statistical noise v_{it} equal to 0, and thus the technical efficiency measuring the distance from observed data to the frontier is simply the ratio of observed data to optimal output given inputs, denoted as $TE_{it}^{DEA} = Y_{it}/f_t(K_{it}, L_{it})$. The frontier $f_t(K_{it}, L_{it})$ and the associated efficiency scores are estimated via mathematical programming algorithms as described in (2.2).

Unlike DEA, LLE accounts for statistical noise v_{it}, i.e., v_{it} is different from 0. The technical efficiency is the ratio of observed output to the corresponding stochastic frontier, denoted as

$TE_{it}^{LLE} = Y_{it}/(f_t(K_{it}, L_{it})e^{v_{it}})$, which can be estimated via an approximation as shown in equation (2.9). The frontier $f_t(K_{it}, L_{it})$ is estimated via the maximum likelihood-based method.

Both DEA and SDEA require mathematical programming algorithms to estimate the individual technical efficiency scores. While DEA uses an 'original' data $\{(K_{it}, L_{it}, Y_{it})\}_{i=1}^{n}$ to construct the frontier $f_t(K_{it}, L_{it})$, SDEA uses a 'filtered' data $\{(K_{it}, L_{it}, \hat{Y}_{it}^{*})\}_{i=1}^{n}$, where $\hat{Y}_{it}^{*}$ is filtered from possible noise using LLE.

It is noted that the frontier, in the set up of LLE, is not required to meet a number of regularity conditions, as is the case in DEA and SDEA. The regularity conditions can be imposed using the constrained regression techniques (see Du *et al.* (2013)). In KR's study, the constant returns to scale can be imposed directly in LLE by normalizing the dependent and independent variables by labor (i.e., $Y_{it}/L_{it} = f_t(K_{it}/L_{it}, 1)$).

2.5. Tripartite decomposition of output per worker growth

Under the constant returns to scale technology, KR decomposed output per worker (labor productivity) growth into three factors: efficiency change (the movement toward to the frontier), technological change (the shift of the frontier) and capital deepening (the movement along the frontier). The basic idea underlying the decomposition is that the countries are assumed to operate according to a common efficient frontier. Accordingly, the countries can be thought of as performing on or within the frontier, and the distance from a country to the frontier reflects the technical efficiency. Over time, a country can perform more efficiently and move toward the frontier, or the frontier itself can shift outward over time, indicating technological progress. A country can also move along the frontier by changing inputs.

Mathematically, the decomposition of labor productivity growth for a country i between a base period $(t = 0)$ and a current period $(t = 1)$ can be represented as

$$\frac{Y_{i1}/L_{i1}}{Y_{i0}/L_{i0}} = \frac{TE_{i1}}{TE_{i0}} \times \left(\frac{f_1(K_{i1}/L_{i1},1)}{f_0(K_{i1}/L_{i1},1)} \times \frac{f_1(K_{i0}/L_{i0},1)}{f_0(K_{i0}/L_{i0},1)}\right)^{1/2} \times \left(\frac{f_1(K_{i1}/L_{i1},1)}{f_1(K_{i0}/L_{i0},1)} \times \frac{f_0(K_{i1}/L_{i1},1)}{f_0(K_{i0}/L_{i0},1)}\right)^{1/2}$$

$$= EFF_i \qquad \times \qquad TECH_i \qquad \times \qquad KACCUM_i, \quad (2.12)$$

where $f_t(.,.)$ is the potential output per unit of labor in period t. An individual score TE_{it} is estimated via DEA (equation (2.2)), SDEA (equation (2.10)) or LLE (equation (2.9)). The first term on the right-hand side of equation (2.12) measures efficiency change, the second term captures technological change and the last term reflects capital deepening.

3. Empirical Results

KR focus on a number of important issues in productivity growth and convergence: What are the main sources of labor productivity growth during 25 periods from 1965-1990? Is there any evidence suggesting the tendency of world convergence—the poor can 'catch up' with the rich? What are the key determinants for world convergence?

3.1. Analysis of tripartite decomposition of labor productivity growth

Table 3.1 presents the percentage change of the tripartite decomposition of labor productivity growth for some selected countries in the sample.[1] Column (i) of Table 3.1 provides the results estimated via DEA, column (ii) reports the values computed via SDEA, and column (iii) presents the estimates obtained via LLE.

Comparing across the three sets of these estimates, we note that although the difference in the mean contribution of technological change is relatively small, there are substantial discrepancies

[1] The results of percentage change of the tripartite decomposition of labor productivity growth for all 57 countries are referred to Table 4.1 in Appendix.

Table 3.1: Estimation results of growth in output per worker from 1965 to 1990 and its resources using DEA, SDEA and LLE

