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Health, Education and Life-Cycle Savings in Different Stages of Development

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Abstract

This paper studies investment in health and education in a life-cycle model. Health investment enhances survival to old age by improving health from its endowed level. The model predicts two distinctive phases of development. When income is low enough, the economy has no health investment and little savings, leading to slow growth. When income grows, health investment will become positive and the saving rate will rise, leading to higher life expectancy and faster growth. A health subsidy can move the economy from the first phase to the next. Subsidies on health and education investments can improve welfare.

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I. INTRODUCTION

Between countries with low and high life expectancy, there are striking differences in their school enrollments, investment-to-GDP ratios, health spending, and growth rates of per capita income. According to the data set in Barro and Lee (1994), in countries with life expectancy in 1960 below 50 (with a mean of 43.4 years), the average ratio of private investment to GDP was 14%, the average secondary school enrollment ratio was 17.6%, and the average growth rate of per capita GDP was 1.4%, for the period 1960-1989. By contrast, in countries with life expectancy in 1960 above 65, the corresponding average figures were 22%, 71% and 2.96%, respectively. With no exception, countries with life expectancy in 1960 below 50 (28 in total) are poor and their very low life expectancy is a consequence of little health investment. On the other hand, those with life expectancy in 1960 above 65 are developed countries (24 in total), and are mostly the members of the OECD. These rich countries with much longer life expectancy have had much higher health spending than those at the other end of the spectrum.[1]

Similar to the cross-country comparison, there were upward trends in the ratios of health and education spending to GDP and in life expectancy in the time-series data of the United States in Table 1 for the period 1870-2000. The postwar average growth rate of per capita GDP also appeared to be higher than the prewar average growth rate in the United States, as in many other developed countries according to Maddison (1991). Though the long-term saving rate did not have a discernable trend in the United States, it typically had an upward trend in other developed countries as documented in Maddison (1992). It is thus important to explore the interaction between life expectancy and growth by investigating household decisions on health investment, education investment and life-cycle savings.

Moreover, in many developed countries health and education expenditures are heavily subsidized or publicly provided through distortionary taxes. To a lesser extent, health and education expenditures are also subsidized in some less developed countries. Thus, it is also important to investigate the impacts of these subsidies on household decisions about health spending, education spending and life-cycle savings. Through this investigation, we can learn how these subsidies affect life expectancy, output growth and welfare.

Recently, the relationship between longevity and household decisions on savings and education investment has received a great deal of attention. The typical view is that rising longevity or declining mortality encourages savings and education investment and hence promotes economic growth. See, e.g., Skinner (1985), Ehrlich and Lui (1991), Barro and Sala-i-Martin (1995), de la Croix and Licandro (1999), Zhang, Zhang and Lee (2001), Boucekkine, de la Croix and Licandro (2002, 2003), Zhang, Zhang and Lee (2003), and Zhang and Zhang (2005). However, the rate of survival or death is usually assumed to be exogenous in these papers. Though some studies have considered health investment, e.g. Ehrlich and Chuma (1990), Philipson and Becker (1998) and Leung, Zhang and Zhang (2004), they have not considered education investment at the same time, and therefore their models do not permit sustainable growth in the long run. Intuitively, economic growth promises more resource available for future improvements in health care and life expectancy, while rising life expectancy may in turn motivate savings and education investment.

Some recent studies have considered health and education expenditures together in a life-cycle model. Among them, Chakraborty and Das (2005) focus on how the distribution of wealth interacts with health investment and education investment in accounting for the high intergenerational correlation of economic status and persistent disparities in health status between the rich and the poor. Also, Corrigan, Glomm and Mendez (2005) find large growth effects of an AIDS epidemic and relatively small effects of policies such as the subsidization of AIDS medication.

In this paper we investigate health investment, education investment and life-cycle savings in an endogenous growth model. Health investment improves survival to old age that has a lower bound supported by an endowment of health capital to each young individual. Unlike Chakraborty and Das (2005) and Corrigan, Glomm and Mendez (2005), however, we focus on whether the equilibrium solution can differ significantly in different stages of development in a way that resembles what we observe in the real would. Also, we explore how subsidies on education spending or health spending influence capital accumulation, health investment and welfare in different stages of development.

Our model predicts two distinctive phases of development. When income is sufficiently low, there is no health investment, because the marginal utility of consumption would then exceed the marginal utility of health investment given the lower bound on the expected rate of survival. Corresponding to this minimum rate of survival to old age, the saving rate is at its lowest level, leading to very slow growth. When income grows, health investment will become positive and the saving rate will rise substantially, leading to higher life expectancy and faster growth. These results capture some of the stylized facts mentioned earlier.

Interestingly, a health subsidy can move the economy from the no-health-investment phase to the next, a transition that brings about higher life expectancy, greater savings, and faster growth. A growing economy in this model converges to a unique balanced growth path. A health subsidy does not affect the balanced growth rate, while an education subsidy increases it under plausible conditions. These subsidies through a wage income tax can also improve welfare with an externality from average health spending in the formation of health capital, particularly in the short run. Numerically, the ideal subsidy rates are found to be around 60% for both education and health expenditures. The findings support recent reform in some developed countries converting a public health and education system into one with shared financial responsibility between the state and households.

The rest of the paper proceeds as follows. Section II describes the model. Sections III and IV provide analytical and numerical results, respectively. Section V summarizes our findings and discusses further extensions. The last section concludes.

II. THE MODEL

The economy consists of overlapping generations of agents who live for three periods (one period in childhood and two in adulthood). Children learn to embody human capital through education, young adults work, and old adults live in retirement. Each worker gives birth to one child and each working generation has a mass L. Survival from childhood to young adulthood is certain, while survival to old age is uncertain at a rate of $P(h_t)$ that is increasing and concave in health capital h_t. We abstract from child mortality and the choice of the number of children to keep the model tractable. Workers make decisions on life-cycle savings s, investment in health m, and investment in the education of their children q.

Health capital accumulates according to:

(1) $$h_t = A_h m_t^{\alpha} (\overline{m}_t)^{1-\alpha} + \overline{h},$$

where $\overline{m}_t$ refers to average investment in health, $\overline{h}$ refers to an endowment of health capital to each young adult, $A_h > 0$ indicates the effectiveness of the health technology, and $0 < \alpha < 1$ indicates the relative importance of an individual's own health spending versus average health spending in the economy. One rationale for the inclusion of average health investment in the formation of health capital is that when there is little health spending on average, there would be lack of health professionals and health care facilities as in many poor countries. In this scenario, health spending by a single agent can hardly enhance his health status. The endowed component of health capital reflects the fact that there is still a chance expected to survive to old age even without health spending. As we will see, this endowment of health capital can lead to realistic transitional dynamics in this model.

The education of a child is determined by

(2) $$e_{t+1} = A_e q_t^{\eta} e_t^{1-\eta},$$

where e_{t+1} and e_t stand for the human capital of the child and his parent, respectively, and q_t is the amount of education spending. In addition, $A_e > 0$ is an efficiency parameter in education and $0 < \eta < 1$ measures the relative importance of education spending versus parental human capital in education.

The rate of survival to old age is an exponential function of health capital:

(3) $P_t = 1 - 1/\exp(h_t)$,

which is clearly increasing and concave in health capital.

Final production uses physical capital, *K*, and effective labor, Le:

(4) $Y_t = A_y K_t^{\phi} (Le_t)^{1-\phi}$,

where $A_y > 0$ and $0 < \phi < 1$ are the total factor productivity parameter and the share parameter of physical capital, respectively. Production inputs are compensated according to their marginal products:

(5) $1 + r_t = \phi A_y (e_t / k_t)^{1-\phi}$,

(6) $w_t = (1-\phi) A_y (k_t / e_t)^{\phi}$,

where *w* and *r* are the real wage rate and the real interest rate, respectively.

We assume a perfect annuity market through which workers invest their savings in exchange for income for retirement conditional on survival. Under this assumption, savings left by savers who die at the end of working age will be shared by the rest of savers who survive to old age. This assumption implies that the rate of return on savings is equal to $(1 + r_{t+1}) / \overline{P}_t$ where $\overline{P}$ is the average rate of survival (i.e. the portion of the working population surviving to old age). The household budget constraints are given by:

(7) $w_t e_t (1 - \tau_w) = m_t (1 - \pi_m) + q_t (1 - \pi_e) + s_t + c_t$,

(8) $z_{t+1} = s_t (1 + r_{t+1}) / \overline{P}_t$.

