

# Excess sensitivity to targeted fiscal interventions in HANK models with zero liquidity\*

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# Excess sensitivity to targeted fiscal interventions in HANK models with zero liquidity\*

Yuichiro Waki<sup>†‡</sup>

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## Abstract

The *zero liquidity* assumption — nobody can borrow or lend in equilibrium because everyone faces a stringent borrowing constraint and because assets are in zero net supply — simplifies analyses of incomplete-market heterogeneous-agents macroeconomic models, by making the equilibrium distribution of asset holdings degenerate. However, when combined with the heterogeneous-agents New Keynesian (HANK) models, this assumption implies that aggregate variables exhibit excess sensitivity to targeted fiscal interventions. Specifically, an arbitrarily large contraction of output can occur in response to an arbitrarily small-sized redistribution of income across households. Yet, at the same time, this result is fragile: by relaxing the households' borrowing constraints, even slightly, the marginal intervention effect on output becomes finite, thereby eliminating the potential for small interventions to have large effects on output. Although the zero liquidity assumption makes HANK models tractable, it should be used with caution when redistribution of income is concerned.

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## 1 Introduction

This paper examines the consequences of simplifying a heterogeneous-agents New Keynesian (HANK) model (Kaplan, Moll, and Violante, 2018; Gornemann, Kuester,

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and Nakajima, 2021) by imposing the *zero liquidity assumption*: households trade assets subject to a stringent borrowing constraint or a no-short-selling constraint, and, because these assets are in zero net supply, all agents hold zero asset in equilibrium. In general, characterizing non-steady-state equilibria in incomplete-market heterogeneous-agents models such as HANK models and navigating the inner workings of these models are not a simple task. This is because an endogenously evolving distribution of asset holdings, which is high-dimensional, serves as an aggregate state variable. Imposing some simplifying assumptions on such an otherwise complex model is often helpful to understand the inner workings of the model more clearly.

The zero liquidity assumption has been known to have a clear merit: it makes heterogeneous-agents models (not necessarily New Keynesian ones) particularly tractable, without restricting household heterogeneity.<sup>1</sup> Because nobody holds assets in equilibrium, the equilibrium distribution of asset holdings is degenerate and time-invariant. Hence, the aggregate state variables remain low dimensional, thereby making these models tractable and facilitating the understanding of mechanisms at work. For this reason, this assumption has been used in the literature: Krusell, Mukoyama, and Smith (2011) derived a rich set of asset pricing implications in an incomplete-market heterogeneous-agents model without nominal frictions; Werning (2015) derived aggregation results in incomplete market models with and without the zero liquidity assumption; Ravn and Sterk (2017, 2021) used it in New Keynesian models with a search-and-matching friction in labor market, and Challe (2020) considered an optimal monetary policy in such an environment; Broer, Hansen, Krusell, and Öberg (2020) used it to examine the monetary policy transmission mechanisms in HANK models with sticky price and with sticky wage. Given the merit of tractability and a recent rapid development of the HANK literature, it is worthwhile to evaluate its implications and usefulness in HANK models.

Although the zero liquidity assumption makes an otherwise complicated model tractable, the present paper demonstrates that this tractability comes at a cost in HANK models: the zero liquidity assumption implies an unrealistic and fragile policy implication. Aggregate variables exhibit *excess sensitivity to targeted fiscal interventions*:

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<sup>1</sup>In the HANK literature, there are two other approaches to simplify models. A widely used approach is to limit the degree of heterogeneity as in two-agents New Keynesian (TANK) models *a la* Bilbiie (2008, 2020). Another approach maintains rich household heterogeneity and obtains explicit aggregation results under additional assumptions on the households' preferences and the shock structure: Braun and Nakajima (2012) obtain a linear aggregation result under a CRRA utility and a particular structure of idiosyncratic shocks; Acharya and Dogra (2020) assume a CARA utility and obtain a different linear aggregation result in the absence of the borrowing constraint.

a one-time, vanishingly small-sized policy intervention that is targeted to a specific group of households can generate an unrealistically large response of macroeconomic variables. However, the same policy intervention can have only vanishingly small effects in models when the borrowing constraint is relaxed even slightly. The excess sensitivity is therefore specific to the zero liquidity HANK model, and does not survive under small perturbations to the model environment. In other words, the zero liquidity HANK model can yield a misleading implication regarding income redistribution policy.

Aggregate variables exhibit excess sensitivity when the government collects tiny per-capita tax from a large number of households, and when it rebates all the revenue to a small number of households. Such an intervention can bring about an output contraction of any size. The size of the required intervention is small: both the total tax paid by a large fraction of households and the fraction of households who receive the transfer can be made arbitrarily small, while keeping the size of the output decline unchanged. Hence, an arbitrarily small-sized intervention can generate an arbitrarily large output decline.

Why does a small intervention have a large effect on output? Even if the total tax collection is small, as long as it is distributed toward a vanishingly small fraction of households, the per-capita transfer they receive is huge. Their saving incentives are strengthened in response to the intervention, and it is the size of the per-capita transfer that affects their saving incentives. However, because the economy as a whole cannot save and because nobody can borrow, they must have incentives to save exactly zero in equilibrium. If the real interest rate does not fall sufficiently, the before-transfer income of the recipients needs to fall so that their after-transfer income equals the level that would have realized without the intervention. Because their income is an increasing function of the aggregate income, the aggregate income must fall in equilibrium.

The excess sensitivity result is, however, fragile: it disappears once the borrowing constraint is relaxed, no matter how small the relaxation is. In the model in which the borrowing constraint is sufficiently loose, no Ricardian households are borrowing constrained, and the output effect of the aforementioned intervention converges to zero as the size of the total tax goes to zero. Therefore, small interventions can only have small effects on output. The same is true for the model in which the borrowing constraint binds for some but not all Ricardian households, though the marginal intervention effect is larger in absolute value for the latter model than in the former.

The reason why an even slightly relaxed borrowing constraint resolves the excess

sensitivity is that a subset of households increase their borrowing in response to the intervention, thereby at least partially offsetting an increased savings demand from the transfer recipients. These households who increase borrowing face higher taxes due to the intervention and are not borrowing constrained. They find it optimal to borrow when the intervention occurs, because the intervention temporarily reduces their after-tax income. Therefore, at an aggregate level, the saving incentive does not increase as much as under the stringent borrowing constraint. The aggregate income still needs to fall in order to re-equilibrate the saving market, but the size of the required fall of income is much smaller. When both the fraction of households who receive transfer and the size of total transfer are close to zero, the intervention changes the aggregate saving only by a little, and the size of the required adjustment of income is also approximately zero.

