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Quality Job Programs, Unemployment and the Job Quality Mix *

Benoit Julien, John Kennes, Ian King

School of Economics
The University of Queensland,
St Lucia, QLD 4072, Australia

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†Benoit Julien: UNSW, John Kennes: Aarhus University. Ian King: University of Queensland. Corresponding author: Benoit Julien (benoit.julien@unsw.edu.au).
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Quality Job Programs, Unemployment, and the Job Quality Mix*

Benoit Julien John Kennes Ian King[†]

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Abstract

We examine the impact of Quality Jobs Programs (QJPs) on job creation, unemployment, and average job quality, in a directed search model of the labor market where firms bid for labor and can choose to create different qualities of jobs. The government has access to differential tax and subsidy structures, and unemployment benefits. QJPs are defined as job subsidy structures where higher quality jobs are subsidized more generously than lower quality ones. We find that QJPs increase the number of higher quality jobs but decrease the number of lower quality jobs commensurately – raising the average quality of jobs but leaving the unemployment rate unchanged and inducing an inefficient job quality mix. Uniform subsidies, on the other hand, increase the number of lower quality jobs while not affecting the number of higher quality jobs – thereby reducing both the average quality of jobs and the unemployment rate. If uniform subsidies are set equal to unemployment benefit levels then the job quality mix is also efficient.

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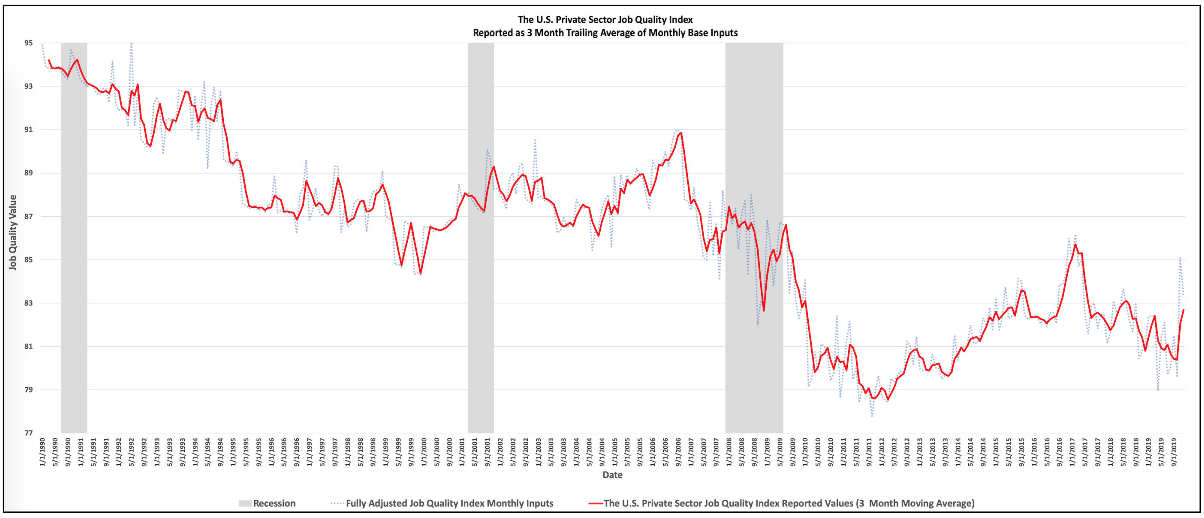
1 Introduction

Optimism over the reduction of the unemployment rate in the US over the past decade has been tempered by the ongoing secular decline of the quality of the jobs being created since the 1980s. This decline, though well known for many years, has been thrown into sharp focus by the recent publication of the new US Private Sector Job Quality Index (JQI), a monthly indicator that keeps track of the ratio of "high-quality" jobs to "low-quality" jobs.



The JQI Index from January 1, 1990 to January 1, 2020 (Source: jobqualityindex.com.)

This index, which defines "high-quality" jobs as those with above average wages and benefits, has seen an approximate 12.5% drop from January 1, 1990 to January 1, 2020, prompting some dramatic commentary in the popular press.[1]

Awareness of this decline pre-dates the publication of this index and policymakers have been arguing for, and implementing, subsidy programs that target "high-quality" jobs for some time. These are commonly known as "Quality Job Programs" (QJPs), and different forms of these programs have been instituted, for example, in many states in the US.[2] Jin (2016) provides an excellent summary of many of these programs – some of which define "high-quality" jobs by above-average wages offered, some by the level of investment associated with them, and some by more complex measures.

Empirical estimates on the overall size of subsidy programs vary significantly. De Simone et al (2019) report: "The aggregate dollar value of tax subsidies awarded amounts to nearly

[1] For example: Denning, S., "Understanding the US Economy: Lots of Rotten Jobs", *Forbes*, December 5, 2019.

[2] For details, see for example, the *Subsidy Tracker* available at www.goodjobsfirst.org.

$11 billion in 2014 alone". The Subsidy Tracker reports "between $45 billion and $70 billion annually". One difficulty, when estimating these numbers, is discerning which subsidies are specifically aimed at high-quality jobs.

Evaluations of the effectiveness of subsidies, to create new jobs, is also mixed. Work by Devereux et al (2007) found positive but small effects of subsidies on firm location decisions. Hanson and Rohlin (2011) found that local subsidies can actually reduce job entry by raising property rental values. De Simone et al (2019) find "little to no effects for over 1,000 subsidies that cost approximately $99.8 million in the aggregate". Theoretically, with mobile firms, the effectiveness of local subsidies is complicated by the issue of tax competition across tax jurisdictions, which can make subsidies a zero-sum game across regions. This issue is explored in an extensive literature going back to the "bidding for firms" framework of Black and Hoyt (1989) and King et al (1993).[3]

In this paper, however, we focus on the specific effectiveness of QJPs, where greater subsidies are promised for the creation of higher quality jobs. We abstract away from the tax competition issues across tax jurisdictions and consider, instead, one government that can potentially influence the mix of different qualities of jobs being created in its jurisdiction through differential subsidies. The model itself is based on the "bidding for labor" framework developed by Julien, Kennes and King (2000, 2006) and Julien *et al* (2009), where homogeneous workers apply to multiple vacancies, vacancies approach workers with offers and workers sell their labor to the highest bidder. Firms can create either low quality jobs (with low output and low costs) or higher quality jobs (with higher output and costs), in a policy environment with (in general) different subsidies for each type of job. Workers choose (and announce) their reserve wages while facing a graduated tax structure and benefits if they are unemployed. The tax structure we consider is progressive and consists of two parameters for each type of job created in equilibrium: a threshold level of income and a tax rate. The thresholds determine the levels of income that are subject to the different tax rates. We identify (and restrict attention to) the region in the parameter space where all possible types of jobs are created in equilibrium. We characterize the symmetric mixed strategy equilibrium in this region and consider the comparative statics of the endogenous variables (in particular, the creation of the different qualities of jobs, and the unemployment rate) as the policy parameters change.

We find that, when benefits are taxed, job creation and unemployment are independent of the tax structure – both the thresholds and the rates themselves. Similarly, the creation of higher quality jobs is independent of unemployment benefits, the tax structure, and is influenced by employment subsidies only when these subsidies are different for high and low quality

[3] See Keen and Konrad (2013) for a survey.

jobs. Moreover, subsidizing the creation of high quality jobs has no effect on unemployment rates. It does, however, have an effect on the *mix* of different jobs created and raises the average quality of jobs. Also, if job subsidies are uniform – the same for all types of jobs – then any increase in these subsidies will stimulate the creation of the lowest quality jobs but leave all other jobs unaffected – thus, lowering the unemployment rate and the average quality of jobs. When benefits are not taxed then the tax structure does play a role in determining outcomes – but only the lowest tax rate and threshold; the remainder of the tax structure, once again, has no effects on the equilibrium outcomes.

We also characterize the (constrained) efficient allocations in this model, and identify the policy configurations whereby the equilibrium allocations are constrained efficient. This is a directed search model, with large numbers of agents, so (with risk-neutral agents) the laissez-faire equilibrium (where all policy parameters are zero) is constrained efficient. We show that many other policy settings also achieve constrained efficiency. In particular, efficiency simply requires that the subsidies are uniform and equated with the unemployment benefits. This allows for a wide range of values for the policy parameters, including those that preserve efficiency but also completely eliminate the *ex post* risk associated with the laissez faire equilibrium, providing *ex post* equity while balancing the government's budget.

Related Literature

Pissarides (1985) used a variant of the Diamond-Mortensen-Pissarides (DMP) model with random matching and generalized Nash bargaining to consider the effect of subsidies and taxes. He introduced lump-sum subsidies to firms, and lump-sum benefits to workers, financed through proportional wage taxes, to this (now standard) environment. Intuitively, job creation and unemployment always move in opposite directions in that model, since job creation is the channel through which equilibrium unemployment changes. In that setting, unemployment is decreasing in firm subsidies, and increasing in unemployment benefits. Somewhat less straightforwardly, since benefits are untaxed in Pissarides' model, increases in the tax rate must be borne, at least partially, by firms. This implies that job creation is decreasing (and unemployment is increasing) in the income tax rate. He did not consider the creation of different types of jobs or QJPs.

Pissarides' paper was concerned solely with positive issues, not with the normative questions of optimal allocations or policies. These were taken up in subsequent work by, for example, Boone and Bovenberg (2002), Mortensen and Pissarides (2003), Hungerbühler *et al* (2006), Lehmann and Van Der Linden (2007), Jiang (2014), and Michau (2015). In all of these studies, which use variants of the DMP model, care needs to be taken about the inherent inefficiency of the equilibrium of the DMP model outside of the Hosios rule, and many of the results hinge on whether or not this rule is imposed.