Here, health and education expenditures are subsidized at respective rates π_m and π_e (funded by a wage income tax τ_w), *c* refers to working-age consumption, *s* refers to life-cycle savings, and *z* stands for old-age consumption. Also, we assume inelastic supply of one unit of labor per worker to keep things simple.

The government budget constraint is balanced in every period:

(9) $\tau_w w_t e_t = \pi_m m_t + \pi_e q_t$.

In this model, the tax rate τ_w will be treated as a variable, while the subsidy rates, π_e and π_m, will be treated as policy parameters.

The preferences are assumed to be:

(10) $$U_t = \frac{c_t^{1-\theta}}{1-\theta} + \delta P_t \frac{z_{t+1}^{1-\theta}}{1-\theta} + \xi \frac{e_{t+1}^{1-\theta}}{1-\theta}, \qquad 0 < \delta,\ \xi,\ \theta < 1;$$

$$U_t = \ln c_t + \delta P_t \ln z_{t+1} + \xi e_{t+1} \text{ if } \theta = 1.$$

That is, an agent derives utility from working-age consumption, old-age consumption (conditional on survival), and the education outcome of his child. The parameters in (10) include two discounting factors, δ and ξ, and one constant coefficient of relative risk aversion, θ. The presence of the product $z^{1-\theta}P/(1-\theta)$ for $\theta \neq 1$, or $P \ln z$ for $\theta = 1$, in (10) introduces non-concavity in the utility function. To ensure the needed concavity for any interior solution to be optimal, we require $0 < \theta \leq 1$. Note that the use of a CES utility function is typical in endogenous growth models to which our model belongs.

The capital market clears when

(11) $$k_{t+1} = s_t,$$

where $k=K/L$ is physical capital per worker. Correspondingly, output per worker is $y_t = A_y k_t^{\alpha} e_t^{1-\alpha}$. In equilibrium, we expect $e = \overline{e}$, $m = \overline{m}$ and $P = \overline{P}$ by symmetry, because workers in each generation are identical ex ante in this model.

III. EQUILIBRIUM AND RESULTS

Taking $(r_{t+1}, w_t, e_t, \overline{m}_t, \overline{P}_t)$ as given, the household problem in period t may be formulated as

(12) $$\max \mathrm{L}_t = \frac{c_t^{1-\theta}}{1-\theta} + \delta \frac{z_{t+1}^{1-\theta}}{1-\theta}[1-\exp(-A_h m_t^{\alpha} \overline{m}_t^{1-\alpha} - \overline{h})] + \xi \frac{(A_e q_t^{\eta} e_t^{1-\eta})^{1-\theta}}{1-\theta} +$$

$$\lambda_t [w_t e_t (1-\tau_w) - m_t(1-\pi_m) - q_t(1-\pi_e) - z_{t+1}\overline{P}_t/(1+r_{t+1}) - c_t],$$

by choice of nonnegative variables $(c_t,\ z_{t+1},\ m_t,\ q_t)$, where λ is the Lagrange multiplier.

The first-order conditions of the household problem are given below for $\theta \in (0,1)$:

(13) $$\frac{\partial \mathrm{L}_t}{\partial c_t} = c_t^{-\theta} - \lambda_t \le 0, \quad c_t \frac{\partial \mathrm{L}_t}{\partial c_t} = 0,$$

(14) $$\frac{\partial \mathrm{L}_t}{\partial z_{t+1}} = \delta\, z_{t+1}^{-\theta} - \frac{\lambda_t}{1 + r_{t+1}} \le 0, \quad z_{t+1} \frac{\partial \mathrm{L}_t}{\partial z_{t+1}} = 0,$$

(15) $$\frac{\partial \mathrm{L}_t}{\partial m_t} = \delta \frac{z_{t+1}^{1-\theta}}{1-\theta} \alpha A_h \exp(-h_t) - \lambda_t (1 - \pi_m) \le 0, \quad m_t \frac{\partial \mathrm{L}_t}{\partial m_t} = 0,$$

(16) $$\frac{\partial \mathrm{L}_t}{\partial q_t} = \xi\, e_{t+1}^{-\theta}\, \eta\, A_e\, (e_t / q_t)^{1-\eta} - \lambda_t (1 - \pi_e) \le 0, \quad q_t \frac{\partial \mathrm{L}_t}{\partial q_t} = 0.$$

For the log utility case, we can simply set $\theta = 1$ in (13), (14) and (16), and replace $z_{t+1}^{1-\theta} /(1-\theta)$ with $\ln z_{t+1}$ in (15). The first-order conditions indicate that the net marginal benefits of all the choice variables are non-positive. If the net marginal benefit of a household variable is strictly negative, then this variable must be equal to zero. Obviously, when consumption and education investment approach zero, their marginal benefits will approach infinity. However, when health investment approaches zero, its marginal benefit does not approach infinity because $h_t \ge \bar{h}$ provides a lower bound on $\exp(h_t)$ or an upper bound on $\exp(-h_t)$ in (15). In other words, there may be a corner solution for health investment.

We give the condition for a corner solution for health investment below and relegate the proof to the Appendix:

PROPOSITION 1. *For small enough initial capital stocks* (k_0, e_0) *relative to a given* $\bar{h}$*, we have* $m_t = 0$ for some $t \ge 0$. *Subsidizing health investment can avoid such a corner solution.*

Proposition 1 is consistent with the fact that many poor countries suffer from lack of health services with little health spending. The intuition is that when households are so poor that their marginal utility of consumption exceeds their marginal utility of health investment, they are unwilling to divert their small income for health investment. This is mainly because the expected rate of survival is bounded below by endowed health capital such that the marginal utility of health investment is bounded above. Another interesting result in Proposition 1 is that governments can induce positive health investment by subsidizing it.

With a corner solution for health investment, equations $m_t = 0$, (13), (14), (16) and the budget constraints lead to the evolution of human and physical capital below:

$$(17) \qquad e_{t+1} / e_t = \Theta_0 P(\bar{h})^{\frac{-\theta\eta}{1-\eta}} \left(k_{t+1} / e_{t+1}\right)^{\frac{\eta[1-\phi(1-\theta)]}{1-\eta}},$$

$$(18) \qquad A_y(1-\phi)(k_t / e_t)^{\phi} = A_e^{\frac{-1}{\eta}} \Theta_0^{\frac{1}{\eta}} P(\bar{h})^{\frac{-\theta}{1-\eta}} (k_{t+1} / e_{t+1})^{\frac{1-\phi(1-\theta)}{1-\eta}} + \Theta_0 P(\bar{h})^{\frac{-\theta\eta}{1-\eta}} (k_{t+1} / e_{t+1})^{\frac{1-\eta\phi(1-\theta)}{1-\eta}}$$

$$+ \delta^{\frac{-1}{\theta}} (A_y \phi)^{\frac{\theta-1}{\theta}} \Theta_0 P(\bar{h})^{\frac{-[1-\eta(1-\theta)]}{1-\eta}} (k_{t+1} / e_{t+1})^{\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\theta)(1-\eta)}{\theta(1-\eta)}},$$

with

$$\Theta_0 = \left[(\phi A_y)^{\theta-1} A_e^{1/\eta} \left(\frac{\eta\xi}{1-\pi_e} \right) \frac{1}{\delta} \right]^{\eta/(1-\eta)} > 0.$$

For some parameterizations (e.g. a large A_y), there exists sustainable growth in this corner solution without health investment, as in endogenous growth models with both human and physical capital in the literature. In a growing economy, health investment will eventually become positive.

With an interior solution for health investment, the first-order conditions of the household problem imply:

$$(19) \qquad s_t (\phi A_y)^{\frac{\theta-1}{\theta}} (e_{t+1} / k_{t+1})^{\frac{(1-\phi)(\theta-1)}{\theta}} = P_t \delta^{\frac{1}{\theta}} c_t,$$

$$(20) \qquad \xi e_{t+1}^{-\theta} \eta A_e (e_t / q_t)^{1-\eta} = (1-\pi_e) c_t^{-\theta},$$

$$(21) \qquad s_t \alpha A_h = (1-\pi_m)[\exp(h_t) - 1](1-\theta),$$

for $0 < \theta < 1$. Further, the budget constraints (7) to (9) imply $w_t e_t = m_t + q_t + s_t + c_t$.

