Subsidies in an R&D growth model with elastic labor

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Abstract

This paper compares different subsidies in an R&D growth model with competitive suppliers of a final good and monopolistic suppliers of intermediate goods. Unlike existing studies with lump-sum taxes and fixed labor, we assume distortionary taxes and elastic labor, finding some new insights. First, subsidizing R&D investment is more effective than subsidizing final output or subsidizing the purchase of intermediate goods in terms of promoting growth. Second, in terms of raising welfare, the R&D subsidy may also be more effective than the other subsidies and all of them are dominated by their mix, but none can achieve the social optimum.

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1. Introduction

R&D activities for innovations, a major driving force for growth, are subsidized in many industrial countries and receive increasing attention in economic studies. The rationale for government intervention involving R&D activities originates from the fact that innovators of new goods face knowledge spillovers and difficulties in appropriating the benefits of innovations (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Jones and Williams, 2000). Due to this externality, there would be too little incentive to engage costly innovations without government intervention, as innovators do not internalize the gains from their innovations. A typical form of government intervention dealing with this R&D externality is to grant monopoly rights to innovators in such forms as patents and trademarks. However, even if monopoly protection is granted permanently to successful innovators selling their goods in the model of Barro and Sala-i-Martin (1995) with expanding varieties of intermediate goods via R&D, the decentralized equilibrium always suffers too little R&D investment and thus too slow growth.[1] In such a model, pricing above marginal costs is necessary for R&D to break even on the one hand, but reduces the demand for intermediate goods below the first-best level on the other hand. That is, granting monopoly rights alone does not eliminate under-investment in R&D in the presence of the R&D externality.

In order to internalize the R&D externality and correct the distortion of the monopoly pricing, various types of subsidies have been examined in the literature with lump-sum taxes and fixed labor supply. As expected, the R&D subsidy can indeed promote R&D investment and growth (e.g., Barro and Sala-i-Martin, 1995; Davidson and Segerstrom, 1998). Much less obvious is that the R&D subsidy is dominated by other types of subsidies in terms of social welfare (Barro and Sala-i-Martin, 1995): Subsidizing either final output produced by competitive firms or the purchase of intermediate goods produced by monopolistic firms can achieve the social optimum, but subsidizing R&D, though also welfare improving, cannot. This is somewhat surprising as the actual government policy has tended to rely on R&D subsidies, e.g. the United States has long had an R&D subsidy

[1] Over-investment in R&D may also occur in models with different settings but much empirical evidence supports under-investment in R&D (e.g., Cohen and Levin, 1991; Griliches, 1992; Nadiri, 1993; Jones and Williams, 1998).

in place. One reason seems to be that the R&D subsidy is an “inexpensive” tool in terms of lost revenue, which can only be made up with distortionary taxes, since lump-sum taxes assumed in the related studies mentioned above are hard to implement.

Once limiting our funding options to distortionary taxes and allowing for elastic labor supply, several interesting questions arise: First, do these subsidies still stimulate growth and improve welfare? Second, are the subsidies to final or intermediate products still better than the R&D subsidy? Third, can these subsidies completely eliminate the distortion of the monopoly pricing and internalize the R&D externality to achieve the social optimum in a decentralized setting? Finally, if the social optimum cannot be achieved, can different subsidies be combined to generate a better outcome than using a single subsidy?[2]

The objective of this paper is to answer these questions. Specifically, using distortionary taxes we examine the different types of subsidies and their combinations in terms of their effects on growth and welfare. In order to do so, we extend the R&D growth model of Barro and Sala-i-Martin (1995) with variety expansion by incorporating elastic labor supply.

Our different approach brings to light several new insights. First, subsidizing R&D investment is more effective than subsidizing final output or subsidizing the purchase of intermediate goods in terms of promoting growth. This is because the former directly reduces the cost of R&D investment at a lower tax cost compared to the latter forms of subsidies. The lower tax revenue for the R&D subsidy to achieve any given growth target than other subsidies does give the R&D subsidy an advantage when the tax has to be distortionary.[3] Second, in terms of raising welfare, the R&D subsidy may also be more effective than the other subsidies and all of them are dominated by their mix, but none can achieve the social optimum, because of the relative strength and weakness associated with the different types of subsidies. As mentioned above, the R&D subsidy tends to be more effective in engendering dynamic gains at a lower tax cost than the other types of subsidies, in

[2]This is not an issue in Barro and Sala-i-Martin (1995) because a single subsidy, either to final output or to the purchase of intermediate goods, can achieve the social optimum with the aid of lump-sum taxes.

[3]When the tax is lump-sum in Barro and Sala-i-Martin (1995), this lower tax revenue needed to finance one subsidy than the others does not matter.

a direction of mitigating the under-investment caused by the R&D externality. As in the literature, however, the R&D subsidy is less effective than the other subsidies in reducing the efficiency loss associated with monopoly pricing.

The rest of this paper is organized as follows. The next section sets up the model and solves firms and households' optimization problems. Section 3 describes the social planner's problem, which is to be compared with decentralized solutions. Section 4 derives the results. The last section concludes.

2. The model

The model is an extension of the endogenous growth model with variety expansion in Barro and Sala-i-Martin (1995, Chapter 6) by considering a labor-leisure choice and distortionary taxes. In this model, technological progress results from intentional investment in R&D that aims at creating new types of intermediate goods for final production.

2.1 Households

The economy is populated with a continuum of identical infinitely-lived households with a (constant) mass L. Each household has one unit of time which is allocated between leisure l and production $(1-l)$. The representative household's preferences are defined over an infinite horizon:

$$U_0 = \int_0^\infty \left[\frac{(cl^\epsilon)^{1-\theta} - 1}{1-\theta} \right] e^{-\rho t} dt, \quad \theta > 0, \quad \rho > 0, \quad \epsilon > 0 \tag{1}$$

where c is consumption, ρ the rate of time preference, ϵ the taste for leisure, and θ the inverse of the constant elasticity of intertemporal substitution. To economize notations, we omit time subscript t whenever no confusion may arise.

Household income, from assets and work, is allocated between consumption and saving:

$$\dot{a} = ar + (1-\tau)w(1-l) - c, \tag{2}$$

where τ is the tax rate on labor income, a the amount of assets, $\dot{a}$ the time derivative of a (or investment), r the real interest rate, and w the wage rate. Here, we abstract from taxing interest income because it would further complicate the difficult welfare analysis. If interest income were taxed at the same rate as wages are taxed, the tax base would be broadened, but the after-tax rate of return on investment in R&D would fall. To generate the same level of tax revenue for subsidies, the broadened tax base reduces the tax rate on wage income and thus may lead to higher labor supply (hence higher demand for the intermediate goods), while the reduced rate of return on R&D investment reduces R&D investment. The net effects on growth and welfare of the uniform income tax are therefore unclear. However, with this uniform income tax, all the subsidies that we consider would still stimulate R&D activities and growth, in a direction to mitigate the efficiency loss originating from the R&D externality. Thus, the results in this alternative setting would be essentially similar to what the wage tax could achieve, though quantitatively different.

The household chooses consumption c and leisure l to maximize its utility in (1) subject to the budget constraint (2), taking the interest and wage rates as given. Solving this problem yields:

$$\gamma \equiv \dot{c}/c = (r - \rho)/\theta, \tag{3}$$

$$c = (1 - \tau)wl/\epsilon. \tag{4}$$

Equation (3) is standard in the literature, linking consumption growth positively to the rate of return on assets (r) and the willingness of intertemporal substitution ($1/\theta$) but negatively to the rate of time preference (ρ). Equation (4) captures the relationship between consumption and leisure. Finally, the transversality condition is $\lim_{t\to\infty}\left\{a \cdot \exp\left[-\int_0^t r_v dv\right]\right\} = 0$, i.e. neither debt nor asset will be left at the end of the planning horizon.

2.2 Final production

A final good is produced by a large number of identical competitive firms. A firm i uses X_{ij} units of intermediate good j and L_i units of labor to produce Y_i units of the final good according to:

$$Y_i = F(X_{ij}, L_i) \equiv AL_i^{1-\alpha} \int_0^N {X_{ij}}^{\alpha} dj, \quad A > 0, \quad 0 < \alpha < 1, \tag{5}$$

where A is a productivity parameter, N is the number of available intermediate goods, and α measures the importance of intermediate good j relative to labor in final production. Since $X_{ij} =$

X_i, $\forall j$, in equilibrium by symmetry, the production function in (5) becomes

$$Y_i = ANL_i^{1-\alpha} X_i^{\alpha}, \tag{6}$$

where output growth is driven by expanding the variety of intermediate goods N.

The profit function of firm i in the final-good sector is defined as

$$\Pi_i = (1+s_y)AL_i^{1-\alpha} \int_0^N {X_{ij}}^{\alpha} dj - wL_i - (1-s_x) \int_0^N p_j X_{ij} dj, \quad 0 \;\Box\; s_x < 1, \quad s_y \geq 0, \tag{7}$$

where the price of the final good is normalized to unity, p_j is the price of intermediate good j measured in units of the final good, and s_y and s_x are respectively subsidies to final output and to the purchase of intermediate goods. In the final-good sector, factors are paid by their marginal products: $(1+s_y)F_{X_{ij}} = (1-s_x)p_j$ and $(1+s_y)F_{L_i} = w$. The optimal condition $(1+s_y)F_{X_{ij}} = (1-s_x)p_j$ gives firm i's demand for an intermediate good, X_{ij}, leading to the aggregate demand X_j as

$$X_j = \Gamma \sum_i L_i (\alpha p_j)^{1/(\alpha-1)}, \quad \Gamma \equiv \left[\frac{\alpha^2 A(1+s_y)}{1-s_x} \right]^{\frac{1}{1-\alpha}}, \tag{8}$$

where Γ is a function of the subsidy rates s_x and s_y. The optimal condition $(1+s_y)F_{L_i} = w$ gives firm i's demand for labor, $L_i = (1-\alpha)(1+s_y)Y_i/w$. Equating aggregate labor demand and supply, i.e., $\sum_i L_i = L(1-l)$, the equilibrium quantity of labor is equal to

$$L(1-l) = (1-\alpha)(1+s_y)Y/w. \tag{9}$$

2.3 Expansions of the variety of intermediate goods

We adopt several assumptions in the literature to simplify the analysis. First, the R&D process is deterministic, i.e. investing η fixed units of the final good in R&D creates a new type of intermediate good. Also, innovators are given permanent monopoly rights over the production and sale of their invented intermediate goods, and one unit of any intermediate good can be produced using one unit of the final good (i.e., a unit marginal cost). Finally, there is free entry in the R&D sector.

