Non-linear revenue evaluation in oligopoly

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Abstract

In this article we investigate oligopolies where firm decision makers have multiple objectives. We focus on cases where the decision maker is incentivized by profit, and by revenue. Our innovation—motivated by the internal scrutiny that is often placed on revenue—is that managers derive utility from revenue in a potentially non-linear fashion. This allows for incremental changes in revenue to have different incentive effects when it comes to production choice, depending on the amount of revenue generated by the firm. We show that this intuitively appealing extension to the revenue maximisation model reverses some conventional results of that model: we derive conditions where decision makers may actually increase output in the presence of demand contractions. Whether a decision maker increases or decreases output in the presence of a demand shock depends on the concavity of their utility function with respect to revenue. Our findings help us in understanding cases of output growth in the presence of negative demand shocks.

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1 Introduction

The traditional approach in industrial organization is to view firms as profit-maximizing entities. Even though this seems a reasonable assumption from the shareholders' or owner's perspective, critical strategic decisions that determine a firm's performance are more often than not in the hands of CEOs or managers who may pursue different objectives. This feature has been recognized early on, and integrated in the development of the theory of the firm and organization economics (e.g., Baumol, 1958; Williamson, 1963). Under this alternative paradigm, the firm is still seen as maximizing profits, but the agents contracted for fulfilling this goal can be endowed with complementary objectives such as sales (i.e., total revenue) or market share maximization, or production cost minimization. This, admittedly more realistic, modeling strategy alters the predictions relating to the degree of competition between firms, as well as the effect of policy measures, or changes in market conditions.

While the original contributions conceived of firm decision makers having general preferences over multiple objectives, existing approaches that account for strategic behavior among firms have assumed linear evaluation of these alternative objectives (e.g., Sklivas, 1985; Fershtman and Judd, 1987), essentially assuming decision makers care about a weighted average of profit and alternative objectives. Such an assumption, however, is neither innocuous nor necessarily realistic. A manager maximizing a combination of profits and some other performance measure may seek to achieve higher sales or market share because of internal scrutiny on these measures of firm 'success', to improve the likelihood of being promoted, to improve the chances of obtaining a better job, or simply to aim at building an empire (e.g., Baker et al., 1988). There is no *a priori* justification for restricting analysis to linear evaluations and, indeed, it is reasonable in such circumstances to consider the manager's utility to be more general in their alternative objective(s), as originally proposed within Williamson (1963) but where his focus was not on the alternative objective of sales. The rationale for decision-makers deriving more general—non-linear—utility from revenues can stem from the managers' need to increase their power, status, or prestige, which can be symptomatic of a manager's desire for empire-building considerations (Baumol, 1959; Williamson, 1963; Jensen, 1986; Hope and Thomas, 2008), to reduce their unemployment risk (Amihud and Lev, 1981), or to enhance promotion objectives (Cao et al., 2018). Incorporating such non-linearities in a manager's objective function has important consequences to the extent that the marginal utility derived from fulfilling this alternative objective will vary with the firm's performance in that objective, and will therefore directly impact the manager's incentives. In this article we explore such consequences by considering an oligopolistic framework with decision makers endowed

with non-linear preferences over sales.

Baumol (1958) considered that managers are motivated by sales maximization subject to a minimum profit constraint, meaning the manager's payoff is linear in sales. When managers value sales non-linearly, negative demand shocks may generate highly counter-intuitive results since we demonstrate that production may increase in some circumstances. The intuition of this result is clear. If the sole objective of a decision maker is either to maximize profits or sales, a negative demand shock will always lead to lower production. Indeed, a negative demand shock reduces marginal sales, and thus incentivizes both the profit-maximizing and the revenue-maximizing decision makers to reduce their production. If, however, a manager values sales non-linearly, a negative demand shock may reduce marginal profits while *increasing* the marginal utility of sales (since they have reduced). When the latter effect is stronger than the former, the manager will opt for higher production. The same result cannot be obtained in restrictive linear valuation models of alternative objectives since marginal utility of sales would always track the changes in demand.