Countries	**% change in output per worker**	**Contribution to % change in output per worker of** (EFF-1)	(TECH-1)	(KACCUM-1)	(EFF-1)	(TECH-1)	(KACCUM-1)	(EFF-1)	(TECH-1)	(KACCUM-1)
			DEA(i)			**SDEA(ii)**			**LLE(iii)**	
Australia	42.67	8.06	14.03	15.78	-14.62	8.33	54.26	-9.24	8.17	54.49
Belgium	78.36	22.04	12.94	29.41	-11.46	8.58	85.52	-6.65	8.56	85.57
France	78.29	3.59	16.72	47.45	-23.34	8.65	114.04	-12.48	7.16	117.03
Germany, West	70.75	13.24	14.49	31.70	-29.86	17.76	106.72	-21.53	6.62	128.32
Hong Kong	251.08	120.17	2.36	55.79	83.51	14.23	67.48	50.68	7.83	57.99
Japan	208.52	3.27	15.16	159.42	-28.19	6.49	303.45	-21.92	6.47	303.54
Italy	117.45	31.76	13.45	45.46	-0.68	9.16	100.57	-0.37	7.59	103.49
Luxembourg	78.47	31.88	24.55	8.65	11.04	17.92	36.29	7.48	5.44	52.42
Netherlands	51.45	5.38	11.21	29.24	-20.88	9.20	75.29	-9.59	9.15	75.37
United Kingdom	60.74	-3.70	1.39	64.63	-11.50	2.00	78.05	-7.54	7.87	95.28
United States	31.29	0.00	10.02	19.33	-12.55	0.34	49.62	-8.42	8.78	62.36
Zimbabwe	11.38	37.24	2.47	-20.80	36.77	5.64	-22.92	30.22	5.62	-22.92
Mean over 57 countries	**75.06**	**5.24**	**6.17**	**58.47**	**-8.50**	**5.31**	**86.04**	**-5.40**	**6.96**	**86.03**

Notes:

- See equation (2.12) for details of the abbreviations.
- Some observations, in the case of SDEA, yield efficiency scores greater than 1. For these observations, we consider two possible scenarios: 'fully efficient' ($\widehat{TE}_{it}^{SDEA} = 1$), and 'super efficient' observations (Timmer (1971)) ($\widehat{TE}_{it}^{SDEA} \geq 1$). Here we only present the fully efficient case for the sake of brevity. We note that the conclusions remain the same across the cases.

in the contributions of efficiency change, and of capital deepening. In particular, both stochastic approaches (LLE and SDEA) suggest a decrease in the mean contribution of efficiency scores to labor productivity growth (-8.5% in the case of SDEA, and around -5.4% in the case of LLE), while the reverse holds in the case of deterministic DEA (5.2%). One explanation is statistical noise. DEA considers that any deviation of observations from the frontier is explained by inefficiencies. On the other hand, the deviations are due to both inefficiencies and statistical noise in LLE and SDEA. The negative contribution of efficiency change to labor productivity growth (e.g., Australia, Belgium, Germany, Italy, United Kingdom, etc.) should not be interpreted as lowering the well-being of these countries, rather, it means that these countries have not exploited their resources as efficiently as other countries with higher technical efficiency scores over the time period, given similar levels of inputs. Also, a negative contribution of technical efficiency scores might indicate that the countries can improve their respective labor productivity further if there is an improvement in technical efficiency (e.g., better utilization of resources, labor management, etc.).

Capital deepening is confirmed as the primary source of worldwide productivity growth over 25 years, yet the magnitude of the mean contribution of capital deepening seems to be underestimated when statistical noise is not considered in the estimation of the world frontier. Its magnitude is only 59% in the case of DEA whereas it is 86% for both LLE and SDEA. The differences across the approaches are more pronounced when we have a closer look at individual countries such as Belgium, France, Japan, Luxembourg. For Belgium, the contribution of capital deepening estimated in SDEA, LLE (approximately 85.5%) is almost three times higher than that obtained in DEA (29.41%).

3.2. Analysis of labor productivity distributions

By employing a kernel-based density estimator, KR observed that labor productivity distribution had one peak in 1965, and contained a twin-peak in 1990. They went on to explore the underlying factors driving the bimodal transformation. Following KR, we first construct the counter-factual distribution by sequentially multiplying the labor productivity ($y_L = Y/L$) using the actual data for 1965 by each component of the tripartite decomposition. For instance, the counter-factual distribution based on capital deepening, denoted as $pdf(y_{L65} \times KACCUM)$, is obtained by adjusting labour productivity based on the 1965 data by the effect of capital deepening.

Table 3.2: Testing for the significant contributions of various sources to the bimodal transformation of output per worker

	DEA (i)		**SDEA (ii)**		**LLE (iii)**	
H_0 : Distributions are equal H_1 : Distributions are not equal	**Test statistic (ia)**	**Bootstrap p-value (ib)**	**Test statistic (iia)**	**Bootstrap p-value (iib)**	**Test statistic (iiia)**	**Bootstrap p-value (iiib)**
$pdf(y_{L90})$ vs. $pdf(y_{L65})$	3.52	0.00	3.52	0.00	3.52	0.00
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times EFF)$	2.78	0.00	4.69	0.00	4.26	0.00
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times TECH)$	2.06	0.01	2.27	0.01	2.79	0.01
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times KACCUM)$	0.77	0.19	0.80	0.16	1.26	0.07
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times EFF \times TECH)$	1.61	0.04	3.76	0.00	3.71	0.00
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times EFF \times KACCUM)$	0.68	0.27	0.37	0.59	0.10	0.90
$pdf(y_{L90})$ vs. $pdf(y_{L65} \times TECH \times KACCUM)$	-0.06	0.93	1.83	0.03	2.13	0.01

Notes:

- $pdf(z)$ stands for probability density function of Z.
- We use the bootstrapped Li (1996) tests with 2000 bootstrap replications and Silverman's(1986) rule of thumb bandwidth.