With the permanent monopoly right, the value of a new technology (the discounted present value of the gross profit from producing a new intermediate good) is

$$V_t(p_j) = \int_t^{\infty} (p_j - 1) X_j e^{-\int_t^v r(s)ds} dv, \tag{10}$$

where r is the interest rate. Without any state variable in (10), the problem $\max_{p_j} V_t$ is equivalent to

$$\max_{p_j}[(p_j-1)X_j] = \max_{p_j}\left\{(p_j-1)\Gamma L(1-l)(\alpha p_j)^{1/(\alpha-1)}\right\}. \tag{11}$$

Since an individual supplier of intermediate good j is negligible compared to a continuum of intermediate goods with a mass N, we assume that it takes the quantity of aggregate labor $L(1-l)$, the wage, and the prices of other intermediate goods and the final good as given when making its own price decision. The problem in (11) gives the monopoly pricing rule:

$$p_j = p = 1/\alpha > 1. \tag{12}$$

That is, the monopoly sets a constant markup on the unit marginal cost. Also note that the price is the same for all intermediate goods by symmetry.

Combining (8) and (12) yields the equilibrium quantity of an intermediate good

$$X_j = X \equiv \Gamma L(1-l), \tag{13}$$

which is also constant over time and the same for all intermediate goods. With free entry into the R&D sector, the profit from R&D must be zero: $\eta(1-s_n) = V_t = (1-\alpha)X/(\alpha r)$. Rewrite it as

$$r = \frac{(1-\alpha)X}{(1-s_n)\alpha\eta} = \frac{(1-\alpha)\Gamma L(1-l)}{(1-s_n)\alpha\eta}. \tag{14}$$

By (14) and the definition of Γ, the rate of return on R&D investment r depends on the subsidies (s_y, s_x, s_n) and leisure l. Also, this rate of return is increasing with the size of the labor force L. In other words the model suffers from a "level effect" as discussed in Jones (1995).[4]

2.4 Government

The government taxes labor income at a flat rate τ to finance the subsidies:

$$\tau w(1-l)L = s_y Y + s_x N X/\alpha + s_n \eta \gamma_N N, \tag{15}$$

where $\gamma_N \equiv \dot{N}/N$ is the growth rate of the variety of intermediate goods. In (15), the left-hand side is the total revenue from labor income taxes and the right-hand side is the total expenditure

[4]When the "level effect" is removed, the welfare differences for different policies may be smaller in the light of Jones (1995). In our simulation later, we will remove the level effect by normalizing L to unity in the benchmark parameterization, and explore the remaining welfare differences across different policies.

on subsidizing final output ($s_y Y$), the purchase of intermediate goods ($s_x NX/\alpha$) and investment in R&D ($s_n \eta \gamma_N N$). As other types of taxes only lead to quantitative rather than qualitative alterations, we ignore them for simplicity. However, we will explicitly have a consumption tax later when focusing on whether subsidies funded by distortionary taxes can achieve the social optimum.

3. The social planner's problem

In order to see whether the socially optimal outcome can be achieved in a decentralized setting, we first solve the social planner's problem:

$$\max_{c,l} \int_0^\infty \left[\frac{(cl^\epsilon)^{1-\theta}-1}{1-\theta}\right] e^{-\rho t} dt, \tag{16}$$

subject to the following resource constraint

$$Y = A[L(1-l)]^{1-\alpha} N X^\alpha = Lc + \eta \dot{N} + NX.$$

The solution is given by:

$$l_{sp} = \frac{\epsilon \eta \rho (A\alpha)^{-\alpha/(1-\alpha)} - \epsilon AL(1-\alpha)(1-\theta)}{\theta AL(1-\alpha) - \epsilon AL(1-\alpha)(1-\theta)}, \tag{17}$$

$$X_{sp} = (A\alpha)^{1/(1-\alpha)} L(1-l_{sp}), \tag{18}$$

$$\gamma_{sp} = (1-\alpha)X_{sp}/(\alpha\eta\theta) - \rho/\theta, \tag{19}$$

$$c_{sp} = c_0 e^{\gamma_{sp} t} \text{ with } c_0 \equiv A[L(1-l_{sp})]^{1-\alpha} N_0[(1-\alpha)(\theta-1) + \alpha\eta\rho/X_{sp}]. \tag{20}$$

The welfare function is:

$$U_0 = \max_{\{c,l\}} \int_0^\infty \left(\frac{(cl^\epsilon)^{1-\theta}-1}{1-\theta}\right) e^{-\rho t} dt = \frac{\left(c_0 l_{sp}^\epsilon\right)^{1-\theta}}{(1-\theta)[\rho-(1-\theta)\gamma_{sp}]} - \frac{1}{\rho(1-\theta)}. \tag{21}$$

The transversality condition implies $\rho - (1-\theta)\gamma_{sp} > 0$. For later comparisons with the decentralized equilibrium, Table 1 shows the social optimum in terms of the ratio of R&D investment to output, the growth rate, and welfare for various parameterizations. The benchmark parameterization is briefly noted in Table 1 and is to be discussed with more details later.

4. The decentralized equilibrium with subsidies

The decentralized economy in this type of model is known to be always in a balanced equilibrium for any constant subsidy rates set by the government, whereby the proportional allocations of output and time and the growth rate are all constant over time, and the growth rate is the same for final output, consumption, and the rate of innovation. Because of the labor-leisure choice, the derivation of the growth rate is much more involved than in a standard R&D model with fixed labor supply.

We first rewrite (3) as: $r = \theta\gamma + \rho$. Substituting it into (14) provides $X = \alpha\eta(1 - s_n)(\theta\gamma + \rho)/(1-\alpha)$, which, together with (13), gives $l = 1-\alpha\eta(1-s_n)(\theta\gamma+\rho)/[(1-\alpha)\Gamma L]$. Also, combining (9), (13) and the final-output function $Y = A[L(1-l)]^{1-\alpha}NX^{\alpha}$ yields $w = (1-\alpha)(1+s_y)AN\Gamma^{\alpha}$. Finally, substituting (4) and the above expressions of (X, l, w, Y) into the resource constraint for the economy, $C = Lc = Y - \eta\gamma_N N - NX$, leads to the solution for the growth rate:

$$\gamma = \frac{[(1-\alpha)\Gamma L - (1-s_n)\alpha\eta\rho](1-\alpha)(1-\tau)(1+s_y)A\Gamma^{\alpha-1} - \epsilon\alpha\eta\rho(1-s_n)(A\Gamma^{\alpha-1}-1)}{\theta\alpha\eta(1-\alpha)(1-s_n)(1-\tau)(1+s_y)A\Gamma^{\alpha-1} + \epsilon\theta\alpha\eta(1-s_n)(A\Gamma^{\alpha-1}-1) - \epsilon\eta(1-\alpha)} \tag{22}$$

where the rates of the tax and subsidies must satisfy the following transformed version of the government budget constraint

$$\tau = \frac{s_y}{(1-\alpha)(1+s_y)} + \frac{\alpha s_x}{(1-\alpha)(1-s_x)} + \frac{\alpha\gamma s_n}{(1-s_n)(1-s_x)(\theta\gamma+\rho)} \tag{23}$$

which arises from dividing both sides of the government budget constraint (15) by final output Y and arranging terms. Therefore, (22) and (23) jointly determine the equilibrium solution for the growth rate. Now we can see that the subsidies (s_x, s_y, s_n) affect the growth rate γ. The effects of subsidies (s_x, s_y) on growth also go through the factor Γ defined in (8).

To facilitate the welfare analysis, we also derive the solution for the equilibrium welfare level as a function of the tax and subsidies. Given the initial number of intermediate goods N_0, the subsequent number is determined by the exponential expansion $N = N_0 e^{\gamma t}$. Also, given the solution for γ, we can obtain the solution for consumption and leisure:

$$c = c_0 e^{\gamma t}, \quad c_0 = \frac{\eta N_0}{L}\left\{\frac{[1 - s_x - \alpha^2(1+s_y)](1-s_n)(\theta\gamma+\rho)}{\alpha(1-\alpha)(1+s_y)} - \gamma\right\}, \tag{24}$$

$$l = 1 - \frac{\alpha\eta(1-s_n)(\theta\gamma+\rho)}{(1-\alpha)\Gamma L}. \tag{25}$$

As in the social planner's problem, the welfare function is of the form:

$$U_0 = \max_{\{c,l\}} \int_0^\infty \left(\frac{(cl^\epsilon)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt = \frac{(c_0 l^\epsilon)^{1-\theta}}{(1-\theta)[\rho - (1-\theta)\gamma]} - \frac{1}{\rho(1-\theta)}, \tag{26}$$

where γ, c_0 and l are respectively given by (22), (24) and (25). Moreover, the transversality condition implies $\rho - (1-\theta)\gamma > 0$. We can now compare different types of subsidies.

We begin with the equivalence between subsidies provided to final output s_y and to the purchase of intermediate goods s_x and any combination of them. Define $s_f \equiv (s_x+s_y)/(1-s_x)$ as the effective subsidy rate to final output (hereafter, the *production* subsidy), then $(1+s_y)/(1-s_x) = 1+s_f$ and $\Gamma = [\alpha^2 A(1+s_f)]^{1/(1-\alpha)}$. It suffices to show that it is the effective subsidy rate s_f, not the decomposition between s_x and s_y, that matters for both the growth rate γ and welfare U. Using the definition of s_f, we substitute (23) into (22) and solve it to obtain

$$\gamma = \frac{-\Psi_2 + (\Psi_2^2 - 4\Psi_1\Psi_3)^{1/2}}{2\Psi_1}, \tag{27}$$

where

$$\Psi_1 \equiv \theta \left\{ \theta\epsilon(1-s_n)[1-\alpha^2(1+s_f)] - (\epsilon+s_n)(1-\alpha)\alpha(1+s_f) + \theta(1-s_n)[1-\alpha(1+s_f)] \right\},$$

$$\Psi_2 \equiv -\left[\frac{(1-\alpha)L\Gamma}{\alpha\eta(1-s_n)} - \rho \right] \{\theta(1-s_n)[1-\alpha(1+s_f)] - (1-\alpha)s_n\alpha(1+s_f)\}$$
$$+ 2\epsilon\theta\rho(1-s_n)[1-\alpha^2(1+s_f)] - \epsilon\rho(1-\alpha)\alpha(1+s_f) + \theta\rho(1-s_n)[1-\alpha(1+s_f)],$$

$$\Psi_3 \equiv -\left[\frac{(1-\alpha)L\Gamma}{\alpha\eta(1-s_n)} - \rho \right] \rho(1-s_n)[1-\alpha(1+s_f)] + \epsilon\rho^2(1-s_n)[1-\alpha^2(1+s_f)].$$

To guarantee that the growth rate γ given by (27) is nonnegative, we assume that $\Psi_1 > 0$, $\Psi_2 < 0$ and $\Psi_3 < 0$.[5] From (27), we can see that the growth rate depends on the combined effective subsidy rate s_f rather than individual subsidy rates s_x and s_y. Also, leisure l is a function of the effective subsidy rate s_f in (25), and so is consumption c_0 when writing (24) as

$$c_0 = \frac{\eta N_0}{L} \left\{ \frac{[1-\alpha^2(1+s_f)](1-s_n)(\theta\gamma+\rho)}{\alpha(1-\alpha)(1+s_f)} - \gamma \right\}. \tag{28}$$

[5] This assumption holds true if $\theta > \alpha$ and if ρ is sufficiently small in the absence of any subsidy. The need of a small enough ρ is standard for $\gamma > 0$ by (3). Also, the restriction $\theta \geq \alpha$ is not binding in the real world since most related empirical evidence suggested $\theta > 1$. With these assumptions, we ignore the negative root $\gamma = \frac{-\Psi_2 - (\Psi_2^2 - 4\Psi_1\Psi_3)^{1/2}}{2\Psi_1}$.