This paper's results suggest that one needs to be cautious about using the zero-liquidity HANK model to analyze a policy that redistributes income across households. The zero liquidity model can be thought of as a limit of models with less stringent borrowing constraints. It has a clear merit: it is more tractable than the models with non-stringent borrowing constraints. In many cases it may provide a good approximation of latter models. However, for the particular intervention considered in this paper, the policy effect in the limit model is vastly different from the limit of policy effects in models with less stringent borrowing constraints, and the zero-liquidity model does not provide a good approximation of more general models.

The remainder of this paper is organized as follows. Section 2 introduces the New Keynesian model with discount factor heterogeneity and demonstrates the excess sensitivity result. Section 3 illustrates the key mechanism and argues that the transmission mechanism is the same as that of monetary policy. Section 4 discusses the role of assumptions other than the zero liquidity assumption. In Section 5, I examine models with less stringent borrowing constraints and show that the excess sensitivity is specific to the model with the zero liquidity assumption. Section 6 provides the conclusion.

## 2 The model: discount factor heterogeneity

I employ a deterministic New Keynesian model to study the effect of an unexpected targeted fiscal intervention that redistributes income across households. Time is discrete and goes from  $t = 0$  to infinity. The intervention is introduced unexpectedly

in period 0 and in effect only in that period. Households are heterogeneous in period 0, but are assumed to be homogeneous from period 1 onward. The homogeneity assumption simplifies the equilibrium from period 1 onward. Households have diverse saving incentives in period 0: they differ in their preference discount factors and there are some hand-to-mouth households. In the baseline model, the before-tax-and-transfer flow income distribution is assumed to be degenerate among the non-hand-to-mouth households. A model with income heterogeneity is discussed later in Section 4.1. Finally, the firms' price setting behavior is summarized by a reduced-form, yet standard New Keynesian Phillips curve, and monetary policy follows a standard Taylor rule.

There are a continuum of households with measure  $1 + H$ . Among them, the measure  $H$  of households are hand-to-mouth: in period  $t$ , each receives  $y_t^H$  units of income, pays  $\tau_t^H$  units of lump-sum tax, and consumes all of them contemporaneously: i.e.,  $c_t^H = y_t^H - \tau_t^H$  at an individual household level. Defining their total consumption, income, and tax as  $C_t^H = Hc_t^H$ ,  $Y_t^H = Hy_t^H$ , and  $T_t^H = H\tau_t^H$ , the same equation also holds at the aggregate level:

$$C_t^H = Y_t^H - T_t^H.$$

There is a unit measure of households that have access to a saving/borrowing market. I call them the Ricardian households. The Ricardian households have a common discount factor from period 1 onward,  $\beta_{**} \in (0, 1)$ , but are heterogeneous in their period-0 discount factor. The period-0 discount factor of household  $i \in [0, 1]$  is denoted by  $\beta(i) \in [0, 1)$ . Without loss of generality it is assumed that the function  $\beta(\cdot)$  is non-decreasing. I assume that  $\max_i \beta(i)$  exists and that it is strictly less than one. The maximal discount factor is denoted by  $\beta_*$ .

The Ricardian household  $i$ 's utility from a consumption sequence,  $\{c_t^R(i)\}_{t=0}^\infty$ , is given by:

$$u(c_0^R(i)) + \beta(i) \sum_{t=1}^{\infty} \beta_{**}^{t-1} u(c_t^R(i)).$$

The period utility function  $u$  satisfies a standard set of assumptions: it is continuously differentiable, strictly concave, and strictly increasing, and satisfies the Inada condition.

The Ricardian households face the budget and the borrowing constraints every period. The period- $t$  budget constraint is

$$c_t^R(i) + a_{t+1}^R(i) = y_t^R - \tau_t^R(i) + (1 + r_{t-1})a_t^R(i).$$

All the Ricardian households receive the same before-tax income,  $y_t^R$ . In contrast, I allow the lump-sum transfer to depend on the Ricardian household's index  $i$  in order to examine the targeted intervention to the Ricardian households. The beginning-of-period asset level in period  $t$  is denoted by  $a_t^R(i)$ , and the initial asset level is assumed to be zero,  $a_0^R(i) = 0$ . The households face a stringent borrowing constraint,  $a_{t+1}^R(i) \geq 0$ , i.e., they are not allowed to borrow at all.

Instead of specifying the source of income for two types of households, I assume that their incomes are determined by simple incidence functions.<sup>2</sup> The total (before-tax-and-transfer) income received by the Ricardian households,  $Y_t^R$ , and by the Hand-to-Mouth households,  $Y_t^H$ , are determined by the aggregate income,  $Y_t$ , according to the incidence functions,  $\gamma$  and  $\delta$ , respectively:  $Y_t^R = \gamma(Y_t)$ , and  $Y_t^H = \delta(Y_t)$ . The functions  $\gamma$  and  $\delta$  satisfy a consistency condition:  $\delta(Y_t) + \gamma(Y_t) = Y_t$ . The incidence functions are continuously differentiable, non-negative, and strictly increasing.

New Keynesian Phillips curve (NKPC) is specified as:

$$\pi_t = \Pi(Y_t, \pi_{t+1}).$$

The function  $\Pi$  is strictly increasing in its arguments. It is assumed that zero inflation is achieved when output equals the natural output. The natural output is assumed to be constant and is denoted by  $Y^n$ . Hence,  $\Pi(Y^n, 0)$  equals zero. The inflation dynamics are assumed to depend only on the aggregate variables, but not directly depend on the distribution of income. This formulation is consistent with a standard log-linearized New Keynesian Phillips curve,  $\pi_t = \kappa \ln(Y_t/Y^n) + \beta\pi_{t+1}$ , which obtains under the nominal price stickiness.

Regarding monetary policy, a simple interest rate rule is assumed. Let  $r_* = 1/\beta_* - 1$  and  $r_{**} = 1/\beta_{**} - 1$ . The nominal interest rate follows the Taylor rule from period 1 on:

$$R_t = r_t + \pi_{t+1} = r_{**} + \phi\pi_t$$

with  $\phi > 1$  for  $t \geq 1$ , and the central bank sets a constant nominal rate in period 0:

$$R_0 = r_0 + \pi_1 = r_*.$$

As will be described later, I will focus on an equilibrium in which inflation equals zero from period 1 on, and the above specification implies that the real rate in period

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<sup>2</sup>The same approach is taken in e.g. [Alves, Kaplan, Moll, and Violante \(2020\)](#).

0 is constant at  $r_*$  and that it remains at  $r_{**}$  from period 1 onward. The assumption of constant nominal and real rates in period 0 is made for the sake of simplicity for policy experiments. I will discuss in Section 4 what would happen if the central bank follows the same Taylor rule in period 0.