From these equations, plus $k_{t+1} = s_t$ and the technologies in production and education, we can determine the evolution of (e_t, k_t) for $0 < \theta < 1$ with positive health investment as:

$$\ln\Theta_1 + \theta A_h A_y(1-\phi)(k_t/e_t)^{\phi} e_t - \theta A_h A_e^{-1/\eta}(e_{t+1}/e_t)^{1/\eta} e_t - \theta A_h k_{t+1} - \tag{22}$$

$$\theta\Theta_2(e_{t+1}/e_t)^{[1-\eta(1-\theta)]/(\theta\eta)} e_t + \theta\bar{h} + (1-\phi)(1-\theta)\ln(k_{t+1}/e_{t+1})$$

$$= \left(\frac{1-\eta}{\eta}\right)\ln(e_{t+1}/e_t) + \theta\ln e_{t+1},$$

where

$$\Theta_1 = \left[\frac{(1-\pi_m)(1-\theta)}{\alpha A_h}\right]^{\theta} \frac{(\phi A_y)^{\theta-1}\xi\eta A_e^{1/\eta}}{\delta(1-\pi_e)} > 0,$$

$$\Theta_2 = A_h(\xi\eta)^{-1/\theta}(1-\pi_e)^{1/\theta} A_e^{-1/(\theta\eta)} > 0,$$

$$\ln[\alpha A_h k_{t+1} + (1-\pi_m)(1-\theta)] = \ln(1-\pi_m)(1-\theta) + A_y A_h(1-\phi)(k_t/e_t)^{\phi} e_t - \tag{23}$$

$$A_h A_e^{-1/\eta}(e_{t+1}/e_t)^{1/\eta} e_t - A_h k_{t+1} - \Theta_2(e_{t+1}/e_t)^{[1-\eta(1-\theta)]/(\theta\eta)} e_t + \bar{h}.$$

In the log utility case, the evolution of human and physical capital is a special case of (22) and (23) with $\theta = 1$.

According to (21), in a growing economy in which $k_{t+1} = s_t$ grows over time, $\exp(h_t)$ will also grow over time and therefore the rate of survival $P(h_t)$ will converge to 1. We now establish the convergence of a growing economy to its unique balanced growth path below and relegate the proof to the Appendix.

PROPOSITION 2. *If* $1-\phi(1-\theta)-\phi(1-\eta) > 0$, *a growing economy starting with either* $m = 0$ *or* $m > 0$ *converges to a unique balanced growth path with* $\lim_{t\to\infty} P(h_t) = 1$. *The balanced growth rate is positive if* A_y *is large enough.*

The condition in Proposition 2 for a growing economy to converge to its unique balanced growth path is satisfied if the share parameter associated with effective labor exceeds that associated with physical capital in final production, as is widely accepted in the literature, i.e. if $1-\phi > \phi$. When total factor productivity, measured by A_y, is high enough, the balanced growth rate is positive.

We now investigate how subsidies on health and education spending affect the economy. The proof is relegated to the Appendix.

PROPOSITION 3. *With* $0 < \theta < 1$ *and an initial state* $(e_{t,} k_t)$, *a health subsidy raises health spending but has ambiguous effects on savings and education spending; it has no effect on the balanced growth rate. An education subsidy increases education spending but has ambiguous effects on savings and health spending; it also increases the balanced growth rate if* A_y *is sufficiently large under* $1-\phi(1-\theta)-\phi(1-\eta)$ >0.

The results in Proposition 3 are intuitive. Concerning the spending that is directly subsidized, the positive net effect indicates that the substitution effect of each of the subsidies dominates its income effect. For other types of spending that are not directly subsidized, the net effects of the subsidies will depend on parameterizations and are likely to be negative because the substitution effects may be negative. On the balanced growth path, the balanced growth rate is dependent on the ratio of physical to human capital. Because the rate of survival on the balanced growth path is equal to one, the health subsidy can no longer increase the survival rate. In this case, the health subsidy may have proportionate effects on physical and human capital accumulation, as it does in this model. Therefore, the health subsidy has no effect on the capital ratio and the growth rate on the balanced growth path in our model. On the other hand, by promoting education spending the education subsidy can increase the balanced growth rate as long as total factor productivity A_y is high enough and the share parameter of labor exceeds the share parameter of capital in production.

In the case of the log utility with $\theta = 1$, the equilibrium solution with positive health spending can be determined by the following equations:

(24) $\quad w_t e_t = m_t + \{1 + [1 + \delta(1 - \exp(-h_t(m_t)))]\frac{1 - \pi_e}{\xi\eta}\}q_t$,

(25) $\quad \delta\alpha A_h (1 - \pi_e) q_t \{\phi \ln \delta(1 - \pi_e) - \phi \ln \xi\eta + \ln A_y \phi A_e^{1-\phi} + [\phi + \eta(1 - \phi)] \ln q_t +$

$$+ (1 - \eta)(1 - \phi) \ln e_t - (1 - \phi) \ln[1 - \exp(-h_t(m_t))]\} = \xi\eta(1 - \pi_m) \exp(h_t(m_t)),$$

where the large expression inside the bracket $\{\ldots\}$ on the LHS of (25) is equal to $\ln z_{t+1}$ and $h_t(m_t) = A_h m_t + \overline{h}$. We establish the following result and relegate the proof to the Appendix:

PROPOSITION 4. *With the log utility and an initial state* (e_t, k_t), *a health subsidy raises health spending, reduces education spending, and has an ambiguous effect on savings; it has no effect on the balanced growth rate. An education subsidy raises education spending, but has ambiguous effects on savings and health spending. If* $1 - \phi > \phi$, *the education subsidy raises the balanced growth rate unless the subsidy rate is too high.*

The results with the log utility are similar to those with $\theta \in (0,1)$ in most aspects. One difference is that in the case of the log utility, a health subsidy reduces education spending. Another difference is that the education subsidy can increase the balanced growth rate if the subsidy rate is not too high under $1 - \phi > \phi$.

IV. NUMERICAL RESULTS

In order to reveal the quantitative implications of the model, we now turn to numerical simulations. We want to see how significant is the difference in life expectancy and growth rates across the two distinctive stages with or without health investment on an equilibrium path over 15 periods or generations (with 25 years in one period). We also want to see the welfare implication of subsidies on health and education spending, and find the preferred range of the subsidy rates.

On the technical side of our numerical simulation, the system of the nonlinear equations in the previous section determines the following 13 variables:

$(c_t, z_t, m_{t,} \ q_t, s_t, h_t, e_{t+1}, k_{t+1}, y_t, P_t, r_t, w_t, \tau_w)$,

where $z_t = k_t(1+r_t)/P_{t-1}$ from backdating (8) and (11) by one period. Using these equations, we can update the state variables from (k_t, e_t) to (k_{t+1}, e_{t+1}) and then calculate $1+r_{t+1} = \phi A_{yt}(e_{t+1}/k_{t+1})^{1-\phi}$ to evaluate old-age consumption z_{t+1} and welfare U_t. We can also do so by solving the evolution equations of human and physical capital from (k_t, e_t) to (k_{t+1}, e_{t+1}) first and then finding solutions for the other variables. To distinguish between the corner and interior solutions concerning health investment, we compute the marginal utility of consumption and that of health investment in each period to determine whether the condition for a corner solution for health investment is satisfied. If it is satisfied, we replace (15) by $m = 0$; otherwise, equation (15) holds in strict equality with $m > 0$.