Other research has considered both positive and normative questions, using directed search models of the labor market, where the inherent inefficiency of equilibria is not generically an issue. Broadly speaking, directed search models follow two different modelling traditions. In one, a market is defined with finite numbers of players, and results from this finite game are examined in the limit, as the scale of the market becomes arbitrarily large. The model is solved for exact equilibria of a strategic game in which one side post terms of trade and the other selects trading partners using mixed strategies. This generates an endogenous (Binomial) matching process. In the limit as the number of agents is arbitrarily large, keeping the ratio of vacancies to unemployed constant, this generates a Poisson matching process (also known as urn-ball). Early examples of these types of models are Peters (1991), Montgomery (1991), Burdett, Shi, and Wright (2001) (hereafter, BSW), who have individual firms posting wages, and workers choosing which firm to approach. Julien, Kennes, and King (2000) (hereafter, JKK), have a similar framework, but where individual workers sell their labor through auctions and announce reserve wages. Firms, in that setting, choose which worker to approach, and bid for their labor.[4]

The second approach, based on Moen (1997) and Shimer (1996), starts with the initial assumption that markets are large. They consider measures of agents and an environment in which submarkets can be opened freely, each of which has random matching at the local level, but workers are able to choose which island to approach, directed by the wages posted by either market makers, firms or workers. This is known as the competitive search approach. The main difference between the finite and large market approach is that the former generates a matching technology endogenously, while the latter can use any matching technology in any submarkets, (although, often, pairwise matching is assumed). It is well known that if the matching process used in submarkets is Poisson, the two approaches generate the same equilibrium allocation.[5]

In this paper we use the large market approach with multilateral meetings generated by endogenous visit probabilities, while assuming that the terms of trade in these submarkets are posted by workers, as auctions with reserve wages (as in JKK). This approach is a natural one to use, given McAfee's (1993) result that, in these environments, auctions of this type are optimal for sellers to post. Julien *et al* (2009) used the JKK approach, with homogeneous workers and

[4]For a synthesis of the BSW and JKK approaches, see Albrecht, Gautier, and Vroman (2006).

[5]For a survey of competitive and directed search models, see Wright, Kircher, Julien, and Guerrieri (2021).

firms, to analyse a policy structure similar to the one found in Pissarides (1985). They pointed out, (among other things) that unemployment benefits will pull the economy away from efficiency unless they are paired with employment subsidies of equal value. Golosov, Maziero, and Menzio (2013) used the large market approach, but where each worker's application strategy is private information – policymakers cannot discern whether a worker is unemployed due to bad luck or to an unwillingness to apply. Workers are risk averse in this setting, and the optimal policy involves positive benefits and a regressive labor income tax. Geromichalos (2015) used the BSW approach with firms posting wages, homogeneous firms and workers, and considered different ways of taxing firms to finance benefits. He showed (for example) that lump-sum payments are, in fact, distortionary in this environment, because they lead to firms being too aggressive in their wage competition. None of these studies, however, focus on the main issues that we address in this paper: how the mix of job qualities, unemployment, and welfare, respond to policy changes.

The paper is organized as follows. Section 2 details the structure of the model. Section 3 analyzes the decisions of the workers and firms and the equilibrium. Section 4 presents the comparative statics of the equilibrium. Section 5 analyses efficient allocations and Section 6 provides concluding remarks. The appendix provides proofs of all of the propositions.

2 The Model

Consider a static economy with a measure N of homogeneous workers and a measure M of homogeneous firms. Each firm can open only one vacancy but can choose over a finite number $Q \in \mathbb{N}$ of productivity types for this vacancy, with corresponding outputs y_q, $q = 1, 2, ..., Q$ where $0 < y_1 < y_2 < ... < y_Q$. Once a vacancy is created its type is common knowledge. Each type of vacancy creation costs an amount k_q, $q = 1, 2, ..., Q$, with $k_1 \in (0, y_1)$, and $k_q \in (k_{q-1}, y_q)$ for all $q = 2, 3, ..., Q$. Let M_q be the measure of firms creating type q vacancies. The value of each M_q is determined endogenously by free entry. We define $\Theta_q = M_q/N$ as market tightness for vacancies of type q and aggregate market tightness as $\Theta = \sum_{q=1}^{Q} \Theta_q$.

The economy has a government which provides employment subsidies σ_q to firms that fill a vacancy of type q, where $\sigma_1 \leq \sigma_2 \leq ... \leq \sigma_Q$. The government levies labor income taxes on workers, with the following progressive structure. All income less than ω_1 is counted as non-taxable. Income between ω_q and ω_{q+1} is taxed at rate $\tau_q \in [0, 1]$, where $0 \leq \omega_1 \leq \omega_2 \leq ... \leq \omega_Q$ and $0 \leq \tau_1 \leq \tau_2 \leq ... \leq \tau_Q$. The output levels from operating different jobs are assumed

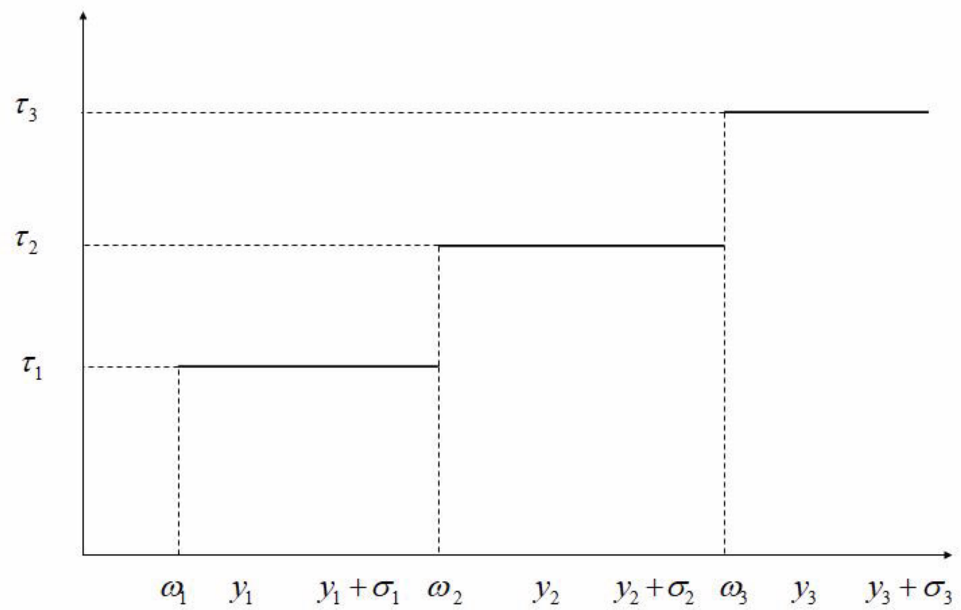


Figure 1: The Tax Structure when $Q = 3$

to relate to the tax thresholds in the following way:

$$\omega_1 < y_1 \leq y_1 + \sigma_1 < \omega_2 < y_2 \leq y_2 + \sigma_2 < ... < \omega_Q < y_Q \leq y_Q + \sigma_Q \tag{1}$$

The government also provides benefits b to any worker who is unemployed in equilibrium. We assume that $b \in [0, y_1 + \sigma_1)$. To be general enough to consider different policies across existing economies, we consider the possibilities that the unemployment benefits are either taxed or not. When $b \geq \omega_1$ the benefits are taxed and the outside option for workers is $(1 - \tau_1)b + \tau_1\omega_1$. When $b < \omega_1$, the benefits are not taxed and the outside option for workers is simply b. Thus, in general, after tax benefits, when unemployed, are given by $\min\{(1 - \tau_1)b + \tau_1\omega_1,\ b\}$.

We consider a competitive search framework along the lines of Moen (1997). However, this model differs from his because the heterogeneity of vacancies (and hence, productivities) reflects *choices* made by firms before their entry decisions.[6] We also differ in the use of the

[6] In Moen's model, productivity comes from random draws from a known distribution upon incurring an entry cost k, which is the same for all vacancies.

terms of trade mechanism we use. Moen uses wage posting, we use *reserve* wage posting with a second-price auction. This allows for the existence, in equilibrium, of residual wage dispersion within job types – as observed empirically. In addition workers, (rather than market makers, in Moen), are posting the terms of trade. A continuum of submarkets can be opened freely by a positive measure of workers posting the same reserve wage $w_r \in \mathbb{R}_+$, which they commit to. Workers can exploit ex post opportunities by allowing bidding if a worker gets more than one job offer. This is done via a second-price auction.[7] Firms observe all reserve wages posted in all submarkets, and decide in which submarket to search for a worker. Before doing so, though, firms choose which type of vacancy to open, pay the cost associated with the vacancy type, and choose a submarket.[8] We focus on symmetric equilibria in which workers and firms are indifferent (*ex ante*) regarding which submarket to participate in, and where all types of firms participate in all submarkets. We restrict the posted reserve wage to be anonymous, in the sense that workers cannot post a menu of reserve wages contingent on the type of firms they meet.[9]

The focus on symmetric equilibria allows us to focus the analysis on only one submarket, where all workers post the same reserve wage w_r. There is a measure $n(w_r) \leq N$ of workers active in this submarket, and measures $m_q(w_r)$ of active firms with vacancies of type q, in the submarket in which all workers post the same w_r. We then write $\theta_q(w_r) = m_q(w_r)/n(w_r)$ as the market tightness of a type q vacancy active in the submarket in which n workers post a reserve wage w_r. For ease of notation, we suppress the argument and write $\theta_q \equiv \theta_q(w_r)$ only. We use Θ_q to denote the ratio of vacancies of type q to workers, in equilibrium. In equilibrium, all workers post the same reserve wage across submarkets, workers and firms are indifferent across submarkets, and $\theta_q = \Theta_q$.

Within any submarket, each firm searches for a worker, and we assume that firms of each vacancy type meet workers according to a Poisson distribution with mean θ_q. This process allows multilateral meetings, and is often referred to as urn-ball meeting.[10] The probability that

[7] In essence this is the structure used in Julien, Kennes and King (2000) where workers post reserve wages and firms choose over workers, in a setting with finite numbers of agents, and where this finite number is taken to the limit, holding market tightness fixed. Here, instead, we consider the large market directly, for simplicity.

[8] Because we focus on symmetric equilibria, the timing of vacancy creation is innocuous. Firms could choose a submarket first and then choose a vacancy type before searching for workers in the submarket.

[9] Allowing for this type of reserve wage posting could lead to separation, that is, low and high type firms participate in different submarkets, not competing with each other. While this may be an interesting avenue to explore, here we wish to draw implication in environments in which different types of jobs coexist in the same market.

[10] Peters (1991), Julien, Kennes and King (2000), and Burdett, Shi and Wright (2001) show that in fact the Poisson meeting process is indeed the limit of a sequence of games, for which one set of agents selects the others using a mixed strategy.

a worker receives the number z_q of job offers from type q vacancies is $\Pr\{z_q\} = \frac{\theta_q^{z_q} e^{-\theta_q}}{z_q!}$. Since all vacancies search simultaneously, the expected number of types of offers are independent. The market tightness in the submarket defines all of the relevant probabilities of the meeting process. For the rest of the analysis, the only relevant probabilities are $\Pr\{z_q = 0\} = e^{-\theta_q}$, $\Pr\{z_q = 1\} = \theta_q e^{-\theta_q}$ and $\Pr\{z_q > 1\} = 1 - \theta_q e^{-\theta_q} - e^{-\theta_q}$. The government operates in all open submarkets, setting the values of policy parameters $\Omega = (\omega_q, \tau_q, \sigma_q, b)_{q=1,2,\dots,Q}$ These, along with the productivity parameters, y_q, and the vacancy creation costs, k_q, constitute the entire set of parameters.