Thus, the welfare function given by (26) depends on the effective subsidy rate s_f as does the growth rate. Summarizing our discussion, we have

Proposition 1: *With elastic labor supply and distortionary taxes, the subsidies to final output or to the purchase of intermediate goods are equivalent concerning their effects on growth and welfare.*

The equivalence between these two types of subsidies was seen in Barro and Sala-i-Martin (1995) with lump-sum taxes and fixed labor supply. Here, we extend it to the case with a labor-leisure choice and distortionary taxes. With Proposition 1, we can now focus on the production subsidy s_f and the R&D subsidy s_n in the rest of the paper.

Setting $s_x = 0$ and $s_f = s_y$, the government budget constraint (23) becomes

$$\tau(1-\alpha)(1+s_f) = \frac{s_n\eta\gamma\alpha^2(1+s_f)}{X} + s_f. \tag{29}$$

Using (22) and (25), we have the solution for leisure:

$$l = \Upsilon_1/\Upsilon_2, \tag{30}$$

where

$$\Upsilon_1 \equiv \epsilon\eta\rho(1-s_n)\Gamma^{-\alpha}/\alpha + \epsilon AL\{\theta(1-s_n)[1-\alpha^2(1+s_f)]/\alpha - (1-\alpha)(1+s_f)\},$$

$$\Upsilon_2 \equiv \epsilon AL\{\theta(1-s_n)[1-\alpha^2(1+s_f)]/\alpha - (1-\alpha)(1+s_f)\}$$

$$+ AL\theta(1-s_n)(1-\alpha)(1+s_f)(1-\tau)/\alpha.$$

We can then express c_0, X and γ as

$$c_0 = \frac{\Gamma N_0 l(1-\tau)(1-\alpha)}{\alpha^2\epsilon}, \tag{31}$$

$$X = \Gamma L(1-l), \tag{32}$$

$$\gamma = \frac{(1-\alpha)X}{(1-s_n)\alpha\eta\theta} - \frac{\rho}{\theta}. \tag{33}$$

Before comparing the subsidies, it is interesting to compare the decentralized equilibrium solution without any subsidies and taxes with the social planner's solution. First, from (30) and (17), leisure is higher in the former solution than in the latter. Conversely, from (32) and (18), the equilibrium quantity of intermediate goods is lower in the former than in the latter, partly also because at $s_y = s_n = 0$, $\Gamma = [\alpha^2 A]^{1/(1-\alpha)} < (\alpha A)^{1/(1-\alpha)}$ due to the monopoly pricing $p = 1/\alpha$ allowed in the former. As a result, the growth rate is lower in the former than in the latter, according to (33) and (19). The intuition is that in the presence of the externality of the variety expansion, the perceived rates of return to working and innovation in this decentralized economy without subsidies are lower than the social rates. Granting monopoly rights to innovators alone does not close the gap in the rates of return as pricing intermediate goods above their marginal costs reduces the demand for intermediate goods.

It may appear that financing subsidies through a lump-sum tax can achieve the socially optimal solution. This was indeed the case in Barro and Sala-i-Martin: With inelastic labor supply in their model, setting the final production subsidy rate as $1 + s_y = 1/\alpha$ (i.e. equalizing the user cost and the marginal cost of intermediate goods) would imply $\Gamma = [\alpha^2 A(1 + s_y)]^{1/(1-\alpha)} = (\alpha A)^{1/(1-\alpha)}$ and hence would achieve the socially optimal quantity of intermediate goods X. But in our model, this is insufficient to obtain the socially optimal quantity of $X = \Gamma L(1 - l)$ since the level of labor per worker $1 - l$ at this particular subsidy rate remains below its socially optimal level. That is, lowering the user cost of intermediate goods to their marginal cost by the production subsidy to correct the efficiency loss of monopoly pricing is not enough to achieve the social optimum with elastic labor. Subsidizing R&D investment by a lump-sum tax cannot achieve the socially optimal solution either, since when X becomes the same as in the social planner's solution, a positive s_n in (33) will lead to excessive growth compared to that in the social planner's solution in (19).

4.1 Growth-maximizing subsidies

Before turning to the analysis of optimal subsidies, we compare the two types of subsidies and their mix in terms of their effectiveness in promoting growth, for at least two reasons. First, growth is important in its own right, both in theory and in practice. Second, the type of subsidy that is more conducive to growth can gain more dynamic efficiency in dealing with the R&D externality and monopoly pricing, and thus is a possible candidate to improve welfare.

The growth rate under the production subsidy can be obtained from (27) by setting $s_n = 0$:

$$\gamma(s_f) = \frac{\left[\frac{(1-\alpha)\Gamma L}{\alpha\eta} - \rho\right][1-\alpha(1+s_f)] - \epsilon\rho[1-\alpha^2(1+s_f)]}{\theta[1-\alpha(1+s_f)] + \epsilon\theta[1-\alpha^2(1+s_f)] - \epsilon(1-\alpha)\alpha(1+s_f)}. \tag{34}$$

We then observe the following (see Appendix A for the proof):

Proposition 2: *For $\theta \geq \alpha$ and a small enough ρ, the growth rate is globally concave with respect to the production subsidy, and reaches its global maximum at a finite positive value of the subsidy.*

Intuitively, the production subsidy exerts opposing forces on growth. The positive force on growth is standard and obvious. Missing in models with lump-sum taxes is the negative force arising from the accompanying rise in the tax distortion that reduces labor supply and in turn lowers the demand for intermediate goods by (13). Further, the positive effect falls with the subsidy rate due to diminishing marginal product of intermediate goods in final production. Thus, when the subsidy rate is low (high), the positive (negative) effect dominates. When the opposing effects exactly cancel out at a positive rate of the subsidy, the growth rate peaks.

Under the R&D subsidy s_n, the growth rate is determined by (27) with $s_f = 0$, i.e.,

$$\gamma(s_n) = \frac{-\Psi_2(s_n) + [\Psi_2(s_n)^2 - 4\Psi_1(s_n)\Psi_3(s_n)]^{1/2}}{2\Psi_1(s_n)}, \tag{35}$$

where

$$\Psi_1(s_n) \equiv \theta\{\theta(1-s_n)[1+\epsilon(1+\alpha)] - \alpha(\epsilon+s_n)\},$$

$$\Psi_2(s_n) \equiv -\left[\frac{(1-\alpha)L(\alpha^2 A)^{1/(1-\alpha)}}{\alpha\eta(1-s_n)} - \rho\right][\theta(1-s_n) - \alpha s_n] + \theta\rho(1-s_n)[1+2\epsilon(1+\alpha)] - \epsilon\rho\alpha,$$

$$\Psi_3(s_n) \equiv -\rho\left\{\frac{(1-\alpha)L(\alpha^2 A)^{1/(1-\alpha)}}{\alpha\eta} - \rho(1-s_n)[1+\epsilon(1+\alpha)]\right\}.$$

The growth effects of the R&D subsidy are summarized below (see Appendix A for the proof).

Proposition 3: *For $\theta \geq \alpha$ and a small enough ρ, the growth rate is globally concave with respect to the R&D subsidy, and reaches its global maximum at a finite positive value of the subsidy.*

Like the production subsidy, the tax distortion has a negative effect on growth. Unlike the production subsidy, however, the R&D subsidy stimulates growth by directly reducing the cost of R&D investment. There is a positive rate of the R&D subsidy that maximizes the growth rate.

An important question can then be posed: Which of these subsidies can lead to a higher growth rate? An analytical investigation into this question is complicated, because of the complex expressions of the growth rate in (34) and (35). We thus appeal to numerical simulations with various parameterizations. A benchmark parameterization is set as: $\alpha = 0.3$, $\theta = 1.5$, $\epsilon = 0.5$, $\eta = 2.02$, $\rho = 0.05$ and $A = L = N_0 = 1$. Here, the values of $(\alpha, \epsilon, \theta, \rho)$ are within the standard ranges used in the literature,[6] while those of (A, L, η) are chosen such that the (welfare-maximizing) growth rate would equal 3.0% in the decentralized equilibrium with the R&D subsidy. This scenario is calibrated to the United States which has an average growth rate around 3.0% for the last 30 years and has used an R&D subsidy, not the production subsidy. In other parameterizations, we allow the values of the parameters to vary around their benchmark levels, and see whether the results are sensitive to the variations in parameterization. We report the results in Table 2 which also gives the numerical solution for the social planner's problem for comparisons.

For all the parameterizations with which we have experimented, the R&D subsidy always leads to a higher growth rate than does the production subsidy. And the tax rate (for growth-maximizing) is lower under the R&D subsidy than under the production subsidy unless the latter cannot achieve positive growth. With the benchmark parameterization, for example, the growth rate is only 0.35% (with a tax rate of 90%) under the production subsidy, but is 5.94% (with a tax rate of 84%) under the R&D subsidy. Further, the growth-maximizing combinations of subsidies are the same as the growth-maximizing R&D subsidies for all the parameterizations.

To provide a global view, Figures 1(a) and 1(b) depict the relationship between the combinations of the two types of subsidies and the resulting growth rates under the same benchmark parameterization (viewed from *different* angles). From these figures, it is clear that only the R&D

[6] The value of the key parameter $\epsilon = 0.5$ is taken directly from Lucas (1990), while the values $\theta = 1.5$, $\rho = 0.05$ and $\alpha = 0.3$ (implying a labor's share of 0.7) are based on the growth calibration exercises in Lucas (1990), King and Rebelo (1990), and Stokey and Rebelo (1995).

subsidy should be used to maximize the growth rate, corresponding to the mix of subsidies (s_f, s_n) $= (0, 86.15\%)$ in Table 2.

