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Slack-based directional distance function in the presence of bad outputs: Theory and Application to Vietnamese Banking

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Abstract

In this paper we extend the slack-based directional distance function introduced by Färe and Grosskopf (2010) to measure efficiency in the presence of bad outputs and illustrate it through an application on data of Vietnamese commercial banks. We also compare results from the slack-based directional distance function relative to the directional distance function, the enhanced hyperbolic efficiency measure (Färe et al., 1989) and the Farrell-type technical efficiency and confirm that it has greater discriminative power.

Key words: Banking, Bad outputs, Data Envelopment Analysis, Directional distance function, Slack-based efficiency, Performance analysis.

JEL classification: C14, C15, C44, D24, G21

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1 Introduction

It is widely conceded that financial system has a strong influence on the functioning of economy. Indeed, a comment of William Gladstone, the former British Prime Minister, in 1858 is a stark example: "Finance is, as it were, the stomach of the country, from which all the other organs take their tone"(Ratcliffe, 2011). Another illustration is the 2008 Global Financial Crisis that led to the disruption of a huge number of companies all over the world, illustrating how adversely economies can suffer from the instability of financial systems. As banks play a key role for the health of financial systems, measuring efficiency of banking industry is essential for devising relevant policies and regulations.

In their recent survey, Fethi and Pasiouras (2010) pointed out that Data Envelopment Analysis (DEA) is the most commonly used technique in assessing bank performance. When looking at the world of DEA, it is well-known that popular radial efficiency measures, e.g., the Farrell-type technical efficiency (Farrell, 1957), possess a number of drawbacks. First, as pointed out in several works, e.g., Färe and Lovell (1978), radial measures compute efficiency scores on the basis of the isoquant, not the efficient subset of the technology. Second, traditional efficiency measures cannot deal with negative values in the data without transformations which, as Portela et al. (2004) argued, make it hard to interpret the results since different types of transformations may lead to different estimates of efficiency scores.

Furthermore, in the banking sector, undesirable outputs such as non-performing loans (NPLs) require special attention as they can adversely affect banks' performance. Indeed, not only do NPLs lower profit but they may also jeopardize cash flows from operation, putting banks in a liquidity crisis or even more severely, an insolvent situation.[1] However, only a relatively small portion of studies extensively consider the undesirable outputs in modeling bank efficiency. For example, Fukuyama and Weber (2010) examined the efficiency of Japanese banks for a two-stage system with bad outputs using a slack-based measure. Recently, Lozano (2016) summarized network DEA applications to bank efficiency measurement and also proposed a general network slack-based approach to assess efficiency at the bank level and at the branch level where bad outputs are taken into consideration.

[1]If we view banks as financial intermediaries who take deposits from customers to fund for loans, NPLs are bad outputs that incur losses since banks cannot collect related interests/fees as scheduled whereas they still have to pay interests to depositors. In addition, banks have to put aside an amount of their money to make provisions for credit losses regarding loans made. This will be discussed further in section 4.

In this paper we are particularly interested in the slack-based directional distance function (SD) proposed by Färe and Grosskopf (2010). More recently, Färe et al. (2015) pointed out that SD approach satisfies four important properties: (i) units invariant, (ii) monotone, (iii) translation invariant, and (iv) reference set invariant. Besides, as the SD has been recently introduced, theoretical frameworks for applying this measure in the DEA context have not been developed relative to the traditional Farrell-type measure, particularly where bad outputs are taken into consideration. Hence, we intend to fill this gap here to some extent.

In a nutshell, this study aims to achieve three major goals: (i) develop a theoretical framework for measuring efficiency by the SD in the presence of bad outputs, (ii) investigate the differences between the SD and other efficiency measures, focusing on their ability in distinguishing individual firms, and (iii) apply the developed framework to study the efficiency of Vietnamese commercial banks.[2]

The rest of the paper is structured as follows. Section 2 provides a general picture of the Vietnamese banking system. Section 3 establishes the methodology for computing efficiency scores using the SD measure in the presence of bad outputs. Section 4 presents decompositions of revenue efficiency using the SD measure. Section 5 present an empirical illustration of the developed methodology using a dataset on Vietnamese commercial banks. Section 6 summarizes important points in this paper and give some hints to policy-makers.

2 Background on the Vietnamese banking industry

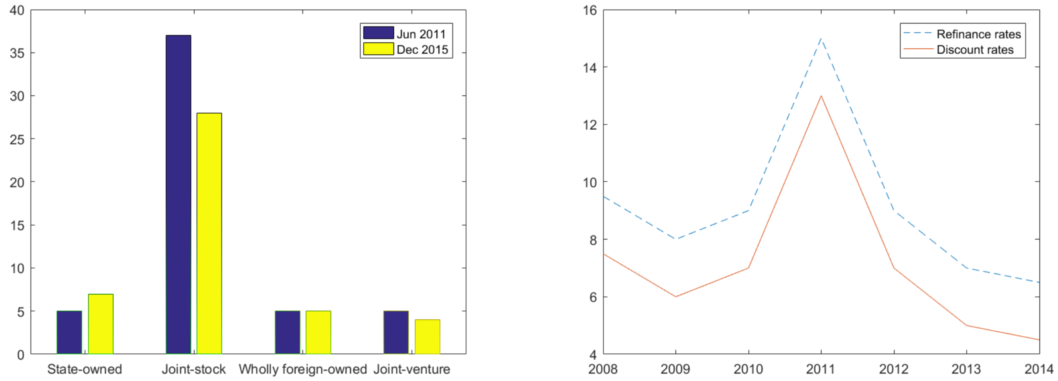
The Vietnamese banking industry has undergone about seven decades of construction and development since the first establishment of The State Bank of Vietnam (SBV) in 1951. According to The State Bank of Vietnam (2016b), there are 44 commercial banks operating in Vietnam as of 31 December 2015, including: (i) 7 state-owned commercial banks,[3] (ii) 28 joint-stock commercial banks, (iii) 5 wholly foreign-owned banks and (iv) 4 joint-venture banks. There are also 2 policy banks and one co-operative bank in the system.

In general, it is observed that the Vietnamese banking system has been exposed to some weaknesses in recent years. First, the problem of NPLs is particularly puzzling in Vietnam. On 12 July 2012, SBV informed that the NPL ratio of the whole Vietnamese

[2] There is also a wide variety of other approaches for studying productivity and efficiency in the banking industry, e.g., Berger and Mester (2003); Du et al. (2015); Curi et al. (2015); Zelenyuk and Zelenyuk (2015).

[3] These are banks where state ownership is over 50% of charter capital.

banking system as of 31 March 2012 was 8.6%, significantly higher than a common reference rate 3%.[4] Loans involving real estates and stocks might be one of the reasons behind this situation. In fact, the increasing amounts of loans for investments in real estates and stocks and loans collateralized by real estates in the past years might lead to "bubbles" which adversely affected banks when the real estate and stock market underwent a downturn.



(a) Number of banks (2011 and 2015) (b) Interest rates in 2008-2014 (%/year)

Figure 1: Number of banks and interest rates in the Vietnamese banking industry.
Note: The figures area drawn based on information from SBV (2008; 2009; 2010; 2011; 2012; 2013; 2014)

Second, the Vietnamese banking system also witnessed a rapid increase in the number of credit institutions together with complicated cross-ownership and severe competition. Escalations in interest rates for deposits from customers well illustrate the competition in the Vietnamese banking industry in that period of time. Indeed, the rates for deposits in Vietnam Dong gradually climbed to approximate 15.6% at the end of June 2011, higher than at the end of 2010 (12.44%) and the cap set by the government at that time (14%) (The State Bank of Vietnam, 2011).[5]

In order to strengthen the Vietnamese financial system, the government determined to restructure the whole banking system, focusing on reducing the number of commercial banks by a package of solutions including eliminating unhealthy banks and merging several banks. This aim was materialized in the Scheme on "Restructuring the credit institutions system in the 2011-2015 period".[6] On implementing the Scheme,

[4]For example, see Circular No. 36/2014/TT-NHNN dated November 20, 2014 of The State Bank of Vietnam where the NPL rate of 3% was set as one of requirements for some banking activities. Another example can be found in Directive No. 02/CT-NHNN dated January 27, 2015 of The State Bank of Vietnam.

[5]For more details, see Circular No. 02/2011/TT-NHNN dated March 03, 2011, Circular No. 30/2011/TT-NHNN dated September 28, 2011 and Directive No. 02/CT-NHNN dated September 07, 2011 of The State Bank of Vietnam.

[6]The Scheme was approved by the Vietnamese Prime Minister in the Decision No. 254/QD-TTg

several actions have been taken by the government, resulting in: (i) the nine weakest commercial banks being identified and restructured during 2011-2012, (ii) the number of pairs of banks who cross-own each other reduced from 7 pairs in 2012 to 3 pairs in June 2015, (iii) 493,000 billions VND of NPLs were tackled from 2012 to 2015, reducing the NPL ratio to 2.55% at the end of 2015 (The State Bank of Vietnam, 2016a).

It is also noteworthy that since the 2008 Global Financial Crisis, there has been some other events that might had influences on banks' performance. For example, 2012 saw a number of disadvantageous events for the Vietnamese banking system, notably: (i) growth of credit hit the lowest point in the recent years (8.85%–according to The State Bank of Vietnam (2012)), (ii) several bank officers were detected in illegal activities, (iii) a number of banks were inspected by SBV, especially nine banks were identified and restructured in 2011-2012 (The State Bank of Vietnam, 2016a). Therefore, it is of interest to investigate the performance of Vietnamese commercial banks, in terms of their efficiency, and its link with changes in the operating environment. In addition, we also want to explore efficiency of individual banks in accordance with some available qualitative information in the industry. These facts are our motivations for applying the theoretical framework proposed in section 3 to the data on Vietnamese commercial banks.

3 Methodology

3.1 Background

In the initial stage, we reiterate foundations for studies on efficiency where undesirable outputs are not taken into consideration. We assume that all banks operate with the same technology T defined as:

$$T = \{(x, y) \in \mathbb{R}_+^N \times \mathbb{R}_+^M : y \text{ is producible from } x\} \tag{1}$$

where $x = (x_1, \ldots, x_N)'$ denotes a column vector of N inputs and $y = (y_1, \ldots, y_M)'$ denotes a column vector of M outputs. Equivalently, T can be expressed via the output set as:

$$P(x) = \{y \in \mathbb{R}_+^M : y \text{ is producible from } x\}, \; x \in \mathbb{R}_+^N. \tag{2}$$

In a majority of studies, T is assumed to satisfy standard regularity axioms as described below (for details, see Färe and Primont, 1995).

Axiom 1 *No free lunch:*

$$y \notin P(0_N), \quad \forall y \geq 0_M. \tag{3}$$

dated March 01, 2012.

Axiom 2 *Producing nothing is possible:*

$$0_M \in P(x), \quad \forall x \in \mathbb{R}^N_+. \tag{4}$$

Axiom 3 Boundedness of the output sets: *P(x) is a bounded set for all* $x \in \mathbb{R}^N_+$.

Axiom 4 Closeness of the technology set: *T is a closed set.*

Axiom 5 *Strong (Free) disposability of inputs:*

$$(x, y) \in T \Rightarrow (x^*, y) \in T, \quad \forall x^* \geqq x, y \in \mathbb{R}^M_+. \tag{5}$$

Axiom 6 *Strong (Free) disposability of outputs:*

$$(x, y) \in T \Rightarrow (x, y^*) \in T, \quad \forall y^* \leqq y, x \in \mathbb{R}^N_+. \tag{6}$$

Among efficiency measures which have been proposed to date, the Farrell's (1957) technical efficiency can be considered as one of the most commonly used measures. Its output-oriented version is defined as:

$$TE_o(x, y) = \sup_{\alpha}\{\alpha : \alpha y \in P(x)\}. \tag{7}$$

Although the Farrell-type technical efficiency measure has been employed in a large number of empirical works, it is also criticized for having several limitations, e.g., measuring in a radial direction, difficulty in dealing with non-positive data, ignoring slacks of inputs and outputs, etc. (for more details, see Färe and Lovell, 1978; Färe et al., 1994; Tone, 2001). In the search for more advantageous measures, a number of new ones have been introduced. Chambers et al. (1996, 1998) proposed the directional distance function (DDF):

$$DDF(x, y; g_x, g_y) = \max_{\beta}\{\beta : (x - \beta g_x, y + \beta g_y) \in T\} \tag{8}$$

where $g = (g_x, g_y) \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$ is the directional vector selected by researchers. One can set $g_x = 0_N$ to obtain the output-oriented version of the DDF measure.

Furthermore, slack-based measures of efficiency have been introduced and developed in a number of works, e.g., Charnes et al. (1985); Tone (2001); Fukuyama and Weber (2009). More recently, a new efficiency measure called slack-based directional distance function (SD) was introduced by Färe and Grosskopf (2010) and has been further developed in the works of Färe et al. (2015, 2016). In essence, SD is generalized upon from the DDF measure where all elements of the directional vector are equal to 1 ($g_x = 1_N$,

$g_y = 1_M$) but input and output components are allowed to vary asymmetrically, with different slacks. Formally, SD efficiency is defined as:

$$SD(x,y) = \max_{\substack{\beta_1,\dots,\beta_N \\ \gamma_1,\dots,\gamma_M}} \Big\{ \sum_{i=1}^{N} \beta_i + \sum_{j=1}^{M} \gamma_j : \\ (x_1 - \beta_1 \cdot 1, \dots, x_N - \beta_N \cdot 1, y_1 + \gamma_1 \cdot 1, \dots, y_M + \gamma_M \cdot 1) \in T, \\ \beta_i \geq 0 \ \forall i = 1, \dots, N, \gamma_j \geq 0 \ \forall j = 1, \dots, M \Big\}. \tag{9}$$

If we are interested in the output orientation only, we can simply set $\beta_i = 0$ ($i = 1, \dots, N$) and solve the problem (9) with respect to γ_1, γ_2,..., γ_M.

As Färe and Grosskopf (2010) pointed out, the advantage of setting all elements of the directional vector equal to 1 is that $\beta_1, \dots, \beta_N, \gamma_1, \dots, \gamma_M$ become scalars and independent of units of measurements of inputs and outputs. As a consequence, SD(x, y) is also a scalar and independent of the units of measurement (for a proof and more details, see Färe et al., 2015, 2016). In their paper, Färe and Grosskopf (2010) also pointed out that the Tone's (2001) measure is a special case of the SD measure.