Given the multilateral meeting process workers may receive only one offer (in which case they get their reserve wage w_r); or many offers, (where bidding among firms occurs, and the worker chooses the highest offer). The winning vacancy, at any given worker, is the highest productivity vacancy, paying a price equal to the productivity of the second-highest vacancy. This is a second-price auction without private information, and the dominant strategy for bidders is to bid their valuations. Thus, here, vacancies bid their surplus.

2.1 Worker Payoffs

Given the progressive income tax structure, when workers choose a reserve wage, that wage can fall between any threshold of the structure and be taxed at different rates. In the case of only one offer, the after tax wage w_a is given by

$$w_a = \begin{cases} w_r & \text{if } w_r < \omega_1 \\ \omega_1 + (1-\tau_1)(w_r - \omega_1) & \text{if } \omega_1 \le w_r < \omega_2 \\ \omega_1 + (1-\tau_1)(\omega_2 - \omega_1) + (1-\tau_2)(w_r - \omega_2) & \text{if } \omega_2 \le w_r < \omega_3 \\ . & . \\ . & . \\ . & . \\ \omega_1 + \sum_{q=1}^{Q} (1-\tau_q)(\omega_{q+1} - \omega_q) + (1-\tau_Q)(w_r - \omega_Q) & \text{if } w_r > \omega_Q \end{cases} \tag{2}$$

The winning bid is an after-tax wage w_q^j offered by firm of vacancy type q with the second highest vacancy being j. The following equation summarizes the possible (winning) after-tax wage bids, which depend on the composition of different vacancy types.

$$
w_q^j = \begin{cases} w_0^0 = & \min\{(1-\tau_1)b + \tau_1\omega_1,\ b\} \\ w_q^0 = & w_a \ \ \forall\ q = 1,2,...,Q \\ w_q^1 = & w^1 = \omega_1 + (1-\tau_1)(y_1+\sigma_1-\omega_1)\ \forall\ q=1,2,...,Q \\ w_q^2 = & w^2 = \omega_1 + (1-\tau_1)(\omega_2-\omega_1) + (1-\tau_2)(y_2+\sigma_2-\omega_1)\ \forall q = 2,3,...,Q \\ & . \\ & . \\ & . \\ w_Q^j = & w^j = \omega_1 + \sum_{i=1}^{j-1}(1-\tau_i)(\omega_{i+1}-\omega_i) + (1-\tau_j)(y_j+\sigma_j-\omega_j)\ \forall\ j = 2,3,...,Q \end{cases} \tag{3}
$$

If a worker is not approached by any firm he receives w_0^0: his unemployment benefit less any tax incurred. If the worker is approached by only a single firm (w_q^0) the firm extracts the full surplus from production, leaving the worker with his after tax reserve wage w_a. If the worker is approached by more than one firm, then he is able to extract the surplus of the second highest bidder. For example, if at least two type 1 firms approach the worker then the worker obtains the full surplus from a type 1 job (which includes σ_1) with after-tax payoff equal to $w_1^1 = \omega_1 + (1-\tau_1)(y_1+\sigma_1-\omega_1)$. More generally, if one type $q \geq 1$ firm and at least one type 1 firm (but no other types of firm) approach the worker, then he is paid the full surplus from a type 1 job, with after-tax payoff $w_q^1 = \omega_1 + (1-\tau_1)(y_1+\sigma_1-\omega_1)$. Similarly, if one type $q \geq 2$ firm and at least one type 2 firm approach the worker (regardless of how many type 1 firms approach, but no other types of firm approach) then he is paid the full surplus from a type 2 job, with after-tax payoff $w_q^2 = \omega_1 + (1-\tau_1)(\omega_2-\omega_1) + (1-\tau_2)(y_2+\sigma_2-\omega_2)$, and so on. Given our restrictions, we know that $w_0^0 \leq w_q^0 \leq w_q^1 \leq w_q^2 \leq ... \leq w_Q^Q$. The luckiest workers are approached by at least two type Q firms, extract the full surplus from a type Q job, and receive the after-tax payoff w_Q^Q.

If we use $\boldsymbol{\theta}$ to denote the vector of market tightness values, the (after-tax) expected payoff for a worker is given by

$$
\begin{aligned}
V(w_r, \boldsymbol{\theta}) \quad = \quad & \exp\left(-\sum_{q=1}^{Q}\theta_q\right) w_0^0 + \theta_1 \exp\left(-\sum_{q=1}^{Q}\theta_q\right) w_1^0 + \left(1 - e^{-\theta_1} - \theta_1 e^{-\theta_1}\right) \exp\left(-\sum_{q=1}^{Q}\theta_q\right) w_1^1 \\
& + \theta_2 \exp\left(-\sum_{q=1}^{Q}\theta_q\right) w_2^0 + \left(1 - e^{-\theta_1}\right) \theta_2 \exp\left(-\sum_{q=2}^{Q}\theta_q\right) w_2^1 \\
& + \left(1 - e^{-\theta_2} - \theta_2 e^{-\theta_2}\right) \exp\left(-\sum_{q=3}^{Q}\theta_q\right) w_2^2 + ... + \theta_Q \exp\left(-\sum_{q=1}^{Q}\theta_q\right) w_Q^0 \\
& + \theta_Q \sum_{i=1}^{Q-1}\left(1 - e^{-\theta_i}\right) \exp\left(-\sum_{q=i+1}^{Q}\theta_q\right) w_Q^i + \left(1 - e^{-\theta_Q} - \theta_Q e^{-\theta_Q}\right) w_Q^Q \qquad (4)
\end{aligned}
$$

where the w_i^j are given in (2) and (3). The payoff reflects the different state of offers for a worker. If no offers arrive, which occurs with probability $\exp\left(-\sum_{q=1}^{Q}\theta_q\right)$, the payoff is w_0^0: the unemployment benefit (which may or may not be taxed). If an offer comes from only one type 1 vacancy, and no type 2 vacancies, which occurs with probability $\theta_1 \exp\left(-\sum_{q=1}^{Q}\theta_q\right)$, the worker receives the payoff of w_1^0 (which may or may not be taxed). If offers come from at least two type 1 vacancies, and no other types of vacancies, which occurs with probability $\left(1 - e^{-\theta_1} - \theta_1 e^{-\theta_1}\right) \exp\left(-\sum_{q=2}^{Q}\theta_q\right)$, then the worker's payoff is w_1^1, which is taxed at rate τ_1 for all income above ω_1. If an offer comes from only one type 2 vacancy, and no other vacancies, which occurs with probability $\theta_2 \exp\left(-\sum_{q=1}^{Q}\theta_q\right)$, the worker receives payoff w_2^0 (which may or may not be taxed). If an offer from exactly one type 2 vacancy and at least one type 1 vacancy, which occurs with probability $\left(1 - e^{-\theta_1}\right) \theta_2 \exp\left(-\sum_{q=2}^{Q}\theta_q\right)$, the worker receives the payoff of w_2^1, (which is taxed at rate τ_1 for all income above ω_1). If offers come from at least two type 2 jobs, but from no other jobs of a higher type, which occurs with probability $\left(1 - e^{-\theta_2} - \theta_2 e^{-\theta_2}\right) \exp\left(-\sum_{q=3}^{Q}\theta_q\right)$, the worker receives the maximum payoff of w_2^2, (which is taxed at the rate τ_1 for all income between ω_1 and ω_2 and at rate τ_2 for all income greater than ω_2). Analogous terms apply for all other intermediate types of jobs.

If an offer comes from only one type Q vacancy, and no other vacancies, which occurs with probability $\theta_Q \exp\left(-\sum_{q=1}^{Q}\theta_q\right)$, the worker receives payoff w_Q^0 (which may or may not be taxed). If an offer arrives from exactly one type Q vacancy and at least one type $i < Q$ vacancy which occurs with probability $\left(1 - e^{-\theta_i}\right) \theta_Q \exp\left(-\sum_{q=i+1}^{Q}\theta_q\right)$, the worker receives the payoff of w_Q^i. If offers come from at least two type Q jobs, which occurs with probability $\left(1 - e^{-\theta_Q} - \theta_Q e^{-\theta_Q}\right)$, the worker receives the highest after-tax payoff of w_Q^Q.

2.2 Firm Payoffs

Once the vacancy cost has been paid, the expected payoff from opening a type 1 vacancy is

$$\pi_1(w_r, \boldsymbol{\theta}) = [y_1 + \sigma_1 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right). \tag{5}$$

This payoff reflects the fact that if a type 1 vacancy is alone when approaching a worker, which occurs with probability $\exp\left(-\sum_{q=1}^{Q} \theta_q\right)$, it receives the output y_1 and the subsidy σ_1 minus the reservation wage paid to the worker, w_r. If not alone (ie., in all other cases) then the firm either is unsuccessful in hiring the worker or it is successful but all of the surplus is bid away. In either case, if not alone, the firm's *ex post* payoff is zero.

For a type 2 vacancy:

$$\begin{aligned} \pi_2(w_r, \boldsymbol{\theta}) &= [y_2 + \sigma_2 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right) \\ &\quad + [y_2 + \sigma_2 - y_1 - \sigma_1] \left(1 - e^{-\theta_1}\right) \exp\left(-\sum_{q=2}^{Q} \theta_q\right) \end{aligned} \tag{6}$$

If a type 2 vacancy is alone when approaching a worker, which, again, occurs with probability $\exp\left(-\sum_{q=1}^{Q} \theta_q\right)$, it receives the output y_2 and the subsidy σ_2 minus the reservation wage paid to the worker, w_r. If the type 2 vacancy faces no other vacancies of type 2 or higher, but faces at least one type 1 vacancy, when approaching a worker, which occurs with probability $\left(1 - e^{-\theta_1}\right) \exp\left(-\sum_{q=2}^{Q} \theta_q\right)$, it receives the surplus $(y_2 + \sigma_2)$ minus the surplus from the type 1 job $(y_1 + \sigma_1)$, due to the bidding process. If the type 2 vacancy faces at least one other vacancy of type 2 or higher, when approaching the worker (with the remaining probability), then the firm is either unsuccessful in hiring the worker or it is successful but all of its surplus is bid away. In either case, when facing at least one vacancy of type 2 or higher, the firm's *ex post* payoff is zero.