Concerning parameterization, we first consider the balanced growth path and then consider initial conditions for rich and poor countries. As in Proposition 2, the rate of survival from working age to old age equals 1 on the balanced growth path, which exceeds the observed rates of survival in all countries. In developed countries, the rate of survival from age 20 to 65 is close to 80-90% in recent years according to the Life Tables from the World Health Organization.[2] For example, this rate of survival was 79% for males and 87% for females in 2000 in the United States and higher in some other OECD countries like Australia, Canada, Japan and Sweden. Since the rate of survival to old age in these countries is close to 1, we set the balanced growth rate as the average annual growth rate of GDP per capita 2.9% for the period 1960-1689 in countries with 1960 life expectancy above 65; see Barro and Sala-i-Martin (1994) that used purchasing power adjusted output growth data in the Penn World Table. In addition, we set a 20% saving rate, a 10% ratio of education spending to output and 60% subsidies on both education and health expenditures for the balanced growth path.[3] We calibrate our model to this balanced growth path with 60% subsidy rates on education spending and health spending. We also consider cases with no subsidies or with different subsidy rates. A common parameterization in all of our reported numerical results is given below:

$$\alpha = \frac{1}{4},\ \theta = 0.85,\ \delta = 0.5,\ \xi = 0.3,\ \bar{h} = 0.1,\ A_h = 0.5,\ A_e = 3.3,\ A_y = 4,\ \eta = \phi = \frac{1}{3}.$$

In this parameterization, setting the share parameter of physical capital at 1/3 is widely used in the literature. Setting $\delta = 1/(1+\rho)$ =0.5 and assuming one period as 25 years, the corresponding annual rate of time preference is equal to $\rho = 0.028$, which is within its usual range used in the literature. The values of the other parameters, which are either unavailable or vary significantly in the literature, are chosen so as to calibrate our model to the particular balanced growth path specified above.

We select three cases of numerical results to report. All of these cases start from the same initial capital stocks $e_0 = 0.9506$ and $k_0 = 0.0481$ which are low enough to allow for the corner and interior solutions to mimic poor and rich countries over 15 periods (generations) or more. The first case has no subsidies at all, the second case has a 60% health subsidy and the third case has 60% subsidies on both health and education spending. These cases are reported in Figures 1 to 3. Each of these figures has four panels. Panel (a) reports proportional allocations of output to savings, health and education. Panel (b) reports the growth rate of output and the rate of survival from working age to old age. Panel (c) reports health spending and log output per worker. Panel (d) gives the level of welfare.

In the first three periods in Figure 1, there is no health investment (Figures 1a and 1c), as output per worker is initially low at 1.42. When income becomes higher, health investment becomes positive (Figures 1b and 1c) and rises over time. The ratio of health investment to output increases in period four through to seven, peaks at a level exceeding 10%, and then falls gradually in the long run. Corresponding to the time path of health investment, the rate of survival is very low initially (below 0.1) and then rises toward its long-run level (equal to one) in Figure 1b. Matching a low rate of survival, the saving rate is very low initially and therefore the growth rate of output per worker is low as well (below 10% and 1%, respectively). The ratio of education spending to output is relatively smooth throughout the entire equilibrium path (about 4-6%). When health investment raises the rate of survival at higher income levels, the saving rate and the growth rate of output all converge to their long run levels (20% and 2%, respectively), that are higher than in the first phase without health investment. These patterns of movements of the key variables over time are similar to those in the cross-country comparison between poor and rich countries.

The patterns of movements of the variables in the several periods with a rising ratio of health spending to output in Figure 1 also capture some features in the time series data in the United States and other developed countries. According to Table 1, there were upward trends in the ratio of health spending to GDP, the ratio of education spending to GDP, and life expectancy in the United States. The substantial rises in health investment relative to output and in life expectancy are echoed in periods 4 to 7 in Figure 1. Also, the postwar average growth rate appeared to be higher than the prewar average growth rate in the United States, as in other 15 developed countries from 1870 to 1990 according to Maddison (1991). This overall rise in the long-term average growth rate is reflected in Figure 1b.

As mentioned earlier, according to Maddison (1992) there was a discernable upward trend in the long-term saving rates of 11 developed countries for the period 1870-1987 (with the United States as the only exception), as captured in periods from four through to seven in our Figure 1. For example, the average saving rate of Canada rose from 9.1% in 1870-1889 to 14.4% in 1914-1938, and further to 23.4% in 1960-1973. Since the mid-1970s, it had declined slightly to 20.4% in 1981-1987. It appears that Figure 1a has a better match with the saving rate in Canada than in the United States. This is perhaps because Canada started with much lower per capita GDP in 1870 than the United States: 1330 versus 2244 dollars (the 1985 US$), respectively. With such a low level of per capita GDP, Canada was in an early stage of development in the 1870s, which fits better into the first few periods in our Figure 1.

In Figure 2 with a 60% health subsidy, the economy jumps to the phase with positive health investment immediately in the first period as predicted in Propositions 1 and 2.

Compared to Figure 1, now the health subsidy raises the rate of survival and the saving rate substantially on the transitional path. As a result, the growth rate is higher in the first few periods on the transitional path with the health subsidy than without. Also greater is the short-run level of welfare in Figure 2d with the health subsidy (8.99656 in period one) than in Figure 1d without the subsidy (8.98961 in period one). The intuition for this welfare improvement arises from the positive externality from average health spending to the formation of health capital of every worker. Intuitively, the externality leads to under-investment in health in the first place, and hence leaves room for welfare improvements. In the long run, the balanced growth rate in Figure 2b with the health subsidy is the same as that in Figure 1b without any subsidy, as shown in Proposition 3.

In Figure 3 with a 60% education subsidy and a 60% health subsidy, the economy initially has just one period with zero health investment and moves to the phase with positive health investment in the second period. Compared to Figure 2 with the health subsidy alone, the addition of the education subsidy reduces health spending, savings and welfare (8.94953) in the initial period but raises them later on by accelerating human capital accumulation and output growth. Compared to Figure 1 without any subsidy, Figure 3 with both subsidies has higher spending on health and education and higher savings, leading to a higher growth rate. But the welfare level in Figure 3d with 60% subsidies on both education and health expenditures is initially lower than in Figure 1d without any subsidy. From the second period onward, the welfare level in Figure 3d is higher than in Figures 1d and 2d.

This welfare comparison suggests that poor countries may benefit more from subsidizing health spending alone than from subsidizing both health and education spending in the short run. In addition to these subsidies, poor countries may also benefit from subsidizing investment in physical capital or savings. The reason lies in the first-order condition with respect to health investment (15). That is, when the externality in health investment causes under-investment in health, a subsidy on savings can help raise the marginal benefit of health investment through increasing expected old-age consumption and hence encouraging more health investment. Also, like the education subsidy, a saving subsidy can promote capital accumulation and economy growth.

Comparing the balanced growth paths in Figures 1 to 3, the case with 60% subsidies on both education and health spending has the highest ratio of education spending to output and consequently the highest balanced growth rate. The balanced growth rate is the same in Figures 1 and 2 without subsidy or with a 60% health subsidy as in Propositions 3 and 4.

To focus on the welfare ranking with different combinations of subsidies in rich countries, we select a new initial condition with $e_0 = 11.936$ and 2.425. Without any subsidy, this new initial condition gives a level of output per worker at 28.22, which is almost 20 times as much as the output level 1.42 with the previous initial condition for poor countries. This resembles a comparison between a high income level $20,000 in developed countries and a low income level $1,000 in poor countries. Also, the corresponding rate of survival is equal to 0.79, which is close to the current rate of survival from age 20 to 65 for males in the United States. In this case with no subsidy, the welfare level in the initial period is 16.64372.

When there is an education subsidy at a rate 40%, 60% or 80%, the corresponding welfare level in the initial period changes to 16.6819, 16.66663, or 16.47193 in descending order. Among these welfare levels, the one with a 40% or 60% education subsidy is higher than that without subsidies, while the one with an 80% education subsidy is lower than that without subsidies. When there is a health subsidy at a rate 40%, 60% or 80%, the welfare level changes to 16.83126, 16.89986 or 19.90546 in ascending order, all of which exceed the level without subsidies. When there are equal subsidies on both education and health expenditures at a rate 40%, 60% or 80%, the welfare level changes to 16.88662, 16.97430 or 16.92425, peaking in the middle of the three subsidy rates. Also, when both subsidies are used, the welfare level is higher than when one of the subsidies is used alone. The result indicates that the optimal policy in rich countries is a combination of both subsidies at a rate around 60%.

V. SUMMARIES AND EXTENSIONS

Our model captures some stylized facts over different stages of development. When the initial income level is very low, so are health investment and savings relative to income. As a consequence, the rate of survival and the rate of output growth are low. When the income level rises over time, both health investment and savings will increase rapidly relative to income for some periods, leading to higher life expectancy and faster growth. Eventually, the ratio of health investment to output will fall when survival to old age becomes almost certain, whereas the saving rate and the rate of survival converge to steady-state levels on the balanced growth path. In our numerical results, the balanced growth rate is much higher than in the corner solution without health investment. The contrasting patterns of these variables at low and high levels of income resemble what we observe between poor and rich countries or across different stags of development in time series data in some developed countries.