Overall, the two types of subsidies affect R&D incentives quite differently. The R&D subsidy *directly* lowers the cost of R&D investment, while the production subsidy *indirectly* raises the marginal benefit of R&D investment by strengthening the demand for intermediate goods. It turns out that the more directly a subsidy affects R&D incentives, the more effectively it promotes growth. In fact, as shown in Table 2, the R&D subsidy *may* generate excess growth compared to the growth rate in the social planner's solution (derived in Section 3), while the production subsidy always produces a lower growth rate than the socially optimal growth rate.

It is also interesting to see how the variations in the parameters affect growth in Table 2. As one may expect, a higher rate of time preference ρ or a lower elasticity of intertemporal substitution $1/\theta$ leads to a lower growth rate, because individuals are less willing to save. Higher productivity in final production A or a larger labor force L raises the rate of return on investment in R&D, resulting in a higher growth rate. Similarly, a lower cost of R&D η raises investment in R&D, leading to a higher growth rate. A stronger taste for leisure ϵ decelerates growth by increasing leisure (and thus reducing labor supply). Finally, a higher value of α, i.e. a more important role of intermediate goods relative to labor in final production, decelerates growth by lowering the demand for intermediate goods and hence the price of intermediate goods $1/\alpha$.

4.2 Optimal production subsidies

Setting $s_n = 0$ in (29) leads to $\tau = s_f/[(1-\alpha)(1+s_f)]$. Thus, the welfare function under the production subsidy is given by

$$U_0(s_f) = \frac{\{c_0(s_f)\,[l(s_f)]^{\epsilon}\}^{1-\theta}}{(1-\theta)[\rho-(1-\theta)\gamma(s_f)]} - \frac{1}{\rho(1-\theta)}. \tag{36}$$

where $(c_0(s_f), l(s_f), \gamma(s_f))$ can be found by substituting $s_n = 0$ and $\tau = s_f/[(1-\alpha)(1+s_f)]$ into equations (30)-(33) as follows. First, note that the government budget balance $\tau = s_f/[(1-\alpha)(1+s_f)]$ in this case fully determines τ by the share parameter α and the subsidy rate s_f. Substituting this budget balance and $s_n = 0$ into (30) for τ and s_n allows us to obtain the reduced-form solution for leisure $l(s_f)$. Further, substituting $l(s_f)$, $s_n = 0$ and $\tau = s_f/[(1-\alpha)(1+s_f)]$ into (31), (32) and (33) leads to reduced-form solutions for $c_0(s_f)$, $X(s_f)$ and $\gamma(s_f)$. We thus have:

Proposition 4: *For a large enough θ (e.g. $\theta \geq 1$), the welfare level reaches its global maximum at a finite positive value of the production subsidy.*

Proof: Differentiating (36) with respect to s_f, we have sign $U_0'(s_f) =$ sign $\Theta(s_f)$, where

$$\begin{aligned}\Theta(s_f) \;\equiv\; & \epsilon\alpha AL\left\{AL\theta(\theta-1)(1-\alpha) + \frac{A\epsilon\eta\rho\alpha^2[(1+s_f)(1+\alpha\theta-\alpha)-\theta]+\eta\rho\theta s_f}{(1-\alpha)\Gamma}\right\} \\ & \times\left\{\frac{1+\epsilon}{\Upsilon_1} - \frac{(1-\alpha)\Gamma}{\Upsilon_2\alpha\eta\rho-(1-\theta)(1-\alpha)\Gamma(\Upsilon_2-\Upsilon_1)}\right\} + \frac{\Upsilon_2}{(1-\alpha)(1+s_f)} \\ & + \frac{[\Gamma/(1+s_f)]\Upsilon_2(\Upsilon_2-\Upsilon_1)}{\Upsilon_2\alpha\eta\rho-(1-\theta)(1-\alpha)\Gamma(\Upsilon_2-\Upsilon_1)} - \frac{\Upsilon_2}{[1-\alpha(1+s_f)](1+s_f)}.\end{aligned} \tag{37}$$

When $s_f = \tau = 0$, the expression in the first bracket on the right-hand side of (37) becomes proportional to $\Upsilon_2 - \Upsilon_1$. Thus, by converting Υ_1 and Υ_2 back to l or $1-l$, (37) becomes:

$$\text{sign } \Theta(0) = \text{sign }\left\{(1-l)(1+\epsilon)\left[\frac{\epsilon AL\alpha(\theta-1)}{l} + \frac{AL\theta(1-\alpha)\Gamma}{\alpha\eta\rho-(1-\theta)(1-\alpha)\Gamma(1-l)}\right]\right\}.$$

Note that $\Theta(0) > 0$ at least for $\theta \geq 1$, because the denominator of the second term is positive under the transversality condition and because the rest is obviously positive. If s_f were too high, e.g. pushing τ towards 1, the remaining resource for consumption would be too little. In this scenario, further rises in s_f would surely decrease welfare. Thus, the welfare level must reach its global maximum for some $s_f \in (0, \infty)$ under $\theta \geq 1$, given the underlying continuity of $U_0(s_f)$. ■

Starting with too little R&D investment and too much leisure without any subsidy, an increase in the production subsidy encourages R&D investment and thus stimulates growth as mentioned earlier, moving in the direction toward their socially optimal levels. The accelerated growth thus enhances welfare over time. On the other hand, however, an accompanying increase in the labor income tax tends to raise leisure further above its socially optimal level, leading to a lower level of welfare. When the subsidy rate is low, the tax distortion is weak and the efficiency gain from faster growth dominates, leading to a net gain in welfare. Obviously, if the production subsidy is very high, the tax distortion becomes stronger and eventually dominates the positive welfare effect. In other words, there should be a positive rate of the subsidy at which welfare is maximized.

Although Proposition 4 gives the existence of the value of the production subsidy that maximizes welfare, it is difficult to show analytically whether welfare is globally concave with this subsidy for the welfare-maximizing subsidy to be unique. Numerically, Table 3 gives the simulation results for welfare using the same parameterizations as in Tables 1 and 2. There indeed exists a unique positive welfare-maximizing production subsidy for each of the parameterizations, as shown in Figure 2 (a).

4.3 Optimal R&D subsidies

The welfare function under the R&D subsidy is given by

$$U_0(s_n) = \frac{\{c_0(s_n)\,[l(s_n)]^{\epsilon}\}^{1-\theta}}{(1-\theta)[\rho-(1-\theta)\gamma(s_n)]} - \frac{1}{\rho(1-\theta)}, \tag{38}$$

where

$$c_0(s_n) = \frac{\eta N_0}{L}\left\{\frac{(1+\alpha)(1-s_n)[\theta\gamma(s_n)+\rho]}{\alpha} - \gamma(s_n)\right\}, \tag{39}$$

$$l(s_n) = 1 - \frac{\alpha\eta(1-s_n)[\theta\gamma(s_n)+\rho]}{(1-\alpha)(\alpha^2 A)^{1/(1-\alpha)}L}. \tag{40}$$

Unlike the previous case with $s_f > 0$ and $s_n = 0$, the current case with $s_n > 0$ and $s_f = 0$ has no reduced-form solutions: the five variables c_0, τ, l, X and γ are implicitly determined by five equations (29)-(33). It is thus difficult to derive analytical results concerning the welfare effect of the R&D subsidy. For this reason, we again perform numerical simulations and report the results in Table 3. For each of the parameterizations, there is a unique optimal R&D subsidy rate. With the benchmark parameterization, for instance, the optimal R&D subsidy is 76.33% and the maximum welfare level is -65.30. Clearly, the welfare curve in Figure 2(b) is smooth and single-peaked, and the magnitude of the welfare gain can be substantial. Thus, like the production subsidy, the R&D subsidy can improve welfare by promoting R&D investment and growth but can reduce welfare by raising the tax distortion. When the opposing effects exactly offset each other at a positive rate of the R&D subsidy, the welfare level peaks.

The results in Table 3 also help understand how and why the variations in the key parameters affect welfare, together with the aid of equation (26) that links welfare to leisure, initial consumption and the growth rate. For example, a fall in the elasticity of intertemporal substitution (a rise in θ) indicates less willingness to save. As a result, it may raise leisure and initial consumption and hence raise welfare on the one hand; but it may decelerate growth and hence reduce welfare on the other hand. (Of course, a higher θ also changes welfare without going through any of the variables l, c_0 and γ.) The negative growth effect turns out to dominate and results in a net decline in welfare. Also, a rise in the cost of R&D investment η has opposing effects on both leisure and initial consumption, but certainly has a negative effect on growth. Consequently, it is most likely that the growth effect of a higher cost of R&D dominates, and leads to a net decline in welfare. Further, a stronger taste for leisure ϵ raises welfare by increasing leisure and reduces welfare by decreasing initial consumption and the growth rate, resulting in a net decline in welfare.

Another interesting point is that a change in a particular parameter may change the relative effectiveness of the two subsidies in raising welfare, given the different characteristics of the two subsidies. Through reducing the R&D cost, the R&D subsidy is more effective in removing the *dynamic* efficiency loss of the R&D externality by raising the growth rate, but less effective in reducing the *static* efficiency loss of monopoly pricing. In contrast, the production subsidy directly reduces the user cost of intermediate goods, and thus is more effective in reducing the *static* efficiency loss but less effective in reducing the *dynamic* efficiency loss. Concerning tax distortions, the R&D subsidy requires a much lower labor income tax rate than the production subsidy to achieve any given growth target, since the subsidy base is much smaller in the former case (the R&D investment) than in the latter case (final output). A change in a particular parameter changes the magnitudes of these efficiency gains and losses.

For example, when the taste for leisure is weaker (e.g., $\epsilon = 0.3$), the production subsidy generates a higher welfare level than does the R&D subsidy in Table 3, reversing the ranking with the benchmark parameterization. This is mainly because, when leisure is less important, the labor income tax distortion is weaker. When leisure becomes more important (e.g., $\epsilon = 0.7$), the ranking goes back to that with the benchmark parameterization. When the elasticity of intertemporal substitution is higher (e.g. $\theta = 1.1$) than its benchmark level (greater willingness to save), the R&D subsidy remains better than the production subsidy, as the dynamic efficiency loss, which is better handled by the R&D subsidy, becomes even more important. When the elasticity of intertemporal substitution is lower (e.g. $\theta = 2$, or 3), the welfare ranking can be reversed. When the rate of time preference is higher than the benchmark level ($\rho = 0.08$), the production subsidy cannot generate any positive growth and is thus ranked below the R&D subsidy. When the rate of time preference is much reduced ($\rho = 0.02$), the growth rates are rather high in both cases while the welfare ranking of the subsidies is reversed, indicating that the static efficiency loss is now the chief concern, which is better handled by the production subsidy. Further, when the cost of R&D investment η is much lower (say $\eta = 1$) than its benchmark, the ranking of the subsidies is reversed, because now the static efficiency loss becomes more important. When the cost of R&D investment is higher than its benchmark (say $\eta = 3$), the ranking goes back to that with the benchmark parameterization. We thus conclude: In terms of improving welfare, the R&D subsidy can be more or less effective than the production subsidy depending on parameterizations.