For a type 3 vacancy:

$$\begin{aligned} \pi_3(w_r, \boldsymbol{\theta}) &= [y_3 + \sigma_3 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right) + [y_3 + \sigma_3 - y_1 - \sigma_1] \left(1 - e^{-\theta_1}\right) \exp\left(-\sum_{q=2}^{Q} \theta_q\right) \\ &\quad + [y_3 + \sigma_3 - y_2 - \sigma_2](1 - e^{-\theta_2}) \exp\left(-\sum_{q=3}^{Q} \theta_q\right) \end{aligned} \tag{7}$$

If a type 3 vacancy is alone when approaching a worker, which, again, occurs with probability $\exp\left(-\sum_{q=1}^{Q} \theta_q\right)$, it receives the output y_3 and the subsidy σ_3 minus the reservation wage paid to the worker, w_r. If the type 3 vacancy faces no other vacancies of type 2 or higher, but faces at least one type 1 vacancy, when approaching a worker, which occurs with probability $\left(1-e^{-\theta_1}\right)\exp\left(-\sum_{q=2}^{Q}\theta_q\right)$, it receives the surplus $(y_3+\sigma_3)$ minus the surplus from the type 1 job $(y_1+\sigma_1)$, due to the bidding process. If the type 3 vacancy faces no other vacancies of type 3 or higher, but faces at least one type 2 vacancy, when approaching a worker, which occurs with probability $\left(1-e^{-\theta_2}\right)\exp\left(-\sum_{q=3}^{Q}\theta_q\right)$, it receives the surplus $(y_3+\sigma_3)$ minus the surplus from the type 2 job $(y_2+\sigma_2)$, due to the bidding process. If the type 3 vacancy faces at least one other vacancy of type 3 or higher, when approaching the worker (with the remaining probability), then the firm is either unsuccessful in hiring the worker or it is successful but all of its surplus is bid away. In either case, when facing at least one vacancy of type 3 or higher, the firm's *ex post* payoff is zero.

For a type Q vacancy:

$$\begin{aligned}\pi_Q(w_r,\boldsymbol{\theta}) &= [y_Q+\sigma_Q-w_r]\exp\left(-\sum_{q=1}^{Q}\theta_q\right)+[y_Q+\sigma_Q-y_1-\sigma_1]\left(1-e^{-\theta_1}\right)\exp\left(-\sum_{q=2}^{Q}\theta_q\right)\\ &\quad +...+[y_Q+\sigma_Q-y_{Q-1}-\sigma_{Q-1}](1-e^{-\theta_{Q-1}})\exp\left(-\theta_Q\right) \qquad (8)\end{aligned}$$

which has an analogous interpretation.

With free entry of each type of vacancy we have, in equilibrium:

$$\pi_q(w_r,\boldsymbol{\theta})=k_q \quad \forall\ q=1,2,...,Q \qquad (9)$$

3 The Competitive Search Equilibrium

Definition 1 *A symmetric competitive search equilibrium*[11] *is an allocation defined by the tuple* $(V^*,\pi_1^*,\pi_2^*,...,\pi_Q^*)$*, a choice of reserve wage* w_r^* *by workers, and submarkets* $(\Theta_1^*,\Theta_2^*,...,\Theta_Q^*)$ *such that* $\forall\ q=1,2,...,Q$:

i) $\pi_q^*(w_r,\boldsymbol{\theta})=k_q\ \forall\ w_r^*\in[0,\omega_Q]$.

ii) $V^*=\max_{w_r,\boldsymbol{\theta}}V(w_r,\boldsymbol{\theta})$ *s.t.* $\pi_q(w_r,\boldsymbol{\theta})\leq k_q$ *and* $\theta_q\geq 0$ *with complementary slackness.*

iii) $\Theta_q^*=\theta_q^*$.

Since we restrict attention to symmetric competitive search equilibria, hereafter we will refer to these simply as "equilibria".

[11] A similar formulation can be found in Moen (1997), and Acemoglu and Shimer (1999), and many other competitive search papers. The constraint $\pi_q(w_r,\theta)=k_q$ yields a one-to-one relationship $\theta_q(w_r;k_q)$, and this is the reason we can consider the problem of workers maximizing directly by choosing the vector $\boldsymbol{\theta}$.

Additional Restrictions We now consider four additional restrictions on the parameters which, as we show below, are necessary and sufficient for the existence of a unique equilibrium, in the different cases we consider, where all types of jobs are created in equilibrium.

$$\frac{y_1 + \sigma_1 - b}{k_1} > \frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1}, \tag{10}$$

$$\frac{y_1 + \sigma_1 - b}{k_1} > \frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1} + \ln\left(\frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1}\right)\tau_1 \frac{y_1 + \sigma_1 - \omega_1}{k_1}, \tag{11}$$

$$\frac{y_q + \sigma_q - y_{q-1} - \sigma_{q-1}}{k_q - k_{q-1}} > \frac{y_{q+1} + \sigma_{q+1} - y_q - \sigma_q}{k_{q+1} - k_q} \quad \forall q = 2, 3, ..., Q-1, \tag{12}$$

$$\frac{y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}}{k_Q - k_{Q-1}} > 1. \tag{13}$$

Putting aside the policy parameters for a moment, the restrictions represent concavity requirements on the relationship between y and k for all job qualities to be active in equilibrium.[12] This is seen most easily in (12), when all of the subsidies are set equal to zero, in which case, this condition becomes:

$$\frac{\Delta y_q}{\Delta k_q} > \frac{\Delta y_{q+1}}{\Delta k_{q+1}} \quad \forall q = 2, 3, ..., Q-1,$$

Conditions (10) and (11) have similar interpretations keeping in mind that, implicitly, $y_0 = k_0 = 0$. Once again, ignoring the policy variables, condition (13) can be re-written as

$$y_Q - k_Q > y_{Q-1} - k_{Q-1}.$$

This is a level condition on the surplus of the highest quality job relative to second-highest one. Higher quality jobs have more surplus than others, but the marginal increment of the surplus is diminishing as quality increases, with an infimum of unity at the highest quality.

[12] Indeed, it is easy to show that, if $y = Ak$, for any $A > 0$, then only the highest quality jobs would be created in equilibrium.

Proposition 1 *There exists a unique symmetric competitive search equilibrium, with all types of jobs operating, if and only if (12) and (13) hold and*

a) inequality (10) holds if $b \geq \omega_1$

b) inequality (11) holds if $b < \omega_1$.

In this equilibrium, the values of market tightness for all $q = 2, 3, ..., Q$ *are given by:*

$$\theta_q^* = \ln\left(\frac{y_q + \sigma_q - y_{q-1} - \sigma_{q-1}}{k_q - k_{q-1}}\right) - \ln\left(\frac{y_{q+1} + \sigma_{q+1} - y_q - \sigma_q}{k_{q+1} - k_q}\right) \quad \forall\, q = 2, 3, ..., Q-1 \quad (14)$$

$$\theta_Q^* = \ln\left(\frac{y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}}{k_Q - k_{Q-1}}\right). \quad (15)$$

Moreover:

i) *If* $b \geq \omega_1$ *then the reserve wage is given by:*

$$w_r^* = b. \quad (16)$$

Market tightness for the lowest quality jobs is given by:

$$\theta_1^* = \ln\left(\frac{y_1 + \sigma_1 - b}{k_1}\right) - \ln\left(\frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1}\right). \quad (17)$$

The unemployment rate is given by:

$$U^* = \frac{k_1}{y_1 + \sigma_1 - b} \quad (18)$$

ii) *If* $b < \omega_1$ *then the reserve wage is given implicitly by:*

$$w_r^* = b + \ln\left(\frac{y_1 + \sigma_1 - w_r^*}{k_1}\right)\tau_1\left(y_1 + \sigma_1 - \omega_1\right). \quad (19)$$

Market tightness for the lowest quality jobs is given by:

$$\theta_1^* = \ln\left(\frac{y_1 + \sigma_1 - w_r^*}{k_1}\right) - \ln\left(\frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1}\right). \quad (20)$$

The unemployment rate is given by:

$$U^* = \frac{k_1}{y_1 + \sigma_1 - w_r^*} \quad (21)$$

where w_r^* *in (20) and (21) is determined in (19).*

This proposition provides solutions for the key endogenous variables in this model, and it is useful to analyse their properties before considering their policy implications. As is standard in large markets with competing auctions, the reserve price is driven to the sellers' *ex ante* outside option.[13] When benefits are taxed, this implies that workers set their reserve wage $w_r^* = b$, (i.e., the benefits), in (16). When benefits are untaxed, then this implies that the reserve wage also includes a premium, which is increasing in the tax liability associated with the probability of working, as shown in (19).

Hereafter, we restrict attention to the equilibrium in which (10), (11), (12), and (13) hold, in the relevant cases.

One striking feature of this equilibrium is evident in equations (14), (15), (17) and (20) of the Proposition, and is crucial for many of the policy results that follow. We summarize this feature as a corollary of Proposition 1.

Corollary 1 *For any job type q, equilibrium market tightness θ_q^* depends **only** on the parameters of job types $q-1$, q, and $q+1$.*

That is, parameters for jobs of types below $q-1$ and above $q+1$ are *irrelevant* to firms considering creating a job of type q, in equilibrium. The intuitive reasoning behind this result comes from the first order approach to mechanism design. When considering whether or not to create a type q job, a firm considers 2 problems: creating a type q job versus creating a type $q-1$ job, and creating a type q job versus creating a type $q+1$ job. Consider the first problem. Due to the auction setup in this model (whereby the wage paid by a firm, if it hires a worker, is equal to the productivity (plus any subsidy) of the second highest productivity firm that approaches the worker) the influence on *ex post* profits for this firm of jobs of types $1, 2, ..., q-2$ is the *same* whether the firm chooses a type $q-1$ job or a type q job. Hence, information about the parameters values of jobs of types $1, 2, ..., q-2$ is *irrelevant* when choosing between a type $q-1$ job or a type q job. Consider now the second problem. Again, due to the auction structure, the influence on *ex post* profits for this firm of jobs of types $q+2, q+3, ..., Q$ is the *same* whether the firm chooses a type q job or a type $q+1$ job. (In this case, if any job of type $q+2$ or higher approaches the same worker as the firm in question approaches, *ex post* profits for this firm will be zero.) Hence, information about the parameters values of jobs of

[13] See, for example, McAfee (1993), Peters and Severinov (1997) in the context of large markets. Julien, Kennes and King (2000) demonstrate that, in small markets, the reserve price is above the outside option, and converges monotonically to the outside option as the market size increases. For a more general discussion in settings informational asymmetries, see Albrecht, Gautier, and Vroman (2012, 2014).

types $q+2, q+3, ..., Q$ is irrelevant when choosing between a type q job or a type $q+1$ job.[14]

4 The Policy Results

In this section we consider the comparative static policy results. These results follow from Proposition 1 and, accordingly, are summarized in a set of corollaries. In the subsequent section we consider efficiency and equity issues.

