

# Donations in a recursive dynamic model

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## Abstract

This paper studies how donations respond to unexpected permanent changes in income and tax rates in a recursive dynamic model. The dynamic approach yields several interesting insights. If marginal tax rates are progressive, a permanent jump in a household's income *increases* its consumption and donations in the short run, but has no effect in the long run. The permanent income elasticity of current donations is likely to exceed one. If the marginal tax rate is flat, the jump in income raises consumption and donations in both the short and the long run. A permanent marginal tax rate cut *raises* consumption and donations in the long run if marginal tax rates are progressive, while it *reduces* donations in the short run if it has little direct impact on tax payments. If the marginal tax rate is flat, a tax cut has a *positive* effect on consumption in both the short and the long run, but has an ambiguous effect on donations.

*Keywords:* Donations; Recursive preferences; Progressive taxes

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# 1. Introduction

Donations are a major source of revenue for charitable organizations and have attracted a great deal of attention in the economic literature. Numerous empirical studies have been devoted to estimating the elasticities of donations with respect to changes in both income and the tax price (for tax-deductible donations, the tax price, commonly defined as one minus the marginal tax rate, moves in the opposite direction to the change in the tax rate). This research has been accompanied by progress in the theoretical analysis of charitable giving behavior.<sup>1</sup> As stated in Feldstein and Clotfelter (1976), one of the main motivations of these studies is to provide parameters for evaluating the impact on donations of various proposals for changes in tax policy.

Most early studies carried out only reduced-form estimations of the determination of charitable contributions using cross-section data, with their specifications building on static models. Since the nature of cross-section data makes it difficult to untangle the effects of permanent vs. transitory changes, recent studies have turned to panel data, using multi-period dynamic models for their theoretical guidance. For example, Randolph (1995) used a two-period life cycle model and derived the short-run responses of donations to permanent and temporary changes in income and in the tax system. To capture a household's expectations of future changes in income and in tax policy, Auten et al. (2002) used a standard recursive model with an infinite planning horizon and derived optimal conditions from this framework.

However, an important question regarding the theoretical analysis of donations remains unexplored: is there any difference between the short-run and the long-run responses of donations? In particular, is the response of donations to a permanent in-

come or price shock only a temporary deviation from its current state or a permanent switch to a new long-run state?

A key task in exploring these issues is to capture the fact that a household can respond to changes in its income and in the tax system by adjusting savings at the margin in order to best allocate its income across periods within its entire planning horizon. When saving changes, so do planned expenditures on consumption and donations. For instance, if a household faces a permanent reduction in the tax rate, a static model predicts that the household will cut its donations due to the increase in the price of donations. However, in a dynamic model, the tax reduction raises households' incentive to save via increasing the after tax rate of return to saving, which then increases households' lifetime wealth. As a result, donations could rise in the long-run after a permanent tax cut in a dynamic model.

The pressing need for a dynamic model to deal with the issues at hand also arises from the fact that it is very difficult to explain the large price and income elasticities for donations in a static model. For example, Auten et al. (2002) found that the short-run elasticity of donations with respect to a persistent change in the tax price lies between -0.31 and -2.13; Randolph (1995) documented that the short-run elasticity of donations with respect to a permanent change in income ranges from 1.14 to 1.69.<sup>2</sup> A static model can hardly reconcile the large values of the elasticity with the U.S. tax system. With realistic progressivity in the marginal tax rate, the income elasticity of donations at a level of 1.69 in a single-period model would imply that, following a rise in income, a household would substantially reduce the fraction of its income spent on private consumption. This implication is inconsistent with the conventional

view on consumption behavior vs. permanent income change. But a dynamic model with many periods would show a household responding to a permanent rise in income through reducing savings, since with the increased income, it would face a higher marginal tax rate and hence a lower rate of return on savings. When saving falls as a result of increased income, a household can spend more on both private consumption and donations in the short run.

Exploring these questions theoretically can give charitable organizations better tools to assess the impact of tax reform on donations. It can also help resolve the debate on the sign and magnitude of the short-run elasticity of donations, in particular shedding light on the question of whether any important factor is missing in the estimation specification. Ultimately, it can help predict the long-run elasticity of donations.

Our purpose in this paper is to examine the effects of unexpected permanent changes in income and in the tax price on charitable giving in a recursive dynamic framework. We will explicitly analyze these effects in both the short run and the long run. Given the fact that Auten et al. (2002) found that donations are rather inelastic to temporary changes in income and in the tax system, we will ignore these temporary changes in the analysis.<sup>3</sup>

The recursive dynamic approach yields several interesting results when the marginal tax rates are progressive. First, a permanent rise in a household's income reduces its asset holding, but has no effect on its consumption and donations in the long run. However, in the short run, the levels of both consumption and donations rise in response to the income increase when the reduced amount of saving further adds

to short-run expenditures. The elasticity of donations with respect to a permanent income change in the short run is likely to exceed unity, resulting from a combination of increases in short-run expenditures and reductions in the tax price of donations. Second, in the long run, a permanent marginal tax-rate cut raises consumption and wealth. While the tax cut also raises donations in the long run, it reduces donations in the short run if it has little direct impact on tax payments that arises from the tax code change for any given amount of disposable income. Last but not least, because fluctuations in saving are negatively correlated with changes in tax payments but positively correlated with variations in the tax price of donations, failing to control for the fluctuations in savings levels could introduce an upward bias to the estimate of price elasticity and a downward bias to the estimate of income elasticity.

If the marginal tax rate is flat, our model predictions on how wealth responds to a change in income or taxes are less clear since there is only one particular value of the flat tax rate that can lead to a steady state for a household in the long run. However, we can still show that a permanent rise in income increases consumption and donations in both the short and the long run. This is because the relative price of donations is constant under the flat tax system, a permanent rise in income only works through the income effect. A permanent tax cut has a positive effect on consumption but an ambiguous effect on donations in both the short and the long run, because a tax cut raises not only disposable income but also the relative price of donations.

The rest of the paper is organized as follows. The next section describes the model. Section 3 focuses on the response of donations to a permanent change in income, and Section 4 on the response of donations to a permanent change in the tax system.

Section 5 provides a numerical illustration of our model predictions. The last section provides some concluding remarks.