We perform a statistical test proposed by Li (1996, 1999) to test the equality between the 1990 labor productivity distribution and its counter-factual distributions based on efficiency change, technological change, and capital deepening. We find that the conclusions of the hypotheses are identical across the approaches (see the second, third and fourth rows of columns (ib), (iib), (iiib) in Table 3.2), confirming that capital deepening is the key source of the bimodal transformation of labor productivity distribution.[2] In addition, we note that p-values are smaller than 0.05 in the hypothesis tests where the counter-factual distributions are based on technological change or its

[2]For the sake of space, we only present the distribution of labour productivity based on the actual 1990 data and

combination with other components (see the third, fifth and seventh rows of columns (iib), (iiib) in Table 3.2). This might indicate that technological change dampens the twin-peak transformation, and that it might be a source impeding the division of the world into two clubs: the poor and the rich.

3.3. Convergence hypothesis

Another important issue investigated by KR is to identify the determinants of world convergence. Following KR, we regress each of the decompositions on the output per worker in the base period (1965). The signs and significance of the resulting coefficients of these regressions shed light on catch-up convergence (if they are negative and significant), or divergence (if they are positive and significant).

Table 3.3: Growth regressions of the percentage change in output per worker 1965 and the three decompositions

Independent variable (Output per worker in 1965)	**DEA (i)**	**SDEA (ii)**	**LLE (iii)**
Dependent Variables	**Coefficient (p-value)**	**Coefficient (p-value)**	**Coefficient (p-value)**
$(EFF-1)\times 100$	0.09	-0.87	-0.47
	(0.81)	(0.01)	(0.07)
$(TECH-1)\times 100$	0.77	0.60	0.05
	(0.05)	(0.01)	(0.70)
$(KACCUM-1)\times 100$	-2.37	-0.83	0.26
	(0.00)	(0.46)	(0.82)

Table 3.3 summarizes the estimated coefficients and the p-values of their respective regressions.[3] The coefficient, reflecting the relationship between the contribution of efficiency to the

its counter-factual distributions following the order $KACCUM-TECH-EFF$ for all approaches. These are referred to Figure 1, 2, 3 in Appendix. For other orders, the results are available from the authors upon request.

[3]Figure 4 depicting the patterns of the least-square regressions is presented in Appendix.

productivity growth and the output per worker in 1965, was positive and statistically insignificant in KR's study. On the other hand, we find the coefficient is significantly negative at the 5% level of significance in the case of SDEA, and at 10% in the case of LLE. The result favors the conclusion that efficiency change is one of drivers of world convergence. In other words, the low capital-labor ratio countries have benefited significantly from the improvement in their own efficiency, and poor countries are able to 'catch up' with the rich. The result is in line with the findings of Badunenko *et al.* (2008).

We now turn into the impact of technological change on world convergence. The coefficients of the least-square regressions are positive and statistically significant in the cases of DEA and SDEA, yet this does not hold in the case of LLE. It is therefore inconclusive to conclude whether technological change makes a contribution in reducing the gap between the rich and the poor.

For capital deepening, our results do not suggest capital deepening contributes to convergence of income per worker across the sample as found in KR's study. We find the coefficients are insignificantly negative (positive) in SDEA (LLE). The insignificant results should not be interpreted to mean that capital deepening does not contribute to labor productivity growth. In fact, the results in Table 4.1 reveal capital deepening has a significant impact on labor productivity growth of most countries in the dataset such as Austria, Ecuador, Greece, Ireland, Spain, Portugal, etc.

4. Conclusion

This paper first set out to replicate the KR's results and that was achieved. Secondly, the robustness of their conclusions was investigated when statistical noise is explicitly modeled in the estimation of the production frontier. For this purpose, a deterministic approach (DEA) and stochastic non-parametric approaches (LLE, SDEA) were used. Capital deepening is confirmed as the primary source driving the bimodal transformation of the labor productivity distribution. The magnitude of its contribution to labor productivity growth seems to be underestimated when statistical noise is not accounted for. For instance, deterministic DEA suggests the U.S. capital deepening only contributes around 19% to the growth of U.S. labor productivity whereas its magnitude is about two and half times lower than those estimated in LLE and SDEA (above 49%). We found efficiency change makes a contribution to world convergence.

Beyond our comparison and replication, the methods (SDEA and LLE) used in this paper might be appealing to some economic growth researchers, who wish to study other issues in economic growth and convergence in the framework, where an assumption about the functional form of the production function is relaxed and statistical noise is modeled in the frontier estimation.

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References

Aigner, D., Lovell, C., and Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6(1):21 – 37.

Badunenko, O., Henderson, D. J., and Zelenyuk, V. (2008). Technological change and transition: Relative contributions to worldwide growth during the 1990s*. *Oxford Bulletin of Economics and Statistics*, 70(4):461–492.

Battese, G. E. and Coelli, T. J. (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data. *Journal of Econometrics*, 38(3):387 – 399.

Du, P., Parmeter, C., and Racine, J. (2013). Nonparametric kernel regression with multiple predictors and multiple shape constraints. *Statistica Sinica*, 23(3):1347–1371.

Färe, R., Grosskopf, S., Norris, M., and Zhang, Z. (1994). Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries. *American Economic Review*, 84(1):66–83.

Färe, R. and Primont, D. (1995). *Multi-ouput production and duality:theory and applications*. New York: Kluwer Academic Publishers.

Henderson, D. J. and Russell, R. R. (2005). Human capital and convergence: A production-frontier approach. *International Economic Review*, 46(4):1167–1205.