4.1 The Tax Structure

Corollary 2 *i) Job creation, output, and unemployment are independent of the tax structure* $(\tau_q, \omega_q) \forall q = 2, 3, ..., Q$.

ii) If benefits are taxed $(b \geq \omega_1)$ *then job creation, output, and unemployment are also independent of* (τ_1, ω_1).

iii) If benefits are untaxed $(b < \omega_1)$ *then job creation and output are decreasing in* τ_1 *and increasing in* ω_1*, while unemployment is increasing in* τ_1 *and decreasing in* ω_1.

It is clear from Proposition 1 that, when $b \geq \omega_1$, the creation of each quality of job $q = 1, 2, ..., Q$ and the unemployment rate are completely independent of the tax structure. Inutitively, with directed search and auctions, worker reserve wages are driven to their outside options. When benefits are taxed then workers cannot escape the taxes by not working, and the outside option is simply the benefit. Pre-tax wages, in this case, are completely independent of the tax structure. Since firms pay pre-tax wages, the wage tax structure considered here is irrelevant to their entry decisions. (Of course, though, the after-tax wage structure does depend on the tax structure – in a straightforward way.)

When benefits are untaxed ($b < \omega_1$), job creation (and, hence, output and unemployment) are still unaffected by the tax structure for incomes above the lowest level (for reasons analogous to those described in Corollary 1) but, in this case, when hiring a worker, the firm must pay the worker more than b to offset the taxes that must be paid. Thus, in this case, some of the incidence of the tax falls on the firm – which reduces firm profits and, thus, deters entry. (The results themselves, for part (iii), can easily be found by differentiating (19), (20), and (21).)

4.2 Unemployment Benefits

Corollary 3 *Unemployment benefits* b *reduce the creation of the lowest quality jobs* θ_1^* *but have no effect on the creation of any of the other job qualities* θ_q^* $\forall q = 2, ..., Q$. *Consequently, an increase in* b *increases the equilibrium unemployment rate* U^* *and increases average job quality.*

[14] We are very grateful to Andy McLennan for pointing out this intuitive interpretation.

The reason an increase in b reduces the creation of type 1 jobs is entirely straightforward: type 1 jobs can generate positive *ex post* profits for firms if and only if these firms are alone when they approach workers; in this case, the wage that they must pay is equal to the reserve wage announced by the worker – which is equal to b, according to Proposition 1. Any increase in b therefore reduces *ex post* profits for type 1 jobs, which reduces entry of those jobs.

Why do changes in b not affect the creation of jobs of any type greater than 1? The reasoning here is less straightforward, but follows from the reasoning, above, why, for jobs of type q or greater, the parameters associated with jobs of types $1, 2, ..., q-2$ do not affect the entry of these jobs. Intuitively, we can think of unemployment as a "job of type 0". Using the reasoning above, the parameters associated with a job of type 0 (i.e., b) affect only the entry of jobs of type 1. More explicitly, when considering whether to create a job of type 2 versus one of type 1, the influence on *ex post* profits for this firm of a job of type 0 (i.e., being alone when approaching a worker, so that worker's next best "offer" is unemployment with payment b) is the *same* whether the firm chooses a type 1 job or a type 2 job. Hence, the precise value of b is *irrelevant* when choosing between a type 1 job or a type 2 job. This line of argument works for the entry of all types of jobs $q > 1$.

Since an increase in b reduces the entry of type 1 jobs, but does not affect the entry of any of the other types of jobs, overall job creation falls. With a fixed number of workers, N, this implies that unemployment rises. This can also be seen clearly in the formulas for the unemployment rate, given in Proposition 1.

From (2) and (3), it is also clear that increases in b will increase expected wages, both before and after tax, and (by raising wages at the lower tail of the distribution) reduce wage inequality. By reducing the number of type 1 jobs, leaving all other types of jobs unaffected, it will also increase the average quality of jobs in equilibrium.

4.3 Job Creation Subsidies

Proposition 1 makes clear that the effects of job creation subsidies $\sigma_q\ \forall q = 1, ..., Q$ depend crucially on whether or not these subsidies are uniform (i.e., the same value for all types of jobs) or different values for different jobs. We consider both cases.

4.3.1 If Subsidies are Uniform

Corollary 4 *If all jobs are subsidized by the same amount σ then this will increase the creation of the lowest quality jobs θ_1^* but have no effect on the creation rates of any of the other job qualities θ_q^* $\forall q = 2, ..., Q$. Consequently, an increase in σ decreases the equilibrium unemployment rate U^* and decreases average job quality.*

It is quite clear why a uniform job creation subsidy increases the number of type 1 jobs created in equilibrium. The only way a type 1 job can generate a positive *ex post* profit is if that job is alone when it approaches a worker – in which case it must pay the amount b. If it hires the worker, the firm creating this job receives the subsidy σ,which is independent of b. Thus, ex post profits are greater with higher values of σ, inducing more entry of jobs of type 1.

Intuitively, as discussed above, when considering creating a job of type $q > 1$, a firm considers only the parameters associated with jobs of type $q - 1$, q, and $q + 1$. If a job of type q approaches a worker whose second best option is a job of type $q - 1$ then, due to the auction structure, the type q job must pay everything that the type $q - 1$ job could offer the worker: $y_{q-1} + \sigma$. Thus, through this channel, σ *decreases* the ex post profitability of creating a type q job by the exact amount σ. However, the type q job also receives a subsidy of the amount σ, which *increases* the ex post profitability of creating a type q job by exactly the same amount. These two effects exactly cancel each other out and so, when facing competition against a job of type $q - 1$ (or a job of any type lower than $q - 1$), for a job of type q the value of σ is irrelevant. If, on the other hand, a job of type q competes with a job of type $q + 1$ or higher, the type q job will lose the auction when trying to hire the worker, regardless of the value of σ.

Consequently, since the existence of a uniform job creation subsidy σ increases the number of type 1 jobs being created, without changing the number of jobs of any other type, this will decrease the equilibrium unemployment rate U^*, and reduce the average quality of jobs in the economy. From (3) it is also clear that this will increase average wages.

4.3.2 If Subsidies are Heterogeneous and $\sigma_1 < \sigma_2 < ... < \sigma_Q$

Corollary 5 *If $\sigma_1 < \sigma_2 < ... < \sigma_Q$ then*

i) An increase in the subsidy of the lowest quality job σ_1 increases the creation of lowest quality jobs θ_1^*, reduces the creation of the second lowest quality jobs θ_2^*, has no effect on the creation of all other jobs θ_q^* $\forall q = 3, 4, ..., Q$, reduces the unemployment rate U^*, and decreases average job quality.

ii) *An increase in the subsidy to any intermediate job* σ_q $\forall q = 2, 3, ..., Q-1$ *increases the creation of that quality of job* θ_q^**, decreases the creation of jobs of qualities* $q-1$ *and* $q+1$*, but has no effect on the creation of jobs of any other quality and no effect on unemployment.*

iii) *An increase in the subsidy of the highest quality job* σ_Q *increases the creation of jobs of this quality, decreases the creation of jobs of the second highest quality* $Q-1$*, but has no effect on the creation of jobs of any other quality, no effect on unemployment, and increases average job quality.*

Intuitively, the reasoning behind the positive effect of σ_1 on jobs of type 1 is exactly the same as in the uniform subsidy case, above. When a firm creates a job of type 1, it only makes a positive *ex post* profit if it is alone when approaching a worker, in which case it must pay b but it will be subsidized by σ_1, which increases *ex post* profits, inducing more entry of these jobs. For a job of type 2 there are two opportunities for this job to earn positive *ex post* profits: when it is alone when it approaches a worker (in which case it must pay b) or when the only other type of job approaching that worker is of type 1 (in which case it must pay $y_1 - \sigma_1$). An increase in σ_1 clearly increases the wage a type 2 job must pay, in this second case, but not the first. This reduces the *ex post* profits from creating a type 2 job, reducing entry of these jobs. As discussed at length above, the parameters from a type $q-2$ job have no effect on the entry of jobs of type q or higher, so σ_1 has no effect on jobs of type 3 or higher. It is easy to show, from Proposition 1, that an increase in σ_1 increases θ_1 by more than it decreases θ_2. Since all other job types are unaffected, this reduces the equilibrium unemployment rate.

This corollary also tells us that an increase in σ_q for any $q = 2, ..., Q-1$ will increase the number of jobs of type q being created, while decreasing the numbers of jobs of types $q-1$ and $q+1$ being created – but will have no effect on any jobs of types less than $q-1$ or greater than $q+1$, or on equilibrium unemployment. Intuitively, again, when considering creating a job of type $q > 1$, a firm considers only the parameters associated with jobs of type $q-1$, q, and $q+1$. If a job of type q approaches a worker whose second best option is a job of type $q-1$ then, due to the auction structure, the type q job must pay $y_{q-1} + \sigma_{q-1}$.Thus, through this channel, σ_{q-1} *decreases* the ex post profitability of creating a type q job by the exact amount σ_{q-1}. However, the type q job also receives a subsidy of the amount σ_q, which *increases* the ex post profitability of creating a type q job by that amount. Thus, the *ex post* profit of a type q job is positively related to the *difference* between σ_q and σ_{q-1}. On the other side, the probability that a type q competes against a job of type $q+1$ depends on the subsidy σ_{q+1} that a type $q+1$ job gets. Any increase in the *difference* between σ_{q+1} and σ_q increases entry of type $q+1$ jobs, which increases the probability that a type q job will compete against a type $q+1$ job, which reduces the expected profit of a type q job, driving down the entry of this type of job.

An increase in σ_q thus *increases* θ_q but *decreases* both θ_{q-1} and θ_{q+1}, completely offsetting any effect on the total number of vacancies created. Thus, unemployment is unaffected by any change in subsidies to any jobs other than the lowest quality jobs.