## 2. The model

Although there are many possible causes for donations as discussed in Andreoni (2006), we focus on the “warm-glow” theory in which donors derive utility directly from their acts of giving. By doing this, we abstract from the public goods aspect of donations, which has been analyzed in, e.g., Bergstrom et al. (1986), Steinberg (1987), Andreoni (1989), Glazer and Konrad (1996), and Harbaugh (1998). This public goods aspect would entail strategic behavior on the part of both households and the government and thus would complicate a recursive model greatly. Moreover, Andreoni (2006) shows that the “warm-glow” model is most suitable for analyzing the amount of donations. As in Auten et al. (2002), in our model agents allocate their disposable income between private consumption  $C_t$  and donations  $G_t$  over an infinite horizon. Their preferences are defined as

$$V_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, G_s), \quad 0 < \beta < 1, \quad (1)$$

where  $\beta$  is the discounting factor and  $E_t$  the expectation at time  $t$ . We adopt the standard regularities of  $U$ : it is twice differentiable, increasing in all elements, and strictly concave, with  $U_1 > 0$ ,  $U_2 > 0$ ,  $U_{11} < 0$ ,  $U_{22} < 0$  and  $U_{11}U_{22} - U_{12}^2 > 0$ . We also assume that  $U_{12} \geq 0$ , an approach that assumes private consumption and donations to be complementary for their competing uses of income. This assumption is stronger than what we need to prove Propositions 1 and 2 in the next section. We maintain this assumption just for the sake of continuity as it is commonly assumed in

the donation literature. It should be noted that we implicitly rule out the possibility of bequests to children as a normal good by using an infinitely lived agent model.<sup>4</sup>

The intertemporal budget constraint of households is given by

$$C_t + G_t + W_{t+1} = W_t(1 + r_t) + Y_t - T_t(r_t W_t + Y_t - G_t), \quad (2)$$

where  $W_t$  is the amount of wealth at  $t$ ,  $Y_t$  is earnings,  $r_t$  the interest rate, and  $T_t$  the tax as a function of income net of donations. Following the literature on donations, we treat  $r$  and  $Y$  as exogenously given.<sup>5</sup> Adopting this assumption helps us compare the results of the recursive model with those of the conventional static model; it also keeps the model manageable. Finally, the present value of households' assets at the end of their planning horizon is restricted to zero (i.e. the transversality condition):

$$\lim_{t \rightarrow \infty} W_t / [1 + r(1 - \tau)]^t = 0. \quad (3)$$

The problem of a household is to solve the following concave programming:

$$\begin{aligned} V_t(W_t) = \max_{G_t, W_{t+1}} \{ & U[W_t(1+r_t) + Y_t - T_t(r_t W_t + Y_t - G_t) - G_t - W_{t+1}, G_t] \\ & + \beta E_t V_{t+1}(W_{t+1}) \}, \end{aligned} \quad (4)$$

where (2) is used to substitute out  $C_t$ .

Differentiating (4) with respect to  $G_t$ , the corresponding first-order condition is

$$(1 - T'_t) \frac{\partial U(t)}{\partial C_t} = \frac{\partial U(t)}{\partial G_t}. \quad (5)$$

This condition balances the gain in utility from making an additional unit of donations against the loss in utility from reducing private consumption by the same unit. It is valid regardless of the time line on which the decision is based (whether a single

period or an infinite horizon). The derivative  $T'$  stands for the marginal tax rate at a particular level of taxable income, denoted  $Y_t^T \equiv r_t W_t + Y_t - G_t$ . Thus,  $1 - T'$  is the tax price of charitable giving. The tax price appears on the left-hand side alongside the marginal utility derived from private consumption but is absent from the right-hand side of the equation because donations are tax deductible but private consumption is not. Note that, when the marginal tax rate  $T'$  changes as a result of either a change in income or a change in the tax system, the tax price  $1 - T'$  changes in the opposite direction, tending to alter private consumption and donations.

Differentiating (4) with respect to  $W_{t+1}$  produces

$$\frac{\partial U(t)}{\partial C_t} = \beta E_t V'_{t+1}. \quad (6)$$

This condition governs the intertemporal allocation of resources in a typical fashion: the loss in utility via giving up one more unit of private consumption today for saving is compensated by a gain in utility through building up more future assets.

The envelope condition arrives from differentiating (4) with respect to  $W_t$ :

$$V'_t = \frac{\partial U(t)}{\partial C_t} [1 + r_t(1 - T'_t)], \quad (7)$$

which indicates the contribution to utility that results from increasing wealth at the margin. According to (7), this contribution can be measured by the marginal utility derived from spending the increased wealth, plus its interest income net of taxes, on private consumption. Conditions (6) and (7), updated by one period, lead to the following Euler equation:

$$\frac{\partial U(t)}{\partial C_t} = \beta E_t \left( \frac{\partial U(t+1)}{\partial C_{t+1}} \right) [1 + r_{t+1}(1 - T'_{t+1})]. \quad (8)$$



In this equation, a change in the marginal tax rate  $T'$  resulting from either a change in income or a change in the tax system will alter the after-tax rate of return on saving, tending to alter the paths of saving, donations, and private consumption.

In the next two sections, we consider how donations respond to unexpected permanent changes in income and in the tax system, respectively. Since the problem at hand with a recursive structure and with a general utility function has no reduced-form solution, we attempt instead to characterize key features of household responses to these changes in both the long run and the short run. To simplify our analysis, we assume the same level of earnings  $Y$  in all periods after an initial change, and the same interest rate  $r$  in all periods. With these assumptions, in the long run, the implicit solution of the concave programming under  $0 < \beta < 1$  becomes stationary when the marginal tax rate  $T'(Y^T)$  is adjustable to taxable income  $Y^T$ , which will become clear later. Thus, we drop the time subscripts associated with  $C, G, T, T'$ , and  $W$  in the above equations. Specifically, the long-run stationary versions of (2), (5), and (8) are given below:

$$C = rW + Y - T(rW + Y - G) - G, \quad (9)$$

$$(1 - T')U_1 = U_2, \quad \text{where } U_1 \equiv \partial U / \partial C, \quad U_2 \equiv \partial U / \partial G, \quad (10)$$

$$1 = \beta[1 + r(1 - T')]. \quad (11)$$

We are now ready to analyze the responses of donations to changes in income and in the tax system. We begin with the former.

### 3. A permanent change in income

Differentiating equations (9)–(11) with respect to  $Y$ , we derive the following results:

**Proposition 1.** (a) For  $T'' > 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , a permanent rise in  $Y$  has no long-run effects on consumption  $C$  and donations  $G$  but has a negative long-run effect on wealth  $W$ . However, it has positive short-run effects on consumption and donations through reducing saving if  $1/r \geq -W'_t$ . (b) For  $T'' = 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , a permanent rise in  $Y$  has positive effects on consumption  $C$  and donations  $G$  in all periods  $t \geq 0$ .

**Proof.** Part (a). Differentiating (11) with respect to  $Y$  gives

$$0 = -\beta r T'' \times (rW' + 1 - G'). \quad (12)$$

Since  $T'' > 0$  with progressive taxes, we must have

$$rW' + 1 - G' = 0. \quad (13)$$

Differentiating (9) with respect to  $Y$  and using (13), for  $0 < T' < 1$  we obtain

$$C' = rW' + 1 - T'(rW' + 1 - G') - G' = 0. \quad (14)$$

Differentiating (10) with respect to  $Y$  and using  $C' = 0$  in (14), we have

$$G'[U_{12}(1 - T') - U_{22}] = 0. \quad (15)$$

If  $U_{12} \geq 0$  (sufficient but not necessary), then  $G' = 0$  under  $0 < T' < 1$  and  $U_{22} < 0$ .