When it comes to the presence of bad outputs, several works, e.g., Färe et al. (1989), pointed out that it is unreasonable to assume strong disposability of all outputs (Axiom 6) as the bad outputs cannot be freely disposed without any cost. Keeping that in mind, in this paper we replace Axiom 6 by the assumption that good outputs are strongly disposable while good and bad outputs are jointly weakly disposable, which is expressed formally as:

Axiom 7 *Strong disposability of good outputs and jointly weak disposability of good and bad outputs:*

$$(y, w) \in P(x) \Rightarrow (y^*, \theta w) \in P(x), \quad \forall \ y^* \leqq \theta y, 0 \leq \theta \leq 1, x \in \mathbb{R}_+^N.$$

Here y and w are sub-vectors representing M_1 good outputs and M_2 bad outputs, respectively, and the technology is redefined in the presence of bad outputs as:

$$P(x) = \{(y, w) \in \mathbb{R}_+^{M_1} \times \mathbb{R}_+^{M_2} : x \in \mathbb{R}_+^N \text{ can produce } (y, w)\}.$$

It is clear that conventional efficiency measures should be modified to treat good and bad outputs differently because of their contradictory essence. For example, the Farrell-type output-oriented efficiency measure can be modified in the way that maximizes the proportionate increase in all desirable outputs while ignoring undesirable outputs:

$$TE(x,y,w) = \max_{\alpha}\{\alpha : (\alpha y, w) \in P(x)\}. \tag{10}$$

The DDF measure can be extended in the way that maximizes the radial increase in good outputs as well as the radial decrease in both inputs and bad outputs along the directional vector $(g_x, g_y, g_w) \in \mathbb{R}^N_+ \times \mathbb{R}^{M_1}_+ \times \mathbb{R}^{M_2}_+$:

$$DDF(x,y,w;g_x,g_y,g_w) = \max_{\beta}\{\beta : (x-\beta g_x, y+\beta g_y, w-\beta g_w) \in T\}. \tag{11}$$

Similarly, the SD efficiency measure can be extended as:

$$\begin{aligned} SD(x,y,w) = \max_{\substack{\beta_1,\ldots,\beta_N \\ \gamma_1,\ldots,\gamma_{M_1} \\ \delta_1,\ldots,\delta_{M_2}}} \Big\{ & \sum_{i=1}^{N}\beta_i + \sum_{j=1}^{M_1}\gamma_j + \sum_{l=1}^{M_2}\delta_l : \\ & (x_1-\beta_1\cdot 1,\ldots,x_N-\beta_N\cdot 1, y_1+\gamma_1\cdot 1,\ldots,y_{M_1}+\gamma_{M_1}\cdot 1, \\ & w_1-\delta_1\cdot 1,\ldots,w_{M_2}-\delta_{M_2}\cdot 1) \in T, \beta_i \geq 0\ \forall i=1,\ldots,N, \\ & \gamma_j \geq 0\ \forall j=1,\ldots,M_1, \delta_l \geq 0\ \forall l=1,\ldots,M_2 \Big\}. \end{aligned} \tag{12}$$

Färe et al. (1989) also introduced the so-called enhanced hyperbolic output efficiency measure which looks to expand good outputs and contract bad outputs:

$$HTE(x,y,w) = \max_{\alpha}\{\alpha : (\alpha y, \alpha^{-1} w) \in P(x)\}. \tag{13}$$

The SD measure is expected to be more advantageous than the other measures in distinguishing individual banks. In particular, if an individual is recognized as fully efficient by the SD measure, it is also recognized as fully efficient by the DDF, HTE and TE measures whereas the converse does not always hold true, implying that the number of fully efficient banks recognized by the SD measure is less than by the other measures. To illustrate, we use a technology with 1 input, 1 good output and 1 bad output and present firms by points $(x,y,w) \in \mathbb{R}^3_+$ where x is input, y is good output and w is bad output. The technology is represented by all points lying in the space (including the surfaces) bounded by four planes $(OAD), (OAB), (OBE)$ and (OED) where $O = (0,0,0), A = (3,5,4), B = (3,5,7), D = (3,0,0), E = (3,0,7)$. Then it is transparent that for firms B, $DDF(B) = 0$ and $HTE(B) = TE(B) = 1$ whereas $SD(B) = 3$ (Figure 2). In fact, all observations on the flat facets like (OAB) will be identified as fully efficient by the DDF, HTE and TE measures but SD measure would still be able to pick up and measure the slack, thus indicating about some possibly high level of inefficiency.

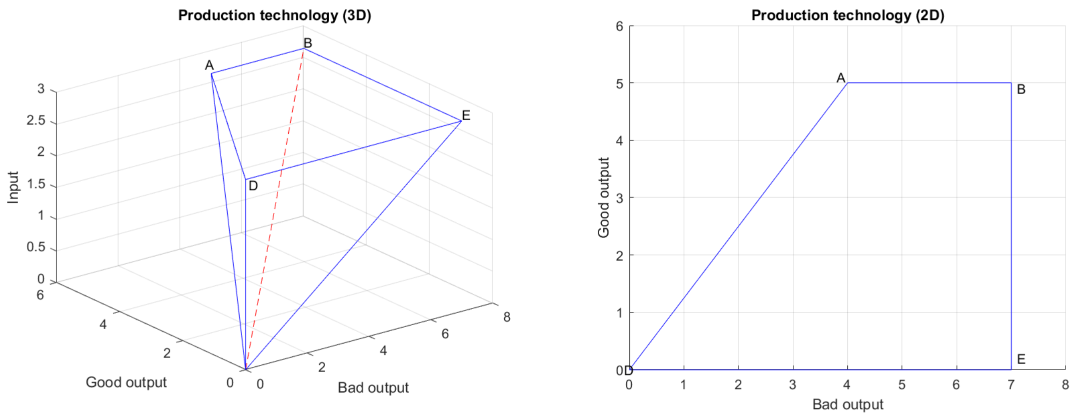


Figure 2: Illustration of a production technology: 1 input, 1 good output and 1 bad output. (The figures are drawn using Matlab).

3.2 DEA implementation in the presence of bad outputs

The true technology T is not observed in reality and must be estimated. Here we employ Data Envelopment Analysis (DEA) to estimate the technology frontier of T, based on which different efficiency measures are applied to compute (in)efficiency scores.[7]

It is worth noting that unlike the manufacturing industries where outputs, which are measured as physical entities, can hardly take negative values, it is common to see negative outputs in the banking industry, e.g., net incomes from trading securities or other activities. Equally important, several components of financial reports of banks, e.g., provision for credit losses on loans (PCL), are always presented in the balance sheet as negative numbers that reduce banks' total assets. As discussed in Portela et al. (2004), the traditional Farrell-type technical efficiency cannot work properly if negative data appears. As such, it is advantageous that the SD measure can handle negative data directly without any data transformation required as it is of additive measures. Therefore, we relax the prerequisite of positive values of outputs in our framework developed for the SD measure below.

Assume that we are given a data set of K banks in which the observed inputs and outputs of bank k are denoted by a triple of column vectors $(x^k, y^k, w^k) \in \mathbb{R}_+^N \times \mathbb{R}^{M_1} \times \mathbb{R}^{M_2}$ $(k = 1, \ldots, K)$ where x^k represents inputs, y^k represents good outputs and w^k represents bad outputs.[8] The DEA approximation of technology T under Axiom 7 has been discussed in a number of works, e.g., Färe et al. (1989); Färe et al. (1994); Färe and Grosskopf (2003); Kuosmanen (2005); Kuosmanen and Podinovski (2009); Färe and

[7] In this paper efficiency scores are presented and estimated as numbers which vary from a lower bound to infinity. As a score goes to infinity, the corresponding decision-making-unit is considered to be more inefficient. Thus, we use the term "inefficiency score" henceforward for more precise interpretation.

[8] $M_1 + M_2 = M$ is the total number of outputs.

Grosskopf (2009). In this study we follow the approach of Färe and Grosskopf (2009) to estimate T under constant returns to scale (CRS) environment as:

$$T^{CRS} = \Big\{(x, y, w) \in \mathbb{R}_+^N \times \mathbb{R}^{M_1} \times \mathbb{R}^{M_2} : \sum_{k=1}^{K} \lambda^k x^k \leqq x, \theta \sum_{k=1}^{K} \lambda^k y^k \geqq y,$$
$$\theta \sum_{k=1}^{K} \lambda^k w^k = w, 0 \leq \theta \leq 1, \lambda^k \geq 0 \ \forall k = 1, \ldots, K\Big\} \tag{14}$$

where λ^k, $k = 1, \ldots, K$ are intensity variables and θ is the abatement factor. The DEA approximations of technology T under variable returns to scale (VRS) and non-increasing returns to scale (NIRS) are obtained by adding the contraints $\sum_{i=1}^{K} \lambda^k = 1$ and $\sum_{i=1}^{K} \lambda^k \leq 1$ to (14), respectively. In their paper, Färe and Grosskopf (2009) also noticed that the parameter θ is unnecessary under CRS and NIRS, i.e., we can set $\theta = 1$ under these two types of returns to scale. This result also carries over to the estimation of SD (e.g., see Appendix A for a proof).

Thus, based on the reference technology sets, SD inefficiency score for an arbitrary bank associated with the data (x^o, y^o, w^o) can be estimated by solving the optimization problem:

$$\widehat{SD}(x^o, y^o, w^o) = \max_{\substack{\beta_1,\ldots,\beta_N,\gamma_1,\ldots,\gamma_{M_1} \\ \delta_1,\ldots,\delta_{M_2},\lambda^1,\ldots,\lambda^K,\theta}} \left(\sum_{i=1}^{N} \beta_i + \sum_{j=1}^{M_1} \gamma_j + \sum_{l=1}^{M_2} \delta_l \right) \tag{15}$$

subject to:

$$\begin{array}{l}
x_i^o - \beta_i \geq \sum_{k=1}^{K} \lambda^k x_i^k \ \forall i = 1, \ldots, N \\
y_j^o + \gamma_j \leq \theta \sum_{k=1}^{K} \lambda^k y_j^k \ \forall j = 1, \ldots, M_1 \\
w_l^o - \delta_l = \theta \sum_{k=1}^{K} \lambda^k w_l^k \ \forall l = 1, \ldots, M_2 \\
\lambda^k \geq 0 \ \forall k = 1, \ldots, K; \beta_i \geq 0 \ \forall i = 1, \ldots, N \\
\gamma_j \geq 0 \ \forall j = 1, \ldots, M_1; \delta_l \geq 0 \ \forall l = 1, \ldots, M_2 \\
0 \leq \theta \leq 1
\end{array}$$

where the optimization is done over $\beta_1, \ldots, \beta_N, \gamma_1, \ldots, \gamma_{M_1}, \delta_1, \ldots, \delta_{M_2}, \lambda^1, \ldots, \lambda^K, \theta$, for the case of CRS technology. SD inefficiency scores under VRS and NIRS are estimated by adding $\sum_{k=1}^{K} \lambda^k = 1$ and $\sum_{k=1}^{K} \lambda^k \leq 1$ to the set of constraints in (15), respectively.

It should be noted that the problem (15) is constructed on the basis that data on bad outputs are stored as positive values since it aims to minimize the absolute values of bad outputs for optimal solutions. If the undesirable outputs in the data have negative sign, e.g., PCL, we must reverse their signs so as to ensure the problem (15) produces appropriate solutions.

In this paper, we also compare estimates from the SD measure with those from other three efficiency measures which are the DDF, HTE and TE.[9] Approaches using these three measures in the presence of bad outputs were introduced in numerous studies, e.g., Färe et al. (1989); Chung et al. (1997); Seiford and Zhu (2002); Färe et al. (2005), to mention just a few. We reiterate the optimization problems used to compute inefficiency score for an arbitrary bank having data (x^o, y^o, w^o) with regards to these three measures below.

For the DDF measure under CRS, the inefficiency score of an arbitrary bank associated with the data (x^o, y^o, w^o) can be estimated by solving the optimization problem:

$$\widehat{DDF}(x^o, y^o, w^o) = \max_{\beta,\lambda^1,\dots,\lambda^K,\theta} \beta \tag{16}$$

subject to:

$$\begin{aligned}
& x_i^o - \beta \cdot 1 \geq \textstyle\sum_{k=1}^K \lambda^k x_i^k \ \forall i = 1, \dots, N \\
& y_j^o + \beta \cdot 1 \leq \theta \textstyle\sum_{k=1}^K \lambda^k y_j^k \ \forall j = 1, \dots, M_1 \\
& w_l^o - \beta \cdot 1 = \theta \textstyle\sum_{k=1}^K \lambda^k w_l^k \ \forall l = 1, \dots, M_2 \\
& \lambda^k \geq 0 \ \forall k = 1, \dots, K \\
& 0 \leq \theta \leq 1
\end{aligned}$$

where the optimization is done over $\beta, \lambda^1, \dots, \lambda^K, \theta$.[10] Estimations under VRS and NIRS are obtained by adding $\sum_{k=1}^K \lambda^k = 1$ and $\sum_{k=1}^K \lambda^k \leq 1$ to the set of constraints in (16), respectively.

For the HTE measure under CRS, the inefficiency score of an arbitrary bank associated with the data (x^o, y^o, w^o) can be estimated by solving the optimization problem:

$$\widehat{HTE}(x^o, y^o, w^o) = \max_{\alpha,\lambda^1,\dots,\lambda^K,\theta} \alpha \tag{17}$$

subject to:

[9] It is worth noting that in this paper we use a data set on Vietnamese commercial banks where all outputs selected for the DEA estimation have positive values. Thus, we can compute and interpret the estimated HTE and TE inefficiency scores in comparison with the SD and DDF measures without any data transformation needed.

[10] Here we choose the directional vectors having all elements equal to 1.

$$
\begin{aligned}
&x_i^o \geq \textstyle\sum_{k=1}^{K} \lambda^k x_i^k \ \forall i = 1, \ldots, N \\
&\alpha y_j^o \leq \theta \textstyle\sum_{k=1}^{K} \lambda^k y_j^k \ \forall j = 1, \ldots, M_1 \\
&w_l^o / \alpha = \theta \textstyle\sum_{k=1}^{K} \lambda^k w_l^k \ \forall l = 1, \ldots, M_2 \\
&\lambda^k \geq 0 \ \forall k = 1, \ldots, K \\
&0 \leq \theta \leq 1
\end{aligned}
$$

where the optimization is done over $\alpha, \lambda^1, \ldots, \lambda^K, \theta$. Estimations under VRS and NIRS are obtained by adding $\sum_{k=1}^{K} \lambda^k = 1$ and $\sum_{k=1}^{K} \lambda^k \leq 1$ to the set of constraints in (17), respectively.

For the TE measure under CRS, the inefficiency score of an arbitrary bank associated with the data (x^o, y^o, w^o) can be estimated by solving the optimization problem:

$$
\widehat{TE}(x^o, y^o, w^o) = \max_{\alpha, \lambda^1, \ldots, \lambda^K, \theta} \alpha \tag{18}
$$

subject to:

$$
\begin{aligned}
&x_i^o \geq \textstyle\sum_{k=1}^{K} \lambda^k x_i^k \ \forall i = 1, \ldots, N \\
&\alpha y_j^o \leq \theta \textstyle\sum_{k=1}^{K} \lambda^k y_j^k \ \forall j = 1, \ldots, M_1 \\
&w_l^o = \theta \textstyle\sum_{k=1}^{K} \lambda^k w_l^k \ \forall l = 1, \ldots, M_2 \\
&\lambda^k \geq 0 \ \forall k = 1, \ldots, K \\
&0 \leq \theta \leq 1
\end{aligned}
$$

where the optimization is done over $\alpha, \lambda^1, \ldots, \lambda^K, \theta$. Estimations under VRS and NIRS are obtained by adding $\sum_{k=1}^{K} \lambda^k = 1$ and $\sum_{k=1}^{K} \lambda^k \leq 1$ to the set of constraints in (18), respectively.

3.3 Computational issues

The DEA problems in the presence of bad outputs are non-linear optimization problems due to the appearance of the single abatement factor θ, which makes it harder to solve in comparison with conventional DEA models that are based on linear programming problems. Fortunately, we can set $\theta = 1$ under CRS and NIRS without affecting the optimal value of the objective function (e.g., see Färe and Grosskopf, 2009).[11] Note that even setting $\theta = 1$, the problem (17) for HTE is still non-linear because of the constraint on bad outputs. Thus, in this study we are facing: (i) linear optimization problems when solving for SD, DDF, TE inefficiency scores under NIRS and CRS, and (ii) non-linear optimization problems in the other cases.

To circumvent the non-linearity of HTE, Färe et al. (1989) solved the problem (17)

[11] We also present another proof confirming this argument in Appendix A.

with $\theta = 1$ by transforming it to a convenient form for linear programming using the approximation $1/\alpha \approx 2 - \alpha$ for α close to one. To some extent, this approach might not ensure high accuracy, particularly if firms' efficiencies in the data are not close to one, e.g., $\alpha > 2$. In this paper we solve the non-linear optimization problems directly for more accurate results although this comes at the expense of the time needed to obtain the solutions. We explored various alternatives, and the most effective, in terms of higher speed and accuracy, appears to be using a sequential quadratic programming method where a quadratic programming subproblem is generated and solved at each iteration (e.g., see Han (1977); Powell (1978a,b); Gill et al. (1981); Hock and Schittkowski (1983); Fletcher (1987) for details). In Matlab, this approach is implemented via the algorithm "sqp" of the "fmincon" solver.[12]

Finally, it is also important to note that the solutions to these non-linear problems, especially in high dimensions, often depend on the starting values (as indeed happened frequently for our application) and so it is imperative to use several different starting values to ensure that global rather than local optima are found. In Matlab, this can be implemented using "multistart" algorithm in combination with "fmincon" that randomly generates many starting values, besides any starting values sent by a user. Also, note that while any starting values can be provided by the user, after various experiment we found that for solving the VRS models, the most effective user-provided starting values appear to be the vector of solutions from optimizing under NIRS and $\theta = 1$, complemented by other random values generated by the Multistart.