This corollary also tells us that an increase in the subsidy of the highest quality jobs, σ_Q, will stimulate the creation of the best jobs, depress the creation of the second-best jobs, but have no effect on jobs of qualities $q < Q - 1$ or on unemployment. It is also clear that these subsidies will affect the wage distribution, using (3). Any increase in σ_q for all q will clearly increase expected wages. An increase in σ_1 will increase also reduce wage inequality, since the wage increase comes exclusively from the lower-paid jobs. Similarly, an increase in σ_Q will increase wage inequality.

5 Efficiency and Equity

In this section we consider two normative criteria: *ex ante* constrained efficiency and *ex post* equity. The constrained efficiency criterion is a standard one typically applied in search models of this type: a planner maximizing expected surplus, given constraints on the instruments the planner can use. The equity criterion is less standard but, we believe, relevant in this setting. The equilibrium in this model trivially exhibits *ex ante* equity because all workers are identical *ex ante*, and make their choices accordingly. *Ex post*, however, potentially, there is significant wage dispersion, due to the mixed strategies of the firms, and the luck of the workers.

5.1 Constrained Efficiency

We consider a planner who wishes to maximize *ex ante* surplus per worker. Although the planner is capable of choosing the number of vacancies created, of each type, she is constrained by being required to conform to the urn-ball meeting technology present in the economy. This implies that each vacancy approaches each worker with equal probability. The planner solves the following problem.

$$\begin{aligned}\max_{\boldsymbol{\theta}} Y &= \left(1-e^{-\theta_Q}\right) y_Q + e^{-\theta_Q}\left(1-e^{-\theta_{Q-1}}\right) y_{Q-1} + e^{-\theta_Q-\theta_{Q-1}}\left(1-e^{-\theta_{Q-2}}\right) y_{Q-2} \\ &\quad +\ldots+\exp\left(-\sum_{q=2}^{Q}\theta_q\right)(1-e^{-\theta_1})y_1 - \sum_{q=1}^{Q}\theta_q k_q \qquad (22)\end{aligned}$$

Given the urn-ball matching process, the first term is the product of the matches between the highest quality jobs $(1-e^{-\theta_Q})$ and the output from those matches y_Q. The second term is the product of the matches that did not occur with the highest quality jobs, but did occur with the second-highest quality jobs $e^{-\theta_Q}(1-e^{-\theta_{Q-1}})$ and the output from those matches $y_{Q-1.}$ The third term is the product of the matches that did not occur with either the highest or second-highest quality jobs but did occur with the third-highest quality jobs $e^{-\theta_Q-\theta_{Q-1}}(1-e^{-\theta_{Q-2}})$ and the output from those matches $y_{Q-2.}$, and so on, down to the lowest quality jobs with output y_1. The associated costs of creating each type of job, $\theta_q k_q$ $\forall q = 1, 2, ..., Q$ are then subtracted off from the surplus.

Proposition 2 *The constrained efficient values of market tightness* $\widetilde{\theta}_q$, $q = 1, 2, ..., Q$ *and the unemployment rate* $\widetilde{U}$ *are given by:*

$$\widetilde{\theta}_1 = \ln\left(\frac{y_1}{k_1}\right) - \ln\left(\frac{y_2 - y_1}{k_2 - k_1}\right). \tag{23}$$

$$\widetilde{\theta}_q = \ln\left(\frac{y_q - y_{q-1}}{k_q - k_{q-1}}\right) - \ln\left(\frac{y_{q+1} - y_q}{k_{q+1} - k_q}\right) \quad \forall\ q = 2, 3, ..., Q-1 \tag{24}$$

$$\widetilde{\theta}_Q = \ln\left(\frac{y_Q - y_{Q-1}}{k_Q - k_{Q-1}}\right) \tag{25}$$

$$\widetilde{U} = \exp\left(-\textstyle\sum_{q=1}^{Q} \widetilde{\theta}_q\right) = k_1/y_1 \tag{26}$$

5.2 Implementing Constrained Efficient Equilibria

From Propositions 1 and 2, it is easy to see the following result.

Proposition 3 *Any policy setting satisfying*

$$\sigma_1 = \sigma_2 = ... = \sigma_Q = b \tag{27}$$

is constrained efficient.

Clearly, the planner's solution coincides exactly with the equilibrium allocations when all of the policy parameters are set to zero – the laissez-faire equilibrium. The efficiency of this equilibrium stems from the way the auction process delivers to each firm the expected social benefits from creating a new vacancy, for each type of vacancy, once they have incurred the entry costs. Consider, for example, a firm that creates a new lowest quality job. If the firm is alone when it approaches a worker, then the social benefit from creating that vacancy is y_1, which is precisely the payoff the firm gets in this case. If, however, the firm is not alone when approaching the worker, then the social benefit from producing the vacancy is zero – which, again, is the payoff the firm gets in this case. Consider now a firm that creates a new second-lowest quality job. If the firm is alone when it approaches the worker then the social benefit from creating the vacancy is y_2, which is exactly the payoff the firm gets in this case. If, alternatively, the firm finds that at least one low quality vacancy but no other high quality vacancy has approached the worker, then the social benefit from creating the vacancy is $y_2 - y_1$, which is what is delivered by the auction mechanism in this case. Finally, if a high quality vacancy faces at least one other second-lowest or higher quality vacancy when approaching the worker, then the social benefit from creating that vacancy is zero – once again, this is the payoff delivered by the auction in this case. In each case, the expected payoff to the firm is equated to the social benefit from the creation of the vacancy. In equilibrium, these expected payoffs (i.e., social benefits) are equated to the private costs of vacancy creation, which are also the social costs: k_q, $q = 1, 2, ..., Q$.

It is, of course, no surprise that the decentralized equilibrium of this directed search model, in the absence of a government, is constrained efficient. However, Proposition 3 also implies that many other policy configurations are also efficient, as long as all firms are paid a subsidy that is equal to unemployment benefits. Given this restriction, the tax rates themselves can take any value.

5.3 Ex Post Equity

Notice that, when we impose the efficiency condition $\sigma_1 = \sigma_2 = ... = \sigma_Q = b$, identified in Proposition 3, from (3), the equilibrium wage distribution becomes:

$$
w_q^j = \begin{cases} w_0^0 = & \min\{(1-\tau_1)b + \tau_1\omega_1,\ b\} \\ w_q^0 = & \min\{(1-\tau_1)b + \tau_1\omega_1,\ b\}\ \forall\ q = 1,2,...,Q \\ w_q^1 = & w^1 = \omega_1 + (1-\tau_1)(y_1 + b - \omega_1)\ \forall\ q = 1,2,...,Q \\ w_q^2 = & w^2 = \omega_1 + (1-\tau_1)(\omega_2 - \omega_1) + (1-\tau_2)(y_2 + b - \omega_1)\ \forall q = 2,3,...,Q \\ & . \\ & . \\ & . \\ w_Q^j = & w^j = \omega_1 + \sum_{i=1}^{j-1}(1-\tau_i)(\omega_{i+1} - \omega_i) + (1-\tau_j)(y_j + b - \omega_j)\ \forall\ j = 2,3,...,Q \end{cases} \tag{28}
$$

This wage distribution is constrained-efficient, but allows a considerable amount of *ex post* after-tax wage inequality among workers that are identical *ex ante.*

Proposition 4 *The following policy configuration implements equilibrium allocations that satisfy constrained efficiency, ex post equity, and balance the government's budget:*

$$
\sigma_1 = \sigma_2 = ... = \sigma_Q = \omega_1 = b \tag{29}
$$

$$
\tau_1 = \tau_2 = ... = \tau_Q = 1. \tag{30}
$$

In this allocation:

$$
w_q^j = b \quad \forall q, j = 1,2,...,Q \tag{31}
$$

where b is given by

$$
\begin{aligned}
b = & \left(1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\right) y_Q + \left[\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}}\right) + e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}} - \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\right] y_{Q-1} \\
& + \left[\left(\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q} + \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\left(1 - e^{-\widetilde{\theta}_{Q-2}}\right) + e^{-\widetilde{\theta}_Q - \widetilde{\theta}_{Q-1}}\left(1 - e^{-\widetilde{\theta}_{Q-2}} - \widetilde{\theta}_{Q-2} e^{-\widetilde{\theta}_{Q-2}}\right)\right] y_{Q-2} \\
& + ... + \left[\sum_{q=2}^{Q} \widetilde{\theta}_q e^{-\widetilde{\theta}_q}\left(1 - e^{-\widetilde{\theta}_1}\right) + \exp\left(-\sum_{q=2}^{Q}\widetilde{\theta}_q\right)\left(1 - e^{-\widetilde{\theta}_1} - \widetilde{\theta}_1 e^{-\widetilde{\theta}_1}\right)\right] y_1
\end{aligned} \tag{32}
$$

and $\widetilde{\theta}_q$ $\forall q = 1,2,...,Q$ are given in Proposition 2.

6 Discussion and Conclusions

In this paper we have examined the effectiveness of "Quality Job Programs" (QJPs) in the context of a directed search model of the labor market, where firms can choose the quality of the jobs that they create, in an environment that also includes taxes and unemployment benefits. Here, the quality of a job is defined by its productivity and the investment needed to create it. We allowed for an arbitrary number of qualities of jobs, Q, where each quality of job can, in principle, be given a unique subsidy. We found that if the subsidy amounts are strictly increasing according to job qualities, as long as unemployment benefits are no greater than the maximal wage from the lowest quality job, then the only subsidies that affect the unemployment rate are those for the lowest quality jobs. These reduce the unemployment rate and the average quality of jobs. Subsidies to the highest-quality jobs, on the other hand, do increase the creation of that type of job, but induce a commeasurate reduction in the number of lower quality jobs. This raises the average quality of jobs but leaves the unemployment rate unchanged. The subsidy for any intermediate quality job, between the lowest and the highest, raises the number of that type of job being created, but at the expense of the jobs directly above and below it – leaving, once again, unemployment unaffected. Heterogeneous subsidies of this type, however, also induce inefficiency of the job quality mix – reducing total output. In that sense, QJPs are generically inefficient.

Uniform subsidies, on the other hand, increase the number of the lowest quality jobs but leave the number of other types of jobs unaffected – this reduces both the unemployment rate and the average quality of jobs. If the value of these uniform subsidies is chosen carefully, to be equal to the value of the unemployment benefits, then these subsidies can be consistent with (constrained) efficiency. We also show that, moreover, *ex post* equity can be achieved through the tax structure in this case, without sacrificing efficiency.