These results are in sharp contrast with those in Barro and Sala-i-Martin (1995). In their model with lump-sum taxes and fixed labor supply, the production subsidy always dominates the R&D subsidy, because the former can completely recover both the static efficiency loss (low demand for intermediate goods) and the dynamic efficiency loss (the low growth rate) while the latter cannot. With the labor-leisure trade-off in our model, however, the production subsidy is not effective in bringing leisure down to its socially best level, even when the tax is lump sum, as mentioned earlier. When a labor income tax is used, the production subsidy entails more tax distortions than the R&D subsidy, pushing leisure further up from its socially optimal level. Thus, in our model the R&D subsidy can be more or less effective than the production subsidy depending on parameterizations.

4.4 Optimal combination of production and R&D subsidies

Now we want to see whether combining these two subsidies can do better than using one of them alone. To do so, we first rewrite the government budget constraint (23) as:

$$\tau(1-\alpha)(1+s_f) = \frac{s_n\alpha(1+s_f)}{\theta}\left\{\frac{1-\alpha}{1-s_n} - \frac{\alpha\rho\eta}{\Gamma L(1-l)}\right\} + s_f. \tag{41}$$

Then, as shown in Appendix B, the utility-maximizing problem of the government by choice of (τ, s_f, s_n) subject to its constraint in (41) is equivalent to the following unconstrained problem that chooses (s_f, s_n) to maximize:

$$W \equiv l^{(1+\epsilon)(1-\theta)}\left\{1 - \frac{s_n\alpha(1-\alpha)}{(1-\alpha)\theta(1-s_n)} + \frac{s_n\alpha^2\eta\rho}{(1-\alpha)\theta\Gamma L(1-l)} - \frac{s_f}{(1-\alpha)(1+s_f)}\right\}^{1-\theta}$$
$$\times(1+s_f)^{(1-\theta)/(1-\alpha)}\left\{\rho - \frac{(1-\theta)(1-\alpha)\Gamma L(1-l)}{\alpha\eta(1-s_n)}\right\}^{-1}. \tag{42}$$

The first-order conditions for this maximization problem are:

$$\begin{aligned} W_{s_f} \;=\;& \frac{\partial l}{\partial s_f}\left[\frac{1+\epsilon}{l} + \frac{s_n\eta\alpha^2\rho}{(1-\tau)(1-\alpha)\theta\Gamma L(1-l)^2}\right] - \\ & \frac{\partial l}{\partial s_f}\left\{\frac{(1-\alpha)\Gamma L}{\rho\alpha\eta(1-s_n)-(1-\theta)(1-\alpha)\Gamma L(1-l)}\right\} - \\ & \frac{s_n\eta\alpha^2\rho}{(1-\tau)(1-\alpha)^2(1+s_f)\theta\Gamma L(1-l)} - \frac{1}{(1-\tau)(1-\alpha)(1+s_f)^2} + \\ & \frac{1}{(1-\alpha)(1+s_f)} + \frac{\Gamma L(1-l)}{(1+s_f)\{\alpha\rho(1-s_n)\eta-(1-\theta)(1-\alpha)\Gamma L(1-l)\}} = 0, \end{aligned} \tag{43}$$

$$\begin{aligned} W_{s_n} \;=\;& \frac{\partial l}{\partial s_n}\left[\frac{1+\epsilon}{l} + \frac{s_n\eta\alpha^2\rho}{(1-\tau)(1-\alpha)\theta\Gamma L(1-l)^2}\right] - \\ & \frac{\partial l}{\partial s_n}\left\{\frac{(1-\alpha)\Gamma L}{\alpha\rho\eta(1-s_n)-(1-\theta)(1-\alpha)\Gamma L(1-l)}\right\} - \\ & \frac{\alpha}{(1-\tau)\theta(1-s_n)^2} + \frac{\eta\alpha^2\rho}{(1-\tau)(1-\alpha)\theta\Gamma L(1-l)} + \\ & \frac{(1-\alpha)\Gamma L(1-l)}{\{\alpha\rho(1-s_n)\eta-(1-\theta)(1-\alpha)\Gamma L(1-l)\}(1-s_n)} = 0, \end{aligned} \tag{44}$$

where $\partial l/\partial s_f$ and $\partial l/\partial s_n$ are given in Appendix B. Thus we have:

Proposition 5: *The optimal mix of* (s_f, s_n) is implicitly determined by (41), (43), (44) and (53).

To reveal the quantitative implications of the optimal mix of subsidies, we use numerical simulations and report the results in Table 4 with the same parameterizations as in previous tables. For each parameterization, there is a unique welfare-maximizing combination of the two types of subsidies. For example, with the benchmark parameterization, the optimal mix of these subsidies is $(s_f, s_n) = (73.84\%, 53.01\%)$, and the resulting welfare is -58.32, which is substantially higher than the welfare level without subsidies (-73.90). The relationship between the combination of these subsidies and the welfare level with the same benchmark parameterization is depicted in Figures 3(a) and 3(b) for an easier view from *different* angles. According to the simulation results, combining the two types of subsidies gives a higher level of welfare than a single subsidy. For example, in Table 3 with the benchmark parameterization, the production subsidy obtains a maximum welfare level of -67.41 while the R&D subsidy obtains a maximum welfare level of -65.30, both of which are much lower than that (-58.32) achieved by their optimal mix in Table 4.

The key point of using both subsidies simultaneously rather than separately is to take advantage of their relative strengths. While the production subsidy is more effective in removing the *static* distortion of the monopoly pricing by stimulating the demand for intermediate goods, the R&D subsidy tends to be more effective in removing the *dynamic* distortion by promoting growth. As a result, mixing both types of subsidies does better than using them in separation.

We now consider whether there is any first-best combination of (s_n, s_f) financed by a labor income tax at rate τ and a consumption tax at rate τ_c. The government budget constraint becomes

$$\tau_c Lc + Lw(1-l)\tau = s_n \eta\gamma N + s_f Y, \tag{45}$$

and accordingly the household budget constraint is given by

$$\dot{a} = ar + w(1-l)(1-\tau) - c(1+\tau_c). \tag{46}$$

The equilibrium solution is given by:

$$l = \frac{\epsilon\eta\rho(\alpha A)^{-\alpha/(1-\alpha)} + \epsilon AL[\theta(1-\alpha) - (1-\alpha)/(1-s_n)]}{\epsilon AL[\theta(1-\alpha) - (1-\alpha)/(1-s_n)] + AL\theta(1-\alpha)(1-\tau)/[\alpha(1+\tau_c)]}, \tag{47}$$

$$X = \Gamma L(1-l), \tag{48}$$

$$\gamma = \frac{(1-\alpha)X}{(1-s_n)\alpha\eta\theta} - \frac{\rho}{\theta}. \tag{49}$$

To achieve the first-best outcome in a decentralized equilibrium (i.e., $l = l_{sp}$, $X = X_{sp}$ and $\gamma = \gamma_{sp}$), we must satisfy the following three conditions:

$$(A).\ s_f = (1-\alpha)/\alpha;\ \ (B).\ s_n = 0;\ \ (C).\ 1-\tau = \alpha(1+\tau_c).$$

To be feasible, conditions (A)-(C) must also meet the government budget balance:

$$(D).\ \tau = \frac{l_{sp} - \epsilon(1-l_{sp}) - \alpha l_{sp}}{l_{sp} - \epsilon(1-l_{sp})} < 1.$$

Note that, to be incentive compatible, the labor income tax rate should be less than 1, i.e., $\tau < 1$. Under these restrictions, the first best can never be achieved as shown below:

Proposition 6: *No combination of* (s_f, s_n, τ, τ_c) *can achieve the first best outcome for* $\tau < 1$.

Proof. By (D), whether there exists any labor income tax rate less than 1 such that conditions (A)-(D) hold depends on the sign of $l_{sp} - \epsilon(1-l_{sp})$. If $l_{sp} - \epsilon(1-l_{sp}) < 0$, then $\tau > 1$ under condition (D). Now, suppose $l_{sp} - \epsilon(1-l_{sp}) > 0$. Combining this with the solution for l_{sp}, we have: $\alpha(1+\epsilon)\eta\rho > (\alpha A)^{1/(1-\alpha)}L(1-\alpha)$. Also, note that $\gamma_{sp} \geq 0$ has to hold in this model since the worse growth performance is not to innovate at all (zero growth). That is, we have $(\alpha A)^{1/(1-\alpha)}L(1-l_{sp})(1-\alpha) \geq \rho\alpha\eta$. Combining the above two conditions yields

$$(1+\epsilon)\rho > \frac{(\alpha A)^{1/(1-\alpha)}L(1-\alpha)}{\alpha\eta} \geq \frac{\rho}{1-l_{sp}},$$

which implies that $l_{sp} - \epsilon(1-l_{sp}) < 0$, reaching a contradiction. ■

From this proposition, we can see that the optimal combination of subsidies in Proposition 5 is only the second best. This result is illustrated quantitatively by the simulation results in Tables 3 and 4 in which the numerical solutions for the social planner's problem are also provided. The result here is in sharp contrast with that in Barro and Sala-i-Martin (1995) in which using the production subsidy alone (or a subsidy to the purchase of intermediate goods) can induce the decentralized equilibrium to achieve the social optimum with a lump-sum tax. As they speculated, once the subsidy has to be financed by distortionary taxes, the social optimum may not be achievable.

5. Conclusion

We have examined the growth and welfare implications of various subsidies by extending a standard R&D growth model to incorporate elastic labor supply and distortionary taxes. The results differ substantially from those in the literature. With inelastic labor supply and lump-sum taxes in Barro and Sala-i-Martin (1995), the social optimum can be attained by subsidizing either final output or the purchase of intermediate products. However, in our model none of the subsidies can achieve the social optimum, because in the presence of the R&D externality they cannot bring leisure down to its socially optimal level, although they all simulate R&D investment and growth. Also, in Barro and Sala-i-Martin (1995), subsidizing either final output or the purchase of intermediate goods is definitely better than subsidizing R&D investment, as the former, not the latter, can achieve the social optimum. In our model, which type of the subsidies leads to a higher welfare level is unclear and depends on parameterizations; in particular, the R&D subsidy is more effective in promoting growth and may obtain higher welfare levels than the other forms of subsidies. The possibility that the R&D subsidy is better than the production subsidy is largely due to the labor-leisure trade-off and their different requirements for tax revenue in our model. Moreover, in our approach mixing the two types of subsidies does better than using them in separation in maximizing welfare.