Kumar, S. and Russell, R. R. (2002). Technological change, technological catch-up, and capital deepening: Relative contributions to growth and convergence. *The American Economic Review*, 92(3):527–548.

Kumbhakar, S. C., Park, B. U., Simar, L., and Tsionas, E. G. (2007). Nonparametric stochastic frontiers: A local maximum likelihood approach. *Journal of Econometrics*, 137(1):1 – 27.

Li, Q. (1996). Nonparametric testing of closeness between two unknown distribution functions. *Econometric Reviews*, 15(3):261–274.

Li, Q. (1999). Nonparametric testing the similarity of two unknown density functions: local power and bootstrap analysis. *Journal of Nonparametric Statistics*, 11(1-3):189–213.

Mastromarco, C. and Simar, L. (2014). Effect of FDI and time on catching up: New insights from a conditional nonparametric frontier analysis. *Journal of Applied Econometrics*.

Meeusen, W. and van den Broeck, J. (1977). Efficiency estimation from Cobb-Douglas production functions with composed error. *International Economic Review*, 18(2):435–44.

Pagan, A. and Ullah, A. (1999). *Nonparametric Econometrics*. Cambridge University Press. Cambridge Books Online.

Park, B. U., Simar, L., and Zelenyuk, V. (2015). Categorical data in local maximum likelihood: theory and applications to productivity analysis. *Journal of Productivity Analysis*, 43(2):199–214.

Ray, S. C. and Desli, E. (1997). Productivity growth, technical progress, and efficiency change in industrialized countries: Comment. *The American Economic Review*, 87(5):1033–1039.

Shephard, R. W. (1970). *Theory of cost and production functions*. Princeton,NJ:Princeton University Press.

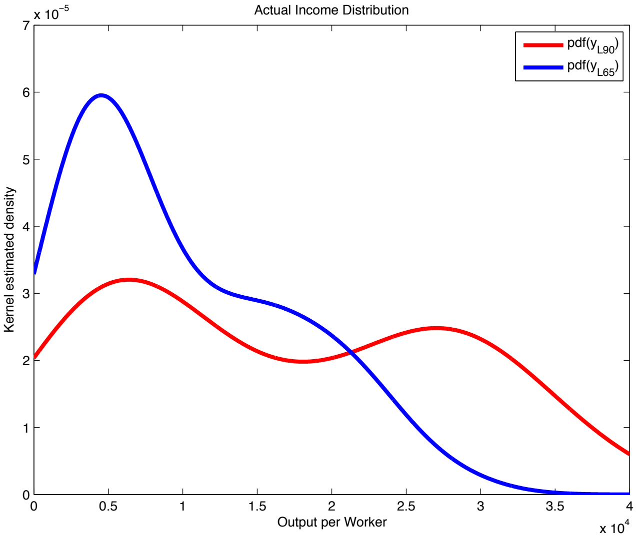
Simar, L. and Zelenyuk, V. (2011). Stochastic FDH/DEA estimators for frontier analysis. *Journal of Productivity Analysis*, 36(1):1–20.

Tibshirani, R. and Hastie, T. (1987). Local likelihood estimation. *Journal of the American Statistical Association*, 82(398):559–567.

Timmer, C. P. (1971). Using a probabilistic frontier production function to measure technical efficiency. *Journal of Political Economy*, 79(4):776–794.

Appendix A

FIGURES & TABLES



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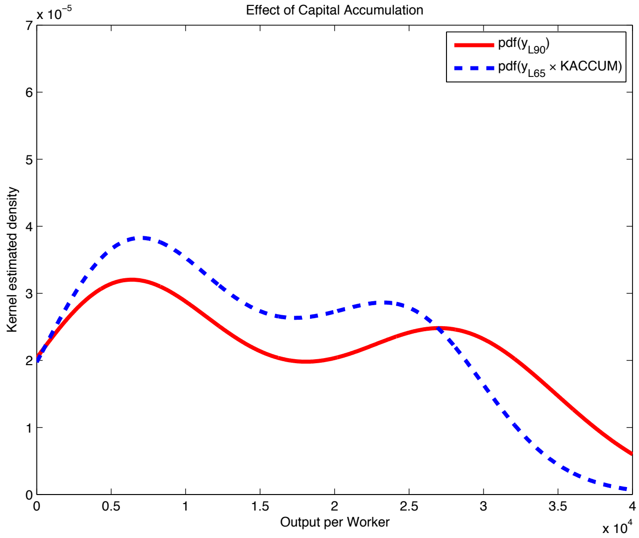

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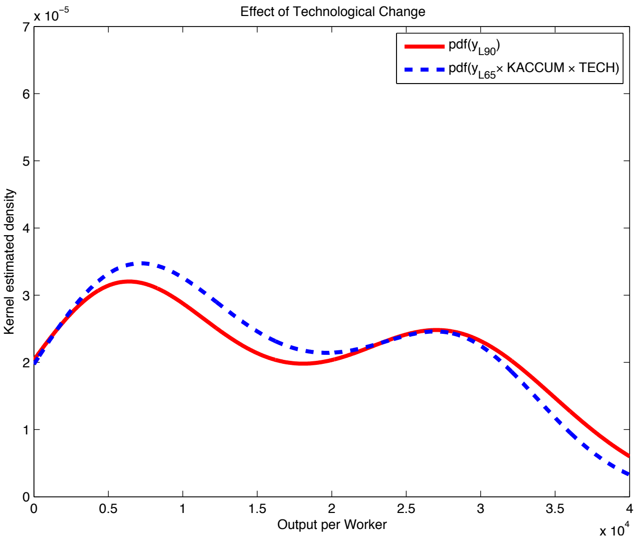