How robust should we expect these results to be? The wage determination mechanism used here – where firms bid for labor – is clearly a key driver for many of the results. How seriously should we take this mechanism as representing how wages are determined in the real world? Although it is true that few workers literally conduct auctions when selling their labor, it is also true that successful wage offers from firms must respect a worker's outside option. Once a firm approaches a worker, to make him/her an offer, the outside option of this worker depends on the number and quality of other offers he/she worker recieves – and that is the essential element that this mechanism captures (more so than other standard mechanisms: bilateral bargaining, or wage posting). Thus, we feel that there is reason to believe that the economic forces at work in this model, driving the policy results, do represent forces that actually exist in labor markets – albeit, arguably, not as starkly as in the model.

The model in this paper is very simple: it is static and it has homogeneous workers. Extending the model, to make it dynamic, would be an interesting exercise – particularly to consider issues of the optimal length of benefits, and the influences of policy parameters on on-the-job search, raiding, and counteroffers from incumbent employers.[15] However, if policy parameters are stationary, we believe that it is reasonable to presume that the main results here would be preserved. Allowing for different types of workers would also complicate things since, in settings like this, worker productivities enter into the probabilities that firms assign when choosing workers to approach.[16] We consider all of these extensions to be interesting and worth exploring.

7 Appendix: Proofs of the Propositions

7.1 Proof of Proposition 1

Suppose all types of jobs are operating (i.e., $\theta_q^* > 0 \; \forall q = 1, 2, ..., Q$). From (5) and (9) we have:

$$[y_1 + \sigma_1 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right) = k_1. \tag{33}$$

From (6) and (9) we have:

$$[y_1 + \sigma_1 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right) + [y_2 + \sigma_2 - y_1 - \sigma_1] \exp\left(-\sum_{q=2}^{Q} \theta_q\right) = k_2$$

Substitution of (33) into the above equation yields:

$$[y_2 + \sigma_2 - y_1 - \sigma_1] \exp\left(-\sum_{q=2}^{Q} \theta_q\right) = k_2 - k_1. \tag{34}$$

From (7) and (9) we have:

$$[y_1 + \sigma_1 - w_r] \exp\left(-\sum_{q=1}^{Q} \theta_q\right) + [y_2 + \sigma_2 - y_1 - \sigma_1] \exp\left(-\sum_{q=2}^{Q} \theta_q\right)$$
$$+ [y_3 + \sigma_3 - y_2 - \sigma_2] \exp\left(-\sum_{q=3}^{Q} \theta_q\right) = k_3$$

Substitution of (33) and (34) into the above equation, and rearranging, yields:

[15] Julien, Kennes, and King (2006) consider a model with these features, but without policy parameters. Most of the essential properties of the static model are preserved in the dynamic extension, including the efficiency results.

[16] See Basov, King, and Uren (2014), for example.

$$[y_3 + \sigma_3 - y_2 - \sigma_2] \exp\left(-\sum_{q=3}^{Q} \theta_q\right) = k_3 - k_2. \tag{35}$$

By induction:

$$[y_{Q-1} + \sigma_{Q-1} - y_{Q-2} - \sigma_{Q-2}]\, e^{-\theta_{Q-1}-\theta_Q} = k_{Q-1} - k_{Q-2} \tag{36}$$

and

$$[y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}]\, e^{-\theta_Q} = k_Q - k_{Q-1} \tag{37}$$

Notice that this equation determines the equilibrium value

$$e^{-\theta_Q^*} = \frac{k_Q - k_{Q-1}}{y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}} \tag{38}$$

from which we obtain (15).

Now, substitution of (38) into (36) yields:

$$[y_{Q-1} + \sigma_{Q-1} - y_{Q-2} - \sigma_{Q-2}] \frac{k_Q - k_{Q-1}}{y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}} e^{-\theta_{Q-1}} = k_{Q-1} - k_{Q-2}.$$

This determines the equilibrium value

$$e^{-\theta_{Q-1}^*} = \frac{k_{Q-1} - k_{Q-2}}{y_{Q-1} + \sigma_{Q-1} - y_{Q-2} - \sigma_{Q-2}} \frac{y_Q + \sigma_Q - y_{Q-1} - \sigma_{Q-1}}{k_Q - k_{Q-1}} \tag{39}$$

Now, by induction, we obtain:

$$e^{-\theta_q^*} = \frac{k_q - k_{q-1}}{y_q + \sigma_q - y_{q-1} - \sigma_{q-1}} \frac{y_{q+1} + \sigma_{q+1} - y_q - \sigma_q}{k_{q+1} - k_q} \quad \forall q = 2, 3, ..., Q-1$$

(from which we obtain (14)). We also obtain:

$$e^{-\theta_1} = \frac{k_1}{y_1 + \sigma_1 - w_r} \frac{y_2 + \sigma_2 - y_1 - \sigma_1}{k_2 - k_1}, \tag{40}$$

This equation makes it clear that the equilibrium value of θ_1 depends on the worker choice of reserve wage w_r. We now consider the worker's optimization problem.

Using (4) and (3), the worker's problem can be re-written as

$$\max_{w_r,\boldsymbol{\theta}} V(w_r,\boldsymbol{\theta}) = \exp\left(-\sum_{q=1}^{Q}\theta_q\right)w_0^0 + \left(\sum_{q=1}^{Q}\theta_q\right)\exp\left(-\sum_{q=1}^{Q}\theta_q\right)w_a$$
$$+\left[\left(1-e^{-\theta_1}-\theta_1 e^{-\theta_1}\right)+\left(1-e^{-\theta_1}\right)\sum_{q=2}^{Q}\theta_q\right]\exp\left(-\sum_{q=2}^{Q}\theta_q\right)w^1$$
$$+\left[\left(1-e^{-\theta_2}-\theta_2 e^{-\theta_2}\right)+\left(1-e^{-\theta_2}\right)\sum_{q=3}^{Q}\theta_q\right]\exp\left(-\sum_{q=3}^{Q}\theta_q\right)w^2$$
$$+...+\left(1-e^{-\theta_Q}-\theta_Q e^{-\theta_Q}\right)w_Q^Q \quad s.t.\ \pi_q(w_r,\theta^q)=k_q\ \forall q=1,2,...,Q$$

Now, using the solutions $(\theta_2^*,\theta_3^*,...,\theta_Q^*)$ above, recognizing that w_q^j are constants (given in (3) for all $j=1,2,...,Q$ and $q=2,3,...,Q$), and defining $W = V\exp\left(\sum_{q=2}^{Q}\theta_q^*\right)$ we can re-write the problem as:

$$\begin{aligned}\max_{w_r,\theta_1} W(w_r,\theta_1) &= w_0^0 e^{-\theta_1} + \theta_1 e^{-\theta_1}w_a + \left(\sum_{q=2}^{Q}\theta_q^*\right)e^{-\theta_1}w_a \\ &+\left[\left(1-e^{-\theta_1}-\theta_1 e^{-\theta_1}\right)+\left(1-e^{-\theta_1}\right)\sum_{q=2}^{Q}\theta_q^*\right]w^1 \\ &+ \textit{constants, subject to } (40)\end{aligned} \tag{41}$$

We have different cases to consider for this problem, according to (2) and (3).

Case 1: $w_r < \omega_1$ (so $w_a = w_r$, from (2)) and $b < \omega_1$ (so $w_0^0 = b$, from (3)). In this case, (41) becomes:

$$\begin{aligned}\max_{w_r,\theta_1} W(w_r,\theta_1) &= be^{-\theta_1} + \theta_1 e^{-\theta_1}w_r + \left(\sum_{q=2}^{Q}\theta_q^*\right)e^{-\theta_1}w_r \\ &+\left[\left(1-e^{-\theta_1}-\theta_1 e^{-\theta_1}\right)+\left(1-e^{-\theta_1}\right)\sum_{q=2}^{Q}\theta_q^*\right]w^1 \\ &+ \textit{constants, subject to } (40)\end{aligned}$$

From the first order conditions to this problem, we obtain:

$$w_r = b + \left(\theta_1 + \sum_{q=2}^{Q}\theta_q^*\right)\tau_1\left(y_1+\sigma_1-\omega_1\right) \tag{42}$$

Now, note, from (38) and (39), that

$$\sum_{q=2}^{Q}\theta_q^* = \ln\left(\frac{y_2+\sigma_2-y_1-\sigma_1}{k_2-k_1}\right). \tag{43}$$

Using this in (42) we obtain:

$$w_r = b + \left(\theta_1 + \ln\left(\frac{y_2+\sigma_2-y_1-\sigma_1}{k_2-k_1}\right)\right)\tau_1\left(y_1+\sigma_1-\omega_1\right) \tag{44}$$

Now, from (40), in equilibrium, we have (20). Using this to substitute out θ_1 in the above equation, we obtain (19).

Substitution of (42) into (40) gives us:

$$y_1+\sigma_1-b-\left(\theta_1^*+\sum_{q=2}^{Q}\theta_q^*\right)\tau_1\left(y_1+\sigma_1-\omega_1\right) = e^{\theta_1^*}\frac{k_1}{k_2-k_1}\left(y_2+\sigma_2-y_1-\sigma_1\right) \tag{45}$$

This equation determines θ_1^*. Clearly, from this equation, since the lhs is monotonically decreasing in θ_1^* and the rhs is monotonically increasing in θ_1^*, the existence of a unique $\theta_1^* > 0$ requires that

$$y_1+\sigma_1-b-\sum_{q=2}^{Q}\theta_q^*\tau_1\left(y_1+\sigma_1-\omega_1\right) > \frac{k_1}{k_2-k_1}\left(y_2+\sigma_2-y_1-\sigma_1\right). \tag{46}$$

Now, substitution of (43) into this inequality yields (11).

Finally, the unemployment rate, with this urn-ball matching process, is given by:

$$U^* = \exp\left(-\sum_{q=1}^{Q}\theta_q^*\right). \tag{47}$$

Using (38), (39), and (40) in (47) we get (21).