Government runs a balanced budget every period:

$$H\tau_t^H + \int_0^1 \tau_t^R(i)di = G_t.$$

Fiscal interventions I will consider are period-0 redistribution within the group of the Ricardian households. Therefore, I assume that the government purchases, the lump-sum tax for the Hand-to-Mouth households, and the total lump-sum taxes for the Ricardian households are constant over time:  $G_t = \bar{G}$  (with  $\bar{G} < Y^n$ ),  $\tau_t^H = \bar{\tau}^H$ , and  $\int_0^1 \tau_t^R(i)di = \bar{\tau}^R$  for all  $t \geq 0$ . For simplicity, I also assume that from period 1 on, the lump-sum tax for the Ricardian households is constant across households and over time, i.e.,  $\tau_t^R(i) = \bar{\tau}^R$  for all  $i$  and for all  $t \geq 1$ .

Because the lump-sum taxes are assumed to be exogenous, households cannot pay taxes if their income are too low. I will focus on a situation in which all households earn sufficient income to pay taxes, i.e., both  $H\bar{\tau}^H \leq \delta(Y_t)$  and  $\max_i \tau_t^R(i) \leq \gamma(Y_t)$  hold for all  $t$ . These inequalities imply the following lower bound for the aggregate income:

$$Y_t \geq Y_t^{LB} := \max \left\{ \delta^{-1} (H\bar{\tau}^H), \gamma^{-1} \left( \max_i \tau_t^R(i) \right) \right\}, \quad (1)$$

where  $\gamma^{-1}$  and  $\delta^{-1}$  denote respectively inverse functions of  $\gamma$  and of  $\delta$ . When equation (1) is violated, it cannot be an equilibrium because some households are unable to pay their taxes. At the lower bound, either the Hand-to-Mouth households or some Ricardian households experience zero consumption, because their income equals their tax obligation.

I assume further that if the aggregate income equals the natural output from period 1 onward, households can pay their respective lump-sum taxes:  $\bar{\tau}^R < \gamma(Y^n)$  and  $H\bar{\tau}^H < \delta(Y^n)$ .



## 2.1 Characterization of an equilibrium

Because nobody can lend or borrow in equilibrium, the Euler equation holds with equality only for the households who have the strongest savings motives.<sup>3</sup> After substituting in the incidence function, their Euler equation is given by: for all  $t$ ,

$$1 = \max_{i \in [0,1]} \left( \beta(i) \frac{u'(\gamma(Y_{t+1}) - \tau_{t+1}^R(i))}{u'(\gamma(Y_t) - \tau_t^R(i))} \right) (1 + r_t). \quad (2)$$

An equilibrium given monetary and fiscal policy is  $\{(Y_t, \pi_t, r_t)\}_{t=0}^{\infty}$  such that the Euler equation (equation 2), the income lower bound (equation 1), NKPC, the monetary policy rule, and the government budget constraint are satisfied.

It is straightforward to show that all variables are constant at their natural levels from period 1 onward:  $Y_t = Y^n$ ,  $\pi_t = 0$ ,  $r_t = r_{**}$ ,  $C_t^R = Y_t^R - \bar{\tau}^R = \gamma(Y^n) - \bar{\tau}^R$ ,  $C_t^H = Y_t^H - H\bar{\tau}^H = \delta(Y^n) - H\bar{\tau}^H$  for all  $t \geq 1$ .

Then, the period-0 output and inflation are determined solely by the dynamic IS equation (i.e., the Euler equation and the monetary policy rule combined):

$$1 = \left[ \max_i \left( \frac{\beta(i)}{u'(\gamma(Y_0) - \tau_0^R(i))} \right) \right] u'(\gamma(Y^n) - \bar{\tau}^R) (1 + r_*), \quad (3)$$

the Phillips curve,  $\pi_0 = \Pi(Y_0, 0)$ , and the income lower bound (equation 1). Here the benefit of assuming a constant nominal rate in period 0 is clearly seen. The Euler equation solely pins down the period-0 output and the Phillips curve the period-0 inflation.

## 2.2 A targeted intervention generates an arbitrarily large contraction

Now I show that it is possible to generate a large contraction in period 0 by a small targeted intervention that redistribute income to the most patient households. The contraction is arbitrarily large in the sense that it hits the lower bound in equation (1).

The intervention has two parameters,  $\varepsilon \in (0, 1)$  and  $\Delta > 0$ . The parameter  $\varepsilon$  specifies a measure of households who receive transfers. The other parameter  $\Delta$  specifies the size of per-capita tax that is paid by  $1 - \varepsilon$  Ricardian households.

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<sup>3</sup>As [Krusell, Mukoyama, and Smith \(2011\)](#) and [Werning \(2015\)](#) stated, there are equilibria in which the Euler equation holds with strict inequality for everyone. In these equilibria, changes in the real interest rate have no effect on inflation or on the output; i.e., monetary policy shocks to the nominal interest rate are neutral. Given that a main focus of HANK models is on monetary policy, I view these equilibria not suitable for analyses using these models.

Importantly, the recipient households are chosen from the top of the distribution of  $\beta$ . Hence, the Euler equation holds for those recipient households that have the highest discount factor,  $\beta(i) = \beta_*$ , both before and after the intervention. Their tax equals

$$\tau^R(i) = \bar{\tau}^R - (1 - \varepsilon)\Delta/\varepsilon$$

after the intervention, because the intervention collects  $(1 - \varepsilon)\Delta$  and redistributes it equally across  $\varepsilon$  households.

The income lower bound is then given by

$$Y_0^{LB}(\Delta) = \max \{ \delta^{-1} (H\bar{\tau}^H), \gamma^{-1} (\bar{\tau}^R + \Delta) \}. \quad (4)$$

The lower bound is smaller than the natural output for small  $\Delta$ , because by continuity  $Y_0^{LB}(\Delta) \rightarrow Y^{LB}(0) < Y^n$  as  $\Delta \rightarrow 0$ .

The proposition below shows that the income lower bound can be achieved by an arbitrarily small, targeted intervention.

**Proposition 1 (A targeted intervention generates an arbitrarily large contraction)** *For any small  $\Delta > 0$ , we can find  $\varepsilon_\Delta \in (0, 1)$  such that when  $\varepsilon = \varepsilon_\Delta$ , the period-0 output hits the lower bound,  $Y_0 = Y_0^{LB}(\Delta)$ , and that  $\varepsilon_\Delta \rightarrow 0$  as  $\Delta \rightarrow 0$ .*

An implication of this proposition is that the government can engineer a small-sized intervention so as to make the period-0 output arbitrarily closer to  $Y^{LB}(0)$ .<sup>4</sup> It is worth emphasizing that Proposition 1 does not require any assumptions on households' heterogeneity. The distribution of  $\beta(i)$  can even be degenerate.