Increasing production in the presence of a demand contraction has been observed in reality. For example, the strategy of IBM's CEO, Samuel J. Palmisano, during the 2009 global economic downturn is illustrative of our theoretical finding. The high tech industry suffered a major blow during the economic crisis, with most actors (Apple being the exception) suffering important profit losses. Nevertheless, in the midst of the crisis, Palmisano implemented a very aggressive strategy that aimed at IBM's expansion. While both the revenues and profits of the firm did decrease in 2009 compared to a year earlier, the company launched a series of acquisitions that allowed it to expand production. IBM's CEO greatly benefited from this strategy since on top of the $21 million of income he managed to secure in 2009, he equally became widely acclaimed in the business world as an excellent manager.[1]

2 Related literature

A large literature has explored the consequences of endowing firms (or their managers) with alternative incentives and objectives other than simply profit. These objectives include maximizing market share (Jansen et al., 2007; Ritz, 2008), or maximizing the relative performance of the firm (Salas Fumas, 1992; Miller and Pazgal, 2002; Vroom, 2006; Matsumura and Matushima, 2012; Nakamura, 2014). Alternatively, scholars have developed principal-agent theories in strategic settings with effort-minimizing managers (e.g., Hart, 1983). Yet, the majority of the literature has focused on sales (i.e., total revenue) maximization as an alternative or

[1] See https://hbr.org/2012/01/sam-palmisanos-transformation.html

complementary objective to profit maximization, following the initial contribution of Baumol (1958).

The empirical evidence testifying that the evaluation of managers and CEOs performance is indeed a function of both the firm's profit—through their effect on shareholders—and sales is compelling (Murphy, 1985; Baker et al., 1988; Huang et al., 2015; Bloomfield, 2016). Conyon (2014), for instance, estimates the elasticity of executive pay to firm sales to be at approximately 35%. Given this background, it is not surprising that an important theoretical literature, known as the *strategic delegation* literature, studied the strategic implications of endowing managers with such incentives. This literature explores the firm owners' incentives to endogenously assign managers strategic decisions and corresponding contracts to alter their firm's strategy by committing via delegation, and to thereby expand profits. Hence, by rewarding managers according to a combination of profit and sales, a firm can be rendered more aggressive in a Cournot competition setting to potentially gain a competitive edge. All the major initial contributions to this literature consider contracts featuring linear combinations of profits and sales, as well as a capacity on the firm's behalf to commit to the announced contracts concluded with the agents/managers (Sklivas, 1985; Vickers, 1985; Fershtman, 1985; Fershtman and Judd, 1987).

More recently, the works of Kaneda and Matsui (2003) and Cornes and Itaya (2016) both consider an oligopoly model where the firms' managers maximize the weighted average of profit and some other alternative objective. Kaneda and Matsui (2003) consider how firms' behaviour changes when they put different weights on alternative objectives. Cornes and Itaya (2016) characterize the equilibria of such games with weaker conditions on the functional forms and also allow for asymmetric firms. Again, their focus is on understanding the relative behaviour of firms when they care more or less about alternative objectives.

In this article we focus on the manager's payoff function and work with a model that incorporates non-linear evaluation of revenue. By relaxing this linearity assumption—in line with the initial intuition of Williamson (1963)—we explore the consequences of exogenous demand shocks on the game's equilibrium. Allowing for this more general framework shows that firm output can either increase or decrease in the presence of an exogenous demand shock. Our framework, then, has the ability to nest the existing literature within our general model as a special linear case. Therefore our major contribution is the development of a framework that provides a broader understanding of production choices under the presence of demand shocks.[2]

[2]Interestingly, other scholars have concluded that changing market conditions can produce results going against the common wisdom: in the context of product differentiation (Chen and Riordan, 2008); in the presence of search

The remainder of the article is organized as follows. In Section 3 we outline the model, in Section 4 we derive the results, and in Section 5 we provide a worked example. Section 6 provides some concluding remarks. The appendix generalizes the model to the case of heterogeneous firms.