Regarding public policies, we find that a health subsidy increases health investment and may raise or reduce the growth rate of output on the transitional path. However, it has no effect on the balanced growth rate in the long run. In terms of its welfare effect, economies with little health investment and low income may benefit more from subsidizing health investment than from subsidizing both health and education investment comprehensively. On the contrary, economies with high income may benefit more from subsidizing both than from subsidizing just one of them. This also captures the fact that rich countries have more public spending on education and health as fractions of GDP than poor countries.

Our model can be extended in several directions. First, one can assume that health capital may contribute to utility directly in addition to its role in enhancing survival as in Corrigan, Glomm and Mendes (2005). Similarly, one can assume that health capital can enhance productivity as in Corrigan, Glomm and Mendez (2005) and Chakraborty and Das (2005). These additional motives for health investment may induce more health spending. However, since health capital is bounded below by its endowed level $h_t \geq \bar{h}$, a corner solution for health investment may still occur in this extended version when income is sufficiently low such that the desire for consumption dominates the desire for health investment. Even with positive health investment at low income levels, the ratio of health investment to output is likely to be low because of the endowed component of health capital.

Second, one may assume idiosyncratic health shocks in the form of terminal diseases like AIDS or cancer that may make survival to old age impossible as in Corrigan, Glomm and Mendez. Correspondingly, one may assume another component of health expenditure that contributes to utility by easing the suffering from such terminal diseases but does not contribute to the rate of survival to old age. In this second extension, average health spending is expected to be always positive due to the new component of health expenditure. However, the component of health spending aiming at enhancing survival to old age is expected to behave in the same way as in the original version of the model.

VI. CONCLUSION

In this paper we have investigated how health investment interacts with education investment and life-cycle savings in an endogenous growth model. We have found that the equilibrium solutions for some key variables depend critically on the initial level of income per capita. When initial income is sufficiently low, the desire for consumption is stronger than the desire for health, resulting in zero health investment and hence a low rate of survival. The low rate of survival in turn leads to little savings for old age and slow growth in output per worker. When income becomes high enough in a growing economy, households will be willing to strike a balance among health, education and savings, leading to higher life expectancy and faster growth than in the early stage without health investment. The findings capture some stylized facts in cross-country comparison between poor and rich countries as well as in time series data in the United States.

Interestingly, subsidizing health spending can move an economy from the no-health-spending equilibrium to the other, a transition that brings about higher life expectancy, greater savings, higher welfare and perhaps faster growth on the transitional path.

Subsidizing both education and health spending may reduce welfare in the short run for poor countries but will lead to higher life expectancy, faster growth and higher welfare in the future. An example of this transition in recent history is the development in the last several decades in China compared to the rest of the developing world. Starting with one of the lowest levels of income and life expectancy but with substantial state funding for education and health services, China has achieved not only phenomenal economic growth but also one of the highest levels of life expectancy in the developing world. [4]

Starting with high income and positive health investment, we have also found that the initial generation of workers are better off from subsidizing both health and education expenditures at realistic rates around 60%. This result is consistent with the practice of substantial government spending on health and education in many developed countries. The welfare gain is attributed to the externality of average health spending in the formation of health capital and the short-sightedness of agents in a typical overlapping-generations model concerning education. However, further rises in the subsidy rate on both education and health expenditures, say 80% or over, are found to reduce welfare in our numerical results, although the higher subsidy rates may yield higher welfare compared to cases without any subsidies. This result supports recent reforms in public funding for education and health, from one with almost free public access to education and health services to one with some sort of shared financial responsibility between the state and households.

APPENDIX

Proof of Proposition 1

From the first-order conditions, we must have

$$\frac{\delta z_{t+1}^{1-\theta}}{1-\theta} c_t^{\theta} \alpha A_h \le (1-\pi_m)\exp(h_t), \quad m_t[\frac{\delta z_{t+1}^{1-\theta}}{1-\theta} c_t^{\theta} \alpha A_h - (1-\pi_m)\exp(h_t)] = 0.$$

In particular, if the initial stocks (k_0, e_0), and hence $y_0 = A_y k_0^{\phi} e_0^{1-\phi}$, are so low that

$$\frac{\delta z_1^{1-\theta}}{1-\theta} c_0^{\theta} \alpha A_h < (1-\pi_m)\exp(\bar{h}),$$

we must have:

$$\frac{\delta z_1^{1-\theta}}{1-\theta} c_0^{\theta} \alpha A_h < (1-\pi_m)\exp(h_0), \quad \text{since } h_0 \ge \bar{h}.$$

Together with $m_t[\frac{\delta z_{t+1}^{1-\theta}}{1-\theta} c_t^{\theta} \alpha A_h - (1-\pi_m)\exp(h_t)] = 0$, we must have $m_t = 0$ for some $t \ge 0$. Obviously, one can always increase the health subsidy rate π_m such that the user cost of health investment (the RHS of the above inequalities) is equal to or below the marginal benefit (the LHS) in order to induce positive health investment. Finally, it is easy to verify that the result holds for the log utility case when setting $\theta = 1$ and replacing $z_{t+1}^{1-\theta}/(1-\theta)$ with $\ln z_{t+1}$ in the above inequalities.

Proof of Proposition 2

Consider first that the economy starts from a corner solution with m = 0. The convergence of this economy is based on equation (18). For convenience, let Γ_i be the coefficient on the ratio k/e in (18), denote $x = k/e$ and rewrite (18) as

(A-1) $$\Gamma_1 x_t^{\phi} = \Gamma_2 x_{t+1}^{\frac{1-\phi(1-\theta)}{1-\eta}} + \Gamma_3 x_{t+1}^{\frac{1-\eta\phi(1-\theta)}{1-\eta}} + \Gamma_4 x_{t+1}^{\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\theta)(1-\eta)}{\theta(1-\eta)}}, \quad \Gamma_i > 0 \ \forall i.$$

Differentiating it with respect to x_t yields:

$$\frac{dx_{t+1}/x_{t+1}}{dx_t/x_t}=\frac{F(x_{t+1})}{B(x_{t+1})}$$, where

$$F(x_{t+1})=\Gamma_2 x_{t+1}^{\frac{1-\phi(1-\theta)}{1-\eta}}+\Gamma_3 x_{t+1}^{\frac{1-\eta\phi(1-\theta)}{1-\eta}}+\Gamma_4 x_{t+1}^{\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\theta)(1-\eta)}{\theta(1-\eta)}}>0,$$

$$B(x_{t+1})=\frac{[1-\phi(1-\theta)]}{\phi(1-\eta)}\Gamma_2 x_{t+1}^{\frac{1-\phi(1-\theta)}{1-\eta}}+\frac{[1-\eta\phi(1-\theta)]}{\phi(1-\eta)}\Gamma_3 x_{t+1}^{\frac{1-\eta\phi(1-\theta)}{1-\eta}}+$$

$$\frac{\{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\eta)(1-\theta)\}}{\phi\theta(1-\eta)}\Gamma_4\, x_{t+1}^{\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\theta)(1-\eta)}{\theta(1-\eta)}}>0.$$

If $1-\phi(1-\theta)-\phi(1-\eta)>0$, we have

$$\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\eta)(1-\theta)}{\phi\theta(1-\eta)}>\frac{1-\eta\phi(1-\theta)}{\phi(1-\eta)}>\frac{1-\phi(1-\theta)}{\phi(1-\eta)}>1,$$

and therefore $F<B$. We thus have

$$0<\frac{dx_{t+1}/x_{t+1}}{dx_t/x_t}=\frac{F(x_{t+1})}{B(x_{t+1})}<1,$$

which implies the convergence of x_t, that is, $\lim_{t\to\infty} x_t = x_\infty$. Taking $t\to\infty$ in (A-1) and dividing it by x_∞, x_∞ is determined by

$$\text{(A-2)}\quad \Gamma_1=\Gamma_2 x_\infty^{\frac{1-\phi(1-\theta)-\phi(1-\eta)}{1-\eta}}+\Gamma_3 x_\infty^{\frac{1-\eta\phi(1-\theta)-\phi(1-\eta)}{1-\eta}}+\Gamma_4 x_\infty^{\frac{\theta[1-\eta\phi(1-\theta)]+(1-\phi)(1-\theta)(1-\eta)-\phi\theta(1-\eta)}{\theta(1-\eta)}}.$$

Here, the LHS is a positive constant, while the RHS is increasing monotonically with x_∞ (starting below Γ_1) because all the exponents of x_∞ in (A-2) are positive under $1-\phi(1-\theta)-\phi(1-\eta)>0$. Thus, x_∞ is unique under this condition. Corresponding to the unique $x_\infty = k_\infty / e_\infty$, there is a unique balanced growth rate from (17) with $m = 0$. From Proposition 1, since a growing economy will eventually have $m_t > 0$, it will not converge to the balanced growth path with $m = 0$.