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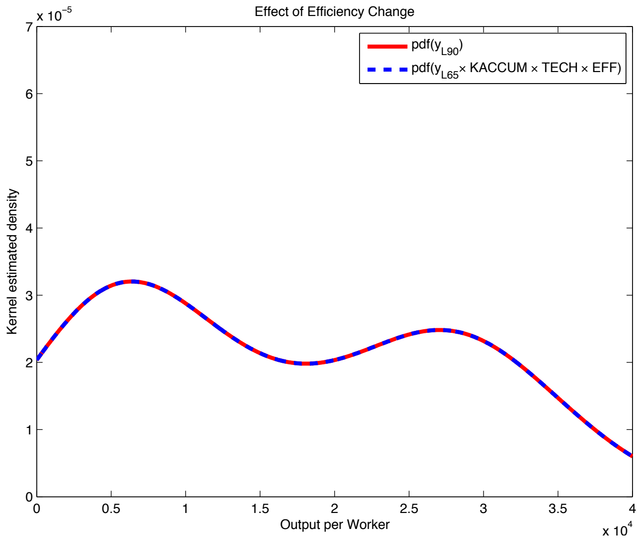
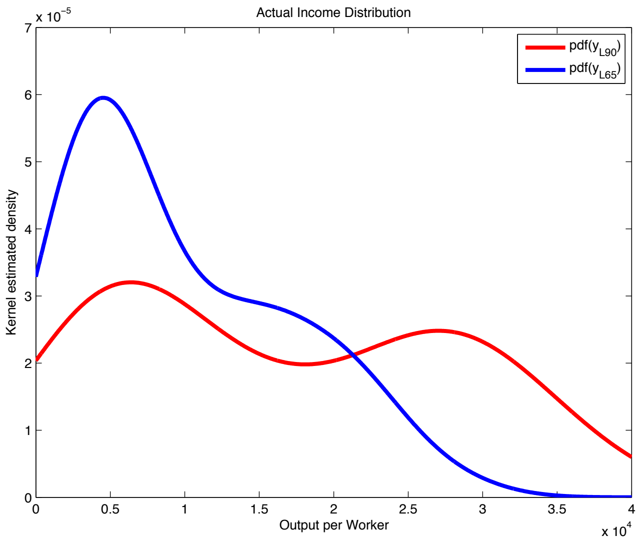
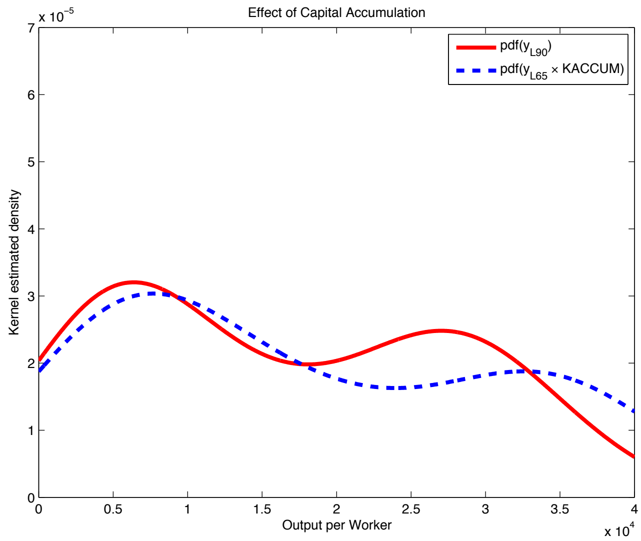


(d)

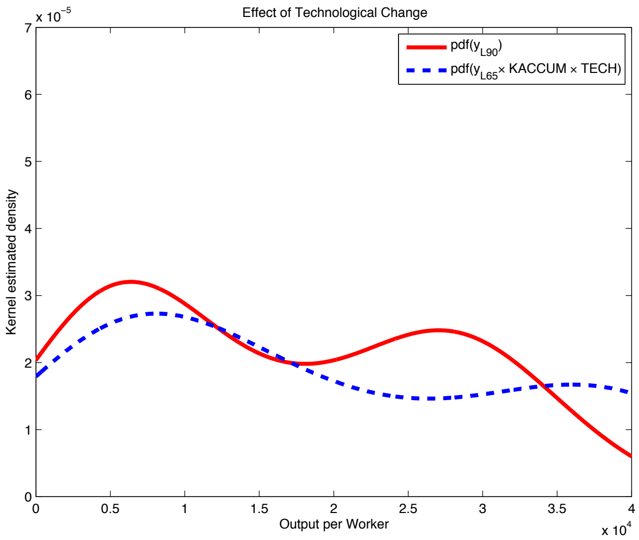
Figure 1: The counter-factual distributions of output per worker are constructed following the order $KACCUM - TECH - EFF$ using DEA