**Proof of Proposition 1.** Because the above intervention cuts the tax paid by the  $\varepsilon$  most patient households, the maximand in the dynamic IS equation (equation 3) satisfies:

$$\max_i \left( \frac{\beta(i)}{u'(\gamma(Y_0) - \tau_0^R(i))} \right) = \frac{\beta_*}{u'(\gamma(Y_0) - \bar{\tau}^R + (1 - \varepsilon)\Delta/\varepsilon)}.$$

Because  $\beta_*(1 + r_*) = 1$ , the dynamic IS equation reduces to

$$u'(\gamma(Y_0) - \bar{\tau}^R + (1 - \varepsilon)\Delta/\varepsilon) = u'(\gamma(Y^n) - \bar{\tau}^R),$$

implying that  $\gamma(Y_0) = \gamma(Y^n) - (1 - \varepsilon)\Delta/\varepsilon$ .

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<sup>4</sup>For a given  $\Delta$ , if the government chooses  $\varepsilon < \varepsilon_\Delta$ , then the level of  $Y_0$  that satisfies the IS equation can be reduced further, but it violates the income lower bound inequality (equation 1).

For any  $\Delta > 0$ , one can find  $\varepsilon > 0$  such that

$$(1 - \varepsilon)/\varepsilon = (\gamma(Y^n) - \gamma(Y^{LB}(\Delta))) / \Delta.$$

Let  $\varepsilon_\Delta$  denote such  $\varepsilon$ . Because the right-hand side goes to infinity as  $\Delta \rightarrow 0$ , it follows that  $\varepsilon_\Delta \rightarrow 0$  as  $\Delta \downarrow 0$ . Moreover, by setting  $\varepsilon = \varepsilon_\Delta$ , the IS equation (equation 3) is satisfied with  $Y_0 = Y^{LB}(\Delta)$ . ■

Can a policy generate an arbitrarily large output expansion by doing the opposite of Proposition 1? No. Suppose that the government takes away resources from the most patient households and redistributes to the other households. Then the most patient households' savings incentives decline, and can become weaker than the savings incentives of some of the recipient households. Then the period-0 income in the dynamic IS equation is that for the latter households, and depends only on  $\Delta$  but not on  $\varepsilon$ . As  $\Delta$  is taken to zero, the policy effect vanishes. Hence, there is an asymmetry in the policy effect. This asymmetry is closely related to [Bernstein \(2021\)](#) that finds a state-dependence of monetary policy effects in a TANK model with stockholders and non-stockholders. In his model, the identity of households for whom the dynamic IS equation holds switches with the state of the economy, due to a difference in their income cyclicalities.

### 3 Inspecting the mechanism

How can an arbitrarily small-sized intervention generate an arbitrarily large output contraction? Does the intervention operate through some non-standard mechanisms?

The key is to understand that under the zero liquidity assumption the dynamic IS equation only concerns saving incentives of the households for whom the Euler equation holds with equality. More specifically, holding the future variables fixed, it requires that *their* after-tax-and-transfer income is constant at a level such that their optimal savings equal zero. When they receive large per-capita transfer, their before-tax-and-transfer income needs to fall sufficiently to meet this requirement, thereby calling for a decline in the aggregate income. The same mechanism is at work if monetary policy is concerned instead of a targeted fiscal intervention.

### 3.1 Why does a small-sized intervention have a large consequence?

The targeted intervention I considered above takes away tiny amount of resources from many households and redistributes the sum to a small number of households. Among the recipient households are those for which the Euler equation holds with equality. I call these households the *EE households* (the *Euler-Equation households*). With extra resources at hands, the EE households have stronger incentives to save than they do without the intervention. If nothing else changes, the intervention makes the optimal savings for the EE households strictly positive, violating the equilibrium condition.<sup>5</sup> Hence, something must adjust to equilibrate the savings market.

Because the real rate is assumed to be constant and because what happens from period 1 onward is independent of the period-0 only intervention, what has to adjust is the period-0 income of the EE households. Because the transfer they receive is exogenous and because their before-tax-and-transfer income is determined only by the aggregate income, their after-tax-and-transfer income declines only if the aggregate income declines. In other words, the aggregate income declines only to bring a small number of the EE households' optimal savings back to zero.

### 3.2 Transmission mechanism is the same as monetary policy

The transmission channel of the targeted fiscal intervention is identical to that of monetary policy. Suppose that there is no fiscal intervention in period 0 (i.e.,  $\tau_0^R(i) = \bar{\tau}^R$  for all  $i$ ) but that the real interest rate is raised from  $r_*$  to  $r_* + \Delta_r$  with  $\Delta_r > 0$ . This is a temporary, contractionary monetary policy shock.

How does the monetary policy shock affect the period-0 output? Because all Ricardian households receive the same after-tax income, the EE households are the most patient ones. Hence, the dynamic IS equation in period 0 is now given by:

$$1 = \beta_* \frac{u'(\gamma(Y^n) - \bar{\tau}^R)}{u'(\gamma(Y_0) - \bar{\tau}^R)} (1 + r_* + \Delta_r). \quad (5)$$

According to this equation, the period-0 output  $Y_0$  declines in response to an increase in the real interest rate.

What is the transmission mechanism? At first look, the above dynamic IS equation may appear to pin down the total income received by all the Ricardian households as

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<sup>5</sup>Suppose that a strictly positive measure of Ricardian households have  $\beta(i) = \beta_*$ . Then for small  $\varepsilon$ , the aggregate savings become positive under the intervention if nothing else changes, because optimal savings for households  $i \in [1 - \varepsilon, 1]$  are strictly positive.

in a standard TANK model.<sup>6</sup> Recall, however, that this equation is derived from the Euler equation for the most patient households. Hence, the term  $\gamma(Y_0) - \bar{\tau}^R$  in equation (5) represents *the EE households'* after-tax income, which in this case happens to be equal to the average income of the Ricardian households. Therefore, as in the case of the targeted fiscal intervention, the IS equation requires that the after-tax income of the EE households be such that they have incentives to save exactly zero, and the aggregate income adjusts to meet that requirement.

## 4 Role of the assumptions other than the zero liquidity

In this section I argue that the assumptions other than the zero liquidity do not play a major role in the main result.

### 4.1 Income heterogeneity

Although the model considered so far in this paper only features preference heterogeneity and not income heterogeneity, the main result is not limited to the preference heterogeneity model. In a HANK model with uninsurable idiosyncratic shocks to income, households that are hit by a temporary good shock have stronger savings incentives than those hit by a temporary bad shock. As a result, the former will be the EE households, and a targeted redistribution towards these households generates a contraction, as in the present paper's model.