Also different from the literature are the policy implications of our results given the real-world tax system that consists of mainly income and consumption taxes. If growth is a chief concern as in many nations, subsidizing R&D is surely better than subsidizing either final output or the purchase of intermediate goods. Even if social welfare is the sole criterion, our results are not against the common practice of subsidizing R&D investment as observed in industrial nations, since the R&D subsidy can still improve on a decentralized equilibrium with or without other forms of subsidies.

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Appendix A

Proof of Proposition 2. To see the response of $\gamma(s_f)$ to a change in s_f, we differentiate (34) with respect to s_f. Then we have: sign $\gamma'(s_f) =$ sign $\Phi(s_f)$, where

$$\Phi(s_f) \equiv \alpha\eta\{\Gamma L[1-\alpha(1+s_f)]/(1+s_f) - \alpha[(1-\alpha)\Gamma L - \alpha\eta\rho] + \epsilon\alpha^3\eta\rho\}$$

$$\times\{\theta[1-\alpha(1+s_f)] + \epsilon\theta[1-\alpha^2(1+s_f)] - \epsilon\alpha(1-\alpha)(1+s_f)\}$$

$$+\alpha^2\eta\{[(1-\alpha)\Gamma L - \alpha\eta\rho][1-\alpha(1+s_f)] - \epsilon\alpha\eta\rho[1-\alpha^2(1+s_f)]\}[\theta + \epsilon\alpha\theta + \epsilon(1-\alpha)].$$

When $s_f = 0$, we have: $\Phi(0) = \alpha\eta\Gamma L(1-\alpha)^2[\epsilon(\theta-\alpha)(1-\alpha) + \epsilon\alpha(1-\alpha) + \theta(1+\epsilon\alpha)] - \epsilon\alpha^3\eta^2\rho(1+\epsilon)(1-\alpha)$. For any meaningful solution, we must have $\gamma(s_f) \geq 0$ at $s_f = 0$ which can usually be guaranteed by a small enough ρ, implying $(1-\alpha)\Gamma L \geq \alpha\eta\rho[1+\epsilon(1+\alpha)]$ by (34). Substituting this into $\Phi(0)$ and noting that the coefficient on ΓL is positive under $\theta \geq \alpha$, we have $\Phi(0) \geq \rho\alpha^3\eta^2(1-\alpha)[\epsilon(1-\alpha) + 1 + 2\alpha\epsilon + \alpha\epsilon^2] > 0$. We then show $\gamma'(s_f) < 0$ at a high level of s_f. Suppose $s_f = (1-\alpha)/(2-\alpha)$ such that $\tau = 1$. It is obvious by (22) that $\gamma(s_f)$ cannot be positive at such a level of s_f, starting with any $\gamma(s_f) > 0$ for $s_f = 0$. Thus, $\gamma'(s_f) < 0$ must occur before s_f is raised to the level $(1-\alpha)/(2-\alpha)$. So $\gamma'(s_f) = 0$ must hold for some $s_f > 0$. Since

$$\text{sign } \Phi'(s_f) = \text{sign } \frac{\Gamma L}{\alpha\eta}\left\{\frac{1-\alpha(1+s_f) - (1-\alpha)([1+\alpha(1+s_f)]}{(1-\alpha)\alpha^2(1+s_f)^2}\right\}$$

$$= \text{sign } \{-\alpha\,[\alpha s_f + (1-\alpha)(1+s_f)]\} < 0,$$

$\Phi(s_f)$ is monotonically decreasing in s_f and thus the solution for $\Phi(s_f) = 0$ is unique. Thus, for θ □ α and a small enough ρ (such that $\gamma > 0$), γ is globally concave with respect to s_f and reaches a maximum level at $0 < s_f < \infty$. ■

Proof of Proposition 3. We set $s_f = 0$ and use (22) and (23) to rewrite the growth rate in terms of both the R&D subsidy s_n and the tax rate τ:

$$\gamma(s_n) = \frac{[(1-\alpha)\Gamma L/(\alpha\eta) - (1-s_n)\rho](1-\tau) - \epsilon\rho(1+\alpha)(1-s_n)}{\theta(1-s_n)(1-\tau) + \epsilon\theta(1+\alpha)(1-s_n) - \epsilon\alpha}, \tag{50}$$

where τ satisfies

$$\tau = \frac{\alpha\gamma s_n}{(1-s_n)(\theta\gamma+\rho)}.$$

Substituting it into (50) for τ and differentiating the resulting equation with respect to s_n, we obtain $\gamma'(s_n) = \Lambda_1/\Lambda_2$, where

$$\begin{aligned}
\Lambda_1 \equiv\ & \alpha\eta\gamma\theta[\theta\gamma + \alpha\gamma + \rho](1-s_n) + \alpha\eta\gamma\theta[(1-s_n)(\theta\gamma+\rho) - \alpha\gamma s_n] \\
& + \gamma\alpha\eta\epsilon\theta(1+\alpha)(1-s_n)(\theta\gamma+\rho) + \gamma\alpha\eta[\epsilon\theta(1+\alpha)(1-s_n) - \alpha\epsilon](\theta\gamma+\rho) \\
& - (1-\alpha)\Gamma L(\theta\gamma+\alpha\gamma+\rho) + \alpha\eta\rho[(1-s_n)(\theta\gamma+\rho) - \alpha\gamma s_n] \\
& + \alpha\eta\rho(1-s_n)(\theta\gamma+\alpha\gamma+\rho) + 2\alpha\eta\epsilon\rho(1+\alpha)(1-s_n)(\theta\gamma+\rho),
\end{aligned}$$

$$\begin{aligned}
\Lambda_2 \equiv\ & \alpha\eta\theta(1-s_n)[(1-s_n)(\theta\gamma+\rho) - \alpha\gamma s_n] + \alpha\eta\gamma\theta(1-s_n)[(1-s_n)\theta - \alpha s_n] \\
& + \alpha\eta(1-s_n)[\epsilon\theta(1+\alpha)(1-s_n) - \alpha\epsilon](\theta\gamma+\rho) + \alpha\eta\gamma\theta(1-s_n) \\
& \times [\epsilon\theta(1+\alpha)(1-s_n) - \alpha\epsilon] - (1-\alpha)\Gamma L[\theta(1-s_n) - \alpha s_n] \\
& + \alpha\eta\rho(1-s_n)[\theta(1-s_n) - \alpha s_n] + \alpha\eta\epsilon\rho\theta(1+\alpha)(1-s_n)^2.
\end{aligned}$$

By $1-\tau = [(1-s_n)(\theta\gamma+\rho) - \alpha\gamma s_n]/[(1-s_n)(\theta\gamma+\rho)]$, it is clear that $1-\tau = 1$ if $s_n = 0$ and that if $s_n = 1$ then $1-\tau$ □ 0 for any non-negative γ. Using (50) to express $(1-\alpha)\Gamma L = \alpha\eta\gamma[\theta(1-s_n)(1-\tau) + \epsilon\theta(1+\alpha)(1-s_n) - \epsilon\alpha]/(1-\tau) + \epsilon\alpha\eta\rho(1+\alpha)(1-s_n)/(1-\tau) + \alpha\eta\rho(1-s_n)$. Substituting out $(1-\alpha)\Gamma L$ in Λ_1 and Λ_2, we have: if $s_n = 0$ and if $\theta \geq \alpha$, then $\Lambda_1, \Lambda_2 > 0$, leading to $\gamma'(s_n) > 0$. If $s_n = 1$, then γ cannot be positive by (50) and by the expression of $(1-\tau)$. Thus, starting with $s_n = 0$ and any positive γ, $\gamma'(s_n) < 0$ must occur before s_n rises to 1. Thus, $\gamma'(s_n) = 0$ for some $s_n > 0$.

The rest of the proof is to show that $\Phi'(s_n) < 0$, paralleling the proof of Proposition 2. From (35), we have: sign $\gamma'(s_n) =$ sign $\Phi(s_n)$, where

$$\begin{aligned}\Phi(s_n) \equiv\ & \Psi_1(s_n)[\Psi_2(s_n)^2 - 4\Psi_1(s_n)\Psi_3(s_n)]^{-1/2} \\ & \times \left[\Psi_2(s_n)\Psi_2'(s_n) - 2\Psi_3(s_n)\Psi_1'(s_n) - 2\Psi_1(s_n)\Psi_3'(s_n)\right] - \Psi_1(s_n)\Psi_2'(s_n) \\ & - \left\{[\Psi_2(s_n)^2 - 4\Psi_1(s_n)\Psi_3(s_n)]^{1/2} - \Psi_2(s_n)\right\}\Psi_1'(s_n). \qquad (51)\end{aligned}$$

From (51), we have

$$\begin{aligned}\text{sign } \Phi'(s_n) = \text{sign } & [\Psi_2(s_n)^2 - 4\Psi_1(s_n)\Psi_3(s_n)]^{-\frac{1}{2}}\left[\Psi_2'(s_n)^2 + \Psi_2(s_n)\Psi_2''(s_n) - 4\Psi_1'(s_n)\Psi_3'(s_n)\right] \\ & -\Psi_2''(s_n) - \left\{[\Psi_2(s_n)^2 - 4\Psi_1(s_n)\Psi_3(s_n)]^{-3/2}\right. \\ & \left.\times \left[\Psi_2(s_n)\Psi_2'(s_n) - 2\Psi_3(s_n)\Psi_1'(s_n) - 2\Psi_1(s_n)\Psi_3'(s_n)\right]\right\}, \qquad (52)\end{aligned}$$

where

$$\begin{aligned}\Psi_1'(s_n) &= -\theta\{\theta[1+\epsilon(1+\alpha)] + \alpha\} < 0, \\ \Psi_2'(s_n) &= \left[\frac{(1-\alpha)L(\alpha^2 A)^{1/(1-\alpha)}}{\eta(1-s_n)^2} - \frac{\alpha\rho}{1-s_n}\right] + \frac{\alpha\rho s_n}{1-s_n} - 2\theta\rho[1+\epsilon(1+\alpha)], \\ \Psi_2''(s_n) &= \left[\frac{2(1-\alpha)L(\alpha^2 A)^{1/(1-\alpha)}}{\eta(1-s_n)^3} - \frac{\alpha\rho}{(1-s_n)^2}\right] + \frac{\alpha\rho}{(1-s_n)^2}, \\ \Psi_3'(s_n) &= -\rho^2[1+\epsilon(1+\alpha)] < 0.\end{aligned}$$

Substituting $\Psi_k(s_n)$, $k = 1, 2, 3$, and their derivatives into (52), we have: if ρ is sufficiently small, then $\Phi'(s_n) < 0$, i.e. $\Phi(s_n)$ is monotonically decreasing in s_n. ■