Now we consider the case with $m > 0$. According to (21), when $k_{t+1} = s_t$ grows over time, $\exp(h_t)$ will also grow over time and therefore the rate of survival P will converge to 1. Thus, when $t\to\infty$, equations (19) and (20) plus the education technology imply

(A-3) $\lim_{t\to\infty} e_{t+1}/e_t = \Theta_0 (k_\infty / e_\infty)^{\eta[1-\phi(1-\theta)]/(1-\eta)}$, where

$$\Theta_0 = \left[(\phi A_y)^{\theta-1} A_e^{1/\eta} \left(\frac{\eta\xi}{1-\pi_e} \right) \frac{1}{\delta} \right]^{\eta/(1-\eta)} > 0.$$

This equation links the long-run growth rate of human capital to the long-run ratio of physical to human capital. Given $e_t \in (0,\infty)$ and $k_t \in (0,\infty)$ in period t, e_{t+1} must be bounded according to the education technology and the household constraint on education spending q_t. That is, the growth rate of human capital is bounded in every period. By (A-3), the capital ratio k_∞ / e_∞ must be bounded as well. Rewrite (22) as

$$\ln \Theta_1 + \theta A_h A_y (1-\phi)(k_t / e_t)^\phi e_t - \theta A_h A_e^{-1/\eta} (e_{t+1}/e_t)^{1/\eta} e_t - \theta A_h (k_{t+1}/e_{t+1})(e_{t+1}/e_t) e_t -$$

$$\theta \Theta_2 (e_{t+1}/e_t)^{[1-\eta(1-\theta)]/(\theta\eta)} e_t + \theta \bar{h} + (1-\phi)(1-\theta)\ln(k_{t+1}/e_{t+1})$$

$$= \left(\frac{1-\eta}{\eta} \right) \log(e_{t+1}/e_t) + \theta \log(e_{t+1}/e_t) + \theta \log e_t .$$

Divide both sides of this version of (22) by e_t, let $t \to \infty$ and $e_t \to \infty$, and use (A-3) to replace the growth rate by the ratio of physical to human capital. In doing so, the ratios of all the constant terms in (22) to e_t will converge to zero as $e_t \to \infty$ in the long run. Because we have noted that the long-run growth rate and the long-run capital ratio are all bounded, the last term on the LHS and the first term on the RHS of (22) will also be driven to zero when they are divided by a rising e_t. As e_t grows, $(\ln e_t)/e_t$ also converges to zero since $\lim_{x\to\infty} (\log x)/x = \lim_{x\to\infty} 1/x = 0$. In the long run, the resultant equation containing the remaining terms in (22) governs the evolution of the capital ratio:

$$A_y (1-\phi)(k_t / e_t)^\phi_{t\to\infty} = A_e^{\frac{-1}{\eta}} \Theta_0^{\frac{1}{\eta}} (k_{t+1}/e_{t+1})_{t\to\infty}^{\frac{1-\phi(1-\theta)}{1-\eta}} + \Theta_0 (k_{t+1}/e_{t+1})_{t\to\infty}^{\frac{1-\eta\phi(1-\theta)}{1-\eta}} +$$

$$(\xi\eta)^{\frac{-1}{\theta}} (1-\pi_e)^{\frac{1}{\theta}} A_e^{\frac{-1}{\theta\eta}} \Theta_0^{\frac{1-\eta(1-\theta)}{\theta\eta}} (k_{t+1}/e_{t+1})_{t\to\infty}^{\frac{[1-\phi(1-\theta)][1-\eta(1-\theta)]}{\theta(1-\eta)}} .$$

As in (A-1), the capital ratio in the above equation will converge to k_∞ / e_∞:

$$\text{(A-4)} \quad A_y(1-\phi) = A_e^{\frac{-1}{\eta}} \Theta_0^{\frac{1}{\eta}} (k_\infty / e_\infty)^{\frac{1-\phi(1-\theta)-\phi(1-\eta)}{1-\eta}} + \Theta_0 (k_\infty / e_\infty)^{\frac{1-\eta\phi(1-\theta)-\phi(1-\eta)}{1-\eta}}$$

$$+ (\xi\eta)^{\frac{-1}{\theta}} (1-\pi_e)^{\frac{1}{\theta}} A_e^{\frac{-1}{\theta\eta}} \Theta_0^{\frac{1-\eta(1-\theta)}{\theta\eta}} (k_\infty / e_\infty)^{\frac{[1-\phi(1-\theta)][1-\eta(1-\theta)]-\theta\phi(1-\eta)}{\theta(1-\eta)}} .$$

Note that all the terms in (A-4) are positive. The LHS is a constant, while the RHS depends on k_∞ / e_∞ which will have a unique solution if the RHS is increasing with it monotonically. To this end, we only need to show that all the exponents of k_∞ / e_∞ are positive. The last one is positive because $[1-\phi(1-\theta)][1-\eta(1-\theta)]-\theta\phi(1-\eta) \equiv F(\theta)$ >0 as

$F(0) = (1-\phi)(1-\eta) > 0$,

$F'(\theta) = \phi[1-\eta(1-\theta)] + \eta[1-\phi(1-\theta)] - \phi(1-\eta) = \phi\eta\theta + \eta[1-\phi(1-\theta)] > 0$,

under $0 < \theta \le 1$. The other exponents of k_∞ / e_∞ are positive under $1-\phi(1-\theta)-\phi(1-\eta)$ >0. Given this condition there is a unique finite solution for k_∞ / e_∞, implying that physical and human capital (hence also output) must share the same balanced growth rate in the long run. Combining this with (A-3), the balanced growth rate must also be finite and unique.

In order to see whether the balanced growth rate can be positive, we set $\pi_e = 0$ and substitute (A-3) into (A-4) to replace k_∞ / e_∞ by the balanced growth rate g_∞:

$$\text{(A-5)} \quad A_y^{\frac{\theta}{1-\phi(1-\theta)}} \phi^{\frac{\phi(1-\theta)}{1-\phi(-\theta)}} A_e^{\frac{-\phi}{\eta[1-\phi(1-\theta)]}} \left(\frac{\delta}{\eta\xi}\right)^{\frac{\phi}{1-\phi(1-\theta)}} (1-\phi) = A_e^{\frac{-1}{\eta}} A_y^{\frac{-(1-\theta)}{1-\phi(1-\theta)}} (1+g_\infty)^{\frac{1-\phi(1-\theta)-\phi(1-\eta)}{\eta[1-\phi(1-\theta)]}}$$

$$+ \phi^{\frac{1-\theta}{1-\phi(1-\theta)}} A_e^{\frac{-1}{\eta[1-\phi(1-\theta)]}} \left(\frac{\delta}{\eta\xi}\right)^{\frac{1}{1-\phi(1-\theta)}} (1+g_\infty)^{\frac{1-\phi\eta(1-\theta)-\phi(1-\eta)}{\eta[1-\phi(1-\theta)]}} + (\eta\xi)^{\frac{-1}{\theta}} A_e^{\frac{-1}{\eta\theta}}$$

$$A_y^{\frac{-(1-\theta)}{1-\phi(1-\theta)}} (1+g_\infty)^{\frac{[1-\phi(1-\theta)][1-\eta(1-\theta)]-\phi\theta(1-\eta)}{\theta\eta[1-\phi(1-\theta)]}} .$$

The LHS of (A-5) is increasing with A_y, while the RHS is decreasing directly with A_y. Also, the RHS is increasing with g_∞ because all the exponents of $1+g_\infty$ are positive. Thus, we have $dg_\infty / dA_y > 0$. That is, if A_y is large enough then $g_\infty > 0$.

In the log utility case with $\theta = 1$, the analysis of convergence to a unique balanced growth path is similar by imposing $\theta = 1$ in (A-3) and (A-4). Also, if A_y is large enough then $g_\infty > 0$, which can be easily verified by setting $\theta = 1$ in (A-5).