Suppose that the period-0 income is given by

$$y_0(i) = \gamma(Y_0) \times \omega_0(i),$$

where  $\omega_0(i) > 0$  satisfies  $\int_0^1 \omega_0(i) di = 1$  with the maximum of  $\omega_0^{max}$  and the minimum of  $\omega_0^{min}$ .<sup>7</sup> For convenience, assume also that  $\omega_0(i)$  is non-decreasing in  $i$  so that the maximum For  $t \geq 1$ , all Ricardian households receive the same income,  $y_t = \gamma(Y_t)$ .

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<sup>6</sup>The equation resembles that in [Bilbiie \(2020\)](#). In [Bilbiie \(2020\)](#), homogeneous Ricardian households (called Savers) can trade a full set of state-contingent securities. Hence the common preference discount factor appears in the IS equation. Here the Ricardian households are heterogeneous in their discount factor and markets are incomplete. The discount factor in the IS equation is the highest among the heterogeneous households.

<sup>7</sup>The main point of the following discussion remains valid if the shock is additive instead of being multiplicative.

Under the zero liquidity assumption, the dynamic IS equation in period 0 is given by:

$$1 = \beta_{**}(1 + r_0)u'(\gamma(Y^n) - \bar{\tau}^R) \max_i \left( \frac{1}{u'(\gamma(Y_0)\omega_0(i) - \tau_0^R(i))} \right).$$

Choose the value of  $r_0$  so that the above equation holds for  $Y_0 = Y^n$  when there is no fiscal intervention, i.e.,  $\tau^R(i) = \bar{\tau}^R$  for all  $i$ . The EE households are the recipients of transfers that have the highest  $\omega_0(i) = \omega_0^{max}$ . The income lower bound needs be modified to

$$Y_0^{LB}(\Delta) = \max \left\{ \delta^{-1}(H\bar{\tau}^H), \gamma^{-1} \left( \frac{\bar{\tau}^R + \Delta}{\omega_0^{min}} \right) \right\}.$$

Then Proposition 1 holds in this model, by setting  $\varepsilon_\Delta$  so that

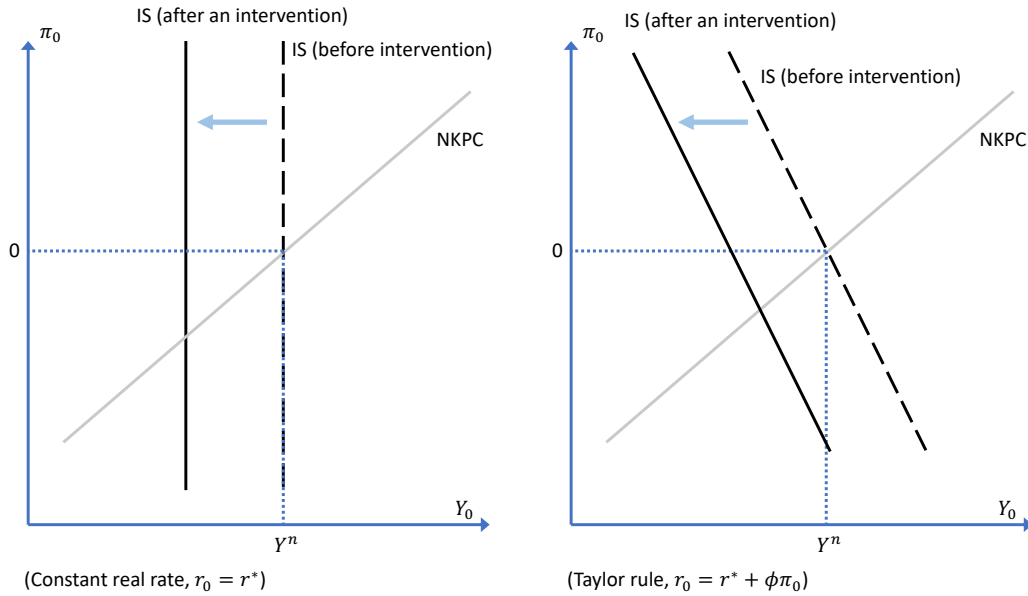
$$\frac{1 - \varepsilon_\Delta}{\varepsilon_\Delta} = \omega_0^{max} \frac{\gamma(Y^n) - \gamma(Y_0^{LB}(\Delta))}{\Delta}.$$

## 4.2 Taylor rule in period 0

I have assumed that the monetary policy achieves a fixed real interest rate,  $r^*$ , in period 0. However, even if the Taylor rule is assumed instead, the main result is unchanged.

To understand that the main result still holds under the Taylor rule in period 0, consider the contractionary targeted intervention. What happens in an equilibrium from period 1 onward is unchanged. What changes is the way inflation and output in period 0 are determined. Figure 1 illustrates this point. The left panel depicts how inflation and output are determined under the constant real interest rate. The graph of the dynamic IS equation is a vertical line and shifts toward left when a targeted intervention is contractionary. The right panel depicts the same situation but under the Taylor rule. The real interest rate is an increasing function of inflation, which makes the IS curve downward-sloping. A leftward shift of the IS curve lowers output and inflation, as in the case of the constant real interest rate.

A contractionary effect on output is weaker under the Taylor rule than the constant interest rate case. This is because the central bank under the Taylor rule cuts the real rate in response to lower inflation, which stimulates the output. Still, for any Taylor rule coefficient, a small-sized targeted intervention can generate an arbitrarily large leftward shift of the IS curve, implying that the resulting contraction of output can be arbitrarily large.



The left panel depicts the dynamic IS and the New Keynesian Phillips curve under a constant interest rate. The right panel depicts the same but under the Taylor rule.

Figure 1: Period-0 output and inflation determination

### 4.3 Hand-to-Mouth households

It is worth emphasizing that the main result of this paper does not exploit the presence of households that are exogenously hand-to-mouth. The targeted intervention I consider above does not redistribute resources from or to these Hand-to-Mouth households. Indeed, Proposition 1 holds even if there is no exogenously Hand-to-Mouth households, i.e., if  $H = 0$ ,  $\gamma(Y) = Y$ , and  $\delta(Y) = 0$ .

### 4.4 Demand-determined output

In the above model the aggregate output is assumed to be demand-determined. Because the real interest rate is pinned down by monetary policy, it does not adjust to clear the asset market, and the transfer recipients' income and hence the aggregate income have to adjust instead. If the sticky price assumption is disposed of and replaced by the neoclassical assumption, then the output is pinned down at the natural (i.e., flexible-price) level, and the real interest rate adjusts to clear the asset market. Hence, the excess sensitivity won't occur for the output, but will occur for the real interest rate. The New Keynesian assumption of demand-determined output is therefore crucial for the aggregate output's excess sensitivity in Proposition 1.