Finally, using the fact that  $rW' + 1 - G' = 0$  in (13) and  $G' = 0$  above, we also have

$$W' = -1/r < 0.$$

The long-run responses of  $W$ ,  $C$ , and  $G$  imply that saving, measured by  $W_{t+1} - W_t$ , should fall in the short run. First, suppose that saving starts to fall in the initial period (time 0) when a household realizes a permanent rise in income. Given a predetermined level of initial wealth  $W_0$ , a decrease in savings means  $d(W_1 - W_0)/dY = W'_1 < 0$ , and we have

$$C'_0 = \frac{[(1 - T'_0)U_{12}(0) - U_{22}(0)](1 - T'_0 - W'_1) - T''_0 U_1(0)W'_1}{2(1 - T'_0)U_{12}(0) - (1 - T'_0)^2 U_{11}(0) - U_{22}(0) + T''_0 U_1(0)}, \quad (16)$$

and

$$G'_0 = \frac{[U_{12}(0) - (1 - T'_0)U_{11}(0)](1 - T'_0 - W'_1) + T''_0 U_1(0)}{2(1 - T'_0)U_{12}(0) - (1 - T'_0)^2 U_{11}(0) - U_{22}(0) + T''_0 U_1(0)}. \quad (17)$$

Clearly, for  $W'_1 < 0$  and  $U_{12} \geq 0$ , together with  $T'' > 0$ ,  $0 < T' < 1$ ,  $U_i > 0$ , and  $U_{ii} < 0$  ( $i = 1, 2$ ), we have  $C'_0 > 0$  and  $G'_0 > 0$ .

Now, suppose that savings start to fall for  $\infty > t > 0$ , i.e.,  $W'_{t+1} - W'_t < 0$ .

Evaluating  $U_{ij}$  and  $U_j$  at time  $t$  for  $i, j = 1, 2$ , we then have

$$C'_t = \frac{[(1 - T'_t)U_{12} - U_{22}][(1 - T'_t)(1 + W'_t r) - (W'_{t+1} - W'_t)] - T''_t U_1(W'_{t+1} - W'_t)}{2(1 - T'_t)U_{12} - U_{22} - (1 - T'_t)^2 U_{11} + T''_t U_1}, \quad (18)$$

and

$$G'_t = \frac{[U_{12} - (1 - T'_t)U_{11}][(1 - T'_t)(1 + W'_t r) - (W'_{t+1} - W'_t)] + (1 + W'_t r)T''_t U_1}{2(1 - T'_t)U_{12} - U_{22} - (1 - T'_t)^2 U_{11} + T''_t U_1}. \quad (19)$$

For  $U_{12} \geq 0$ ,  $T''_t > 0$  and  $0 < T'_t < 1$ , it is clear that  $C'_t > 0$  and  $G'_t > 0$  under  $W'_{t+1} - W'_t < 0$  and  $1/r \geq -W'_t$  (or equivalently  $1 + W'_t r \geq 0$ .)

Part (b). See the proof in the Appendix.  $\square$

It is worth mentioning that part (b) of Proposition 1 still holds when the tax function becomes  $T(Y^T) = \tau Y^T - e$  where  $e$  is a positive parameter. In this case,

while the marginal tax rate is still flat, the average tax rate becomes an increasing function of taxable income  $Y^T$ . This suggests that a permanent change in household income can affect donations in the long run even under a progressive tax system. In short, whether changes in household income can affect donations in the long run depends on whether the marginal, rather than the average, tax rates are progressive.

Moreover, we exclude the case with  $T'' < 0$  (i.e. a regressive tax system) for two reasons. First, a regressive tax system is of little practical relevance in the real world. Second, under this system disposable income becomes a convex function of taxable income  $Y^T$  and hence the optimization problem is not well defined. Specifically, there will be a steady state with  $T'' \neq 0$  but in the steady state the feasible set of  $(C, G)$  satisfying the steady-state budget constraint (9), i.e.  $C \leq Y^T - T(Y^T)$ , is no longer a convex set with  $T'' < 0$ . This is because the steady-state budget constraint line is featured by  $C = Y^T - T(Y^T)$ ,  $dC/dG = -(1 - T') < 0$  and  $d^2C/d^2G = -T'' > 0$  under  $T'' < 0$ . Graphically, the steady-state budget constraint line is bent toward the origin like an indifference curve.

According to the existing empirical literature, most individuals should face a progressive tax system. For example, Akhand and Liu (2002) show that, except for households at the bottom quartile of the income distribution, the effective marginal tax rate almost always increases with income. However, for individuals who have already facing the highest marginal tax rate, they are likely to face a flat tax schedule. The restriction  $U_{12} \geq 0$ , is far more restrictive than what is necessary for the derivation of our results. Even if  $U_{12} < 0$ , both  $C$  and  $G$  increase in the short-run as long as  $U_{12} > \max\{(1 - T')U_{11}, (1 - T')U_{22}\}$ .<sup>6</sup> Lastly, the range of change in wealth

$1/r \geq -W'_t$  that can lead to the ideal result merely means that the change in wealth in any single period does not exceed the needed change in the long run.

The positive short-run responses of private consumption and donations to permanent income changes are in line with the typical empirical evidence found in the literature. What is most intriguing about our findings is the following. Permanent rises in income have no effect on private consumption and donations in the long run, but raise both of them in the short run under a progressive tax system. The explanation for the difference between the short-run and the long-run effects lies in the progressive nature of the tax system.

However, in a dynamic model that includes a progressive tax system, the after-tax rate of return to saving  $r(1 - T'_{t+1})$  in (8) falls as income increases in all periods—unlike earlier static models. The falling after-tax rate of return to saving implies that the marginal rate of substitution between consumption in the current period and the next should fall accordingly to maintain the balance in that equation. Thus, under progressive taxes a household realizing a permanent rise in income will increase its level of consumption in the current period relative to the level of consumption in the next period by reducing saving.<sup>7</sup> Following this argument, the level of current consumption in a household should be greater than the level of its consumption in a more distant future, if the decline in wealth continues. Thus, when the fall in wealth eventually equals the rise in income as seen in (13), long-run consumption need not respond to a permanent income rise in (14). According to (11), the rise in the tax rate resulting from the permanent increase in income will be fully offset by the fall in the tax rate via the reduction in wealth in any long-run stationary solution. Consequently,

the tax price of donations will eventually climb back to its original level. Therefore, the implied relationship between private consumption and donations in (5) suggests that donations, like consumption, need not respond to permanent income changes in the long run.

Like in a static model, in our dynamic model a progressive tax system differentiates among the degrees of the responses of private consumption and donations to a permanent income change, because donations are tax deductible. From (5), when income rises permanently, the tax price of donations  $1 - T'_t$  falls, and so does the marginal rate of substitution between donations and private consumption. In order for this marginal rate of substitution to fall for a given household, it needs to increase its donations by a higher percentage than its consumption as a response to the increased income.

Putting the above discussions together leads us to an important implication of Proposition 1 for the short-run elasticity of donations with respect to permanent income changes. On the one hand, the amount reduced from saving adds to the rise in income for a further increase in spending on private consumption and donations in the short run. On the other hand, the increased short-run spending tends to favor increasing donations against private consumption as the tax price falls. The combination of these two forces enhances the possibility for the permanent-income elasticity of donations to be greater than 1 in the short run. This finding contrasts sharply with the conventional view based on a static model, which focuses only on the decline in the tax price without considering the reduction in savings.