4 Decomposing revenue efficiency using the SD

Parallel to employing the SD measure in computing technical inefficiency scores, recent studies also take advantages of this measure to develop some useful decompositions. Several results in decomposing the cost efficiency and profit efficiency were introduced by Färe et al. (2015, 2016). Nonetheless, to the best of our knowledge, SD-based decompositions for revenue efficiency have not yet been proposed. Although cost efficiency possibly receives special attentions in recessions when banks have concerns about reducing costs in order to maintain on-going operations, it might be more relevant for banks to focus on the output side of efficiency in the normal operating environment. For instance, as banks are financial intermediaries, their success depends crucially on how widely they expand their presence in the population, which explains why establishing new branches and transactional points are often considered in their strategic plans.

[12] Other input arguments used in our Matlab codes are: maximum number of iterations (1000), maximum number of function evaluations (10000), termination tolerance on the function value (1.0e-008), termination tolerance on the current point (1.0e-008) and the number of start points (5).

Even though setting up new branches and transactional points can be considered as cost-inefficient since this may triggers losses in the short-term, banks still accept this strategy for long-term benefits. In other words, output-side efficiencies might be more relevant for banks in normal operating conditions than cost efficiency. This is one of the motivations for us to develop a revenue-oriented version of efficiency decomposition, based on the cost-approach of Färe et al. (2016).

The opinion of considering NPLs as a by-product is very usual in practice and research. For instance, Park and Weber (2006) viewed loan losses as undesirable by-product arising from the production of loans. It is possible that bad outputs do not directly affect firms' benefits in several types of production, e.g., the polluting gas emitted from producing electricity. However, in the banking industry, it is likely that bad outputs negatively affect the benefits of banks. A stark example is NPLs which are subject to high chances of default. In principal, if a borrower defaults on an NPL, the bank can rarely recover all of its money lent and therefore, looses some of its assets. In addition, bank regulators usually require banks to put aside some amount of money in order to make provisions for credit losses of loans to customers. In practice, these provisions are presented separately from operating expenses in income statements. From the balance-sheet point of view, the accumulated provisions are presented as negative numbers which deduct total assets of banks. It is also worth noting that provisions required for NPLs are not insignificant and might have severe impacts on banks' profit.[13]

One possible approach for studying efficiency with the effect of bad outputs taken into account is to consider them as inputs and examine the profit decomposition (for an SD-based profit decomposition, see Färe et al., 2015). However, neither does this approach reflect the true essence of undesirable outputs, e.g., NPLs in the banking industry nor contrast good outputs with bad outputs. Therefore, we construct an SD-based revenue decomposition in the presence of bad outputs, assuming that good outputs generate revenue for banks whereas bad outputs decrease revenue of banks.[14]

Continuing with the notations in subsection 3.2, we assume that prices of good outputs are given by a row vector $p = (p_1, \ldots, p_{M_1}) \in \mathbb{R}^{M_1}_{++}$ and prices of bad output are given by a row vector $q = (q_1, \ldots, q_{M_2}) \in \mathbb{R}^{M_2}_{++}$. Here we also use the output set redefined in

[13]For example, in Vietnam, according to Article 7, Decision No. 493/2005/QD-NHNN of SBV, the allowance rates based on which specific provisions are made is 20%, 50% and 100% for substandard loans, doubtful loans and loss loans, respectively.

[14]It is noteworthy that our approach is a further extension of Färe et al. (2005) where the revenue function was decomposed based on the directional distance function. In our paper, we decompose the revenue function by using the SD measure which is new and has not been presented before, to our best knowledge.

the presence of undesirable outputs:

$$P(x) = \{(y, w) \in \mathbb{R}^{M_1} \times \mathbb{R}^{M_2} : x \text{ can produce } (y, w)\}, \tag{19}$$

the revenue function in the presence of bad outputs is defined as:

$$\begin{aligned} R: \quad & \mathbb{R}^N_+ \times \mathbb{R}^{M_1}_{++} \times \mathbb{R}^{M_2}_{++} \to \mathbb{R}^1 \\ & R(x, p, q) = \max_{y,w}\{py - qw : (y, w) \in P(x)\}, \end{aligned} \tag{20}$$

and the output-oriented SD inefficiency score in the presence of bad outputs is defined as:

$$\begin{aligned} SD_o^B(x, y, w) = \max_{\substack{\gamma_1,\ldots,\gamma_{M_1} \\ \delta_1,\ldots,\delta_{M_2}}} \Big\{ & \sum_{j=1}^{M_1} \gamma_i + \sum_{l=1}^{M_2} \delta_l : \\ & (y_1 + \gamma_1 \cdot 1, \ldots, y_{M_1} + \gamma_{M_1} \cdot 1, w_1 - \delta_1 \cdot 1, \ldots, w_{M_2} - \delta_{M_2} \cdot 1) \in P(x), \\ & \gamma_j \geq 0 \ \forall j = 1, \ldots, M_1, \delta_l \geq 0 \ \forall l = 1, \ldots, M_2 \Big\}. \end{aligned} \tag{21}$$

By definition of $SD_o^B(x, y)$, we can find $\gamma_j^* \geq 0$ $(i = 1, \ldots, M_1)$ and $\delta_l^* \geq 0$ $(l = 1, \ldots, M_2)$ such that $SD_o^B(x, y, w) = \sum_{j=1}^{M_1} \gamma_i^* + \sum_{l=1}^{M_2} \delta_l^*$ and $(y_1 + \gamma_1^*, \ldots, y_M + \gamma_M^*, w_1 - \delta_1^*, \ldots, w_{M_2} - \delta_{M_2}^*) \in P(x)$. Because $(y_1 + \gamma_1^*, \ldots, y_M + \gamma_M^*, w_1 - \delta_1^*, \ldots, w_{M_2} - \delta_{M_2}^*) \in P(x)$, we have

$$R(x, p, q) \geq \sum_{i=1}^{M_1} p_i(y_i + \gamma_i^* \cdot 1) - \sum_{l=1}^{M_2} q_l(w_l - \delta_l^* \cdot 1) \tag{22}$$

which implies

$$R(x, p, q) - (\sum_{i=1}^{M_1} p_i y_i - \sum_{l=1}^{M_2} q_l w_l) \geq \sum_{i=1}^{M_1} p_i \gamma_i^* \cdot 1 + \sum_{l=1}^{M_2} q_l \delta_l^* \cdot 1 \tag{23}$$

or equivalently,

$$R(x, p, q) - (py - qw) \geq \sum_{i=1}^{M_1} p_i \gamma_i^* + \sum_{l=1}^{M_2} q_l \delta_l^*. \tag{24}$$

For $(y, w) \in P(x)$ and $(y, w) \notin \text{Eff}\partial P(x)$, we have $SD_o^B(x, y) > 0$. Thus, we can transform the inequality (24) into the following form:

$$\begin{aligned} R(x, p, w) - (py - qw) \geq SD_o^B(x, y, w) \times & \\ \times \left(\sum_{i=1}^{M_1} \frac{p_i \gamma_i^*}{\sum_{j=1}^{M_1} \gamma_j^* + \sum_{l=1}^{M_2} \delta_l^*} + \sum_{t=1}^{M_2} \frac{q_t \delta_t^*}{\sum_{j=1}^{M_1} \gamma_j^* + \sum_{l=1}^{M_2} \delta_l^*} \right). & \end{aligned} \tag{25}$$

Therefore,

$$\frac{R(x,p,q)-(py-qw)}{\sum_{j=1}^{M_1}p_j+\sum_{l=1}^{M_2}q_l} \geq SD_o^B(x,y,w)\times$$
$$\times\left(\sum_{i=1}^{M_1}\frac{\alpha_i\gamma_i^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}+\sum_{t=1}^{M_2}\frac{\eta_t\delta_t^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}\right) \quad (26)$$

where $\alpha_i = \dfrac{p_i}{\sum_{j=1}^{M_1}p_j+\sum_{l=1}^{M_2}q_l}$ $(i = 1,\ldots,M_1)$ and $\eta_t = \dfrac{q_t}{\sum_{j=1}^{M_1}p_j+\sum_{l=1}^{M_2}q_l}$ $(t = 1,\ldots,M_2)$ represent the price-share weights.

This inequality can be turned into equality by using the (residual) additive allocative inefficiency AIneff(x,y,w,p,q), i.e.,

$$\frac{R(x,p,q)-(py-qw)}{\sum_{j=1}^{M_1}p_j+\sum_{l=1}^{M_2}q_l} = SD_o^B(x,y,w)\times$$
$$\times\left(\sum_{i=1}^{M_1}\frac{\alpha_i\gamma_i^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}+\sum_{t=1}^{M_2}\frac{\eta_t\delta_t^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}\right)+\text{AIneff}(x,y,w,p,q). \quad (27)$$

Equation (27) forms a decomposition of revenue efficiency using the SD model where:

1. $\dfrac{R(x,p,q)-(py-qw)}{\sum_{j=1}^{M_1}p_j+\sum_{l=1}^{M_2}q_l}$ represents the normalized additive revenue efficiency.
2. $SD_o^B(x,y,w)\times\left(\sum_{i=1}^{M_1}\dfrac{\alpha_i\gamma_i^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}+\sum_{t=1}^{M_2}\dfrac{\eta_t\delta_t^*}{\sum_{j=1}^{M_1}\gamma_j^*+\sum_{l=1}^{M_2}\delta_l^*}\right)$ represents the normalized slack-based directional total technical inefficiency term.

Subsequently, we introduce the framework for applying the above revenue decomposition to the DEA context. Similar to subsection 3.2, for $x \in \mathbb{R}_+^N$ the reference technology under CRS is:

$$P_B^{CRS}(x) = \Big\{(y,w)\in\mathbb{R}^{M_1}\times\mathbb{R}^{M_2} : \sum_{i=1}^{K}\lambda^k x^k \leqq x, \theta\sum_{k=1}^{K}\lambda^k y^k \geqq y, \theta\sum_{i=1}^{K}\lambda^k w^k = w,$$
$$\lambda^k \geq 0 \ \forall k=1,\ldots,K, 0\leq\theta\leq 1\Big\} \quad (28)$$

and the reference technology under NIRS and VRS is obtained by adding the constraints $\sum_{k=1}^{K}\lambda^k \leq 1$ and $\sum_{k=1}^{K}\lambda^k = 1$ to the right hand side of (28), respectively.

The output-oriented SD inefficiency score in the presence of bad outputs can be computed as:

$$SD_{Bo}^{CRS}(x,y,w) = \max_{\substack{\gamma_1,\dots,\gamma_{M_1}\\ \delta_1,\dots,\delta_{M_2}}} \Big\{ \sum_{i=1}^{M_1} \gamma_i + \sum_{l=1}^{M_2} \delta_l : \\ (y_1+\gamma_1\cdot 1, ..., y_M+\gamma_M\cdot 1, w_1-\delta_1\cdot 1, \dots, w_{M_2}-\delta_{M_2}\cdot 1) \in P_B^{CRS}(x), \\ \gamma_i \geq 0 \ \forall i, \delta_l \geq 0 \ \forall l \Big\} \tag{29}$$

$$SD_{Bo}^{VRS}(x,y,w) = \max_{\substack{\gamma_1,\dots,\gamma_{M_1}\\ \delta_1,\dots,\delta_{M_2}}} \Big\{ \sum_{i=1}^{M_1} \gamma_i + \sum_{l=1}^{M_2} \delta_l : \\ (y_1+\gamma_1\cdot 1, ..., y_M+\gamma_M\cdot 1, w_1-\delta_1\cdot 1, \dots, w_{M_2}-\delta_{M_2}\cdot 1) \in P_B^{VRS}(x), \\ \gamma_i \geq 0 \ \forall i, \delta_l \geq 0 \ \forall l \Big\}. \tag{30}$$

Since $P_B^{VRS}(x) \subseteq P_B^{CRS}(x)$, we have

$$SD_{Bo}^{VRS}(x,y) \leq SD_{Bo}^{CRS}(x,y). \tag{31}$$

Based on (31), we can define a slack-based scale inefficiency measure as

$$\text{SSIneff}(x,y,w) = SD_{Bo}^{CRS}(x,y,w) - SD_{Bo}^{VRS}(x,y,w). \tag{32}$$

Replacing (32) into the revenue efficiency decomposition (27) under CRS, we have

$$\frac{R^{CRS}(x,p,q) - (py - qw)}{\sum_{i=1}^{M_1} p_i + \sum_{i=1}^{M_2} q_i} = (\text{SSIneff}(x,y,w) + SD_{Bo}^{VRS}(x,y,w)) \times \\ \times \left(\sum_{i=1}^{M_1} \frac{\alpha_i \gamma_i^{CRS*}}{SD_{Bo}^{CRS}(x,y,w)} + \sum_{i=1}^{M_2} \frac{\eta_i \delta_i^{CRS*}}{SD_{Bo}^{CRS}(x,y,w)} \right) + \text{AIneff}^{CRS}(x,y,w,p,q) \tag{33}$$

where $R^{CRS}(x,p,q)$ and $\text{AIneff}^{CRS}(x,y,w,p,q)$ are the revenue function and the allocative inefficiency under CRS technology, respectively, while γ_i^{CRS*} $(i = 1, \dots, M_1)$ and δ_i^{CRS*} $(i = 1, \dots, M_2)$ are the solutions to the problem (29).

In summary, equation (33) can be interpreted as the DEA-based revenue efficiency decomposition using (output-oriented) slack-based directional distance function in the presence of bad outputs where the first term represents normalized technical inefficiency and the last term represents the allocative inefficiency.

5 Empirical application

The goal of this section is to provide an empirical illustration of the theoretical developments defined above when applying to a real data set.

5.1 Data sources

In this paper we use a data set on Vietnamese commercial banks which covers seven years from 2008 to 2014. We constructed this data set by merging annual financial reports that we downloaded for each individual bank wherever it was possible. The data set includes 241 observations in total and is unbalanced because: (i) a small number of banks did not publicly announce their financial reports, and (ii) a series of mergers and acquisitions reduced the number of banks in the recent years.

We set a billion Vietnam Dong as the unit of measurement in our data set. Furthermore, after collecting raw data from financial reports, we smooth the seasonal effects by taking the averages of beginning- and end-year positions with regard to balance-sheet items. Then, we use the GDP deflators (with 2010 as the base year) from World Bank (2015) to adjust for the effect of inflation.[15] To ensure reasonably large number of observations for DEA estimation, we pool the data over seven years.

5.2 Selection of inputs and outputs

It is admitted that measuring efficiency in the banking industry is more difficult than in the manufacturing industries as there is no clear consensus on an appropriate identification of inputs and outputs (Sealey and Lindley, 1977). A simple logic mentioned in Paradi and Zhu (2013) is that inputs are what banks would like to minimize and outputs are, conversely, what banks would like to maximize. Although adapting this logic, we still see some components of financial statements which can be classified as either inputs or outputs based on different viewpoints. An outstanding example is deposits from customers. As Berger and Humphrey (1997) argued, on the one hand, banks have to pay interests for deposits from customers and hence, they need to minimize deposits to reduce interest burden. On the other hand, banks also need to enhance deposits in order to have funds for making loans as well as gain bigger scale, which makes deposits from customers possess the characteristics of both input and output at the same time.

In the banking industry, Kenjegalieva et al. (2009) summarized three popular approaches for selecting inputs and outputs which are the profit/revenue-based approach, the production approach and the intermediation approach. A recent paper of Simper

[15] The base year of GDP deflators is 2010.

et al. (2015) also discussed different choices of inputs and outputs in banking industry. In fact, the production approach which captures only physical inputs such as labor and capital and their costs may be better for evaluating efficiency at the branch level while the intermediation approach, which also includes funds and interest costs, is considered more appropriate for evaluation at the bank level (Berger and Humphrey, 1997). In addition, while the intermediation approach counts some balance-sheet components as outputs, only elements coming from income statements are considered as outputs in the profit/revenue approach. There are also a wide variety of studies in which selections of outputs and inputs do not follow a specific approach but mixed variations of them. Moreover, the number of inputs and outputs in the model should not be neglected so as to avoid the "curse of dimensionality" and ensure good separation and discrimination between decision-making-units (Paradi and Zhu, 2013). Assume that we have N inputs, M outputs and K observations, there are some simple constraints suggested for DEA to work properly. For instance, one common rule is $K \geq 3(N + M)$ (see Jenkins and Anderson, 2003) and another rule is $K \geq \max\{NM, 3(N+M)\}$ (see Cooper et al., 2007).