## 5 Role of the zero liquidity assumption

Now I relax the zero liquidity assumption and examine the consequences. It turns out that even only slightly positive liquidity overturns the excess sensitivity of output in response to the targeted interventions considered above.

### 5.1 A model with the natural borrowing limit

First I allow all Ricardian households to borrow up to the natural borrowing limit. Aggregate savings have to be zero as before. Then I show that the targeted interventions considered above only have muted effects on aggregate variables. As far as an intervention redistributes resources across households with different marginal propensity to consume, its effect on the aggregate output is non-zero. However, the marginal effect is never arbitrarily large, and can be expressed by an intuitive formula.

Assume that either  $u(c) = \ln c$  or  $u(c) = c^{1-\sigma}/(1-\sigma)$  with  $\sigma \neq 1$ . Under this assumption, at each point in time, each Ricardian household consumes a fraction of their financial and human wealth, and the fraction — the marginal propensity to consume (MPC) out of wealth— is determined solely by a sequence of preference discount factors and of the real interest rates from that period onward. From period 1 onward, all households have a common discount factor and a common sequence of MPC's out of wealth,  $\{mpc_t\}_{t=1}^{\infty}$ , and their consumption function is given by: for  $t \geq 1$ ,

$$c_t^R(i) = mpc_t \left\{ (1 + r_{t-1})a_t(i) + \sum_{j=0}^{\infty} \left( \prod_{l=1}^j \frac{1}{1 + r_{t+l-1}} \right) (\gamma(Y_{t+j}) - \bar{\tau}^R) \right\}.$$

Because the aggregate savings,  $\int_0^1 a_t(i) di$ , equal zero every period, the aggregate consumption from period 1 onward is given by: for  $t \geq 1$ ,

$$C_t^R = \int_0^1 c_t^R(i) di = mpc_t \left\{ \sum_{j=0}^{\infty} \left( \prod_{l=1}^j \frac{1}{1 + r_{t+l-1}} \right) (\gamma(Y_{t+j}) - \bar{\tau}^R) \right\}.$$

Importantly, from period 1 onward, the aggregate consumption is independent of the beginning-of-period distribution of asset holdings, because the consumption function is linear and because the MPC is common across households.

Therefore, what happens in equilibrium from period  $t = 1$  onward is the same as in the zero liquidity model, despite that the Ricardian households can borrow or lend: for all  $t \geq 1$ ,  $r = r^{**}$ ,  $Y_t = Y^n$ ,  $\pi_t = 0$ ,  $C_t^R = \gamma(Y^n) - \bar{\tau}^R$ ,  $C_t^H = \delta(Y^n) - H\bar{\tau}^H$ .



In period 0, the MPC's are different across Ricardian households, and individual consumption satisfies:

$$c_0^R(i) = mpc_0(i) \times \left\{ \gamma(Y_0) - \tau_0^R(i) + \underbrace{\frac{1}{1+r_*} \frac{1+r_{**}}{r_{**}} (\gamma(Y^n) - \bar{\tau}^R)}_{\text{Present value of future after-tax income}} \right\}, \quad (6)$$

with  $mpc_0(i) < 1$  and the aggregate consumption satisfies:

$$C_0^R = \overline{mpc}_0 \gamma(Y_0) - \int_0^1 mpc_0(i) \tau_0^R(i) di + \text{t.i.p.}$$

where  $\overline{mpc}_0 := \int_0^1 mpc_0(i) di < 1$  is the average MPC of Ricardian households and t.i.p. collects terms independent of period-0 intervention.<sup>8</sup>

It is straightforward to show that the output is at the natural level without an intervention:  $Y_0 = Y^n$  if  $\tau_0^R(i) = \bar{\tau}^R$  for all  $i$ . What happens under a targeted intervention? Using the resource constraint, I obtain

$$Y_0 - Y^n = \delta(Y_0) - \delta(Y^n) + \overline{mpc}_0 (\gamma(Y_0) - \gamma(Y^n)) - \int_0^1 mpc_0(i) (\tau_0^R(i) - \bar{\tau}^R) di.$$

Because  $Y = \delta(Y) + \gamma(Y)$ , the above equation can be rearranged as:

$$Y_0^R - \gamma(Y^n) = \frac{-\mathbb{C}\mathbb{O}\mathbb{V}_i[mpc_0(i), \tau_0^R(i) - \bar{\tau}^R]}{(1 - \overline{mpc}_0)}, \quad (7)$$

where  $\mathbb{C}\mathbb{O}\mathbb{V}_i[mpc_0(i), \tau_0^R(i) - \bar{\tau}^R]$  is the cross-sectional covariance of MPC and a change in tax. The equality holds because an intervention is purely redistributive, i.e.,  $\int_0^1 (\tau_0^R(i) - \bar{\tau}^R) di = 0$ .

The left-hand side of equation (7) is the effect on the Ricardian households' total income. The right-hand side expression is easily interpretable. Its numerator equals the direct effect of redistribution on the aggregate consumption of the Ricardian households. When tax is increased for high MPC households and is reduced for low MPC households, the covariance term is positive, and the whole numerator is negative. When, instead, a tax-and-transfer system is changed to strengthen redistribution toward high MPC households, the numerator is positive. The direct effect captured by

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<sup>8</sup>Here, the assumption of a constant real interest rate in period 0 simplifies the analysis. This is because the period-0 MPC's and the present value of future after-tax income depend on the period-0 real interest rate.

the numerator is magnified by the standard multiplier effect, captured by the term,  $1 - \overline{mpc}_0$ , in the denominator.

Consider first the simplest case in which the distribution of  $\beta(i)$  is degenerate. Then the MPC distribution is also degenerate, and the covariance in equation (7) is zero. Hence, the intervention considered in Proposition 1 has exactly zero effect on output. This is in a stark contrast to Proposition 1, where an arbitrarily small intervention could generate an arbitrarily large output contraction even if the distribution of  $\beta(i)$  is degenerate. Why the difference? As under the zero liquidity assumption, the intervention increases the savings incentives of the receivers of the transfer. The zero liquidity assumption required their income to fall so that their strengthened savings incentives are fully offset, because nobody can borrow from them. When Ricardian households are allowed to borrow, the givers, which have stronger incentives to borrow due to the intervention, borrow from the receivers. Even without the income adjustment, the aggregate savings stay zero after the intervention if the distribution of  $\beta(i)$  is degenerate.