To see the implication for the size of the permanent-income elasticity of donations

in the short run more specifically, let us start from a static model with a flat-rate tax. By setting  $\beta = 0$ , we reduce our model to a one-period static model. In this version there is no saving, thus the optimal decision is characterized by (5) and a special case of (2) with wealth  $W = 0$ . Correspondingly, the response of donations to a rise in income in (19) becomes

$$G' = \frac{[U_{12} - (1 - T')U_{11}](1 - T') + T''U_1}{2(1 - T')U_{12} - (1 - T')^2U_{11} - U_{22} + T''U_1}. \quad (20)$$

In this static model, if the utility function is log and the tax rate is constant, then the income elasticity of donations would be equal to 1; if the tax is progressive, then the elasticity should exceed 1.<sup>8</sup> Using a progressive tax function regarded by Gouveia and Strauss (1994) as representing the U.S. data very closely, we find that the income elasticity of donations in the static model does indeed exceed unity. Finally, since the short-run response of donations to a permanent change in income  $G'_t$  is greater in a dynamic model than the response  $G'$  in a static model because of changes in savings, the chance for the income elasticity of donations to exceed 1 is even greater in the former than in the latter. A larger-than-one permanent income elasticity of donations in the short run is consistent with the empirical finding in Randolph (1995).

#### 4. A permanent change in the tax system

To facilitate the analysis of the effects of tax reform on donations, let us introduce a transformation of the tax function  $F(Y^T, \theta)$ , where  $Y^T = rW + Y - G$  represents taxable income and  $\theta$  measures the progressivity of the tax code, which has the

following properties:

$$\frac{dF(Y^T, \theta)}{d\theta} = F_1 \frac{\partial Y^T}{\partial \theta} + F_2, \quad 0 < F_1 \equiv \frac{\partial F}{\partial Y^T} < 1, \quad F_2 \equiv \frac{\partial F}{\partial \theta} > 0, \quad (21)$$

$$F_{11} \equiv \frac{\partial F^2}{\partial^2 Y^T} \geq 0, \quad F_{12} \equiv \frac{\partial F^2}{\partial Y^T \partial \theta} > 0, \quad (22)$$

and

$$F(Y^T, \theta)|_{\theta=0} = T(rW + Y - G). \quad (23)$$

By construction,  $F_1$  in (21) is the marginal tax rate, which increases with the tax parameter  $\theta$  and is a nondecreasing function of disposable income  $Y^T$  in (22). Also, when  $\theta$  increases, the overall change in the tax payment of a household in (21) has two components: the direct one,  $F_2$  for the relevant given amount of disposable income, and an indirect one,  $F_1 \partial Y^T / \partial \theta$  which results from the change in disposable income  $Y^T$  (and the corresponding change in donations). Note that since  $F_2 = F_2(Y^T, \theta)$  is a function of disposable income in general, it is not necessarily lump sum. Further, at  $\theta = 0$ , the transformation function  $F$  is the same as the original tax function as seen in (23). In this setting, a reduction in the marginal income tax rate through tax reform can be viewed as a fall in  $\theta$ .

Differentiating equations (9)–(11) with respect to  $\theta$ , we establish the following results:

**Proposition 2.** (a) For  $T'' > 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , a permanent cut in the marginal tax rate raises long-run consumption  $C$ , donations  $G$ , and wealth  $W$ . In the short run, however,  $G$  can fall following such a tax cut if the direct impact on tax payments of the tax cut is small. (b) For  $T'' = 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , a



permanent cut in the marginal tax rate raises consumption  $C$ , but has an ambiguous effect on donations  $G$ .

**Proof.** Part (a). Let us first consider the case when  $T'' > 0$ . Differentiating (11) with respect to  $\theta$  gives

$$0 = -\beta r[F_{11}(Y^T, \theta) \times (rW' - G') + F_{12}(Y^T, \theta)]. \quad (24)$$

Because  $F_{11}|_{\theta=0} = T'' > 0$  under the progressive tax system, and  $F_{12} > 0$ , we have

$$F_{11}(Y^T, \theta) \times (rW' - G') + F_{12}(Y^T, \theta) = 0, \quad (25)$$

and  $rW' - G' < 0$ . Differentiating (9) with respect to  $\theta$  yields

$$C' = rW' - F_1(Y^T, \theta) \times (rW' - G') - F_2(Y^T, \theta) - G'. \quad (26)$$

When  $\theta = 0$ ,  $F_1 = T'$  and  $C'$  can be signed by

$$C' = (1 - T') \times (rW' - G') - F_2|_{\theta=0} < 0, \quad (27)$$

since  $0 < T' < 1$ ,  $rW' - G' < 0$ , and  $F_2 > 0$ .

Differentiating (10) with respect to  $\theta$  produces

$$-[(rW' - G')F_{11}|_{\theta=0} + F_{12}|_{\theta=0}]U_1 + (1 - T')[U_{11}C' + U_{12}G'] = U_{21}C' + U_{22}G'. \quad (28)$$

Equations (25), (27), and (28) imply that

$$G' = \frac{U_{12} - (1 - T')U_{11}}{U_{22} - (1 - T')U_{12}} \left[ (1 - T') \frac{F_{12}|_{\theta=0}}{T''} + F_2|_{\theta=0} \right]. \quad (29)$$

As long as  $T'' > 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , we have  $G' < 0$ , and  $W' < G'/r < 0$ .

To examine the short-run reaction of donations, let us look into the initial period with a predetermined level of initial wealth  $W_0$ . Differentiating (2) and (5) in time 0 with respect to  $\theta$  gives

$$C'_0 = \frac{(F_2 + W'_1)[(1 - T'_0)U_{12}(0) - U_{22}(0) + T''_0 U_1(0)] + (1 - T'_0)F_{12}U_1(0)}{(1 - T'_0)^2 U_{11}(0) - 2(1 - T'_0)U_{12}(0) - T''_0 U_1(0) + U_{22}(0)}, \quad (30)$$

and

$$G'_0 = \frac{(F_2 + W'_1)[U_{12}(0) - (1 - T'_0)U_{11}(0)] - F_{12}U_1(0)}{(1 - T'_0)^2 U_{11}(0) - 2(1 - T'_0)U_{12}(0) - T''_0 U_1(0) + U_{22}(0)}, \quad (31)$$

where all  $F_i$  and  $F_{ij}$  are evaluated at  $\theta = 0$  and  $t = 0$ . Here, the sign of  $C'_0$  is not known in general. However, for  $W'_1 < 0$ ,  $T'' > 0$ ,  $0 < T' < 1$ , and  $U_{12} \geq 0$ , we have  $G'_0 > 0$  if  $F_2$  is sufficiently small.

Part (b). See the derivations in the Appendix.  $\square$

Proposition 2 implies that while a reduction in the marginal income tax rate is likely to reduce donations in the short-run, it raises both private consumption and donations in the long run. Our model implications are consistent with the facts documented in Figure 1 of Andreoni (2006). Andreoni shows that donations as a percentage of personal income jumped in 1986 right before the tax cut in 1987 and then fell considerably in 1987. It kept falling till 1996 and climbed steadily up to 2001.

According to (8), which concerns saving behavior, the tax cut increases the after-tax rate of return to saving  $r(1 - T'_{t+1})$ . To regain the balance of this equation, the marginal rate of substitution between current and future consumption has to increase accordingly. Consequently, a household facing a tax cut will increase its future consumption relative to current consumption through increasing saving. Hence,

consumption increases over time after a tax cut.