Table 1: Description of input-output data (2008-2014) (241 observations)

Unit of measurement: Billion of Vietnam Dong

	Mean	Std. Dev.	Min	Max
Inputs				
Operating expenses	1,360	2,177	21	14,210
Fixed assets	867	1,089	29	5,459
Deposits from customers	47,696	70,435	617	357,237
Good outputs				
Loans to customers net	43,319	71,439	327	391,080
Securities	11,693	13,955	1	61,624
Bad outputs				
Provisions for credit losses of loans to customers	802	1,851	1	13,375

Notes: Balance-sheet items (Fixed assets, Deposits from customers, Loans to customers net, Securities, Provisions for credit losses of loans to customers) are calculated by taking the average of the beginning- and end-year positions. Deflators corresponding to years from 2008 to 2014 used to adjust data for inflation are 84, 89.2, 100, 121.3, 134.5, 140.9, 146.1, respectively (World Bank, 2015).

Considering all points mentioned above, we follow the intermediation approach that uses three inputs (operating expenses, fixed assets and deposits from customers), two good outputs (net loans to customers, securities) and one bad output (provisions for credit losses of loans to customers (PCL)). Two points should be highlighted here. First, it is a common practice to select the number of employees as one type of inputs in the intermediation approach. However, as this number as well as personnel expenses are not always disclosed in published financial reports, we use operating expenses as a proxy for the labor input in each bank. For the same reason, we employ PCL as a proxy for NPLs. In essence, this is reasonable and does not affect the reliability of our results as banks are forced to make provisions for NPLs in accordance with laws enacted by the government as discussed in section 4. A detailed description of these inputs and outputs is shown in Table 1.

Since the goal of this paper is not about particular banks, we avoid finger-pointing at bad or good banks by using the codes "B01", "B02", "B03",..., "B40" to represent individual banks, especially when providing qualitative information regarding some specific banks in our case studies where relevant.

5.3 Efficiency of individual banks

Table 2 presents a summary on individual inefficiency scores computed by different measures (SD, DDF, HTE and TE) in the presence of bad outputs under different types of returns to scale (CRS, NIRS and VRS). Box-plots of estimated inefficiency scores are also provided in Figure 3. Overall, we recognize some notable differences in inefficiency scores computed by the four measures. First, although the SD and DDF measure theoretically have the same range of values which is $[0, +\infty)$, the observed range of estimated inefficiency scores computed by the SD measure (e.g., [0,69496] under VRS) is remarkably larger than by the DDF measure (e.g., [0,517] under VRS). This significant difference stems from the fact that the DDF measure is a restricted version of the SD measure as it does not allow the inputs and outputs to vary asymmetrically but only allows a pre-specified direction. Second, the proportions of fully efficient banks estimated by the DDF, HTE and TE measures are higher than by the SD measure, which is consistent with the theoretical result discussed in subsection 3.1. Third, compared to the HTE and TE measures, the SD and DDF identify much more outlying observations (Figure 3).[16] Thus, a value added from SD relative to the other measures is that it provides further hints on which banks need more detailed investigations to understand why their SD scores are extremely different from scores computed by the other measures. Furthermore, the empirical evidence here also supports our expectation that the SD measure help to differentiate individual banks more than the other measures.

Next, we investigate the correlations between rankings based on different efficiency measures by using the Spearman's ρ statistic. As can be seen in Table 3, the rankings based on the SD and DDF measures are highly correlated as their estimates of ρ are about 0.87 under three types of returns to scale. In contrast, the ranking based on the

[16] Box-plots in Figure 3 are drawn using the default option in Matlab. To be precise, note that the box-plots draw a point as outliers if it lies outside of the range $[q_1 - 1.5(q_3 - q_1), q_3 + 1.5(q_3 - q_1)]$ where q_1 and q_3 are the 25th and 75th percentiles of the sample data, respectively.

SD measure is not well correlated with that based on the HTE and TE measures since the corresponding correlation coefficients are relatively low (between 0.35 and 0.54).

Table 2: Inefficiency scores in the presence of bad outputs

	DEA est.	Mean	Median	Std. Dev.	Max	Fully efficient banks	
						Quantity	Proportion
CRS	SD	39,170	14,781	59,645	343,960	12	5%
	DDF	306	110	513	3,372	17	7%
	HTE	1.482	1.484	0.299	2.648	19	8%
	TE	1.499	1.479	0.307	2.881	17	7%
NIRS	SD	9,146	5,712	11,439	69,496	53	22%
	DDF	81	40	111	517	59	24%
	HTE	1.194	1.167	0.189	1.809	62	26%
	TE	1.235	1.203	0.228	2.036	59	24%
VRS	SD	9,113	5,712	11,403	69,496	56	23%
	DDF	81	40	111	517	61	25%
	HTE	1.190	1.160	0.189	1.809	64	27%
	TE	1.229	1.169	0.228	2.036	62	26%

Source: Authors' calculations using Matlab.
Non-linear optimization problems are solved by the "fmincon" solver with "sqp" option, in combination with "multistart" algorithm using 5 start points.

We also find that the inefficiency scores are quite similar under VRS and NIRS. Regarding the SD measure, the maximum estimated inefficiency scores under NIRS and VRS are both 69496 with the most inefficient bank being B35 in 2014 while under CRS, the maximum estimated score is about five times larger (343960) with the most inefficient banks is B03 in 2012. For all measures, the correlation coefficient of rankings (Spearman's ρ statistic) based on VRS and NIRS are extremely high (above 0.98) whereas the ρ statistics corresponding to the pair CRS-VRS as well as CRS-NIRS are relatively low (between 0.43 and 0.68) (Table 4).

Finally, as can be seen in Figure 4, estimated SD inefficiency scores jumped up in the period 2012-2014 on average, indicating a deterioration in bank efficiency. Interestingly, this jump is consistent with disadvantageous news that appeared in 2012, e.g., some bankers were suspected of illegal banking activities and arrested.[17] In addition, this figure also hints that the Scheme on "Restructuring the credit institutions system in 2011-2015 period" did not bring back immediate positive effects, in terms of efficiency of banks, as was expected.

[17]An arrest in August 2012 created a shock to the whole Vietnamese banking system, which was well illustrated by dramatic falls of Vn-Index and HNX-Index, the benchmark indexes of stocks listed on two stock exchanges in Vietnam, in the next three consecutive days. Specifically, according to Thomson Reuters Datastream, Vn-Index was down 4.7%, 1.6%, and 4.2% and HNX-Index was down 5.2%, 3.4% and 5.3% on 21, 22 and 23 August 2012, respectively.

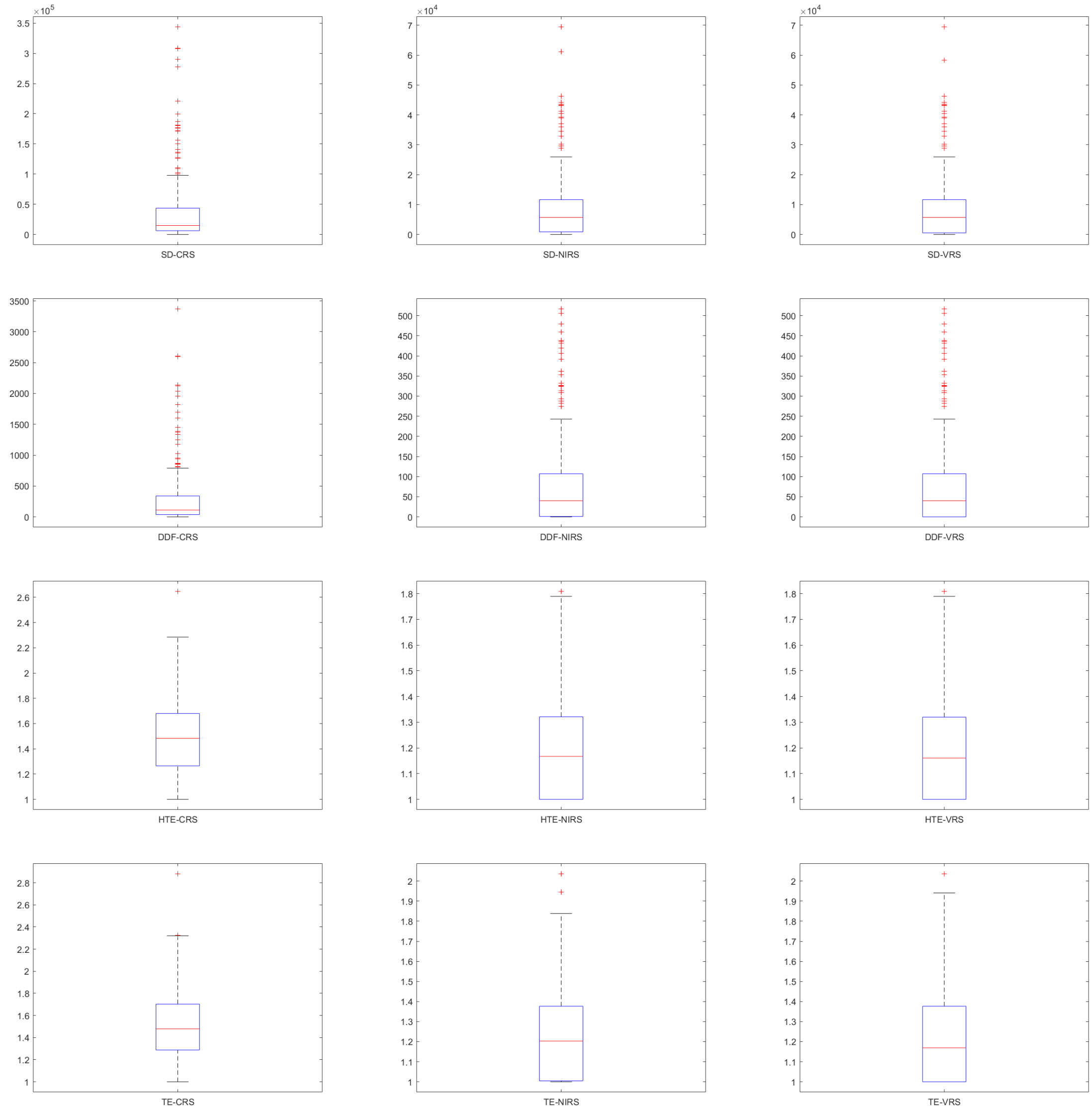


Figure 3: Box-plots of estimated inefficiency scores

5.4 Remarks on some individual banks

In this subsection, we will investigate further the efficiency of individual banks using the SD inefficiency scores computed under VRS. At the first glance, B39 is the only bank having zero inefficiency scores in six out of seven years. Following B39 is B04 which has five out of seven inefficiency scores equal to zero. Thus, these two banks can be considered the most efficient banks in the period 2008-2014. Interestingly, they are both large and state-owned.[18] Conversely, B15, B35 and B08 can be viewed as the most inefficient banks because of their remarkably large inefficiency scores. While B08 and B15 are of medium banks, B35 is in the group of the four largest banks.

[18]In this paper, we divide banks into three categories based on total assets: (i) large banks - the four largest banks, (ii) medium banks - the twelve largest banks, excluding banks in group (i), and (iii) small banks - the remaining banks.

Table 3: Correlation of ranks (Spearman's ρ statistic)

		SD	DDF	HTE	TE
CRS	SD	1.000	0.871	0.367	0.353
	DDF		1.000	0.406	0.366
	HTE			1.000	0.923
	TE				1.000
NIRS	SD	1.000	0.866	0.485	0.510
	DDF		1.000	0.597	0.595
	HTE			1.000	0.980
	TE				1.000
VRS	SD	1.000	0.866	0.508	0.535
	DDF		1.000	0.616	0.618
	HTE			1.000	0.980
	TE				1.000

Source: Authors' calculations.

Table 4: Correlations of rankings (Spearman's ρ) under different returns to scale

		CRS	NIRS	VRS
SD	CRS	1.000	0.430	0.438
	NIRS		1.000	0.999
	VRS			1.000
DDF	CRS	1.000	0.447	0.453
	NIRS		1.000	0.999
	VRS			1.000
HTE	CRS	1.000	0.611	0.633
	NIRS		1.000	0.992
	VRS			1.000
TE	CRS	1.000	0.660	0.681
	NIRS		1.000	0.988
	VRS			1.000

Source: Authors' calculations.

We also observe several banks showing deterioration in efficiency as their ranks are downgraded through years, e.g., B02 (from 1 to 232), B13 (from 63 to 163), B17 (from 1 to 170), B30 (from 121 to 215), B36 (from 1 to 142).[19] On the contrary, B32, a medium bank, demonstrates an admirable improvement in efficiency as its rank climbs from 231 (in 2009) to 1 (in 2014). Overall, we see that deteriorating banks outnumbered the improving banks.

Looking at the inefficiency scores in more details, we see that 30% of fully efficient banks estimated by the SD under VRS are state-owned while the proportion of this type of ownership only accounts for 14% in the whole data set, suggesting that state-owned banks operate more efficient than the others. Similarly, although small banks

[19]Rank 1 means the most efficient bank.

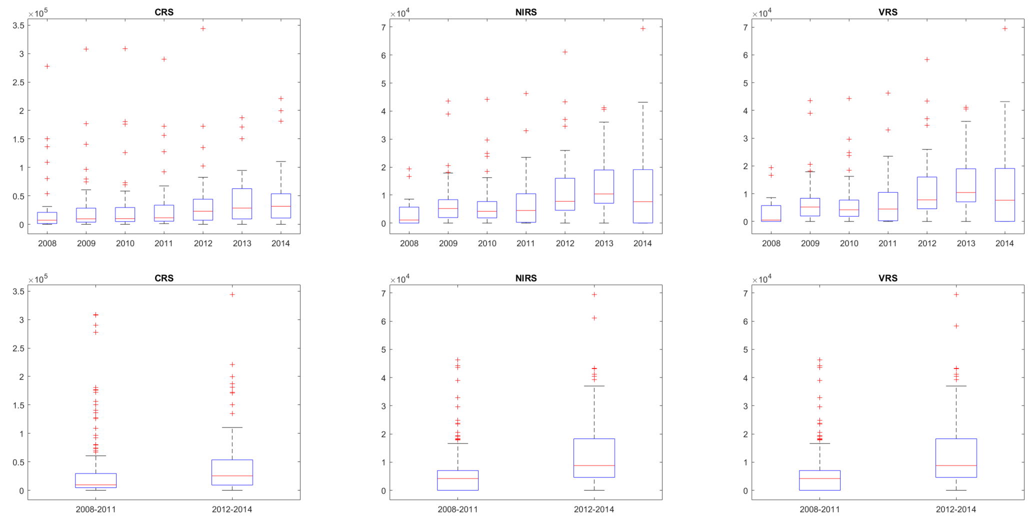


Figure 4: Estimated SD inefficiency scores by years

occupy 66% of the whole sample, its proportion shrinks to 43% in the set of fully efficient banks estimated by the SD under VRS while the opposite is seen in large and medium banks (increase from 11% to 27% and 23% to 30% with respect to large and medium banks). This fact suggests that big banks appear to be more efficient than small banks.

Interestingly, fluctuations in SD inefficiency scores provide useful signals for abnormalities in banking operations. For example, our results show that B39 has all of its SD inefficiency scores equal to zero, except for the year 2010 when the score rocketed to 1,854, ranked 68 in the sample. Then we dig deeper into the financial statements of B39 and see that the growth rate of operating expenses in 2010 soared up to 103%, while this figure is seen at most 6% in the other years. In addition, total liabilities and total assets of the bank only increased by 26% and 25%, hinting that there might be some issues in 2010 which need further exploration. Another example is the case of B02. The event that some people related to the bank were arrested in 2012 put the bank in a difficult situation and adversely affected its operation. In fact, according to Thomson Reuters Datastream, the share price of the bank drop down 6.9%, 6.6% and 6.7% on three consecutive days right after the first arrest (21, 22 and 23 August 2012, respectively). This is well captured and reflected in the inefficiency scores of the bank as its rank, although was number one during 2008-2011, suddenly dropped down to 230 in 2012 and stayed at that level in the following years (229 in 2013 and 232 in 2014).

Moreover, SD inefficiency scores might also be useful in assessing effectiveness of mergers and acquisitions. To illustrate, the ranks of B30, a medium bank, fall in the range [121,181] in the period 2008-2011. Nonetheless, after merging with a small bank in 2012, its rank went up to 223, continuing to rise to 226 in 2013 before a slight decrease to 215 in 2014. We then investigate the financial statements of B30 and see that this bank had to make provisions for credit losses of loans belonging to its merger partner, resulting in a significant reduction in its annual profits. Therefore, changing in the SD-based ranks of the bank appears to be consistent with the bank's operation in reality.