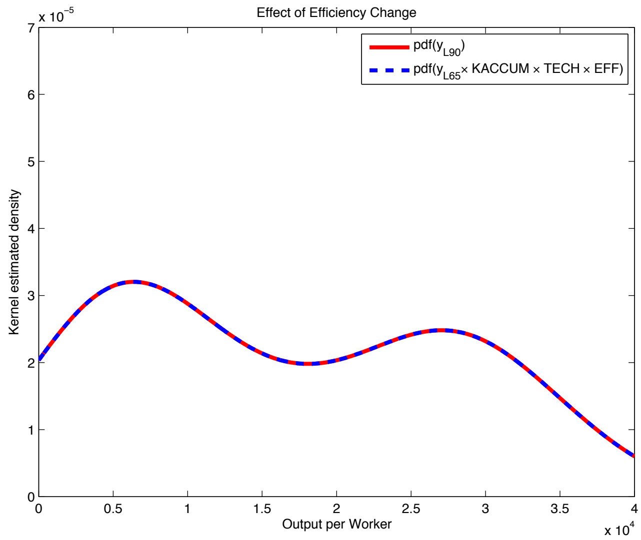
(a)



(b)



(c)



(d)

Figure 2: The counter-factual distributions of output per worker are constructed following the order $KACCUM - TECH - EFF$ using SDEA

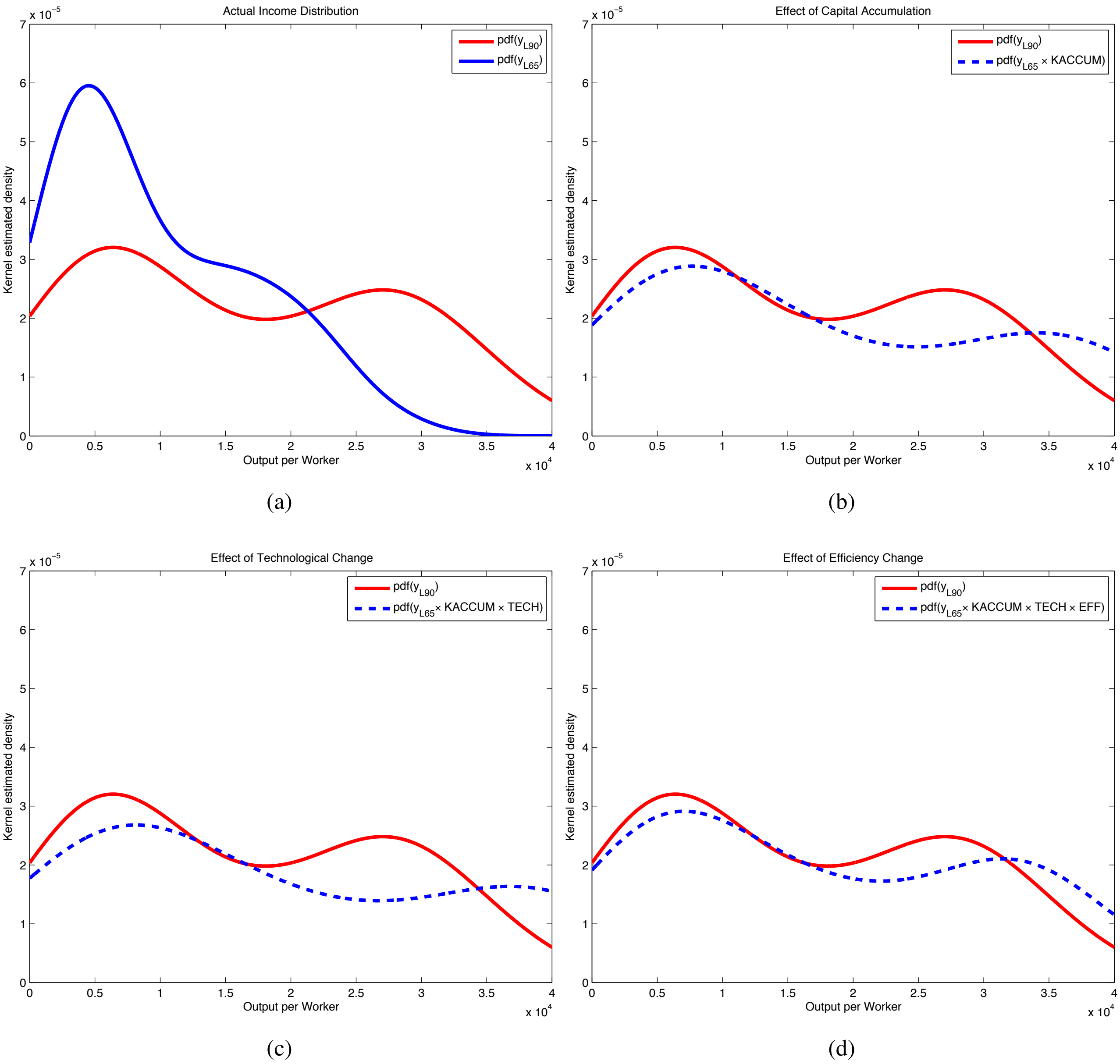


Figure 3: The counter-factual distributions of output per worker are constructed following the order $KACCUM - TECH - EFF$ using LLE