Less intuitive is the long-run increase in donations following the tax cut. As shown in equation (5), the tax cut makes donations more expensive compared with consumption, as in a static model. Hence, the increase in after-tax income resulting from the tax cut does not necessarily lead to a net increase in donations. The two conditions that are essential for the long-run response of donations to the tax cut to be positive are equation (11) and a progressive tax system—i.e., one in which the marginal tax rate increases as households accumulate more assets. Given  $\beta$  and  $r$ , in the long run the rise in the marginal tax rate driven by the increase in  $W$  will eventually cancel out the fall in the marginal income tax rate caused by the tax cut. As a result, the tax cut alters neither the marginal tax rate paid by the household nor the relative price of donations in the long run. Because households accumulate more assets in the new steady state, both consumption and donations increase in the long run. The implied relationship between tax cut and asset accumulation is consistent with the vast majority of the existing literature on taxation and savings surveyed by Bernheim (2002), Boadway and Wildasin (1994) and Kotlikoff (1984). Intuitively, as shown by equation (8), the agent will shift resources from period  $t$  to period  $t + 1$  if he expects the tax rate will decrease in  $t + 1$ . As a result, saving increases in  $t$ .

The positive long-run response of donations to a tax-rate cut appears to be inconsistent with the strong negative price elasticity of donations found by the existing empirical studies. We suggest this is because donations respond differently in the short run than in the long run, and the empirical studies can only document the short-run response. Proposition 2 suggests that donations could indeed decline in the

short run following the tax cut if the direct impact on tax payments of the tax cut is small in the case of revenue-neutral tax reform. As revenue neutrality is usually an important dimension in tax-reform proposals, the prediction in Proposition 2 that donations will decline right after marginal tax rate cuts may turn out to be important in practice. Let us use as an example the Tax Reform Act of 1986, which is said to be roughly revenue neutral by Ballentine (1992). Auten et al. (1992, Table 3) show that, on average, the fraction of household after-tax income donated to charitable organizations was 3.5% in 1986, fell to 3.3% in 1987, and fell further to 3.2% in 1988. The 1987 level was recovered in 1990.

The prediction that household wealth will increase following a tax-rate cut is also supported by data from the U.S. Consumer Expenditure Survey (CES). Approximately, the average value of household assets increased by 18%, from \$14,308 (in current dollars) in 1986 to \$16,894 in 1987.<sup>9</sup> During the same period, the nominal average household annual income increased by only 7%, from \$25,460 to \$27,326.

In contrast to the dynamic model used here, in a static model, households cannot adjust their asset holdings. By setting  $W'_1 = 0$ , we create the static version of equation (31):

$$G'_0 = \frac{F_2[U_{12}(0) - (1 - T'_0)U_{11}(0)] - F_{12}|_{\theta=0}U_1}{(1 - T'_0)^2U_{11} - 2(1 - T'_0)U_{12} - T''_0U_1 + U_{22}}. \quad (32)$$

Respectively,  $F_2$  and  $F_{12}$  are the direct impact on the tax payments and the change in the marginal tax rate due to tax reform; thus, the first part of the numerator of equation (32) represents the response of donations to the direct impact of tax reform on tax payments, and the second part reflects the response of donations to the change in the tax rate itself. This equation is comparable with Randolph's (1995) equation

(12), which decomposes the net elasticity of donations with respect to a permanent proportional change in all marginal tax rates into two parts: the permanent price elasticity and the permanent income elasticity. If the price elasticity dominates, then donations will decrease after a tax cut. Conversely, if the income effect through the change in the tax payment dominates, donations will increase.

However, a comparison between equations (31) and (32) indicates that in a dynamic model, the impact of changes in tax payments is mitigated by the opposite movement in savings. Therefore, a tax-rate cut is more likely to generate a short-run decline in donations in a dynamic model than to reduce donations in a static model. Because  $W'$  is negatively correlated with  $F_2$  but positively correlated with  $-T'_0$ , this observation suggests that the price elasticity could be overestimated while the income elasticity underestimated in a model that does not control for variations in savings  $W'$ .

Finally, if the tax rate is flat, a permanent decline in it raises consumption but has an ambiguous effect on donations. This is because the income effect and price effect of the tax cut are of the same positive sign for consumption, but they are of opposite signs for donations. In addition, there is no general tendency for any difference in the responses between the short run and the long run. These results are very different from those under a progressive tax system.

## 5. Numerical examples

While the results in the case with a flat tax rate is simple, the results in the case with a progressive tax are complicated because of the different patterns of responses

between the short run and the long run. In this section we provide numerical examples to illustrate how households respond to an unexpected permanent income rise as well as to a tax cut over the entire equilibrium path with a progressive tax. In order to do so, let us assume a simple log utility function  $U_t = \alpha \ln C_t + (1 - \alpha) \ln G_t$  and adopt a parameterized tax function from Gouveia and Strauss (1994):  $T(Y_t^T) = bY_t^T - b[(Y_t^T)^{-\rho} + s]^{-1/\rho}$ . In this tax function,  $Y^T \equiv rW + Y - G$  refers to taxable income while  $b$ ,  $s$  and  $\rho$  are positive constants. According to the estimates in Gouveia and Strauss (1994, Table 1, p. 323), we choose the following benchmark parameterization for the tax function:  $b = 0.479$ ,  $s = 0.022$  and  $\rho = 0.817$ . The parameterization of the preferences is assumed to be:  $\alpha = 0.9$  and  $\beta = 0.9753$ . Also, we choose a benchmark level of the exogenous income  $Y = 15$  (the unit is in \$,000) and a constant interest rate  $r = 3\%$ . The steady state solution to the benchmark case is  $C = 12.98239394858$ ,  $G = 1.70873674193$  and  $W = 34.21657257798$ . We focus on small numbers in the examples for ease of computation.

Since the number of equations characterizing the solution of the entire path in the recursive model is infinitely large, various methods of approximation have been used in the literature for numerical solutions in such models. Our method has the following features. First, assume that the solution to the household problem takes  $n$  periods to reach the steady state and solve the system of equations in the  $n$  periods simultaneously. We then consider  $n + 1$  periods and compare the welfare gain over the  $n$ -period solution. We stop the process if the welfare gain, measured by the equivalent amount of consumption added to each period, is less than 1/10,000 in units of consumption.

We consider two cases of changes. The first case is a change of exogenous income  $Y$  from 15 to 15.15, or a one percent rise. The second case is a tax cut by reducing  $b$  and  $\rho$  to new levels 0.47895 and 0.81695 (we have verified these changes indeed cause the marginal tax rate to fall given the parameterizations we use). The changes are assumed to be small in order to make computation manageable because larger changes will take many more periods to adjust (too many to handle on a personal computer). Before each change, we assume that the household initial wealth starts from the benchmark steady state level. We then compute for the transition process toward the new steady state after each change.

The results are plotted in Figures 1 and 2. In Figure 1, we report the transition from an income rise to the new steady state in more than 40 periods. In response to the income rise, the levels of consumption and donations rise initially and eventually fall to the same steady state levels while the level of wealth declines monotonically to its new steady state level. In particular, the income elasticity of donations is found to be over 1.4 for the first ten periods in this simulation. The numerical results are consistent with our analytical results in Section 3. In Figure 2, we report the transition from a tax cut to the new steady state. In response to the tax cut, the levels of consumption and donations fall initially and eventually rise to higher levels in the new steady state while the level of wealth rises monotonically to its new steady state. The numerical results in Figure 2 are also consistent with our analytical results. Notice that, when wealth falls (rises) in the transition, there is dissaving (saving) corresponding to higher (lower) levels of consumption and donations.