Presented evidence support our expectation that SD inefficiency score can be a useful indicator for analyzing the performance of banks in general. To understand performance of individual banks in more details with comprehensive explanations, one could then conduct deeper case studies which are beyond the scope of this paper.

5.5 Analysis of efficiency distributions

In this subsection, we analyze the distributions of inefficiency scores using kernel density estimation and related tests. Our procedure is adapted from the framework constructed for the Farrell-type technical efficiency which was proposed by Simar and Zelenyuk (2006), who in turn adapted the approach of Li (1996, 1999).

First, we do kernel density estimations of inefficiency scores under CRS, NIRS and VRS (Figure 5). Two crucial points are: (i) We overcome the issue of bounded support in density estimation by employing the reflection method proposed by Schuster (1985)-Silverman (1986) (for details, see Simar and Zelenyuk, 2006), and (ii) To calculate the bandwidths, we use the Sheather and Jones's (1991) method.[20] As can be seen in Figure 5, DDF, HTE and TE inefficiency scores follow patterns which are different with that of the SD inefficiency scores. The SD has wide estimated ranges of values and long tails compared to the other measures. In addition, the estimated densities under NIRS and VRS look similar whereas being different with those under CRS. In Figure 5 we also show kernel density estimations of inefficiency scores in the period 2008-2011 and 2012-2014. In all cases, the densities corresponding to period 2008-2011 lie above those corresponding to the period 2012-2014 to the left, suggesting that in 2008-2011 banks operated more efficiently than in 2012-2014.

Figure 6 presents the kernel densities estimations contrasting the different types of ownership (state-owned versus non-state-owned) and bank sizes (large, medium and small). Interestingly, state-owned banks are apparently more efficient than non-state-owned banks by the SD and DDF measures while the opposite is seen by the HTE and TE measures. Analogously, SD and DDF measures recognise large banks as more efficient than small banks while the kernel density estimations corresponding to the HTE and TE measure show the opposite.

[20] Theoretically, SD and DDF inefficiency scores are bounded below by 0 while HTE and TE inefficiency scores are bounded below by 1.

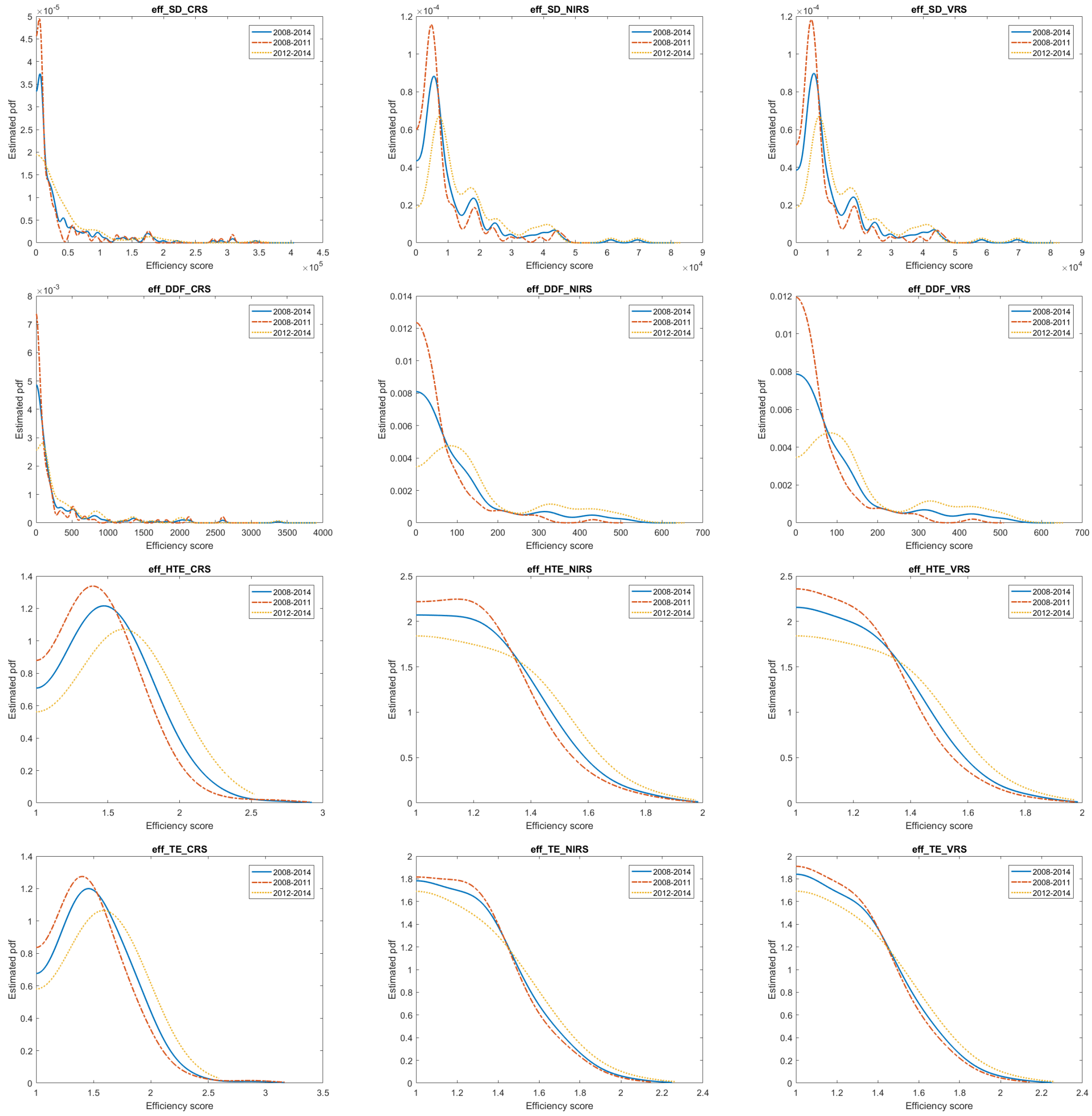


Figure 5: Estimated densities of DEA-estimated inefficiency scores using Sheather and Jones's (1991) bandwidth: Period 2008-2014 and two subperiods 2008-2011 and 2012-2014

Subsequently, we perform tests for equality of densities (adapted from Li, 1996, 1999; Simar and Zelenyuk, 2006) to see whether statistical evidences support the conclusions which we have drawn from kernel density estimations. This aim can be hypothesized as

$$\begin{cases} H_0 : f_A(u) = f_Z(u) & \text{for all } u \text{ in the relevant support} \\ H_1 : f_A(u) \neq f_Z(u) & \text{on a set of positive measures} \end{cases}$$

where A and Z are two types of inefficiency scores which we are going to examine and f_A and f_Z denote their density functions, respectively.

For conducting these tests, we adapt the Algorithm II from Simar and Zelenyuk (2006)

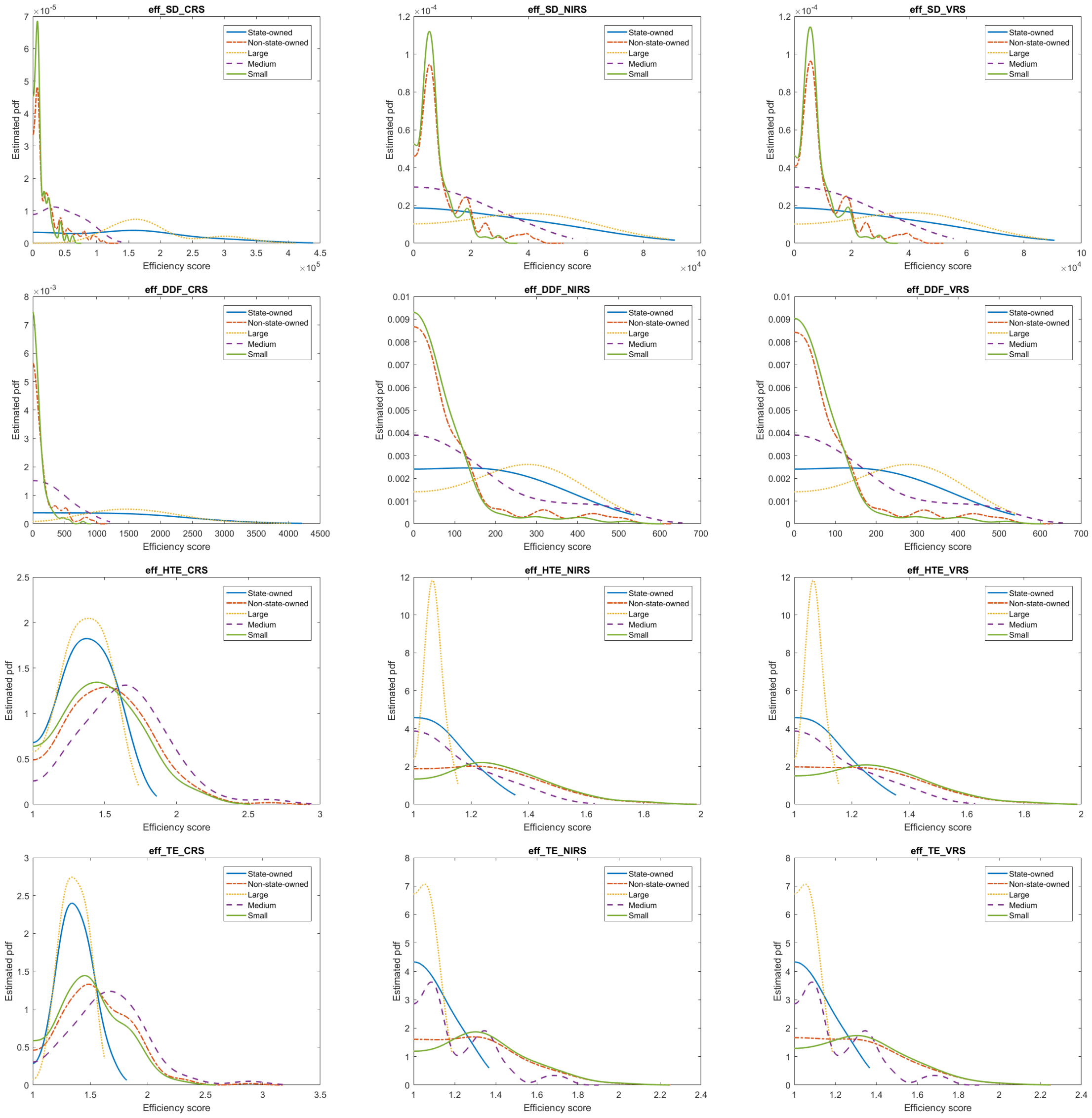


Figure 6: Estimated densities of DEA-estimated inefficiency scores using Sheather and Jones's (1991) bandwidth: State-ownership and bank sizes

Table 5: Adapted Li (1996) test for equality of densities of the DDF and SD inefficiency scores

Returns to scale	Li test statistic	Reject H_0
CRS	94.9897 [0.000]	Yes
VRS	43.2422 [0.000]	Yes
NIRS	45.7596 [0.000]	Yes

Source: Authors' calculations using Matlab. Bootstrapped p-values are provided in brackets. Decisions on rejecting H_0 are based on 5% level of significance.

with 2000 bootstrap replications and Silverman's (1986) rule-of-thumb bandwidth com-

Table 6: Adapted Li (1996) test for equality of densities under different returns to scale

		Li test statistic	Reject H_0
SD	CRS vs VRS	13.7053 [0.0000]	Yes
	CRS vs NIRS	13.6615 [0.0000]	Yes
	VRS vs NIRS	0.0060 [0.9955]	No
DDF	CRS vs VRS	11.0927 [0.0000]	Yes
	CRS vs NIRS	11.0466 [0.0000]	Yes
	VRS vs NIRS	0.0019 [0.9970]	No
HTE	CRS vs VRS	35.0441 [0.0000]	Yes
	CRS vs NIRS	33.5435 [0.0000]	Yes
	VRS vs NIRS	0.0581 [0.9440]	No
TE	CRS vs VRS	30.4120 [0.0000]	Yes
	CRS vs NIRS	28.4234 [0.0000]	Yes
	VRS vs NIRS	0.0738 [0.9260]	No

Source: Authors' calculations using Matlab.
Bootstrapped p-values are provided in brackets. Decisions on rejecting H_0 are based on 5% level of significance.

Table 7: Adapted Li (1996) test for equality of densities of estimated inefficiency scores by different time periods and ownership

Measures	Returns to scale	2008-2011 vs. 2012-2014	State-owned vs. Non-state-owned
SD	CRS	6.3382 [0.0000]	14.0446 [0.0000]
	VRS	8.1510 [0.0000]	6.1749 [0.0000]
	NIRS	8.3161 [0.0000]	6.3671 [0.0000]
DDF	CRS	6.2004 [0.0000]	16.4047 [0.0000]
	VRS	9.5654 [0.0000]	3.7819 [0.0005]
	NIRS	9.6411 [0.0000]	3.8626 [0.0010]
HTE	CRS	5.0077 [0.0000]	1.6149 [0.0275]
	VRS	0.8959 $[0.1160]^n$	5.7069 [0.0000]
	NIRS	0.7248 $[0.2950]^n$	6.2554 [0.0000]
TE	CRS	2.9188 [0.0070]	3.2836 [0.0030]
	VRS	0.1538 $[0.8365]^n$	6.1958 [0.0000]
	NIRS	-0.0413 $[0.9610]^n$	6.7491 [0.0000]

Source: Authors' calculations using Matlab.
Bootstrapped p-values are provided in brackets beside Li test statistics. Decisions on rejecting H_0 are based on 5% level of significance. ^{n} Do not reject H_0.

puted in each bootstrap iteration.[21] Table 5 reports results of comparing the SD and DDF inefficiency scores. Based on the bootstrapped p-values, we can reject the null hypothesis of identical densities under all types of returns to scale at 95% level of confidence. In addition, as reported in Table 6, for all types of measures, the densities of inefficiency scores under VRS and NIRS are similar but they are not identical to the

[21] A common practice is to calculate the Silverman's (1986) rule-of-thumb bandwidth as $h = 1.06n^{-1/5}\min\{\hat{\sigma}(\epsilon), \frac{\text{iqr}(\epsilon)}{1.349}\}$ where n is the sample size, $\hat{\sigma}(\epsilon)$ and $\text{iqr}(\epsilon)$ are the estimated standard deviation and interquartile range for the variable of interest ϵ, respectively.

density under CRS at 5% level of significance. For the sake of completeness, we also compare densities of estimated inefficiency scores of different groups using the adapted Li (1996) tests: (i) period 2008-2012 versus 2012-2014 and (ii) state-owned versus non-state-owned banks. All in all, the evidence from the adapted Li (1996) test is generally consistent with our kernel density estimations and analyses in subsection 5.3.

6 Concluding remarks

The first and main goal of this paper is to extend the slack-based directional distance function to the context of measuring efficiency in the presence of bad outputs. The second goal of this paper was to use the SD measure to propose decompositions of revenue efficiency in the presence of bad outputs, which is a further extension of Färe et al. (2005). In essence, our decompositions separate the normalized SD-based revenue efficiency as a sum of two components: (i) the normalized technical inefficiency and (ii) the allocative inefficiency. It is also worth noting that the revenue decompositions proposed in this paper are applicable to not only the banking industry but also a number of other production processes.

The third goal of this paper was to illustrate our theoretical developments by applying the SD to measure the efficiency of Vietnamese commercial banks. In doing so, we find that SD measure helps discriminate individual banks more relative to the DDF, HTE and TE measure. We also find that fluctuations in SD inefficiency scores seem to go closely with operations and performances of banks and also consist with fundamental analyses based on financial reports. We discover that inefficiency scores under VRS and NIRS are quite similar whereas they are significantly different with those under CRS.

Using the SD inefficiency scores, we find some characteristics of Vietnamese commercial banks which might be helpful to policy-makers. First, banks in period 2012-2014 are generally less efficient when compared to the period 2008-2011. Second, large banks appear to be more efficient than the others and so do state-owned banks. Bank regulators can benefit from using rankings of banks based on inefficiency scores to identify inefficient banks or groups of banks to focus on in their regulation and restructuring the Vietnamese banking system. To some extent, the increase in estimated inefficiency scores in 2012-2014 might suggest SBV to consider carefully its Scheme on "Restructuring the credit institutions system in the 2011-2015 period" for appropriate improvements in future schemes.