Proof of Proposition 3

Taking logs on (19)—(21) and making substitutions, we have

(A-6) $\ln[(\Delta_1/\Delta_2)^\theta \Delta_3^{1-\phi(1-\theta)}] + (1-\eta)(1-\theta)\ln e_t + [1-\phi(1-\theta)]\{\ln(1-\pi_m) +$

$\ln[\exp(h_t)-1]\} + (1-\phi)(\theta-1)\ln e_{t+1} = \ln(1-\pi_e) + [1-\eta(1-\theta)]\ln q_t + \theta \ln P_t$,

with $\Delta_1 = (\phi A_y)^{\frac{\theta-1}{\theta}} \delta^{\frac{-1}{\theta}} > 0$, $\Delta_2 = \left[\xi \eta A_e^{1-\theta}\right]^{\frac{-1}{\theta}} > 0$, and $\Delta_3 = (1-\theta)/(\alpha A_h) > 0$,

(A-7) $c_t = \Delta_2 (1-\pi_e)^{\frac{1}{\theta}} q_t^{\frac{1-\eta(1-\theta)}{\theta}} e_t^{\frac{-(1-\eta)(1-\theta)}{\theta}}$,

(A-8) $s_t = \Delta_3 (1-\pi_m)[\exp(h_t)-1]$.

Substituting (A-7) and (A-8) into the constraint $w_t e_t = m_t + q_t + s_t + c_t$ leads to

(A-9) $w_t e_t = m_t + q_t + \Delta_3(1-\pi_m)[\exp(h_t)-1] + \Delta_2(1-\pi_e)^{\frac{1}{\theta}} q_t^{\frac{1-\eta(1-\theta)}{\theta}} e_t^{\frac{-(1-\eta)(1-\theta)}{\theta}}$.

Note that equations (A-6) and (A-9) contain implicit solutions for m_t and q_t via $h_t = A_h m_t + \bar{h}$ and (5). Taking total differentiation, we get:

(A-10) $\begin{bmatrix} \partial m_t/\partial \pi_e \\ \partial q_t/\partial \pi_e \end{bmatrix} = \begin{bmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \end{bmatrix} [a_1 b_3 - a_2 b_1]^{-1}$,

(A-11) $\begin{bmatrix} \partial m_t/\partial \pi_m \\ \partial q_t/\partial \pi_m \end{bmatrix} = \begin{bmatrix} a_4 b_2 - a_2 b_4 \\ a_1 b_4 - a_4 b_1 \end{bmatrix} [a_1 b_3 - a_2 b_1]^{-1}$,

with $a_1 = [1-(1-\theta)\phi - \theta\exp(-h_t)]A_h / P_t > 0$,

$a_2 = -(1-\phi)(1-\theta)\eta A_e (e_t/q_t)^{1-\eta}/e_{t+1} - [1-\eta(1-\theta)]/q_t < 0$,

$a_3 = -1/(1-\pi_e) < 0$, $a_4 = [1-(1-\theta)\phi]/(1-\pi_m) > 0$,

$b_1 = 1 + \Delta_3(1-\pi_m)\exp(h_t)A_h > 0$,

$$b_2 = 1 + \Delta_2 (1-\pi_e)^{\frac{1}{\theta}} [1-\eta(1-\theta)](q_t / e_t)^{\frac{(1-\eta)(1-\theta)}{\theta}} / \theta > 0,$$

$$b_3 = \Delta_2 (1-\pi_e)^{\frac{1-\theta}{\theta}} q_t^{\frac{1-\eta(1-\theta)}{\theta}} e_t^{\frac{-(1-\eta)(1-\theta)}{\theta}} / \theta > 0, \quad b_4 = \Delta_3[\exp(h_t) - 1] > 0.$$

While the signing is obvious for a_2, a_3 and b_j for all j, the signing of a_1 is more involved. The sign of a_1 depends on $[1-(1-\theta)\phi - \theta\exp(-h_t)]$ or $[1-(1-\theta)\phi]\exp(h_t) - \theta$. Since $1 > \theta + (1-\theta)\phi$, or $1 > \theta / [1-(1-\theta)\phi]$, for $0 < \theta < 1$ and $0 < \phi < 1$, it follows:

$$\exp(h_t) \geq 1 > \frac{\theta}{1-(1-\theta)\phi},$$

which implies $a_1 > 0$. Now, it is obvious that $\partial m_t / \partial \pi_m > 0$ and $\partial q_t / \partial \pi_e > 0$. The effects of the health subsidy on savings and education spending are ambiguous, and so are the effects of the education subsidy on savings and health spending.

According to (A-3) and (A-4), the health subsidy has no effect on the balanced growth rate since it does not appear in these two equations. Differentiating (A-3) and (A-4) with respect to $1/(1-\pi_e)$ yields the following condition for $dg_\infty / d\pi_e > 0$:

(A-12) $$\theta(1-\phi) A_e^{\frac{-\phi(1-\theta)}{\eta[1-\phi(1-\theta)]}} (\phi A_y)^{\frac{1-\theta}{1-\phi(1-\theta)}} \left(\frac{\delta}{\xi\eta}\right)^{\frac{1}{1-\phi(1-\theta)}} (1+g_\infty)^{\frac{\phi(1-\theta)(1-\eta)}{\eta[1-\phi(1-\theta)]}} + (1-\phi)(\xi\eta)^{\frac{-1}{\theta}}$$

$$A_e^{\frac{-(1-\theta)}{\theta\eta}} \left(\frac{1}{1-\pi_e}\right)^{\frac{-(1-\phi)(1-\theta)}{\theta[1-\phi(1-\theta)]}} (1+g_\infty)^{\frac{(1-\theta)(1-\eta)}{\theta\eta}} > \theta\phi\left(\frac{1}{1-\pi_e}\right)^{\frac{1}{1-\phi(1-\theta)}},$$

where g_∞ is the balanced growth rate. For $\theta \in (0,1)$, a larger A_y means a greater LHS, both directly and indirectly through raising g_∞ under $1-\phi(1-\theta)-\phi(1-\eta) > 0$, as shown below (A-5). Thus, the education subsidy increases the balanced growth rate if A_y is sufficiently large and if $1-\phi(1-\theta)-\phi(1-\eta) > 0$.

Proof of Proposition 4

Totally differentiating (24) and (25) with respect to π_m and collecting terms, we have:

$$\frac{dm_t}{d\pi_m} = \frac{\xi\eta c_t P_t \exp(h_t) G_3}{G_1 G_2 + c_t A_h G_3 G_4} > 0, \qquad \frac{dq_t}{d\pi_m} = -\left(\frac{G_1}{G_3}\right)\frac{dm_t}{d\pi_m} < 0,$$

with $G_1 = \xi\eta + \delta A_h q_t (1-\pi_e)\exp(-h_t) > 0,$

$$G_2 = P_t(1-\pi_e)\{(1-\pi_m)\exp(h_t) + c_t \delta\alpha A_h[\phi + \eta(1-\phi)]\} > 0,$$

$$G_3 = \xi\eta + (1-\pi_e)(1+\delta P_t) > 0,$$

$$G_4 = \delta\alpha A_h(1-\pi_e)(1-\phi) q_t \exp(-h_t) + \xi\eta P_t(1-\pi_m)\exp(h_t) > 0.$$

In so doing, we used $\ln z_{t+1} = (1-\pi_m)\exp(h_t)/(c_t\delta\alpha A_h)$ and $c_t = (1-\pi_e)q_t/(\xi\eta)$. The sign of the effect on savings is ambiguous as follows:

$$\frac{ds_t}{d\pi_m} = \exp(-h_t)\delta c_t A_h \frac{dm_t}{d\pi_m} - P_t \xi\eta\delta(1-\pi_e)(G_1/G_3)\frac{dm_t}{d\pi_m},$$

where the first term on the RHS is positive but the second one is negative.

Similarly, we also have

$$\frac{dq_t}{d\pi_e} = \frac{P_t G_1 G_5 + A_h q_t^2 (1+\delta P_t)(1-\pi_e) G_4}{\delta\alpha A_h q_t P_t (1-\pi_e)^2 G_1[\phi + \eta(1-\phi)] + A_h q_t (1-\pi_e) G_3 G_4 + G_6} > 0,$$

with $G_5 = \xi\eta\, q_t\, (1-\pi_m)\exp(h_t) + \delta\beta A_h \phi q_t^2 (1-\pi_e) > 0$,

$$G_6 = \xi\eta P_t (1-\pi_e)(1-\pi_m)\exp(h_t) > 0.$$

The effects of the education subsidy on health spending and savings are ambiguous.