## 6. Concluding remarks

Feldstein and Clotfelter (1976, p.25) state that “the key empirical question is the extent to which alternative tax treatments would affect the volume and distribution of charitable contributions.” Clearly, this question cannot be fully answered purely by empirical analysis if we are interested in both the short-run and long-run effects on donations of changes in the marginal income tax rate and in incomes, because the long-run effects are not observable. For many observers, a tax cut that only temporarily reduces donations definitely causes less concern than one that leads to a permanent decrease.

In this paper, we have used a recursive dynamic model to analyze theoretically the long-run and short-run responses of donations to permanent changes in income and in tax rates. The novel contribution of this analysis to the economic literature on donations is that the responses for the long run and the short run are remarkably different. The key factor behind this difference is the fact that permanent changes in income and in tax rates also affect saving behavior, which moves household expenditures across periods. Our analysis shows that, while donations can be very responsive to changes in permanent income in the short run as the empirical literature suggested, over time, they return to their original level. Moreover, although donations may fall following a permanent tax-rate cut in the short run, as suggested by existing empirical studies in the literature, they will eventually rise beyond their original level. The policy implication of this result should be, to some extent, a relief to charitable organizations when it comes to assessing their potential revenues following tax-cut reforms.



## Appendix

**Part (b), Proof of Proposition 1.** With  $T''(\cdot) = 0$ , for simplicity, let us assume  $T(Y^T) = \tau Y^T$  and hence  $T' \equiv \tau$  where  $Y^T$  refers to taxable income. It is important to note that with  $T'' = 0$  there is only one value of the tax rate  $\tau$  that can be consistent with a steady state  $1 = \beta[1 + r(1 - \tau)]$  according to (11). We thus need to incorporate the transversality condition in (3) so as to prevent wealth from running away from finite values in the absence of steady states. Define  $R \equiv R(\tau) = 1 + r(1 - \tau) > 1$  for notational ease. Making successive substitutions on the budget constraint (2) and imposing the transversality condition yields the following lifetime budget constraint:

$$W_0 R(\tau) + \sum_{t=0}^{\infty} \frac{(1 - \tau)Y_t}{R^t(\tau)} = \sum_{t=0}^{\infty} \frac{C_t + (1 - \tau)G_t}{R^t(\tau)}. \quad (\text{A.33})$$

Assign it with a multiplier  $\lambda$  and define the Lagrangian for the household problem below:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t, G_t) + \lambda \left[ W_0 R(\tau) + \sum_{t=0}^{\infty} \frac{(1 - \tau)Y_t}{R^t(\tau)} - \sum_{t=0}^{\infty} \frac{C_t + (1 - \tau)G_t}{R^t(\tau)} \right] \quad (\text{A.34})$$

The first-order conditions are  $\beta^t U_1(t) = \lambda/R^t(\tau)$  and  $\beta^t U_2(t) = \lambda(1 - \tau)/R^t(\tau)$ , leading to the optimal conditions  $(1 - \tau)U_1(t) = U_2(t)$  and  $U_1(t)/U_1(t + 1) = \beta R(\tau)$  paralleling those in (5) and (8). Differentiating these optimal conditions and the constraint (A.33) with respect to  $Y_t$  for all  $t \geq 0$  gives:

$$C'_t = \frac{U_{22}(t) - (1 - \tau)U_{12}(t)}{(1 - \tau)U_{11}(t) - U_{12}(t)} G'_t, \quad (\text{A.35})$$

$$U_{11}(t)C'_t + U_{12}(t)G'_t = \beta R(\tau)[U_{11}(t + 1)C'_{t+1} + U_{12}(t + 1)G'_{t+1}], \quad (\text{A.36})$$

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{C'_t + (1-\tau)G'_t}{R^t(\tau)} &= \sum_{t=0}^{\infty} \frac{1-\tau}{R^t(\tau)} \\
&= \frac{R(\tau)(1-\tau)}{R(\tau)-1} > 0.
\end{aligned} \tag{A.37}$$

Equation (A.35) states that  $C'_t$  and  $G'_t$  should share the same sign for all  $t \geq 0$  given that  $U_{ii} < 0$  and  $U_{12} \geq 0$ . Combining this statement with (A.37), it follows that the present value of the overall spending on consumption and donations must be positive when summing over all times. Further, combining (A.35) and (A.36) and arranging terms leads to

$$\frac{U_{11}(t)U_{22}(t) - U_{12}^2(t)}{(1-\tau)U_{11}(t) - U_{12}(t)} G'_t = \frac{U_{11}(t+1)U_{22}(t+1) - U_{12}^2(t+1)}{(1-\tau)U_{11}(t+1) - U_{12}(t+1)} G'_{t+1}. \tag{A.38}$$

Note that  $U_{11}U_{22} - U_{12}^2 > 0$  is part of the assumption that  $U(C, G)$  is strictly concave, and that the denominators on both sides must have the same sign. It thus implies that  $G'_t$  and  $G'_{t+1}$  and hence all  $G'$ 's in every period  $t \geq 0$  must have the same sign. Clearly, this result applies to  $C'$ 's in all periods too since  $\text{sign}(C') = \text{sign}(G')$ . Combining all arguments together,  $C'_t > 0$  and  $G'_t > 0$  for all  $t \geq 0$ .  $\square$

**Part (b), Proof of Proposition 2.** Now, let us consider the case when  $T'' = 0$ . We make the same assumption about the tax function as we did in part (b) in the proof of Proposition 1. Differentiating the optimal conditions and the constraint (A.33) therein with respect to  $\tau$  yields:

$$C'_t = \frac{-U_1(t) + [(1-\tau)U_{12}(t) - U_{22}(t)]G'_t}{U_{12}(t) - (1-\tau)U_{11}(t)}, \tag{A.39}$$

$$U_{11}(t)C'_t + U_{12}(t)G'_t = -r\beta U_1(t+1) +$$

$$\beta R(\tau)[U_{11}(t+1)C'_{t+1} + U_{12}(t+1)G'_{t+1}], \tag{A.40}$$

$$\begin{aligned}
-rW_0 - \sum_{t=0}^{\infty} \frac{Y_t}{R^t(\tau)} + \sum_{t=0}^{\infty} \frac{rt(1-\tau)Y_t}{[R(\tau)]^{2t-t+1}} &= \sum_{t=0}^{\infty} \frac{C'_t + (1-\tau)G'_t}{R^t(\tau)} - \\
\sum_{t=0}^{\infty} \frac{G_t}{R^t(\tau)} + \sum_{t=0}^{\infty} \frac{[C_t + (1-\tau)G_t]rt}{[R(\tau)]^{2t-t+1}}. &
\end{aligned} \tag{A.41}$$