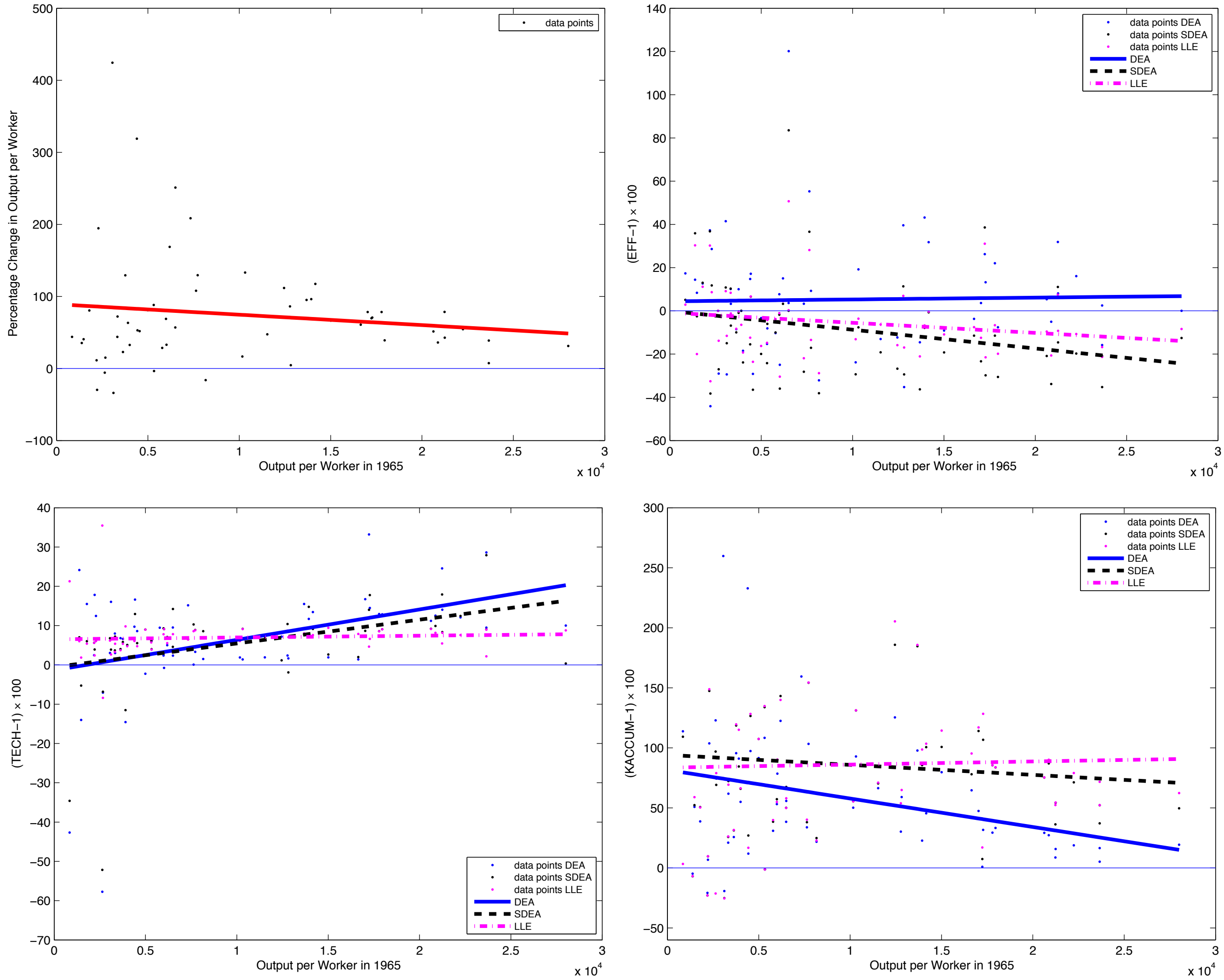


Figure 4: The tripartite decomposition plotted against output per worker in 1965

Table 4.1: Estimation results of the determinants of change in output per worker 1965 to 1990 using deterministic DEA, SDEA and LLE

Countries	**% change in output per worker**	**Contribution to % change in output per worker of**								
		(EFF-1)	(TECH-1)	(KACCUM-1)	(EFF-1)	(TECH-1)	(KACCUM-1)	(EFF-1)	(TECH-1)	(KACCUM-1)
			DEA(i)			**SDEA(ii)**			**LLE(iii)**	
Argentina	4.59	-35.30	1.64	59.04	-29.41	-1.91	51.05	-16.99	7.04	64.93
Australia	42.67	8.06	14.03	15.78	-14.62	8.33	54.26	-9.24	8.17	54.49
Austria	95.15	-14.57	15.49	97.79	-36.34	7.69	184.64	-21.07	8.00	185.77
Belgium	78.36	22.04	12.94	29.41	-11.46	8.58	85.52	-6.65	8.56	85.57
Bolivia	32.71	-18.50	5.02	55.05	-23.86	5.06	65.91	-19.29	4.76	66.38
Canada	54.55	16.06	12.09	18.80	-19.76	12.46	71.27	-10.74	7.59	79.03
Chile	16.57	-23.82	1.88	50.19	-29.35	6.13	55.46	-13.12	6.11	55.41
Colombia	68.78	7.66	2.38	53.12	-1.80	9.31	57.23	-1.07	7.80	54.94
Denmark	39.08	-7.58	12.86	33.33	-30.61	9.08	83.76	-19.86	9.04	83.83
Dominican Rep.	51.81	-29.18	8.61	97.38	-36.51	5.53	126.59	-23.60	4.76	128.26
Ecuador	80.89	-3.35	-2.24	91.46	-19.96	8.98	107.37	-16.26	8.98	107.37
Finland	96.23	43.15	11.70	22.71	-7.87	14.76	85.60	-6.76	7.22	98.66
France	78.29	3.59	16.72	47.45	-23.34	8.65	114.04	-12.48	7.16	117.03
Germany, West	70.75	13.24	14.49	31.70	-29.86	17.76	106.72	-21.53	6.62	128.32
Greece	129.46	9.26	3.31	103.28	-17.11	8.89	154.23	-13.48	8.80	154.44
Guatemala	28.54	-10.30	9.46	30.91	-10.03	3.14	38.52	-3.82	3.04	39.89
Honduras	22.87	-8.52	6.81	25.75	-9.98	4.02	31.21	-8.29	3.74	31.58
Hong Kong	251.08	120.17	2.36	55.79	83.51	14.23	67.48	50.68	7.83	57.99
Iceland	66.42	-9.13	1.92	79.69	-19.24	2.65	100.73	-10.93	9.43	114.36
India	80.52	12.66	15.51	38.72	13.00	6.01	50.69	11.16	5.39	50.37