Appendix B

Derivation of Equation (42). We use (41) to write an implicit solution for leisure in (25) as a function of (s_f, s_n), i.e., $l(s_f, s_n)$:

$$lAL\{\epsilon\theta(1-s_n)[1-\alpha^2(1+s_f)]/\alpha - \epsilon(1-\alpha)(1+s_f) + \theta(1-s_n)[1-\alpha(1+s_f)]/\alpha -$$

$$s_n(1+s_f)(1-\alpha)\} + \frac{lAs_n\alpha\rho\eta(1+s_f)(1-s_n)}{(1-l)\Gamma} =$$

$$\eta\epsilon\rho(1-s_n)\Gamma^{-\alpha}/\alpha + \epsilon AL\{\theta(1-s_n)[1-\alpha^2(1+s_f)]/\alpha - (1-\alpha)(1+s_f)\}. \quad (53)$$

From (53), we derive $\partial l/\partial s_f = \Omega_1^{s_f}/\Omega_2^{s_f}$ and $\partial l/\partial s_n = \Omega_1^{s_n}/\Omega_2^{s_n}$, where

$$\Omega_1^{s_f} \equiv \alpha lAL[(\epsilon+1/\alpha)\theta(1-s_n) + (\epsilon+s_n)(1/\alpha-1)] + \frac{lA\alpha\eta\rho s_n(1-s_n)[1-(1-\alpha)\alpha(1+s_f)]}{(1-\alpha)\alpha(1+s_f)(1-l)\Gamma}$$

$$-\frac{\eta\epsilon\rho(1-s_n)}{(1-\alpha)(1+s_f)\Gamma^\alpha} - \epsilon AL[\theta(1-s_n)+1-\alpha],$$

$$\Omega_2^{s_f} \equiv AL\{\epsilon\theta(1-s_n)(1-\alpha)/\alpha + \theta(1-s_n)[1-\alpha(1+s_f)]/\alpha - (\epsilon+s_n)(1-\alpha)(1+s_f)\}$$

$$+\frac{A\rho\eta s_n(1-s_n)\alpha(1+s_f)}{\Gamma(1-l)^2},$$

$$\Omega_1^{s_n} \equiv lAL\left\{\epsilon\theta(1-s_n)[1-\alpha^2(1+s_f)]/\alpha + \theta[1-\alpha(1+s_f)]/\alpha + (1-\alpha)(1+s_f)\right\}$$

$$-\frac{lA\eta\rho(1-2s_n)\alpha(1+s_f)}{\Gamma(1-l)} - \epsilon\eta\rho\Gamma^{-\alpha}/\alpha - \epsilon AL\theta[1-\alpha^2(1+s_f)]/\alpha,$$

$$\Omega_2^{s_n} \equiv AL\left\{\epsilon\theta[1-\alpha^2(1+s_f)]/\alpha + \theta(1-s_n)[1-\alpha(1+s_f)]/\alpha - (\epsilon+s_n)(1-\alpha)(1+s_f)\right\}$$

$$+\frac{A\rho\eta s_n(1-s_n)\alpha(1+s_f)}{\Gamma(1-l)^2}.$$

Based on the solution for U_0 in (26) and the expression for τ in (41), the utility-maximizing problem of the government by choice of (τ, s_f, s_n) subject to its constraint in (41) is equivalent to choosing (s_f, s_n) to maximize (42).

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Table 1
The social planner's solution

Parameters	R&D Rate (%)	Growth Rate (%)	Welfare
Benchmark parameterization: $\alpha = 0.3$, $\theta = 1.5$, $\epsilon = 0.5$, $\eta = 2.02$, $\rho = 0.05$, $A = L = N_0 = 1$			
	Benchmark parameterization		
	32.35	7.53	-39.90
	Alternative parameterizations		
$\alpha = 0.2$	40.80	10.85	-22.77
$\alpha = 0.4$	23.86	4.93	-59.48
$\epsilon = 0.3$	33.63	8.60	-27.03
$\epsilon = 0.7$	30.99	6.59	-51.92
$\epsilon = 1.0$	28.74	5.34	-68.76
$\theta = 1.1$	45.57	11.47	-20.48
$\theta = 2.0$	23.74	5.27	-74.11
$\theta = 3.0$	15.50	3.30	-270.70
$\rho = 0.02$	41.23	10.11	-16.92
$\rho = 0.08$	22.49	4.96	-33.45
$\eta = 1.0$	39.88	19.59	-14.18
$\eta = 3.0$	24.46	3.67	-51.78
$A = 0.5$	1.43	0.11	-123.62
$A = 2.0$	41.63	27.53	12.98
$L = 0.5$	15.29	1.62	-57.54
$L = 2.0$	39.81	19.35	-14.53

Note: (1) The benchmark parameters are chosen to be consistent with those used in the literature and generate a growth rate of 3.0% in a competitive equilibrium with R&D subsidies (the average growth rate for the past 30 years in the United States). (2) The optimal R&D rate refers to the ratio of R&D investment to output.

Table 2
Growth-maximizing subsidies vs. growth rates

Benchmark parameters: $\alpha = 0.3$, $\theta = 1.5$, $\epsilon = 0.5$, $\eta = 2.02$, $\rho = 0.05$, $A = L = N_0 = 1$							
Parameters	Social Planner's Solution	Production Subsidy (s_f)			R&D Subsidy (s_n)		
	Growth Rate (%)	Tax Rate (%)	Subsidy Rate %	Growth Rate %	Tax Rate (%)	Subsidy Rate (%)	Growth Rate (%)
		Benchmark parameterization					
	7.53	84.59	145.17	0.35	79.67	86.15	5.94
		Alternative parameterizations					
$\alpha = 0.2$	10.85	90.38	261.07	1.78	83.76	89.56	9.13
$\alpha = 0.4$	4.93	77.76	0.00*	0.00	76.14	84.13	3.89
$\epsilon = 0.3$	8.60	88.18	161.29	1.84	82.80	85.65	7.55
$\epsilon = 0.7$	6.59	0.00*	0.00	0.00	77.62	86.73	4.87
$\epsilon = 1.0$	5.34	0.00*	0.00	0.00	75.56	87.62	3.82
$\theta = 1.1$	11.47	84.86	146.30	0.56	80.91	83.53	6.40
$\theta = 2.0$	5.27	84.44	144.56	0.24	78.89	88.38	5.61
$\theta = 3.0$	3.30	84.32	144.03	0.15	78.15	91.14	5.26
$\rho = 0.02$	10.11	88.14	161.09	3.14	81.21	83.03	6.50
$\rho = 0.08$	4.96	0.00*	0.00	0.00	78.85	88.51	5.59
$\eta = 1.0$	19.59	87.58	158.45	5.43	80.89	83.58	12.92
$\eta = 3.0$	3.67	0.00*	0.00	0.00	78.97	88.11	3.80
$A = 0.5$	0.11	0.00*	0.00	0.00	78.10	91.38	1.94
$A = 2.0$	27.53	88.31	161.89	8.83	81.31	82.86	17.60
$L = 0.5$	1.62	0.00*	0.00	0.00	78.50	89.74	2.72
$L = 2.0$	19.35	87.55	158.32	5.33	80.88	83.60	12.78

Note: (1) For all the parameterizations, the growth-maximizing mixes of subsidies are the same as using the R&D subsidy alone. (2) The numbers with $*$ indicate that with these parameterizations, there do not exist production subsidy rates (and tax rates) that can induce R&D investment.

Table 3
Welfare comparisons of production and R&D subsidies

Benchmark parameters: $\alpha = 0.3$, $\theta = 1.5$, $\epsilon = 0.5$, $\eta = 2.02$, $\rho = 0.05$, $A = L = N_0 = 1$								
Parameters	Production Subsidy (s_f)				R&D Subsidy (s_n)			
	Tax Rate (%)	Subsidy Rate (%)	Growth Rate (%)	Welfare	Tax Rate (%)	Subsidy Rate (%)	Growth Rate (%)	Welfare
	Benchmark parameterization							
	77.01	116.95	0.17	-67.41	**30.55**	**76.33**	**3.00**	**-65.30**
	Alternative parameterizations							
$\alpha = 0.2$	85.03	212.72	1.51	-46.66	39.93	82.95	5.33	-43.15
$\alpha = 0.4$	0.00*	0.00	0.00	-90.85	19.03	70.86	1.38	-87.84
$\epsilon = 0.3$	81.64	133.38	1.58	-46.61	36.05	76.38	4.20	-48.43
$\epsilon = 0.7$	0.00*	0.00	0.00	-85.85	25.35	76.38	2.15	-80.03
$\epsilon = 1.0$	0.00*	0.00	0.00	-101.71	17.76	76.54	1.25	-99.01
$\theta = 1.1$	79.14	124.21	0.37	-43.35	41.30	76.24	4.06	-40.64
$\theta = 2.0$	75.23	111.23	0.07	-124.92	23.06	76.40	2.26	-125.41
$\theta = 3.0$	72.99	104.48	0.00**	-516.93	15.48	76.47	1.52	-564.07
$\rho = 0.02$	82.41	136.34	2.97	-78.98	48.86	75.69	4.86	-81.04
$\rho = 0.08$	0.00*	0.00	0.00	-46.19	11.40	76.78	1.11	-45.72
$\eta = 1.0$	81.54	132.99	5.08	-40.81	46.01	75.80	9.22	-41.11
$\eta = 3.0$	0.00*	0.00	0.00	-73.90	15.14	76.71	0.99	-72.48
$A = 0.5$	0.00*	0.00	0.00	-146.87	0.00*	0.00	0.00	-146.87
$A = 2.0$	82.67	137.36	8.37	-1.70	49.72	75.65	13.32	-2.30
$L = 0.5$	0.00*	0.00	0.00	-73.90	0.00*	0.00	0.00	-73.90
$L = 2.0$	81.50	132.82	4.98	-41.23	45.86	75.80	9.10	-41.51

Note: (1) The numbers with $*$ indicate that under these parameterizations, there do not exist subsidy rates (and tax rates) that can induce R&D investment. (2) The number with $**$ is the rounded-up growth rate of 0.0025%.