Group the last terms on both sides of (A.41) together and simplify it using (2) as follows:

$$\begin{aligned}
-\sum_{t=0}^{\infty} \frac{rt[(1-\tau)Y_t - C_t - (1-\tau)G_t]}{[R(\tau)]^{t+1}} &= \sum_{t=0}^{\infty} \frac{rt}{R^t(\tau)} \left( W_t - \frac{W_{t+1}}{R} \right) \\
&= \frac{r}{R} \left( W_1 - \frac{W_2}{R} \right) + \frac{r}{R^2} \left( W_2 - \frac{W_3}{R} \right) + \frac{r}{R^3} \left( W_3 - \frac{W_4}{R} \right) + \dots \\
&\quad + \frac{r}{R^2} \left( W_2 - \frac{W_3}{R} \right) + \frac{r}{R^3} \left( W_3 - \frac{W_4}{R} \right) + \frac{r}{R^4} \left( W_4 - \frac{W_5}{R} \right) + \dots \\
&\quad + \frac{r}{R^3} \left( W_3 - \frac{W_4}{R} \right) + \frac{r}{R^4} \left( W_4 - \frac{W_5}{R} \right) + \frac{r}{R^5} \left( W_5 - \frac{W_6}{R} \right) + \dots \\
&\quad + \dots \\
&= \frac{r}{R} W_1 + \frac{r}{R^2} W_2 + \frac{r}{R^3} W_3 + \dots \quad (\text{cancellation and } W_{\infty}/R^{\infty} \rightarrow 0) \\
&= \sum_{t=1}^{\infty} \frac{r}{R^t(\tau)} W_t.
\end{aligned}$$

Substituting it back into (A.41) and using  $Y_t^T = Y_t - G_t + rW_t$  provides:

$$\sum_{t=0}^{\infty} \frac{C'_t + (1-\tau)G'_t}{R^t(\tau)} = - \sum_{t=0}^{\infty} \frac{Y_t^T}{R^t(\tau)} < 0. \tag{A.42}$$

This equation and (A.39) lead to

$$\sum_{t=0}^{\infty} \frac{1}{R^t} \left( \frac{2(1-\tau)U_{12}(t) - U_{22}(t) - (1-\tau)^2 U_{11}(t)}{U_{12}(t) - (1-\tau)U_{11}(t)} \right) G'_t =$$

$$\sum_{t=0}^{\infty} \frac{1}{R^t} \left( \frac{U_1(t)}{U_{12}(t) - (1-\tau)U_{11}(t)} - Y_t^T \right), \quad (\text{A.43})$$

$$\sum_{t=0}^{\infty} \frac{1}{R^t} \left( \frac{2(1-\tau)U_{12}(t) - U_{22}(t) - (1-\tau)^2 U_{11}(t)}{(1-\tau)U_{12}(t) - U_{22}(t)} \right) C'_t =$$

$$- \sum_{t=0}^{\infty} \frac{1}{R^t} \left( \frac{(1-\tau)U_1(t)}{(1-\tau)U_{12}(t) - U_{22}(t)} + Y_t^T \right) < 0. \quad (\text{A.44})$$

While the right-hand side of (A.44) is negative, the right-hand side of (A.43) is ambiguous. Also note that the coefficients on  $C'$  and  $G'$  in the left-hand sides of these two equations are positive. Thus, a tax cut raises consumption but has ambiguous effects on donations, when considering all time periods together. Also, from (A.40), the signs of  $C'_t$ ,  $G'_t$ ,  $C'_{t+1}$  and  $G'_{t+1}$  may or may not be the same, unlike in the case of a permanent income change.

The results in Propositions 1 and 2 are derived with the assumption  $U_{12} \geq 0$ . However, even if  $U_{12} < 0$ , the results will still remain valid, provided that the magnitude of  $U_{12}$  is relatively small compared to that of  $U_{ii}$ .  $\square$

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## Footnotes

\* We would like to thank three anonymous referees for their valuable comments.

1. Andreoni (2006) and Clotfelter (2002) provide excellent summaries on the current state of the literature, including an extensive set of up-to-date references.
2. There is no consensus in the literature on the sizes of these elasticities, however. These studies use the U.S. tax return data for less than 20 years and the difference in the reported estimates of the elasticities may stem mainly from how the estimation is specified.
3. In their finding, the elasticities of donations to temporary changes in income and in the tax price are far below unity. It is not surprising that a model with an infinite horizon and with a concave utility function shows an inelastic response of donations to such temporary changes. This is because it is optimal for households to spread temporary gains or losses in household resources over many periods through saving or dissaving.
4. However, the recursive model we use here may also be interpreted as an overlapping-generations model whereby agents live one period in childhood and one period in adulthood and are connected by parental altruism in a dynastic family. Accordingly, one may interpret savings in the infinitely lived agent model as bequests parents give to their children in the dynastic, overlapping generations model. The results in this paper will be the same regardless of whether we assume infinitely-lived agents or overlapping generations of two-period lived agents in dynastic families.

5. The tax return data used in the related empirical studies are micro in nature, containing information on income and donations across households and across years. Our treatment of the household problem is carried out with this feature of data in the literature in mind.
6. Clearly, we cannot rule out the possibility that  $U_{12}$  takes a value that leads to  $C'_t \leq 0$  and  $G'_t \leq 0$ . Providing a reasonable value for  $U_{12}$  is beyond the scope of this paper and worth further empirical investigations.
7. This result is similar to the negative effect of the RRSPs on saving found by Ragan (1994) under a progressive tax system. According to him, a tax delay by the RRSPs generates both a substitution effect and an income effect. Under a progressive income tax system, the substitution effect on saving is negative in his model. With the tax delay, individuals may also expect a rise in after-tax lifetime income and hence consume more and save less. Moreover, under a progressive tax system, the increase in permanent income will increase the agent's effective tax rate. Our model's prediction on savings is also broadly consistent with the existing studies on the impact of taxation on savings as surveyed by Boadway and Wildasin (1994) and Bernheim (2002).
8. Suppose that  $U = \alpha \ln C + (1 - \alpha) \ln G$ . Equation (20) then becomes

$$G' = \frac{(1 - \alpha)(1 - T') + \alpha T'' G}{1 - T' + \alpha T'' G} \geq 1 - \alpha \quad \text{with } T'' \geq 0.$$

Also, suppose that  $T'' = 0$  and  $T' = \tau$  where  $\tau$  is a constant. Then, the simplified budget constraint  $C = (Y - G)(1 - \tau)$  and the first-order condition



(5) would imply  $G/Y = (1 - \alpha)$  and  $C/Y = \alpha(1 - \tau)$ . Combining these with the above expression for  $G'$ , we find that the income elasticities of consumption and donations are both equal to unity:  $G'Y/G = C'Y/C = 1$ . With  $T'' > 0$  in the above expression for  $G'$ , however, the income elasticity of donations is greater than 1, i.e.  $G'Y/G > 1$ , so long as  $G/Y$  is close to  $1 - \alpha$ .

9. Because the CES does not give the average value of household assets, we approximate it by dividing incomes from assets (including interest, dividend, and renting) by the annual federal fund rate of the year. Specifically, the nominal values of average incomes from assets are \$973 in 1986 and \$1115 in 1987, and the corresponding federal fund rates are 6.80% and 6.66%. However, the CES does give the year-to-year net changes in household assets, \$1,195 in 1986 and \$3,122 in 1987, which also suggest that households accumulated more wealth after the Tax Reform Act of 1986 was implemented.