Case 2: $w_r < \omega_1$ (so $w_a = w_r$, from (2)) and $b \geq \omega_1$ (so $w_0^0 = (1-\tau_1)b+\tau_1\omega_1$, from (3)). In this case, (41) becomes:

$$\begin{aligned}\max_{w_r,\theta_1} W(w_r,\theta_1) \;&=\; ((1-\tau_1)b+\tau_1\omega_1)e^{-\theta_1}+\theta_1 e^{-\theta_1}w_r+\left(\sum_{q=2}^{Q}\theta_q^*\right)e^{-\theta_1}w_r \\ &\quad+\left[\left(1-e^{-\theta_1}-\theta_1 e^{-\theta_1}\right)+\left(1-e^{-\theta_1}\right)\sum_{q=2}^{Q}\theta_q^*\right]w^1 \\ &\quad+\textit{constants, subject to } (40)\end{aligned}$$

The first order conditions, in this case, imply:

$$w_r = (1-\tau_1)b + \tau_1\omega_1 + \left(\theta_1^* + \sum_{q=2}^{Q}\theta_q^*\right)\tau_1\left(y_1 + \sigma_1 - \omega_1\right) \tag{48}$$

Since, in this case, $b \geq \omega_1$, then the above equation implies that $w_r > \omega_1$, a contradiction. Thus, Case 2 does not apply.

Case 3: $w_r \in [\omega_1, \omega_2)$, so (so $w_a = \omega_1 + (1-\tau_1)(\omega_r - \omega_1)$, from (2)) and $b < \omega_1$ (so $w_0^0 = b$, from (3)).

In this case, using this substitution in (41), the first order conditions imply: $b = \tau_1\omega_1 + (1-\tau_1)w_r$. This implies that $b \geq \omega_1$, a contradiction. Thus, Case 3 does not apply.

Case 4: $w_r \in [\omega_1, \omega_2)$, so (so $w_a = \omega_1 + (1-\tau_1)(\omega_r - \omega_1)$, from (2)) and $b \geq \omega_1$ (so $w_0^0 = (1-\tau_1)b + \tau_1\omega_1$, from (3)). The first order conditions to this problem imply (16). Also, using (16) in (40), we obtain (17). Again, the unemployment rate is given by (47) and, using (38), (39), and (17) in (47) we get (18).

Now we consider the conditions for all types of jobs to be operating in equilibrium in the relevant cases (Cases 1 and 4). (i.e., $\theta_q^* > 0\ \forall q = 1, 2, ..., Q$). Imposing $\theta_1^* > 0$ in (17), in Case 4, implies (10). Imposing $\theta_1^* > 0$ in Case 1, we substitute (43) into (46), which implies (11). In both cases, imposing $\theta_q^* > 0$ in (14) implies (12), and imposing $\theta_Q^* > 0$ in (15) implies (13).

All other cases allowed by (2) and (3) imply contradictions, so do not apply. ■

7.2 Proof of Proposition 2

From the first order condition for the lowest quality job in (22) we have:

$$y_1 \exp\left(-\textstyle\sum_{q=1}^{Q}\theta_q\right) = k_1. \tag{49}$$

From the first order condition for the second-lowest quality job in (22) we have:

$$y_1 \exp\left(-\textstyle\sum_{q=1}^{Q}\theta_q\right) + [y_2 - y_1]\exp\left(-\textstyle\sum_{q=2}^{Q}\theta_q\right) = k_2$$

Substitution of (49) into the above equation yields:

$$[y_2 - y_1]\exp\left(-\textstyle\sum_{q=2}^{Q}\theta_q\right) = k_2 - k_1. \tag{50}$$

From the first order condition for the third-lowest quality job in (22) we have:

$$\begin{aligned} &y_1 \exp\left(-\textstyle\sum_{q=1}^{Q}\theta_q\right) + [y_2 - y_1]\exp\left(-\textstyle\sum_{q=2}^{Q}\theta_q\right) \\ &+[y_3 - y_2]\exp\left(-\textstyle\sum_{q=3}^{Q}\theta_q\right) = k_3 \end{aligned}$$

Substitution of (49) and (50) into the above equation, and rearranging, yields:

$$[y_3 - y_2] \exp\left(-\sum_{q=3}^{Q} \theta_q\right) = k_3 - k_2. \tag{51}$$

By induction:

$$[y_{Q-1} - y_{Q-2}]\, e^{-\theta_{Q-1}-\theta_Q} = k_{Q-1} - k_{Q-2} \tag{52}$$

and

$$[y_Q - y_{Q-1}]\, e^{-\theta_Q} = k_Q - k_{Q-1} \tag{53}$$

Notice that this equation determines the optimal value

$$e^{-\widetilde{\theta}_Q} = \frac{k_Q - k_{Q-1}}{y_Q - y_{Q-1}} \tag{54}$$

from which we obtain (25).
Now, substitution of (54) into (52) yields:

$$[y_{Q-1} - y_{Q-2}] \frac{k_Q - k_{Q-1}}{y_Q - y_{Q-1}} e^{-\theta_{Q-1}} = k_{Q-1} - k_{Q-2}.$$

This determines the optimal value

$$e^{-\widetilde{\theta}_{Q-1}} = \frac{k_{Q-1} - k_{Q-2}}{y_{Q-1} - y_{Q-2}} \frac{y_Q - y_{Q-1}}{k_Q - k_{Q-1}} \tag{55}$$

Now, by induction, we obtain:

$$e^{-\widetilde{\theta}_q} = \frac{k_q - k_{q-1}}{y_q - y_{q-1}} \frac{y_{q+1} - y_q}{k_{q+1} - k_q} \quad \forall q = 2, 3, ..., Q-1 \tag{56}$$

(from which we obtain (24)). We also obtain:

$$e^{-\widetilde{\theta}_1} = \frac{k_1}{y_1 - w_r} \frac{y_2 - y_1}{k_2 - k_1}, \tag{57}$$

(from which we obtain (23)). Finally, from (49) we also obtain:

$$\widetilde{U} = \exp\left(-\sum_{q=1}^{Q} \widetilde{\theta}_q\right) = \frac{k_1}{y_1} \quad \blacksquare$$

7.3 Proof of Proposition 3

Imposing (27) in θ_q^* $\forall q = 1, 2, ..., Q$ and U^* in Proposition 1, one obtains $\widetilde{\theta}_q$ $\forall q = 1, 2, ..., Q$ and $\widetilde{U}$ in Proposition 2. $\blacksquare$

7.4 Proof of Proposition 4

Substitution of (29) and (30) into (28) implies (31).

Consider, now, the government's budget. Let t denote tax revenues per worker. In this equilibrium

$$\begin{aligned} t &= \left(1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\right)(y_Q + \sigma - \omega) \\ &+ \left[\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}}\right) + e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}} - \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\right](y_{Q-1} + \sigma - \omega) \\ &+ \left[\left(\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q} + \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\left(1 - e^{-\widetilde{\theta}_{Q-2}}\right) + e^{-\widetilde{\theta}_Q - \widetilde{\theta}_{Q-1}}\left(1 - e^{-\widetilde{\theta}_{Q-2}} - \widetilde{\theta}_{Q-2} e^{-\widetilde{\theta}_{Q-2}}\right)\right](y_{Q-2} + \sigma - \omega) \\ &+ \ldots + \left[\sum_{q=2}^{Q} \widetilde{\theta}_q e^{-\widetilde{\theta}_q}\left(1 - e^{-\widetilde{\theta}_1}\right) + \exp\left(-\sum_{q=2}^{Q} \widetilde{\theta}_q\right)\left(1 - e^{-\widetilde{\theta}_1} - \widetilde{\theta}_1 e^{-\widetilde{\theta}_1}\right)\right](y_1 + \sigma - \omega) \end{aligned} \tag{58}$$

The first term represents the tax revenue from all the workers that have the highest pre-tax income: $y_Q + \sigma$. This income is attained in only one situation: when at least two of the highest quality jobs approach them. The fraction of workers in this situation is $1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}$. With $\tau = 1$, the government collects all of the income as tax, but only for income above the threshold ω. Thus, the tax revenue from these workers is: $(1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q})(y_Q + \sigma - \omega)$.

The second term represents the tax revenue from all the workers that have the second-highest pre-tax income: $y_{Q-1} + \sigma$. This income is attained in only two situations: when only one of the best jobs and at least one of the second best jobs approach then (which occurs with probability $\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}(1 - e^{-\widetilde{\theta}_{Q-1}})$) and when none of the best jobs but at least two of the second best jobs approach them (which occurs with probability $e^{-\widetilde{\theta}_Q}(1 - e^{-\widetilde{\theta}_{Q-1}} - \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}})$). The fraction of workers in this situation is $1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}$. The government collects all of the income as tax, but only for income above the threshold ω. Thus, the tax revenue from these workers is: $[\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}(1 - e^{-\widetilde{\theta}_{Q-1}}) + e^{-\widetilde{\theta}_Q}(1 - e^{-\widetilde{\theta}_{Q-1}} - \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}})](y_{Q-1} + \sigma - \omega)$. All of the other terms in (58) are analogous, for each possible pre-tax wage income for workers.

Imposing, now, that $\sigma = \omega$, equation (58) becomes:

$$\begin{aligned} t &= \left(1 - e^{-\widetilde{\theta}_Q} - \widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\right) y_Q + \left[\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}}\right) + e^{-\widetilde{\theta}_Q}\left(1 - e^{-\widetilde{\theta}_{Q-1}} - \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\right] y_{Q-1} \\ &+ \left[\left(\widetilde{\theta}_Q e^{-\widetilde{\theta}_Q} + \widetilde{\theta}_{Q-1} e^{-\widetilde{\theta}_{Q-1}}\right)\left(1 - e^{-\widetilde{\theta}_{Q-2}}\right) + e^{-\widetilde{\theta}_Q - \widetilde{\theta}_{Q-1}}\left(1 - e^{-\widetilde{\theta}_{Q-2}} - \widetilde{\theta}_{Q-2} e^{-\widetilde{\theta}_{Q-2}}\right)\right] y_{Q-2} \\ &+ \ldots + \left[\sum_{q=2}^{Q} \widetilde{\theta}_q e^{-\widetilde{\theta}_q}\left(1 - e^{-\widetilde{\theta}_1}\right) + \exp\left(-\sum_{q=2}^{Q} \widetilde{\theta}_q\right)\left(1 - e^{-\widetilde{\theta}_1} - \widetilde{\theta}_1 e^{-\widetilde{\theta}_1}\right)\right] y_1 \end{aligned} \tag{59}$$

Consider, now, the expenditures of the government, per worker. For every matched worker, the government pays the amount σ. For every unmatched worker, the government pays the amount b. With constrained efficient policy, $\sigma = b$. Thus, overall, government expenditures per worker are equal to b. With a balanced budget, $t = b$, and, using (59), we obtain (32). ■

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