Table 4.1 Continued:

Ireland	133.08	19.18	1.41	92.85	-7.63	9.18	131.12	-3.65	9.13	131.21
Israel	86.13	39.58	2.39	30.24	11.34	10.40	51.42	6.92	8.67	53.83
Italy	117.45	31.76	13.45	45.46	-0.68	9.16	100.57	-0.37	7.59	103.49
Ivory Coast	15.00	-29.05	-7.07	74.42	-27.04	-6.83	69.18	-13.86	-8.40	78.95
Jamaica	-3.56	-8.13	6.12	-1.08	-6.06	3.99	-1.28	-4.23	3.88	-1.29
Japan	208.52	3.27	15.16	159.42	-28.19	6.49	303.45	-21.92	6.47	303.54
Kenya	35.29	14.40	24.14	-4.73	35.88	7.02	-6.96	30.29	6.03	-6.83
South Korea	424.45	41.47	3.04	259.76	10.80	6.78	343.30	9.09	6.19	345.74
Luxembourg	78.47	31.88	24.55	8.65	11.04	17.92	36.29	7.48	5.44	52.42
Madagascar	-29.68	-44.14	17.81	6.85	-38.28	3.93	9.62	-32.59	2.40	9.70
Malawi	43.85	17.32	-42.65	113.81	5.10	-34.60	109.27	2.86	21.27	3.34
Mauritius	56.99	3.60	9.51	38.37	0.00	4.60	50.09	0.29	3.18	49.88
Mexico	47.47	-13.06	1.91	66.44	-19.21	7.16	70.34	-6.31	7.55	70.99
Morocco	52.89	17.14	16.62	11.92	6.55	12.95	27.03	6.63	2.28	16.77
Netherlands	51.45	5.38	11.21	29.24	-20.88	9.20	75.29	-9.59	9.15	75.37
New Zealand	7.42	-15.82	9.49	16.54	-35.27	9.02	52.22	-21.31	8.97	52.29
Nigeria	40.58	8.36	-14.00	50.85	-2.56	-5.27	52.30	-19.99	1.86	58.94
Norway	69.72	26.25	33.20	0.92	38.58	14.00	7.43	31.08	4.65	17.03
Panama	32.87	-24.98	-0.77	78.48	-35.97	9.10	90.21	-30.46	9.13	90.17
Paraguay	63.25	0.00	-14.57	91.08	0.00	-11.52	84.50	-6.47	9.80	115.08
Peru	-16.11	-32.15	1.51	21.80	-38.12	8.57	24.86	-28.85	6.90	22.84
Philippines	43.84	10.08	7.98	21.01	10.29	3.67	25.79	8.32	3.29	26.20
Portugal	168.82	15.04	5.02	122.51	3.22	7.11	143.15	4.49	5.52	139.96
Sierra Leone	-5.80	0.00	-57.74	122.92	0.00	-52.17	96.94	0.00	35.46	-21.32

Table 4.1 Continued:

Spain	111.74	-12.35	7.19	125.37	-26.75	1.15	185.81	-15.87	8.05	205.32
Sri Lanka	72.07	3.23	3.00	61.82	-6.85	7.20	72.30	-1.20	5.37	69.36
Sweden	36.03	-5.04	12.54	27.29	-33.84	9.91	87.06	-20.65	8.16	90.08
Switzerland	38.68	2.48	28.61	5.23	-20.95	27.92	37.14	-16.79	2.19	71.67
Syria	107.90	55.26	0.08	33.80	36.58	10.28	38.03	28.12	8.58	40.19
Taiwan	318.96	14.73	9.71	232.84	-15.50	6.35	366.20	-12.44	6.07	367.43
Thailand	194.68	28.61	12.45	103.76	11.76	6.57	147.43	8.63	6.05	148.75
Turkey	129.27	9.99	6.59	95.56	-0.92	5.83	118.66	-0.25	5.38	119.59
United Kingdom	60.74	-3.70	1.39	64.63	-11.50	2.00	78.05	-7.54	7.87	95.28
United States	31.29	0.00	10.02	19.33	-12.55	0.34	49.62	-8.42	8.78	62.36
Yugoslavia	88.10	-15.32	6.62	108.35	-24.22	6.13	133.88	-14.79	5.72	134.78
Zambia	-33.86	-29.40	16.04	-19.27	-14.96	3.90	-25.14	-11.57	2.64	-25.34
Zimbabwe	11.38	37.24	2.47	-20.80	36.77	5.64	-22.92	30.22	5.62	-22.92
Mean	**75.06**	**5.24**	**6.17**	**58.47**	**-8.50**	**5.31**	**86.04**	**-5.40**	**6.96**	**86.03**