Again, the health subsidy has no effect on the balanced growth rate as in Proposition 3. Setting $\theta = 1$ in (A-12), the condition for $dg_\infty / d\pi_e > 0$ becomes:

$$\pi_e < \frac{(1-\phi)(1+\delta) - \phi\eta\xi}{(1-\phi)(1+\delta)}.$$

The RHS of this inequality is positive but less than one under $1-\phi > \phi$.

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FOOTNOTES

1. For example, health spending accounted for 13% of GDP during 1990-1998 in the United States, and nearly 9% of GDP in other advanced countries according to the World Bank (2002), amounting to a per capita health expenditure at 2,000 US dollars or more per year. On the other hand, in countries with very low life expectancy, health expenditures per capita were mostly below 50 US dollars per year, e.g. merely one US dollar in Liberia and below ten US dollars in other eight such countries in 1997.
2. The Life Tables give death rates for each 5 year age gap, e.g. $d_{20,25}$, ... $d_{60,65}$. The rate of survival from age 20 to 65 is computed as the product $(1-d_{20,25})(1-d_{25,30})...(1-d_{60,65})$.
3. The 20% saving rate is close to the 22% ratio of private investment to GDP in countries with life expectancy in 1960 above 65 for the period 1960-1989. In these countries the ratio of public education spending to GDP is about 6%. With a 60% education subsidy, the corresponding figure for the ratio of total education spending to GDP is thus 10% to meet the 6% ratio of public education spending to GDP. Since there are substantial government subsidies on education and health expenditures in many OECD countries, the 60% subsidy rates are plausible figures for these countries on average.
4. Life expectancy at birth in China was only 36 years in 1960, but exceeded 70 in recent years.

TABLE 1

Selected Statistics of the United States from 1870 to 2000

Year	Life expectancy at birth[a]	GDP per capita (1985 \$US)[b]	Average annual GDP per capita growth rate (%)[c]	Average health expenditure (% GDP) (private + public)[d]	Ratio of public to private health expenditure[e]	Average education expenditure (% GDP) (private + public)[f]	Average saving rate (%GDP) (year, rate)[g]	
1870	41.4	2244	--	--	--	--	1870-89	19.1
1890	43.5	3101	1.62	--	--	--	1890-1913	18.3
1910	51.9	4538	1.90	--	--	1.5	1914-38	17.0
1930	59.7	5642	1.09	4.0	0.25	3.2	1939-49	15.2
1950	68.2	8605	2.11	5.3	0.40	4.4	1950-73	19.7
1970	70.8	12815	1.99	8.9	0.70	7.5	1974-87	18.0
1990	75.4	18258	1.77	13.1	0.80	7.4	1990-2000	16.3
2000	77.0	23190	2.39	14.5	0.84	7.6	2000-04	15.1

Notes: Sources of data are as follows. a and f. US Census Bureau (1975, 2004). b and c. Table 12.10, Barro & Sala-i-Martin (1995); Figures for 2000 are calculated using Tables 1.1.6 and CA1-3, National Income and Product Accounts (NIPA) Tables, 2005, US Department of Commerce. d and e. NIPA Tables, and Table B236-247 in US Census Bureau (1975). g. Figures prior to 1987 are from Maddison (1992); Figures for 1990 and 2000 are from NIPA Tables.

FIGURE 1
Numerical Results without Subsidies

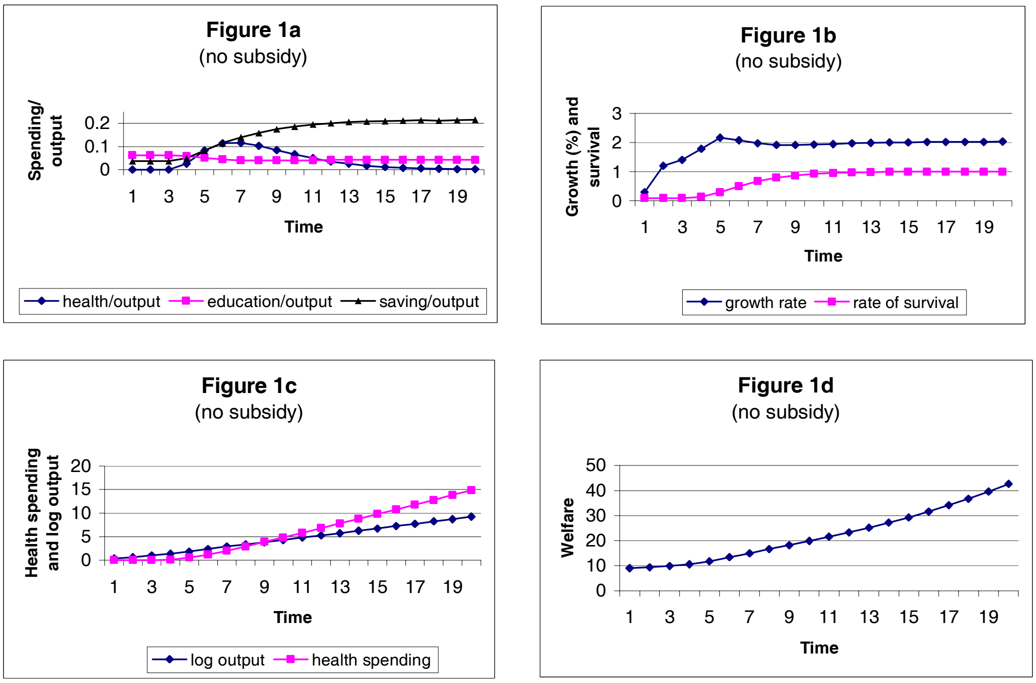

FIGURE 2
Numerical Results with a 60% Health Subsidy

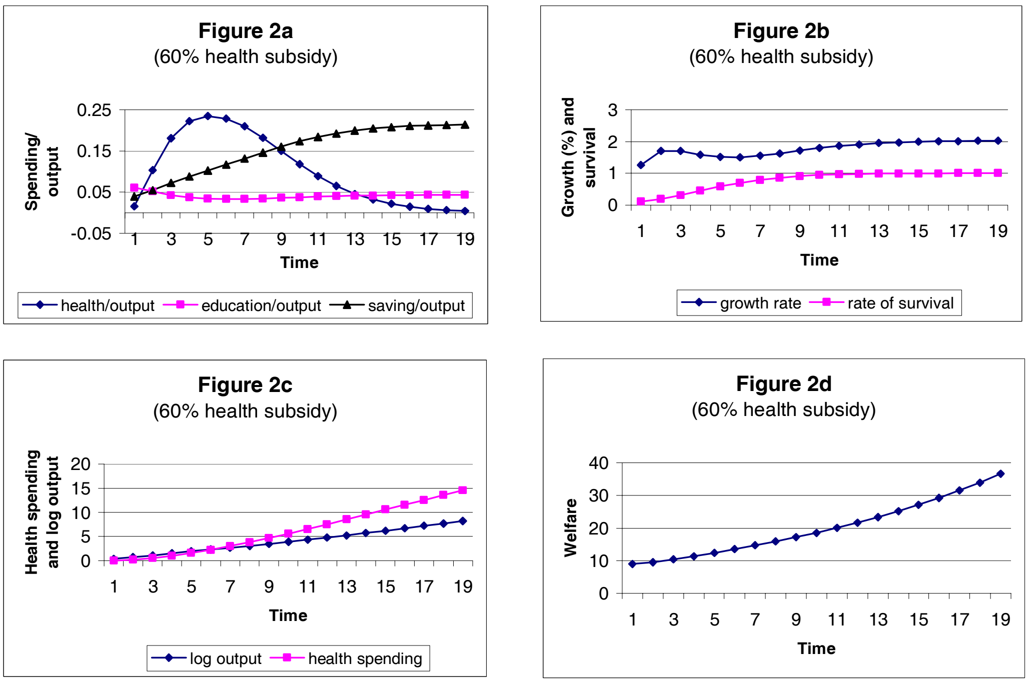

FIGURE 3

Numerical Results with 60% Subsidies on Health and Education Spending