This paper also suggests some directions for future research. First, while our analysis in this paper focuses on using the SD to measure efficiency of banks in the presence of bad outputs, a natural future expansion is to regress the estimated SD inefficiency scores on hypothesized factors which might have potential impacts on bank efficiency.[22] In particular, it is promising to generalize the truncated regression and double bootstrap approach which was originally developed for the Farrell-type measure in the standard DEA context by Simar and Wilson (2007). The second direction would be to adapt the most recent theories from Kneip et al. (2015, 2016) to empower statistical analysis using SD-type measures. Last but not least, it is also interesting to develop the theory of aggregation of inefficiency scores based on the SD measure, which is useful in analyzing efficiency of groups of banks sharing a common property, e.g., ownership or size.

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[22]We also thank an anonymous referee for suggesting a second-stage analysis with regards to the SD measure.

Appendix A

Here we confirm that removing θ from the problem (15) does not change the optimal value of the objective function when computing SD inefficiency scores under NIRS. The proofs for CRS and the other measures are similar.

Assume that $(\beta_1^*, \ldots, \beta_N^*, \gamma_1^*, \ldots, \gamma_{M_1}^*, \delta_1^*, \ldots, \delta_{M_2}^*, \lambda^{1*}, \ldots, \lambda^{K*}, \theta^*)$ is a solution the problem (15) under NIRS:

$$\max_{\substack{\beta_1,\ldots,\beta_N,\gamma_1,\ldots,\gamma_{M_1}\\ \delta_1,\ldots,\delta_{M_2},\lambda^1,\ldots,\lambda^K,\theta}} \left(\sum_{i=1}^{N} \beta_i + \sum_{j=1}^{M_1} \gamma_j + \sum_{l=1}^{M_2} \delta_l \right) \tag{15}$$

subject to:

$$\begin{array}{l} x_i^o - \beta_i \geq \sum_{k=1}^K \lambda^k x_i^k \ \forall i = 1, \ldots, N \\ y_j^o + \gamma_j \leq \theta \sum_{k=1}^K \lambda^k y_j^k \ \forall j = 1, \ldots, M_1 \\ w_l^o - \delta_l = \theta \sum_{k=1}^K \lambda^k w_l^k \ \forall l = 1, \ldots, M_2 \\ \lambda^k \geq 0 \ \forall k = 1, \ldots, K; \beta_i \geq 0 \ \forall i = 1, \ldots, N \\ \gamma_j \geq 0 \ \forall j = 1, \ldots, M_1; \delta_l \geq 0 \ \forall l = 1, \ldots, M_2 \\ \sum_{k=1}^K \lambda^k \leq 1; 0 \leq \theta \leq 1 \end{array}$$

and $(\tilde{\beta}_1, \ldots, \tilde{\beta}_N, \tilde{\gamma}_1, \ldots, \tilde{\gamma}_{M_1}, \tilde{\delta}_1, \ldots, \tilde{\delta}_{M_2}, \tilde{\lambda}^1, \ldots, \tilde{\lambda}^K)$ is a solution of

$$\max_{\substack{\beta_1,\ldots,\beta_N,\gamma_1,\ldots,\gamma_{M_1}\\ \delta_1,\ldots,\delta_{M_2},\lambda^1,\ldots,\lambda^K}} \left(\sum_{i=1}^{N} \beta_i + \sum_{j=1}^{M_1} \gamma_j + \sum_{l=1}^{M_2} \delta_l \right) \tag{34}$$

subject to:

$$\begin{array}{l} x_i^o - \beta_i \geq \sum_{k=1}^K \lambda^k x_i^k \ \forall i = 1, \ldots, N \\ y_j^o + \gamma_j \leq \sum_{k=1}^K \lambda^k y_j^k \ \forall j = 1, \ldots, M_1 \\ w_l^o - \delta_l = \sum_{k=1}^K \lambda^k w_l^k \ \forall l = 1, \ldots, M_2 \\ \lambda^k \geq 0 \ \forall k = 1, \ldots, K; \beta_i \geq 0 \ \forall i = 1, \ldots, N \\ \gamma_j \geq 0 \ \forall j = 1, \ldots, M_1; \delta_l \geq 0 \ \forall l = 1, \ldots, M_2 \\ \sum_{k=1}^K \lambda^k \leq 1 \end{array}$$

then we need to prove that $\sum_{i=1}^N \beta_i^* + \sum_{j=1}^{M_1} \gamma_j^* + \sum_{l=1}^{M_2} \delta_l^* = \sum_{i=1}^N \tilde{\beta}_i + \sum_{j=1}^{M_1} \tilde{\gamma}_j + \sum_{l=1}^{M_2} \tilde{\delta}_l$.

Firstly, it is transparent that $(\tilde{\beta}_1, \ldots, \tilde{\beta}_N, \tilde{\gamma}_1, \ldots, \tilde{\gamma}_{M_1}, \tilde{\delta}_1, \ldots, \tilde{\delta}_{M_2}, \tilde{\lambda}^1, \ldots, \tilde{\lambda}^K, 1)$ satisfies constraints of problem (15). Thus,

$$\sum_{i=1}^{N} \beta_i^* + \sum_{j=1}^{M_1} \gamma_j^* + \sum_{l=1}^{M_2} \delta_l^* \geq \sum_{i=1}^{N} \tilde{\beta}_i + \sum_{j=1}^{M_1} \tilde{\gamma}_j + \sum_{l=1}^{M_2} \tilde{\delta}_l \tag{35}$$

Secondly, since $0 \leq \theta^* \leq 1$, we have

$$x_i^o - \beta_i^* \geq \sum_{k=1}^{K} \lambda^{k*} x_i^k \geq \sum_{k=1}^{K} (\theta^* \lambda^{k*}) x_i^k \quad \forall i = 1, \ldots, N$$

and

$$\sum_{k=1}^{K} (\theta^* \lambda^{k*}) \leq \sum_{k=1}^{K} \lambda^{k*} \leq 1$$

Hence, it is clear that $(\beta_1^*, \ldots, \beta_N^*, \gamma_1^*, \ldots, \gamma_M^*, \delta_1^*, \ldots, \delta_{M_2}^*, \theta^* \lambda^{1*}, \ldots, \theta^* \lambda^{K*})$ satisfies constraints of problem (34). As a consequence,

$$\sum_{i=1}^{N} \beta_i^* + \sum_{j=1}^{M_1} \gamma_j^* + \sum_{l=1}^{M_2} \delta_l^* \leq \sum_{i=1}^{N} \tilde{\beta}_i + \sum_{j=1}^{M_1} \tilde{\gamma}_j + \sum_{l=1}^{M_2} \tilde{\delta}_l \tag{36}$$

From (35) and (36), we have the desired result.

Appendix B Summary of inefficiency scores

Table A1: Average of inefficiency scores: SD and DDF

Banks	CRS		NIRS		VRS	
	SD	DDF	SD	DDF	SD	DDF
B01	19,660	210	13,183	172	13,183	172
B02	88,893	562	16,054	154	16,054	154
B03	305,652	2,566	15,823	0	15,273	0
B04	134,353	802	7,584	76	7,584	76
B05	4,128	17	3,987	17	3,832	16
B06	3,038	32	2,852	31	2,582	30
B07	3,726	27	3,275	22	3,177	22
B08	31,581	356	16,119	193	16,119	193
B09	35,348	338	9,105	74	9,105	74
B10	8,408	28	5,693	22	5,693	22
B11	7,383	87	3,111	64	3,111	64
B12	17,280	100	8,402	61	8,402	61
B13	5,985	49	5,003	46	4,912	45
B14	19,644	137	8,028	76	8,028	76
B15	65,022	610	26,163	296	26,163	296
B16	1,301	8	1,298	7	1,105	6
B17	10,554	171	5,755	86	5,755	86
B18	32,786	221	6,059	55	6,059	55
B19	5,637	33	4,800	31	4,796	31
B20	16,990	44	5,588	21	5,588	21
B21	10,309	68	7,894	60	7,894	60
B22	7,061	108	5,206	77	5,206	77
B23	20,338	46	11,558	21	11,558	21
B24	6,755	78	5,799	68	5,752	67
B25	14,435	70	6,537	53	6,537	53
B26	20,704	136	9,390	116	9,390	116
B27	16,624	97	6,159	56	6,159	56
B28	26,659	225	7,547	116	7,547	116
B29	7,331	82	6,067	75	6,067	75
B30	31,532	254	16,153	191	16,153	191
B31	71,971	566	22,547	191	22,547	191
B32	71,053	460	13,697	82	13,697	82
B33	0	0	0	0	0	0
B34	6,929	76	4,739	66	4,739	66
B35	180,032	990	41,142	248	41,142	248
B36	3,482	32	2,996	30	2,940	30
B37	21,528	113	4,399	42	4,399	42
B38	22,773	128	3,840	18	3,840	18
B39	137,740	1,689	265	2	265	2
B40	4,964	17	4,315	17	4,279	17

Table A2: Average of inefficiency scores: HTE and TE

Banks	CRS		NIRS		VRS	
	HTE	TE	HTE	TE	HTE	TE
B01	1.908	1.882	1.487	1.587	1.487	1.587
B02	1.797	1.847	1.090	1.116	1.090	1.116
B03	1.257	1.267	1.000	1.000	1.000	1.000
B04	1.137	1.138	1.014	1.015	1.014	1.015
B05	1.468	1.575	1.447	1.560	1.442	1.467
B06	1.308	1.348	1.277	1.334	1.240	1.292
B07	1.282	1.338	1.221	1.291	1.188	1.271
B08	1.594	1.604	1.151	1.181	1.151	1.181
B09	1.484	1.509	1.093	1.123	1.093	1.123
B10	1.666	1.722	1.359	1.459	1.359	1.459
B11	1.254	1.215	1.148	1.151	1.148	1.151
B12	1.586	1.589	1.207	1.276	1.207	1.276
B13	1.553	1.617	1.394	1.506	1.377	1.485
B14	1.452	1.451	1.178	1.198	1.178	1.198
B15	2.014	1.936	1.313	1.367	1.313	1.367
B16	1.250	1.280	1.247	1.278	1.210	1.235
B17	1.383	1.365	1.141	1.162	1.141	1.162
B18	1.536	1.529	1.067	1.082	1.067	1.082
B19	1.522	1.571	1.392	1.470	1.392	1.469
B20	1.275	1.264	1.060	1.078	1.060	1.078
B21	1.568	1.609	1.329	1.400	1.329	1.400
B22	1.400	1.400	1.227	1.240	1.227	1.240
B23	1.494	1.383	1.077	1.092	1.077	1.092
B24	1.486	1.481	1.361	1.398	1.349	1.385
B25	1.274	1.336	1.133	1.172	1.133	1.172
B26	1.458	1.413	1.186	1.196	1.186	1.196
B27	1.526	1.450	1.193	1.198	1.193	1.198
B28	1.247	1.242	1.038	1.059	1.038	1.059
B29	1.535	1.551	1.364	1.419	1.364	1.419
B30	1.617	1.654	1.262	1.350	1.262	1.350
B31	1.540	1.845	1.090	1.170	1.090	1.170
B32	1.911	1.897	1.107	1.159	1.107	1.159
B33	1.000	1.000	1.000	1.000	1.000	1.000
B34	1.367	1.366	1.238	1.270	1.238	1.270
B35	1.513	1.458	1.068	1.076	1.068	1.076
B36	1.371	1.363	1.282	1.326	1.271	1.313
B37	1.364	1.334	1.060	1.068	1.060	1.068
B38	1.468	1.500	1.044	1.058	1.044	1.058
B39	1.265	1.391	1.000	1.000	1.000	1.000
B40	1.630	1.750	1.490	1.622	1.462	1.596

References

Berger, A. N. and Humphrey, D. B. (1997). Efficiency of Financial Institutions: International Survey and Directions for Future Research. *European Journal of Operational Research*, 98(2):175–212.

Berger, A. N. and Mester, L. J. (2003). Explaining the dramatic changes in performance of US banks: technological change, deregulation, and dynamic changes in competition. *Journal of Financial Intermediation*, 12(1):57–95.

Chambers, R., Chung, Y., and Färe, R. (1998). Profit, Directional Distance Functions, and Nerlovian Efficiency. *Journal of Optimization Theory and Applications*, 98(2):351–364.

Chambers, R. G., Chung, Y., and Färe, R. (1996). Benefit and distance functions. *Journal of Economic Theory*, 70(2):407–419.

Charnes, A., Cooper, W., Golany, B., Seiford, L., and Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *Journal of Econometrics*, 30(1-2):91–107.

Chung, Y., Färe, R., and Grosskopf, S. (1997). Productivity and Undesirable Outputs: A Directional Distance Function Approach. *Journal of Environmental Management*, 51(3):229–240.

Cooper, W. W., Seiford, L. M., and Tone, K. (2007). *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*. Springer, New York.

Curi, C., Lozano-Vivas, A., and Zelenyuk, V. (2015). Foreign bank diversification and efficiency prior to and during the financial crisis: Does one business model fit all? *Journal of Banking & Finance*, 61:S22–S35.

Du, K., Worthington, A. C., and Zelenyuk, V. (2015). The dynamic relationship between bank asset diversification and efficiency: Evidence from the Chinese banking sector. *Centre for Efficiency and Productivity Analysis Working Paper Series No. WP12/2015.*

Färe, R., Fukuyama, H., Grosskopf, S., and Zelenyuk, V. (2015). Decomposing profit efficiency using a slack-based directional distance function. *European Journal of Operational Research*, 247(1):335–337.

Färe, R., Fukuyama, H., Grosskopf, S., and Zelenyuk, V. (2016). Cost decompositions and the efficient subset. *Omega*, 62:123–130.

Färe, R. and Grosskopf, S. (2003). Nonparametric Productivity Analysis with Undesirable Outputs: Comment. *American Journal of Agricultural Economics*, 85(4):1070–1074.

Färe, R. and Grosskopf, S. (2009). A Comment on Weak Disposability in Nonparametric Production Analysis. *American Journal of Agricultural Economics*, 91(2):535–538.

Färe, R. and Grosskopf, S. (2010). Directional distance functions and slacks-based measures of efficiency. *European Journal of Operational Research*, 200(1):320–322.

Färe, R., Grosskopf, S., and Lovell, C. A. K. (1994). *Production frontiers*. Cambridge University Press.

Färe, R., Grosskopf, S., Lovell, C. A. K., and Pasurka, C. (1989). Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Nonparametric Approach. *The Review of Economics and Statistics*, 71(1):90.

Färe, R., Grosskopf, S., Noh, D.-W., and Weber, W. (2005). Characteristics of a polluting technology: theory and practice. *Journal of Econometrics*, 126(2):469–492.

Färe, R. and Lovell, C. (1978). Measuring the technical efficiency of production. *Journal of Economic Theory*, 19(1):150–162.

Färe, R. and Primont, D. (1995). *Multi-Output Production and Duality: Theory and Applications*. Springer Netherlands, Dordrecht.

Farrell, M. J. (1957). The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society. Series A*, 120(3):253–290.

Fethi, M. D. and Pasiouras, F. (2010). Assessing bank efficiency and performance with operational research and artificial intelligence techniques: A survey. *European Journal of Operational Research*, 204(2):189–198.

Fletcher, R. (1987). *Practical Methods of Optimization*. Wiley.

Fukuyama, H. and Weber, W. L. (2009). A directional slacks-based measure of technical inefficiency. *Socio-Economic Planning Sciences*, 43(4):274–287.

Fukuyama, H. and Weber, W. L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega*, 38(5):398–409.

Gill, P. E., Murray, W., and Wright, M. H. (1981). *Practical Optimization*. Academic Press, Practical Optimization.

Han, S. P. (1977). A globally convergent method for nonlinear programming. *Journal of Optimization Theory and Applications*, 22(3):297–309.

Hock, W. and Schittkowski, K. (1983). A comparative performance evaluation of 27 nonlinear programming codes. *Computing*, 30(4):335–358.

Jenkins, L. and Anderson, M. (2003). A multivariate statistical approach to reducing the number of variables in data envelopment analysis. *European Journal of Operational Research*, 147(1):51–61.