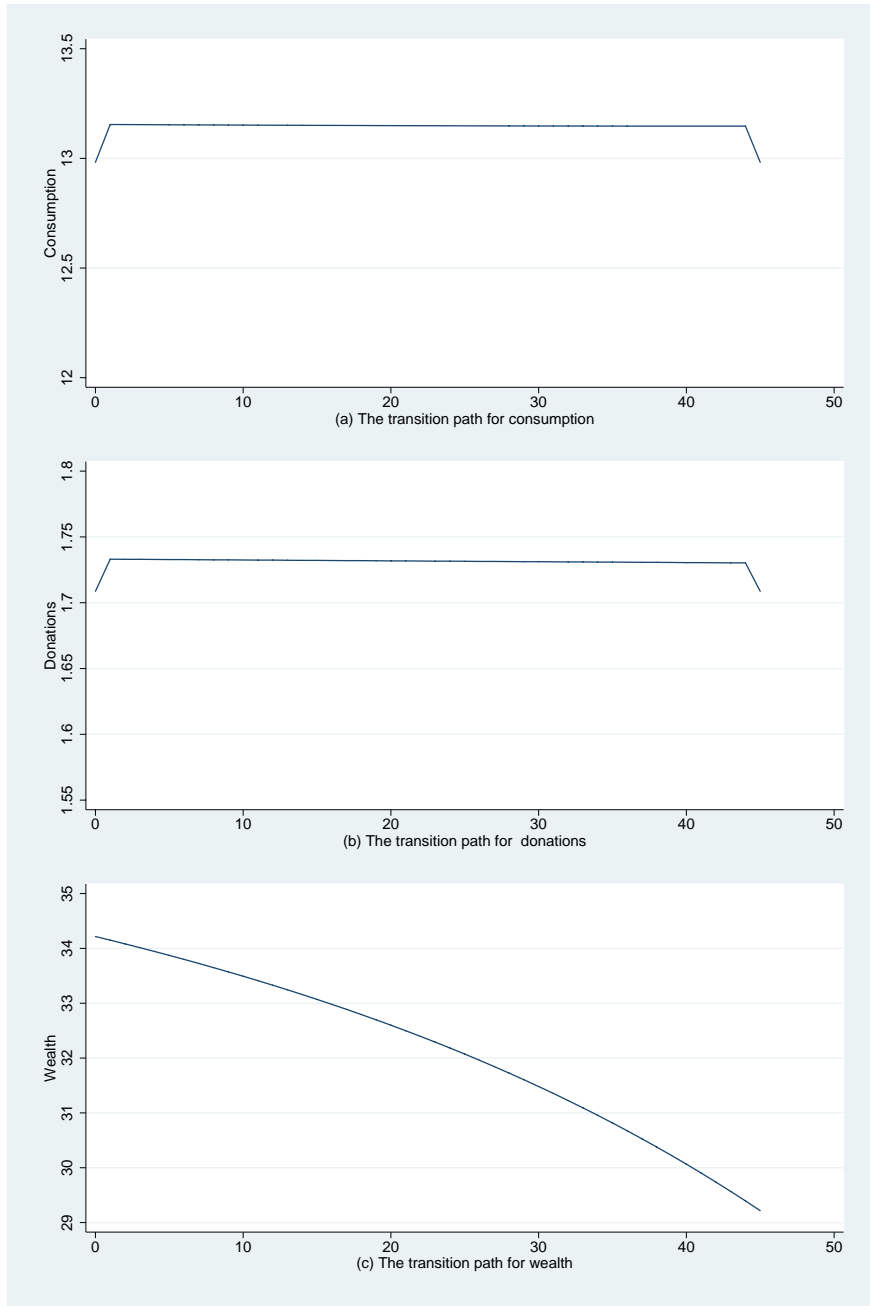


Figure 1: Transition from a permanent income rise

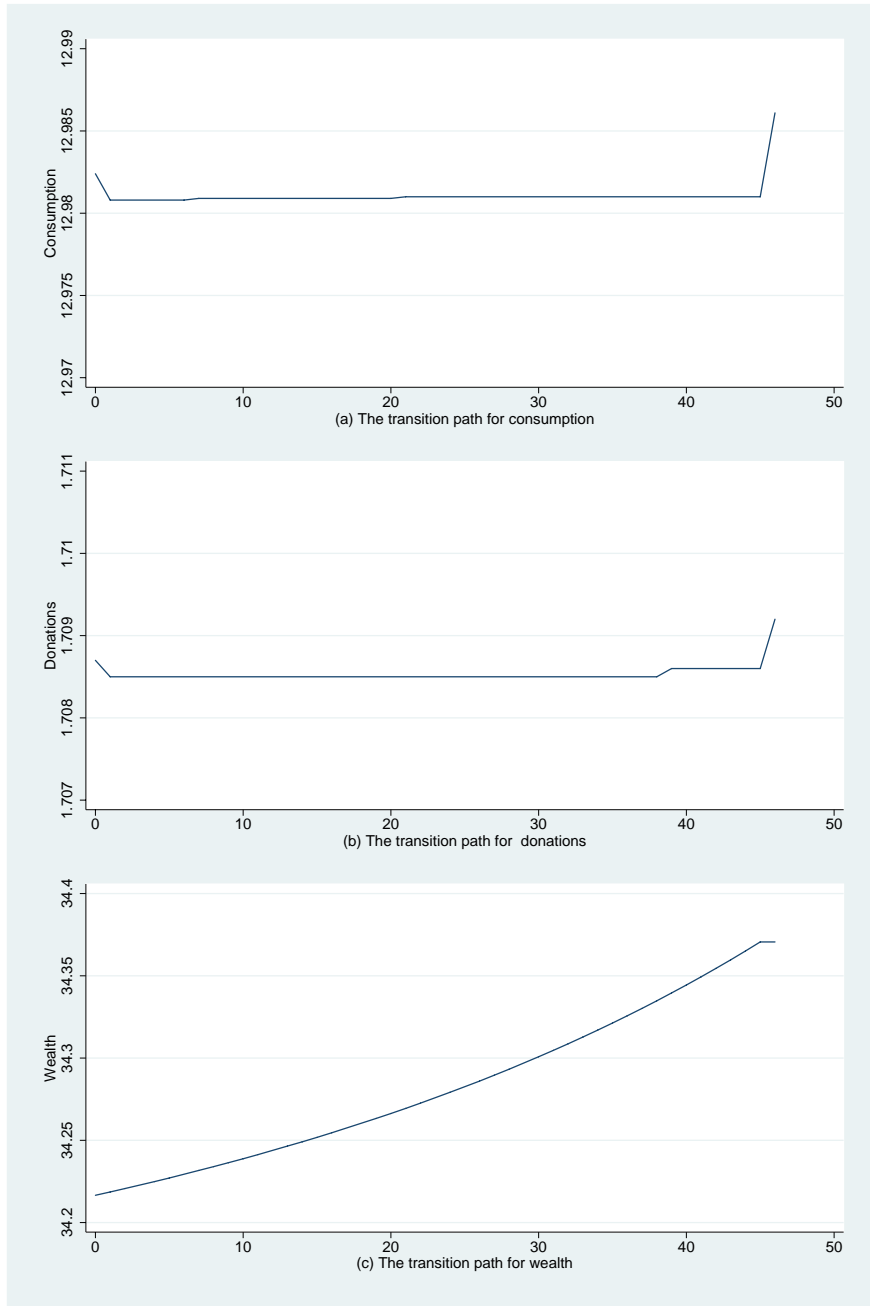


Figure 2: Transition from a tax cut