When the distribution of  $\beta(i)$  is non-degenerate, the output effect of a small-sized intervention is non-zero but cannot be arbitrarily large. The output effect of the intervention considered in Proposition 1 is given by:

$$\gamma(Y_0) - \gamma(Y^n) = \frac{\frac{1}{\varepsilon_\Delta} \int_{1-\varepsilon_\Delta}^1 mpc_0(i) di - \frac{1}{1-\varepsilon_\Delta} \int_0^{1-\varepsilon_\Delta} mpc_0(i) di}{1 - \overline{mpc}_0} (1 - \varepsilon_\Delta) \Delta. \quad (8)$$

Because  $mpc_0(\cdot)$  is a non-increasing function of  $i$ , the size of the numerator is bounded:

$$\left| \frac{1}{\varepsilon_\Delta} \int_{1-\varepsilon_\Delta}^1 mpc_0(i) di - \frac{1}{1-\varepsilon_\Delta} \int_0^{1-\varepsilon_\Delta} mpc_0(i) di \right| \leq mpc_0(0) - mpc_0(1) < \infty.$$

Recall that  $\gamma(Y_{LB}(\Delta)) - \gamma(Y^n) = -(1 - \varepsilon_\Delta) \Delta / \varepsilon_\Delta$  and that  $\varepsilon_\Delta \downarrow 0$  as  $\Delta \downarrow 0$ . It follows that

$$|\gamma(Y_0) - \gamma(Y^n)| < \frac{mpc_0(0) - mpc_0(1)}{1 - \overline{mpc}_0} |\gamma(Y_{LB}(\Delta)) - \gamma(Y^n)| \varepsilon_\Delta \rightarrow 0,$$

as  $\Delta \downarrow 0$ . Because  $\gamma$  is continuous,  $Y_0 \rightarrow Y^n$  as  $\Delta \downarrow 0$ . Therefore, the output effect of a small-sized intervention cannot be arbitrarily large. Again, this implication is in contrast to Proposition 1.

Let us look at the marginal effect of the intervention. As  $\Delta \downarrow 0$ ,  $\varepsilon_\Delta \downarrow 0$  and

$$\frac{dY_0}{\Delta} \rightarrow \underbrace{\frac{1}{\gamma'(Y^n)}}_{\text{Correction}} \times \underbrace{\frac{1}{1 - \overline{mpc}_0}}_{\text{Multiplier effect}} \times \underbrace{\{mpc_0(1) - \overline{mpc}_0\}}_{\text{Change in the aggregate MPC}} < 0. \quad (9)$$

In the limit, the intervention redistributes income from almost everyone (with the average MPC equal to  $\overline{mpc}_0$ ) to the household with the lowest MPC,  $mpc_0(1)$ . This redistribution effectively reduces the aggregate marginal propensity to consume for the transferred income from  $\overline{mpc}_0$  to  $mpc_0(1)$ . Its effect on the Ricardian households' total income is magnified by the multiplier effect. The effect on output is then obtained by multiplying it further with the correction term.

When the distribution of  $\beta(i)$  is non-degenerate, the intervention considered here reduces the aggregate consumption of Ricardian households, by redistributing income from high MPC households to low MPC households. As a result, the aggregate demand becomes less than the natural output, and income falls until the demand and the supply of goods equalize again. Its contractionary effect, however, is much smaller than in Proposition 1.

## 5.2 When a tighter borrowing constraint is imposed

What if the Ricardian households face a tighter borrowing constraint such as  $a_1(i) \geq \underline{a}$ , for some negative  $\underline{a}$ , and it binds for a potentially large fraction of, but not all of, the Ricardian households?

With the borrowing constraint of the form,  $a_1(i) \geq \underline{a}$ , the optimal consumption is given by:

$$\tilde{c}_0^R(i) = \min \{ \gamma(Y_0) - \tau_0^R(i) + \underline{a}, c_0^R(i) \},$$

where  $c_0^R(i)$  is the optimal consumption without the borrowing constraint, which is given in equation (6). Clearly, the borrowing constraint tends to bind for households with high MPC's, i.e., low  $\beta(i)$ 's.

Two things are worth noting. First, when the borrowing constraint binds for a strictly positive fraction of Ricardian households, the period-0 aggregate consumption and output are lower than in the model of the previous section, even without a fiscal intervention, i.e.  $\tau_0^R(i) = \bar{\tau}^R$ . The period-0 output level that would prevail without an intervention is denoted by  $\tilde{Y}_0$ . Second, the set of households that face the binding borrowing constraint is endogenously determined depending on the size

of the intervention. This complicates the analysis because, for each household  $i$ , the period-0 marginal propensity to consume out of *current income* is no longer exogenous and is endogenously determined by whether the borrowing constraint binds or not. In the following, therefore, I focus on the marginal effect of the intervention so that the set changes only marginally. Both  $\Delta$  and  $\varepsilon$  are assumed to be close to zero and  $\varepsilon$  differs from  $\varepsilon_\Delta$ .

Assume that  $\beta(i)$  is not only strictly increasing but also is continuously differentiable in  $i$ . Then the marginal output effect of the intervention considered in Section 2.2 is given as follows: for  $\Delta \approx 0$  and  $\varepsilon \approx 0$ , assuming that the change in output is small ( $dY_0 \approx 0$ ), the change in the Ricardian households' total income is given by:

$$\gamma'(\tilde{Y}_0)dY_0 = \frac{\frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \widetilde{mpc}_0(i) di - \frac{1}{1-\varepsilon} \int_0^{1-\varepsilon} \widetilde{mpc}_0(i) di}{1 - \overline{\widetilde{mpc}_0}} (1 - \varepsilon)\Delta, \quad (10)$$

where  $\widetilde{mpc}_0(i)$  is the period-0 marginal propensity to consume out of current income, not of wealth, and where  $\overline{\widetilde{mpc}_0} = \int_0^1 \widetilde{mpc}_0(i) di$  is its cross-sectional average. The MPC out of current income is identical to the MPC out of wealth,  $mpc_0(i)$ , if the borrowing constraint does not bind for the household  $i$  when there is no intervention (i.e., the taxes are  $\tau_0^R(i) = \bar{\tau}^R$  for all  $i$  and the period-0 output is  $\tilde{Y}_0$ ), and is equal to one if it binds for  $i$  when there is no intervention. An important assumption here is that  $\widetilde{mpc}_0(i) = mpc_0(i) < 1$  for a strictly positive measure of households, so that the denominator in equation (10) is non-zero.

Equation (10) is analogous to equation (8). The left-hand side is the marginal effect on the Ricardian households' total income. The MPC's out of wealth on the right-hand side in equation (8) are replaced by the MPC's out of current income. As before,

$$\left| \gamma'(\tilde{Y}_0)dY_0 \right| \leq \frac{|\widetilde{mpc}_0(1) - \widetilde{mpc}_0(0)|}{1 - \overline{\widetilde{mpc}_0}} (1 - \varepsilon)\Delta \rightarrow 0$$

as  $\Delta \downarrow 0$  and  $\varepsilon \downarrow 0$ . Hence, the effect of small-sized interventions is small.