Kenjegalieva, K., Simper, R., Weyman-Jones, T., and Zelenyuk, V. (2009). Comparative analysis of banking production frameworks in eastern european financial markets. *European Journal of Operational Research*, 198(1):326–340.

Kneip, A., Simar, L., and Wilson, P. W. (2015). When bias kills the variance: Central limit theorems for dea and fdh efficiency scores. *Econometric Theory*, 31(2):394–422.

Kneip, A., Simar, L., and Wilson, P. W. (2016). Testing hypotheses in nonparametric models of production. *Journal of Business & Economic Statistics*, 34(3):435–456.

Kuosmanen, T. (2005). Weak Disposability in Nonparametric Production Analysis with Undesirable Outputs. *American Journal of Agricultural Economics*, 87(4):1077–1082.

Kuosmanen, T. and Podinovski, V. (2009). Weak Disposability in Nonparametric Production Analysis: Reply to Färe and Grosskopf. *American Journal of Agricultural Economics*, 91(2):539–545.

Li, Q. (1996). Nonparametric testing of closeness between two unknown distribution functions. *Econometric Reviews*, 15(3):261–274.

Li, Q. (1999). Nonparametric testing the similarity of two unknown density functions: local power and bootstrap analysis. *Journal of Nonparametric Statistics*, 11(1-3):189–213.

Lozano, S. (2016). Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector. *Omega*, 60:73–84.

Paradi, J. C. and Zhu, H. (2013). A survey on bank branch efficiency and performance research with data envelopment analysis. *Omega*, 41:61–79.

Park, K. H. and Weber, W. L. (2006). A note on efficiency and productivity growth in the Korean Banking Industry, 1992-2002. *Journal of Banking & Finance*, 30(8):2371–2386.

Portela, M. C. A. S., Thanassoulis, E., and Simpson, G. (2004). Negative data in DEA: a directional distance approach applied to bank branches. *Journal of the Operational Research Society*, 55(10):1111–1121.

Powell, M. J. D. (1978a). A fast algorithm for nonlinearly constrained optimization calculations. *Lecture Notes in Mathematics-Numerical Analysis*, 630:144–157.

Powell, M. J. D. (1978b). *Nonlinear Programming 3*, chapter The Convergence of Variable Metric Methods For Nonlinearly Constrained Optimization Calculations. Academic Press.

Ratcliffe, S. (2011). *Oxford Treasury of Sayings and Quotations.* Oxford University Press, 4 edition.

Schuster, E. F. (1985). Incorporating support constraints into nonparametric estimators of densities. *Communications in Statistics - Theory and Methods*, 14(5):1123–1136.

Sealey, C. W. J. and Lindley, J. T. (1977). Inputs, outputs, and a theory of production and cost at depository financial institutions. *The Journal of Finance*, 32(4):1251–1266.

Seiford, L. M. and Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research*, 142(1):16–20.

Sheather, S. J. and Jones, M. C. (1991). A reliable data-based bandwidth selection method for kernel density estimation. *Journal of the Royal Statistical Society. Series B*, 53(3):683–690.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis.* Chapman and Hall, London.

Simar, L. and Wilson, P. W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics*, 136(1):31–64.

Simar, L. and Zelenyuk, V. (2006). On Testing Equality of Distributions of Technical Efficiency Scores. *Econometric Reviews*, 25(4):497–522.

Simper, R., Hall, M. J. B., Liu, W., Zelenyuk, V., and Zhou, Z. (2015). How relevant is the choice of risk management control variable to non-parametric bank profit efficiency analysis? The case of South Korean banks. *Annals of Operations Research.*

The State Bank of Vietnam (2008). Annual report 2008.

The State Bank of Vietnam (2009). Annual report 2009.

The State Bank of Vietnam (2010). Annual report 2010.

The State Bank of Vietnam (2011). Annual report 2011.

The State Bank of Vietnam (2012). Annual report 2012.

The State Bank of Vietnam (2013). Annual report 2013.

The State Bank of Vietnam (2014). Annual report 2014.

The State Bank of Vietnam (2016a). *History of Vietnamese banks 1951-2016*. Labour Publishing House (Vietnam), Hanoi.

The State Bank of Vietnam (2016b). System of Credit Institutions.

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 130(3):498–509.

World Bank (2015). World Bank Open Data.

Zelenyuk, N. and Zelenyuk, V. (2015). Drivers of Efficiency in Banking: Importance of Model Specifications. *Centre for Efficiency and Productivity Analysis Working Paper Series No. WP08/2015*.

Supplement

Table S1: SD inefficiency scores under CRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	6,063	12,203	19,925	17,974	21,324	28,414	31,717
B02	80,178	79,792	73,038	92,047	102,585	94,073	100,540
B03	277,656	307,816	308,503	290,324	343,960		
B04	136,175	140,802	175,755	155,994	0	150,471	181,273
B05		3,040	4,834	4,510			
B06	1,349	3,543	3,840	2,514	3,944		
B07	888	3,516	4,565	5,933			
B08	19,845	24,830	24,407	24,288	30,699	43,682	53,317
B09	31,265	36,640	29,426	21,279	29,965	45,097	53,762
B10		6,466	10,350				
B11	8,156	9,486	7,355	4,534			
B12	0	6,274	8,960	10,283	18,360	33,916	43,170
B13	1,322	3,204	4,791	5,289	7,219	9,008	11,059
B14		3,348	0	9,733	25,379	36,587	42,814
B15	30,828	38,506	52,005	67,411	82,545	85,942	97,918
B16	518	368	3,912	3,610	0	565	134
B17	1,341	4,134	9,343	11,415	14,591	16,004	17,049
B18	11,514	20,380	29,630	34,847	42,902	46,781	43,449
B19	3,699	3,890	3,984	3,724	5,427	7,459	11,276
B20					12,386	22,930	15,654
B21	8,205	7,648	7,884	9,644	9,310	12,369	17,103
B22	5,986	6,775	6,487	4,500	5,691	9,119	10,868
B23	0	11,802	27,123	29,944	27,186	25,973	
B24	868	4,491	7,863	7,871	7,918	8,090	10,185
B25	10,984	7,538	8,261	18,340	27,054		
B26						0	41,408
B27	6,181	10,646	10,874	17,006	25,351	23,093	23,219
B28	17,531	14,781	20,358		44,104	63,177	0
B29	7,129	7,906	7,509	6,174	7,061	7,995	7,542
B30	7,166	13,513	17,789	22,131	46,171	61,041	52,914
B31	53,544	60,529	58,454	56,669	69,616	94,724	110,259
B32		74,516	69,191	57,820	65,788	78,779	80,225
B33	0	0	0				0
B34	6,679	9,467	6,598	2,084	5,928	10,159	7,586
B35	149,928	176,849	180,810	172,690	172,070	187,279	220,596
B36	0	380	1,224	1,122	4,783	8,309	8,556
B37	20,965	28,355	23,302	15,846	20,076	21,508	20,643
B38	14,987	17,147	12,459	8,819	24,647	40,987	40,366
B39	109,193	96,604	125,640	127,522	134,958	170,687	199,578
B40	587	1,969	4,004	8,207	10,054		

Table S2: SD inefficiency scores under NIRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	5,202	8,930	12,246	11,947	15,483	19,038	19,432
B02	0	0	0	0	37,000	36,049	39,332
B03	0	0	0	18,039	61,074		
B04	0	0	29,610	23,479	0	0	0
B05		3,040	4,649	4,271			
B06	1,349	3,538	3,695	2,193	3,485		
B07	888	3,511	4,043	4,659			
B08	8,567	11,685	11,011	12,665	16,121	23,899	28,882
B09	16,604	17,890	0	0	0	10,050	19,192
B10		5,624	5,762				
B11	5,005	5,263	2,174	0			
B12	0	5,138	5,030	5,356	9,203	16,260	17,830
B13	1,322	3,204	4,493	4,495	5,747	7,256	8,507
B14		2,950	0	0	12,996	18,774	13,446
B15	19,323	20,566	24,813	32,959	34,570	25,372	25,539
B16	518	368	3,912	3,610	0	546	134
B17	0	0	4,158	6,429	9,057	10,995	9,647
B18	6,815	6,489	1,997	0	10,290	16,824	0
B19	3,699	3,890	3,834	3,578	4,911	6,040	7,646
B20					7,093	9,671	0
B21	6,777	5,923	5,313	7,001	7,332	10,539	12,374
B22	5,104	5,359	5,145	3,627	4,562	6,329	6,313
B23	0	6,994	18,413	19,199	14,205	10,534	
B24	868	4,391	6,556	7,017	7,026	7,130	7,606
B25	7,384	2,774	0	6,522	16,003		
B26						0	18,780
B27	2,251	7,322	0	0	13,871	10,414	9,258
B28	0	0	7,701		17,311	20,272	0
B29	5,184	6,206	5,894	5,332	6,408	7,066	6,377
B30	5,712	8,365	9,825	11,625	25,951	30,199	21,392
B31	0	18,170	16,240	14,815	24,920	40,535	43,151
B32		38,949	23,789	2,567	0	16,876	0
B33	0	0	0				0
B34	5,767	6,838	4,258	1,666	4,506	6,726	3,409
B35	0	43,574	44,214	46,212	43,317	41,178	69,496
B36	0	380	1,224	1,122	4,114	7,181	6,953
B37	8,011	9,680	0	0	4,580	7,196	1,325
B38	0	9,547	3,675	0	5,505	8,152	0
B39	0	0	1,854	0	0	0	0
B40	587	1,965	4,004	6,802	8,219		

Table S3: SD inefficiency scores under VRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	5,202	8,930	12,246	11,947	15,483	19,038	19,432
B02	0	0	0	0	37,000	36,049	39,332
B03	0	0	0	18,039	58,326		
B04	0	0	29,610	23,479	0	0	0
B05		2,575	4,649	4,271			
B06	0	3,538	3,695	2,193	3,485		
B07	495	3,511	4,043	4,659			
B08	8,567	11,685	11,011	12,665	16,121	23,899	28,882
B09	16,604	17,890	0	0	0	10,050	19,192
B10		5,624	5,762				
B11	5,005	5,263	2,174	0			
B12	0	5,138	5,030	5,356	9,203	16,260	17,830
B13	767	3,121	4,493	4,495	5,747	7,256	8,507
B14		2,950	0	0	12,996	18,774	13,446
B15	19,323	20,566	24,813	32,959	34,570	25,372	25,539
B16	0	0	3,501	3,597	0	546	88
B17	0	0	4,158	6,429	9,057	10,995	9,647
B18	6,815	6,489	1,997	0	10,290	16,824	0
B19	3,679	3,885	3,834	3,578	4,911	6,040	7,646
B20					7,093	9,671	0
B21	6,777	5,923	5,313	7,001	7,332	10,539	12,374
B22	5,104	5,359	5,145	3,627	4,562	6,329	6,313
B23	0	6,994	18,413	19,199	14,205	10,534	
B24	541	4,391	6,556	7,017	7,026	7,130	7,606
B25	7,384	2,774	0	6,522	16,003		
B26						0	18,780
B27	2,251	7,322	0	0	13,871	10,414	9,258
B28	0	0	7,701		17,311	20,272	0
B29	5,184	6,206	5,894	5,332	6,408	7,066	6,377
B30	5,712	8,365	9,825	11,625	25,951	30,199	21,392
B31	0	18,170	16,240	14,815	24,920	40,535	43,151
B32		38,949	23,789	2,567	0	16,876	0
B33	0	0	0				0
B34	5,767	6,838	4,258	1,666	4,506	6,726	3,409
B35	0	43,574	44,214	46,212	43,317	41,178	69,496
B36	0	300	1,076	958	4,114	7,181	6,953
B37	8,011	9,680	0	0	4,580	7,196	1,325
B38	0	9,547	3,675	0	5,505	8,152	0
B39	0	0	1,854	0	0	0	0
B40	412	1,965	3,996	6,802	8,219		

Table S4: DDF inefficiency scores under CRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	69	107	149	190	247	353	357
B02	148	314	479	507	597	953	937
B03	2,121	2,605	2,598	2,134	3,372		
B04	0	0	1,385	1,448	0	1,177	1,601
B05		2	12	37			
B06	3	3	19	37	97		
B07	2	3	38	65			
B08	151	246	306	396	437	492	467
B09	227	366	207	413	359	370	422
B10		28	27				
B11	96	141	112	0			
B12	0	44	57	72	101	193	231
B13	6	15	35	53	76	81	75
B14		8	0	34	161	315	306
B15	204	345	525	677	808	858	853
B16	4	0	19	22	0	7	2
B17	0	156	207	197	240	236	159
B18	53	111	183	203	344	417	238
B19	10	17	31	37	36	41	62
B20					60	58	13
B21	7	51	50	50	65	136	120
B22	48	71	80	91	159	160	150
B23	0	19	85	71	78	25	
B24	8	28	61	104	143	113	86
B25	47	56	13	112	124		
B26						0	272
B27	45	118	83	70	167	106	92
B28	110	40	137		785	278	0
B29	44	81	104	117	115	62	53
B30	9	66	168	221	541	579	195
B31	183	338	545	516	726	866	789
B32		246	480	520	558	520	434
B33	0	0	0				0
B34	35	65	96	100	90	87	61
B35	683	756	624	817	1,336	1,373	1,338
B36	0	4	21	24	37	59	76
B37	152	152	61	35	117	152	122
B38	43	91	144	166	166	148	139
B39	1,248	1,024	1,816	1,698	2,037	2,038	1,960
B40	1	4	13	22	45		

Table S5: DDF inefficiency scores under NIRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	62	86	102	138	211	313	293
B02	0	0	0	0	139	459	479
B03	0	0	0	0	0		
B04	0	0	288	243	0	0	0
B05		2	12	37			
B06	3	3	19	35	97		
B07	2	3	30	52			
B08	21	59	62	181	282	353	392
B09	142	211	0	0	0	24	142
B10		28	16				
B11	82	120	53	0			
B12	0	36	40	45	55	120	133
B13	6	15	35	53	75	74	64
B14		7	0	0	104	219	128
B15	158	239	309	432	506	325	101
B16	4	0	19	22	0	4	2
B17	0	0	82	103	166	166	83
B18	34	24	11	0	133	182	0
B19	10	17	31	37	36	40	47
B20					32	30	0
B21	7	38	40	46	64	122	102
B22	28	52	55	66	123	115	102
B23	0	1	28	42	56	0	
B24	8	28	47	81	129	107	73
B25	34	30	0	80	119		
B26						0	231
B27	14	95	0	0	157	74	50
B28	0	0	127		436	131	0
B29	42	69	92	115	105	56	48
B30	9	47	96	118	406	517	144
B31	0	53	90	108	326	438	324
B32		158	202	0	0	130	0
B33	0	0	0				0
B34	30	59	87	74	88	86	35
B35	0	274	147	204	419	362	332
B36	0	4	21	24	36	59	69
B37	40	32	0	0	88	132	0
B38	0	43	38	0	40	3	0
B39	0	0	13	0	0	0	0
B40	1	4	12	22	45		

Table S6: DDF inefficiency scores under VRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	62	86	102	138	211	313	293
B02	0	0	0	0	139	459	479
B03	0	0	0	0	0		
B04	0	0	288	243	0	0	0
B05		1	12	34			
B06	0	3	19	35	95		
B07	2	3	30	52			
B08	21	59	62	181	282	353	392
B09	142	211	0	0	0	24	142
B10		28	16				
B11	82	120	53	0			
B12	0	36	40	45	55	120	133
B13	2	14	35	53	75	74	64
B14		7	0	0	104	219	128
B15	158	239	309	432	506	325	101
B16	1	0	11	22	0	4	2
B17	0	0	82	103	166	166	83
B18	34	24	11	0	133	182	0
B19	10	17	31	37	36	40	47
B20					32	30	0
B21	7	38	40	46	64	122	102
B22	28	52	55	66	123	115	102
B23	0	1	28	42	56	0	
B24	4	28	47	81	129	107	73
B25	34	30	0	80	119		
B26						0	231
B27	14	95	0	0	157	74	50
B28	0	0	127		436	131	0
B29	42	69	92	115	105	56	48
B30	9	47	96	118	406	517	144
B31	0	53	90	108	326	438	324
B32		158	202	0	0	130	0
B33	0	0	0				0
B34	30	59	87	74	88	86	35
B35	0	274	147	204	419	362	332
B36	0	4	21	23	36	59	69
B37	40	32	0	0	88	132	0
B38	0	43	38	0	40	3	0
B39	0	0	13	0	0	0	0
B40	0	4	12	22	45		