Table 4
Optimal combinations of subsidies vs. welfare

Benchmark parameters: $\alpha = 0.3$, $\theta = 1.5$, $\epsilon = 0.5$, $\eta = 2.02$, $\rho = 0.05$, $A = L = N_0 = 1$					
Parameters	Social Planner's Solution	No Subsidies ($s_f = s_n = 0$)	Combinations of Subsidies ($s_f,\ s_n$)		
	Welfare	Welfare	Tax Rate (%)	Subsidy Rates (%)	Welfare
			Benchmark parameterization		
	-39.90	-73.90	71.16	(73.84, 53.01)	-58.32
			Alternative parameterizations		
$\alpha = 0.2$	-22.77	-58.68	77.70	(109.64, 60.33)	-37.75
$\alpha = 0.4$	-59.48	-90.85	64.23	(51.83, 48.80)	-79.88
$\epsilon = 0.3$	-27.03	-60.21	76.61	(86.11, 48.47)	-40.80
$\epsilon = 0.7$	-51.92	-85.85	67.14	(67.84, 55.55)	-73.35
$\epsilon = 1.0$	-68.76	-101.71	62.72	(64.42, 57.60)	-92.39
$\theta = 1.1$	-20.48	-46.56	74.28	(71.21, 53.76)	-36.94
$\theta = 2.0$	-74.11	-142.16	69.34	(76.06, 52.36)	-107.73
$\theta = 3.0$	-270.70	-647.38	67.75	(78.52, 51.64)	-437.61
$\rho = 0.02$	-16.92	-175.12	77.00	(76.21, 49.39)	-65.03
$\rho = 0.08$	-33.45	-46.19	67.04	(78.79, 53.92)	-41.55
$\eta = 1.0$	-14.18	-73.90	75.95	(75.06, 50.24)	-34.38
$\eta = 3.0$	-51.78	-73.90	67.71	(77.50, 53.88)	-65.74
$A = 0.5$	-123.62	-146.87	0.00*	(0.00, 0.00)	-146.87
$A = 2.0$	12.98	-25.48	77.32	(76.63, 49.11)	1.52
$L = 0.5$	-57.54	-73.90	0.00*	(0.00, 0.00)	-73.90
$L = 2.0$	-14.53	-73.90	75.90	(75.01, 50.28)	-34.77

Note: The numbers with $*$ indicate that for these parameterizations, there do not exist welfare-maximizing mixes of subsidies that can induce R&D investment.

Figure 1: Subsidies vs. Growth

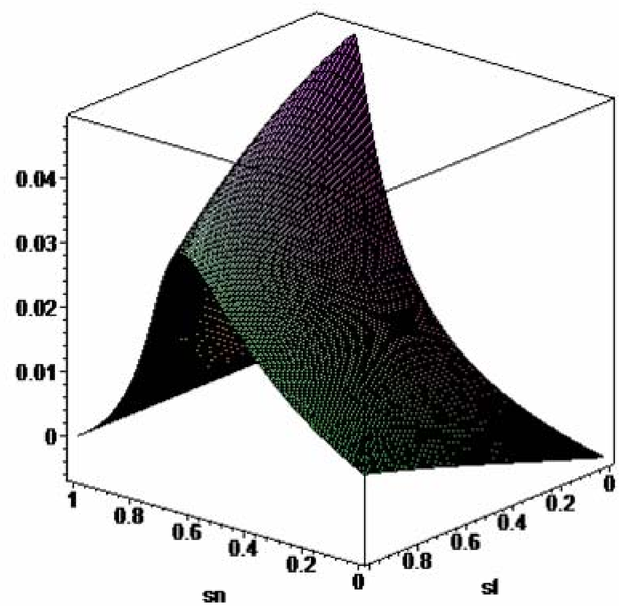

Fig. 1(a)

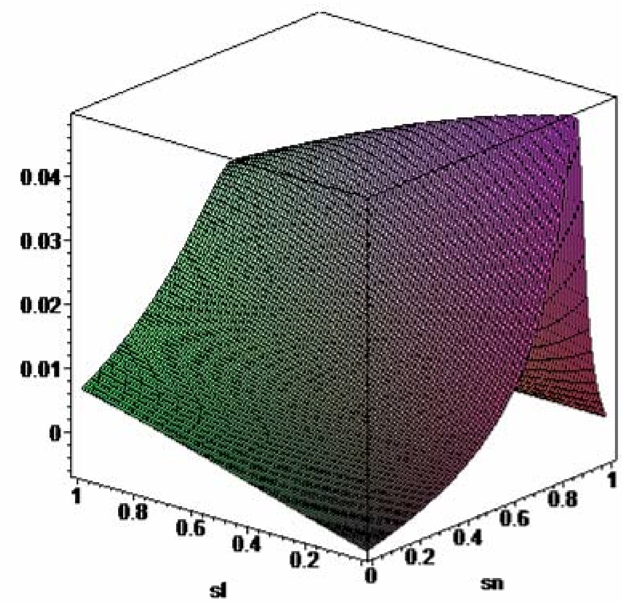

Fig. 1(b)

Parameters: $A = L = N_0 = 1, \; \alpha = 0.3, \eta = 2.38,$

$\theta = 2, \quad \varepsilon = 0.5, \quad \rho = 0.03$

Figure 2: Subsidies vs. Welfare

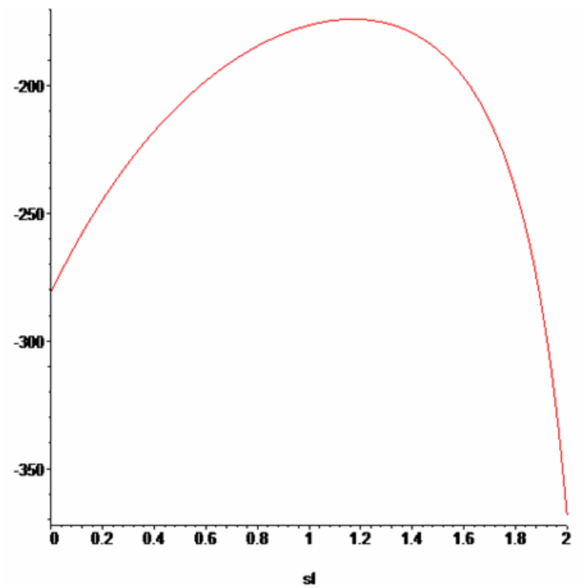

Fig. 2(a): Production Subsidy

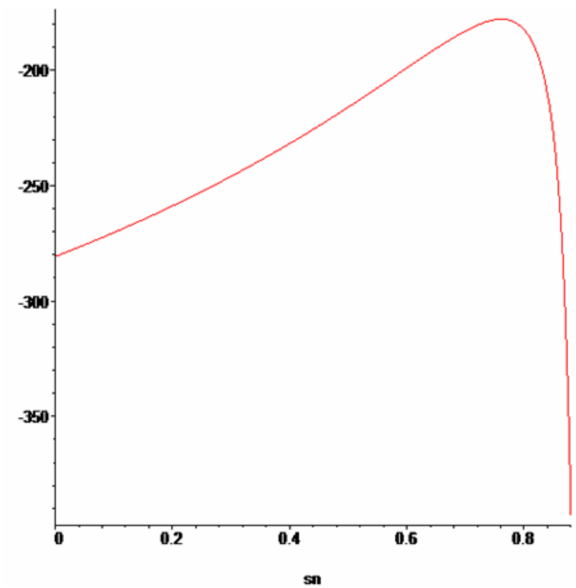

Fig. 2(b): R&D Subsidy

Parameters: $A = L = N_0 = 1,\ \alpha = 0.3,\ \eta = 2.38,$

$\theta = 2, \quad \varepsilon = 0.5, \quad \rho = 0.03$

Figure 3: Mixes of Subsidies vs. Welfare

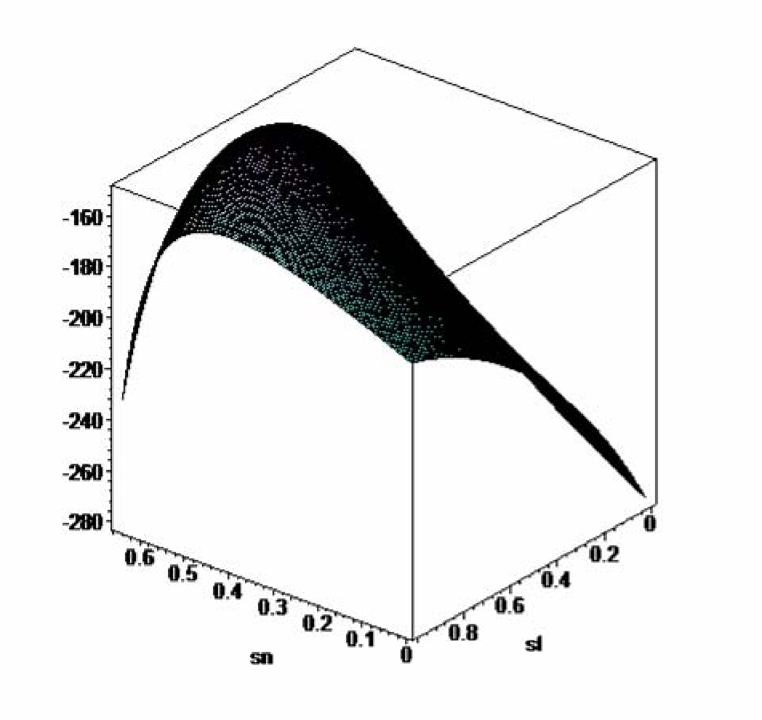

Fig. 3(a)

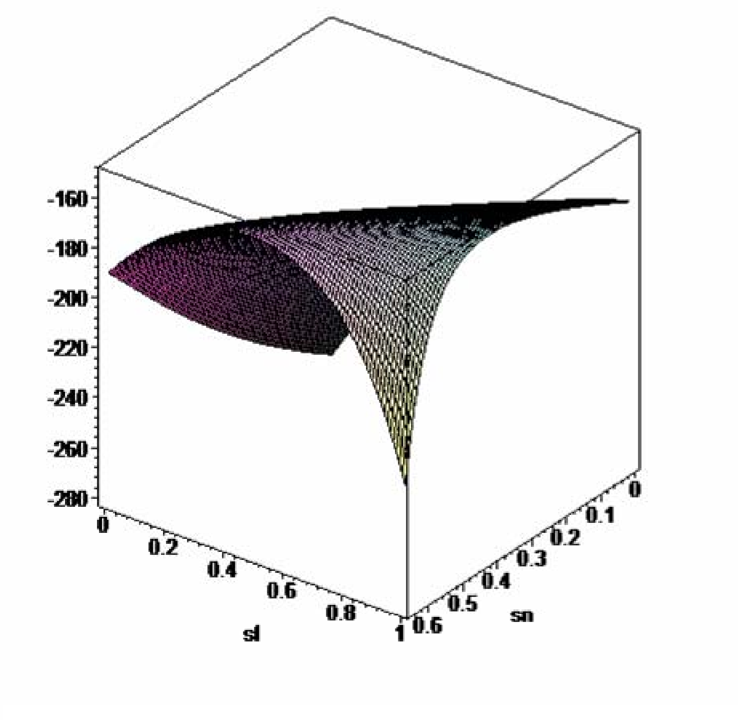

Fig. 3(b)

Parameters: $A = L = N_0 = 1$, $\alpha = 0.3$, $\eta = 2.38$,

$\theta = 2$, $\varepsilon = 0.5$, $\rho = 0.03$