Again, let us derive the marginal effect of the intervention. As  $\Delta \downarrow 0$  and  $\varepsilon \downarrow 0$ , the marginal policy effect on output,  $dY_0/\Delta$ , converges to:

$$\underbrace{\frac{1}{\gamma'(\tilde{Y}_0)}}_{\text{Correction}} \times \underbrace{\frac{1}{(1 - \overline{\widetilde{mpc}_0})}}_{\text{Multiplier effect}} \times \underbrace{\left\{ mpc_0(1) - \overline{\widetilde{mpc}_0} \right\}}_{\text{Change in the aggregate MPC}} < 0. \quad (11)$$

This expression is again analogous to equation (9), with the MPC's out of wealth being

replaced by the MPC's out of current income.

For households with a sufficiently high discount factor, the borrowing constraint does not bind and their MPC's out of income are the same as their MPC's out of wealth, as in the previous section. For households with a sufficiently low discount factor, the borrowing constraint binds. Their MPC's out of current income is one, and is higher than their MPC's out of wealth had the borrowing constraint not been binding. Therefore, a tighter borrowing constraint acts to raise the cross-sectional average of MPC's. As a result, both the multiplier effect and the absolute value of the change in the aggregate MPC in equation (11) are bigger than those in equation (9). However, as in the absence of a borrowing constraint that binds for some households, the whole expression cannot be made arbitrarily large.

### 5.3 The zero liquidity case is special

The discussion so far has shown that the excess sensitivity result no longer holds if the borrowing constraint is relaxed only slightly: small interventions have only small effects on output. Then what is special about the stringent borrowing constraint?

The key is that, with the stringent borrowing constraint of  $\underline{a} = 0$ , the denominator on the right-hand in equation (10) shrinks toward zero as  $\varepsilon$  declines to zero. This is because the MPC's out of current income equals one for all  $i \leq 1 - \varepsilon$ . The coefficient of  $(1 - \varepsilon)\Delta$  in equation (10) is:

$$\frac{\frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \widetilde{mpc}_0(i) di - \frac{1}{1-\varepsilon} \int_0^{1-\varepsilon} \widetilde{mpc}_0(i) di}{1 - \overline{\widetilde{mpc}_0}} = \frac{\frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \widetilde{mpc}_0(i) di - 1}{\varepsilon \left(1 - \frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \widetilde{mpc}_0(i) di\right)} = -\frac{1}{\varepsilon}.$$

Hence the right-hand side of equation (10) diverges to negative infinity if  $\varepsilon$  converges to zero faster than  $\Delta$ , even when both  $\varepsilon$  and  $\Delta$  converge to zero. Hence, small interventions can have large effects on output under the zero liquidity assumption.<sup>9</sup>

### 5.4 Taking stocks

In this section I have examined two models with less stringent borrowing constraints than under the assumption of zero liquidity. In both models, small interventions considered to be highly contractionary in the zero liquidity model are much less effective.

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<sup>9</sup>It cannot be concluded from equation (10) that  $dY_0$  diverges to negative infinity, because the equation is obtained assuming that  $dY_0$  is small. The non-local, global effect of the intervention is given in Proposition 1.

The size of the interventions is characterized by the tax increase for a large number of households,  $\Delta$ , and by the fraction of households who receive transfers,  $\varepsilon$ . When both parameters are taken to zero, the output converges to the level without interventions in both models. The presence of borrowing-constrained households magnifies the marginal output effect of an intervention, but the marginal effect remains finite.

Hence, small interventions can only generate small contraction in output, as far as a strictly positive measure of households can increase their borrowing in response to the intervention. The zero liquidity model can be viewed as a limit of the model with a tight borrowing constraint,  $a_1(i) \geq \underline{a}$ , with  $\underline{a} \uparrow 0$ . Yet, a policy implication that holds in the zero liquidity model does not hold when the model is perturbed so that some borrowing is allowed. As shown above,

$$0 = \underbrace{\lim_{\underline{a} \uparrow 0} \lim_{\Delta \downarrow 0, \varepsilon / \Delta \downarrow 0} dY_0}_{\text{Limit of the policy effects}} \neq \underbrace{\lim_{\Delta \downarrow 0} (Y_0^{LB}(\Delta) - Y^n)}_{\text{Policy effect in the limit model}} = Y_0^{LB}(0) - Y^n < 0,$$

and one cannot exchange the two limits. The limit of marginal policy effects in models with less stringent borrowing constraints does not coincide with the marginal policy effect in the zero liquidity model, which is the limit of these models.

## 6 Conclusion

This paper has shown that the heterogeneous-agent incomplete-market New Keynesian models can exhibit the excess sensitivity to targeted fiscal interventions under the zero liquidity assumption. It is always possible to generate an arbitrarily large contraction of output by an arbitrarily small-sized redistribution. However, the excess sensitivity result is not robust to small changes in the model environment: when the borrowing constraint is relaxed, even slightly, the excess sensitivity result disappears. Hence, small-sized interventions can only have small output effects.

The zero liquidity assumption has a clear merit: tractability. Regardless of the heterogeneity of households, the Euler equation is satisfied for only a small subset of them, resulting in a degenerate wealth distribution in equilibrium. As a result, the characterization of an incomplete-market equilibrium is relatively easily. However, the results in the present paper suggest that it comes at a cost. Targeted interventions may have unrealistically strong effects on the aggregate output, and the strong effects are not at all robust to small perturbations to the model. The intervention considered

in this paper is a fairly simple income redistribution and its transmission mechanism in the zero liquidity model is the same as that of monetary policy. Hence, a lesson is that the zero liquidity assumption should be used with caution in HANK models when redistribution of income is concerned.

What about other approaches to simplify HANK models? Two commonly used approaches result in zero effect of the targeted intervention on output. In the TANK model used in [Bilbiie \(2020\)](#), there are both the Hand-to-Mouth and the Ricardian households, as in the present paper. However, the Ricardian households are homogeneous and trade in complete markets subject to the natural borrowing limit. Hence, as in the model in [Section 5.1](#) with homogeneous households, the output effect of the targeted intervention is zero. In HANK models that permit linear aggregation, such as those used in [Braun and Nakajima \(2012\)](#) and in [Acharya and Dogra \(2020\)](#), one-time redistribution of income has no effect either. As far as the redistribution only uses non-distortionary tax, it is equivalent to redistribution of initial wealth, and initial wealth redistribution has no effect on the aggregate variables under linear aggregation. Although these approaches offer a solution to the excess sensitivity result, they instead result in *extreme insensitivity* to the targeted intervention considered in the present paper. An alternative approach is needed to study the output effect of targeted interventions without trivializing it while maintaining tractability.

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