Table S7: HTE inefficiency scores under CRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.502	1.797	1.808	1.796	2.056	2.248	2.147
B02	1.779	1.902	1.613	1.693	1.721	1.892	1.982
B03	1.328	1.000	1.255	1.228	1.475		
B04	1.000	1.000	1.194	1.276	1.000	1.207	1.286
B05		1.170	1.667	1.569			
B06	1.169	1.312	1.389	1.294	1.378		
B07	1.167	1.233	1.346	1.384			
B08	1.401	1.421	1.484	1.591	1.705	1.777	1.780
B09	1.793	1.777	1.243	1.248	1.395	1.496	1.438
B10		1.792	1.540				
B11	1.416	1.413	1.186	1.000			
B12	1.000	1.460	1.561	1.502	1.823	1.860	1.899
B13	1.188	1.325	1.497	1.560	1.700	1.787	1.814
B14		1.199	1.000	1.179	1.770	2.005	1.561
B15	2.285	2.040	1.898	2.063	2.103	1.882	1.828
B16	1.168	1.000	1.706	1.819	1.000	1.030	1.024
B17	1.000	1.099	1.269	1.380	1.595	1.691	1.646
B18	1.644	1.680	1.524	1.319	1.662	1.661	1.263
B19	1.395	1.347	1.475	1.402	1.643	1.682	1.709
B20					1.265	1.504	1.056
B21	1.294	1.469	1.299	1.395	1.415	2.006	2.097
B22	1.316	1.389	1.432	1.337	1.318	1.503	1.507
B23	1.390	1.335	1.549	1.768	1.680	1.244	
B24	1.182	1.542	1.484	1.542	1.572	1.497	1.584
B25	1.666	1.196	1.029	1.234	1.244		
B26						1.000	1.916
B27	1.278	1.802	1.235	1.247	1.932	1.555	1.635
B28	1.274	1.079	1.235		1.617	1.277	1.000
B29	1.248	1.474	1.588	1.499	1.710	1.639	1.590
B30	1.363	1.763	1.640	1.622	2.024	1.716	1.190
B31	1.705	1.662	1.585	1.619	1.334	1.877	1.000
B32		2.648	1.963	1.613	1.653	1.809	1.781
B33	1.000	1.000	1.000				1.000
B34	1.420	1.366	1.403	1.019	1.453	1.551	1.356
B35	1.587	1.548	1.431	1.430	1.530	1.541	1.524
B36	1.000	1.036	1.300	1.094	1.546	1.892	1.730
B37	1.572	1.545	1.251	1.120	1.335	1.455	1.268
B38	1.411	1.576	1.414	1.169	1.509	1.684	1.512
B39	1.484	1.368	1.000	1.245	1.338	1.002	1.419
B40	1.030	1.512	1.632	1.793	2.183		

Table S8: HTE inefficiency scores under NIRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.352	1.405	1.319	1.365	1.594	1.789	1.583
B02	1.000	1.000	1.000	1.000	1.109	1.255	1.266
B03	1.000	1.000	1.000	1.000	1.000		
B04	1.000	1.000	1.065	1.033	1.000	1.000	1.000
B05		1.170	1.640	1.531			
B06	1.169	1.233	1.337	1.268	1.376		
B07	1.167	1.233	1.227	1.256			
B08	1.030	1.045	1.038	1.048	1.160	1.319	1.416
B09	1.277	1.264	1.000	1.000	1.000	1.019	1.091
B10		1.562	1.156				
B11	1.261	1.259	1.073	1.000			
B12	1.000	1.272	1.207	1.193	1.244	1.272	1.259
B13	1.188	1.325	1.402	1.426	1.488	1.494	1.437
B14		1.164	1.000	1.000	1.327	1.410	1.165
B15	1.497	1.396	1.321	1.400	1.399	1.149	1.034
B16	1.168	1.000	1.706	1.809	1.000	1.019	1.024
B17	1.000	1.000	1.100	1.170	1.249	1.275	1.196
B18	1.213	1.073	1.022	1.000	1.060	1.104	1.000
B19	1.324	1.308	1.444	1.386	1.538	1.420	1.324
B20					1.083	1.098	1.000
B21	1.178	1.230	1.161	1.248	1.300	1.616	1.567
B22	1.162	1.215	1.214	1.175	1.225	1.316	1.282
B23	1.000	1.004	1.108	1.171	1.181	1.000	
B24	1.182	1.524	1.278	1.374	1.414	1.385	1.368
B25	1.289	1.069	1.000	1.096	1.213		
B26						1.000	1.371
B27	1.059	1.449	1.000	1.000	1.468	1.235	1.144
B28	1.000	1.000	1.147		1.000	1.079	1.000
B29	1.232	1.323	1.357	1.365	1.461	1.419	1.393
B30	1.178	1.278	1.232	1.222	1.490	1.357	1.080
B31	1.000	1.049	1.058	1.073	1.198	1.253	1.000
B32		1.370	1.226	1.000	1.000	1.046	1.000
B33	1.000	1.000	1.000				1.000
B34	1.270	1.275	1.204	1.014	1.359	1.413	1.128
B35	1.000	1.087	1.049	1.072	1.126	1.057	1.081
B36	1.000	1.033	1.300	1.094	1.485	1.592	1.468
B37	1.086	1.061	1.000	1.000	1.100	1.170	1.000
B38	1.000	1.164	1.070	1.000	1.068	1.004	1.000
B39	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B40	1.030	1.512	1.632	1.497	1.781		

Table S9: HTE inefficiency scores under VRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.352	1.405	1.319	1.365	1.594	1.789	1.583
B02	1.000	1.000	1.000	1.000	1.109	1.255	1.266
B03	1.000	1.000	1.000	1.000	1.000		
B04	1.000	1.000	1.065	1.033	1.000	1.000	1.000
B05		1.154	1.640	1.531			
B06	1.010	1.209	1.337	1.268	1.376		
B07	1.060	1.209	1.227	1.256			
B08	1.030	1.045	1.038	1.048	1.160	1.319	1.416
B09	1.277	1.264	1.000	1.000	1.000	1.019	1.091
B10		1.562	1.156				
B11	1.261	1.259	1.073	1.000			
B12	1.000	1.272	1.207	1.193	1.244	1.272	1.259
B13	1.091	1.303	1.402	1.426	1.488	1.494	1.437
B14		1.164	1.000	1.000	1.327	1.410	1.165
B15	1.497	1.396	1.321	1.400	1.399	1.149	1.034
B16	1.000	1.000	1.624	1.809	1.000	1.019	1.017
B17	1.000	1.000	1.100	1.170	1.249	1.275	1.196
B18	1.213	1.073	1.022	1.000	1.060	1.104	1.000
B19	1.324	1.308	1.444	1.386	1.538	1.420	1.324
B20					1.083	1.098	1.000
B21	1.178	1.230	1.161	1.248	1.300	1.616	1.567
B22	1.162	1.215	1.214	1.175	1.225	1.316	1.282
B23	1.000	1.004	1.108	1.171	1.181	1.000	
B24	1.099	1.524	1.278	1.374	1.414	1.385	1.368
B25	1.289	1.069	1.000	1.096	1.213		
B26						1.000	1.371
B27	1.059	1.449	1.000	1.000	1.468	1.235	1.144
B28	1.000	1.000	1.147		1.000	1.079	1.000
B29	1.232	1.323	1.357	1.365	1.461	1.419	1.393
B30	1.178	1.278	1.232	1.222	1.490	1.357	1.080
B31	1.000	1.049	1.058	1.073	1.198	1.253	1.000
B32		1.370	1.226	1.000	1.000	1.046	1.000
B33	1.000	1.000	1.000				1.000
B34	1.270	1.275	1.204	1.014	1.359	1.413	1.128
B35	1.000	1.087	1.049	1.072	1.126	1.057	1.081
B36	1.000	1.033	1.247	1.070	1.485	1.592	1.468
B37	1.086	1.061	1.000	1.000	1.100	1.170	1.000
B38	1.000	1.164	1.070	1.000	1.068	1.004	1.000
B39	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B40	1.000	1.400	1.632	1.497	1.781		

Table S10: TE inefficiency scores under CRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.477	1.814	1.945	1.774	1.994	2.126	2.047
B02	2.326	1.873	1.584	1.678	1.700	1.865	1.900
B03	1.312	1.325	1.237	1.171	1.289		
B04	1.000	1.000	1.264	1.249	1.000	1.189	1.265
B05		1.368	1.815	1.540			
B06	1.207	1.393	1.513	1.297	1.327		
B07	1.203	1.393	1.392	1.362			
B08	1.510	1.457	1.475	1.484	1.589	1.839	1.873
B09	1.852	1.775	1.266	1.232	1.392	1.569	1.475
B10		1.801	1.643				
B11	1.353	1.354	1.152	1.000			
B12	1.000	1.602	1.544	1.481	1.790	1.831	1.873
B13	1.253	1.361	1.652	1.572	1.714	1.811	1.958
B14		1.269	1.000	1.247	1.760	1.895	1.538
B15	2.282	1.949	1.859	1.985	2.021	1.761	1.693
B16	1.198	1.000	1.751	1.946	1.000	1.033	1.032
B17	1.000	1.090	1.253	1.357	1.542	1.632	1.679
B18	1.803	1.673	1.495	1.310	1.613	1.561	1.248
B19	1.495	1.457	1.466	1.381	1.665	1.696	1.836
B20					1.256	1.483	1.053
B21	1.562	1.459	1.289	1.381	1.398	1.980	2.197
B22	1.449	1.419	1.425	1.303	1.275	1.457	1.469
B23	1.000	1.336	1.553	1.736	1.526	1.149	
B24	1.207	1.634	1.472	1.524	1.492	1.472	1.566
B25	1.886	1.214	1.032	1.260	1.286		
B26						1.000	1.827
B27	1.288	1.724	1.189	1.201	1.815	1.434	1.497
B28	1.289	1.089	1.274		1.497	1.305	1.000
B29	1.273	1.517	1.543	1.380	1.563	1.883	1.702
B30	1.585	1.838	1.609	1.592	1.932	1.808	1.218
B31	1.907	1.921	1.614	1.645	1.779	1.995	2.052
B32		2.881	1.901	1.562	1.596	1.737	1.704
B33	1.000	1.000	1.000				1.000
B34	1.463	1.397	1.349	1.014	1.388	1.612	1.342
B35	1.519	1.471	1.372	1.389	1.479	1.488	1.489
B36	1.000	1.032	1.288	1.088	1.524	1.908	1.704
B37	1.559	1.532	1.245	1.116	1.313	1.385	1.189
B38	1.515	1.792	1.390	1.155	1.477	1.683	1.486
B39	1.430	1.343	1.402	1.295	1.327	1.485	1.452
B40	1.032	1.649	1.706	2.044	2.320		

Table S11: TE inefficiency scores under NIRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.432	1.575	1.474	1.481	1.704	1.838	1.604
B02	1.000	1.000	1.000	1.000	1.137	1.334	1.338
B03	1.000	1.000	1.000	1.000	1.000		
B04	1.000	1.000	1.073	1.031	1.000	1.000	1.000
B05		1.368	1.778	1.534			
B06	1.207	1.377	1.467	1.293	1.327		
B07	1.203	1.377	1.303	1.281			
B08	1.040	1.053	1.037	1.048	1.158	1.387	1.540
B09	1.376	1.339	1.000	1.000	1.000	1.030	1.118
B10		1.695	1.222				
B11	1.263	1.267	1.075	1.000			
B12	1.000	1.384	1.285	1.258	1.342	1.332	1.329
B13	1.253	1.361	1.567	1.506	1.591	1.630	1.636
B14		1.252	1.000	1.000	1.345	1.415	1.176
B15	1.724	1.444	1.383	1.474	1.377	1.136	1.029
B16	1.198	1.000	1.751	1.946	1.000	1.022	1.032
B17	1.000	1.000	1.126	1.222	1.239	1.269	1.277
B18	1.292	1.105	1.028	1.000	1.055	1.095	1.000
B19	1.424	1.423	1.466	1.381	1.614	1.524	1.455
B20					1.093	1.140	1.000
B21	1.249	1.275	1.176	1.265	1.307	1.721	1.809
B22	1.228	1.245	1.214	1.169	1.210	1.316	1.300
B23	1.000	1.006	1.132	1.238	1.177	1.000	
B24	1.207	1.634	1.339	1.409	1.393	1.394	1.409
B25	1.408	1.083	1.000	1.113	1.258		
B26						1.000	1.393
B27	1.069	1.520	1.000	1.000	1.457	1.205	1.139
B28	1.000	1.000	1.168		1.090	1.096	1.000
B29	1.261	1.374	1.362	1.338	1.448	1.605	1.545
B30	1.252	1.400	1.347	1.334	1.573	1.444	1.102
B31	1.000	1.079	1.096	1.110	1.236	1.366	1.305
B32		1.645	1.267	1.000	1.000	1.043	1.000
B33	1.000	1.000	1.000				1.000
B34	1.366	1.328	1.200	1.011	1.353	1.495	1.138
B35	1.000	1.104	1.066	1.092	1.145	1.048	1.079
B36	1.000	1.030	1.288	1.088	1.519	1.769	1.586
B37	1.121	1.084	1.000	1.000	1.101	1.166	1.000
B38	1.000	1.237	1.081	1.000	1.080	1.007	1.000
B39	1.000	1.000	1.002	1.000	1.000	1.000	1.000
B40	1.032	1.649	1.706	1.688	2.036		

Table S12: TE inefficiency scores under VRS

Banks	Year						
	2008	2009	2010	2011	2012	2013	2014
B01	1.432	1.575	1.474	1.481	1.704	1.838	1.604
B02	1.000	1.000	1.000	1.000	1.137	1.334	1.338
B03	1.000	1.000	1.000	1.000	1.000		
B04	1.000	1.000	1.073	1.031	1.000	1.000	1.000
B05		1.088	1.778	1.534			
B06	1.000	1.377	1.467	1.293	1.324		
B07	1.125	1.377	1.303	1.281			
B08	1.040	1.053	1.037	1.048	1.158	1.387	1.540
B09	1.376	1.339	1.000	1.000	1.000	1.030	1.118
B10		1.695	1.222				
B11	1.263	1.267	1.075	1.000			
B12	1.000	1.384	1.285	1.258	1.342	1.332	1.329
B13	1.120	1.348	1.567	1.506	1.591	1.630	1.636
B14		1.252	1.000	1.000	1.345	1.415	1.176
B15	1.724	1.444	1.383	1.474	1.377	1.136	1.029
B16	1.000	1.000	1.660	1.941	1.000	1.022	1.022
B17	1.000	1.000	1.126	1.222	1.239	1.269	1.277
B18	1.292	1.105	1.028	1.000	1.055	1.095	1.000
B19	1.424	1.423	1.463	1.381	1.614	1.524	1.455
B20					1.093	1.140	1.000
B21	1.249	1.275	1.176	1.265	1.307	1.721	1.809
B22	1.228	1.245	1.214	1.169	1.210	1.316	1.300
B23	1.000	1.006	1.132	1.238	1.177	1.000	
B24	1.117	1.634	1.339	1.409	1.393	1.394	1.409
B25	1.408	1.083	1.000	1.113	1.258		
B26						1.000	1.393
B27	1.069	1.520	1.000	1.000	1.457	1.205	1.139
B28	1.000	1.000	1.168		1.090	1.096	1.000
B29	1.261	1.374	1.362	1.338	1.448	1.605	1.545
B30	1.252	1.400	1.347	1.334	1.573	1.444	1.102
B31	1.000	1.079	1.096	1.110	1.236	1.366	1.305
B32		1.645	1.267	1.000	1.000	1.043	1.000
B33	1.000	1.000	1.000				1.000
B34	1.366	1.328	1.200	1.011	1.353	1.495	1.138
B35	1.000	1.104	1.066	1.092	1.145	1.048	1.079
B36	1.000	1.030	1.225	1.064	1.519	1.769	1.586
B37	1.121	1.084	1.000	1.000	1.101	1.166	1.000
B38	1.000	1.237	1.081	1.000	1.080	1.007	1.000
B39	1.000	1.000	1.002	1.000	1.000	1.000	1.000
B40	1.000	1.569	1.688	1.688	2.036		