3 The model

Consider a market in which there are $i = 1, \ldots, n$ firms who choose their level of output $x^i \geq 0$. We let $X = \sum_{i=1}^{n} x^i$ be total supply and $X^{-i} = \sum_{j \neq i} x^j$. The price is given by $P(X, \alpha)$, where α is a demand parameter. We assume that $P_X < 0$ and $P_X + x^i P_{XX} \leq 0$.[3] The demand parameter α is an exogenous demand shock described by $P_\alpha > 0$ and $P_\alpha + x^i P_{X\alpha} \geq 0$, so an increase in α both increases the price and marginal revenue. If firm i chooses output x^i, it incurs a cost of $C^i(x^i)$, and we assume $C^{i\prime} > 0$ and $C^{i\prime\prime} \geq 0$. A firm's profit is given by $\pi^i \equiv r^i - C^i(x^i)$, where $r^i \equiv x^i P(X; \alpha)$ is the revenue received.

Consider a decision-maker's payoff given by their utility $U^i(\pi^i, r^i)$, which is influenced by both profit and revenue. Following the existing literature, we suppose that the decision-maker's utility is a weighted average of the two components but, in contrast to the existing literature, we allow for the their evaluation of the revenue component to be non-linear with the benefit from revenue given by $v^i(r^i)$, where we assume $v^{i\prime} > 0$ and $v^{i\prime\prime} \leq 0$. This allows us to capture more general preferences, which include cases where incremental gains in revenue may be more valuable when revenue is scarce compared to when it is abundant. As such, for a parameter $\beta^i \in [0, 1]$, we suppose that the decision maker of firm i has the payoff function

$$\begin{aligned} U^i(\pi^i, r^i) &= [1 - \beta^i]\pi^i + \beta^i v^i(r^i) \\ &= [1 - \beta^i][r^i - C^i(x^i)] + \beta^i v^i(r^i). \end{aligned}$$

If $\beta^i = 0$ the decision maker is only concerned about profit, whereas if $\beta^i > 0$ they also have a revenue motive. If $\beta = 1$ they care only about revenue. We also suppose that decision makers within firms are subject to a minimum profit constraint, that we assume to be zero for simplicity.

We assume that each decision maker's β^i is common knowledge and that output decisions are taken simultaneously, and study the n-player simultaneous-move game of complete information where each decision maker chooses their output $x^i \geq 0$ subject to the constraint

costs (Janssen and Moraga-Gonzalez, 2004); or when consumers are endowed with market power (Dickson, 2013).

[3]This latter assumption requires marginal revenue, $2P_X + x^i P_{XX}$, to be less than the slope of the demand function, P_X. Since $P_X < 0$, this implies $2P_X + x^i P_{XX} < 0$. Another interpretation is that the x^i-elasticity of the slope of demand, given by $|x^i P_{XX}/P_X|$, is less than one.

that their profit is non-negative and their payoffs are given by $U^i(\pi^i, r^i)$. We seek a Nash equilibrium in pure strategies $\{x^{i*}\}_{i=1}^n$ in which, for each firm $i = 1, \ldots, n$,

$$x^{i*} \in \arg\max_{x^i \geq 0}\{[1-\beta^i][r^i - C^i(x^i)] + \beta^i v^i(r^i) \text{ s.t. } r^i - C^i(x^i) \geq 0\}$$

where $r^i = x^i P(x^i + X^{-i*}; \alpha)$.

In the analysis that follows, for simplicity, we treat decision makers as symmetric where $v^i(\cdot) = v(\cdot)$, $\beta^i = \beta$ and $C^i(\cdot) = C(\cdot)$ for all $i = 1, \ldots, n$. In the appendix, however, we extend the analysis to account for heterogeneity among firms and derive similar results.

4 Analysis

4.1 The equilibrium

Assuming firms are symmetric, a typical decision-maker's payoff is given by

$$U = [1-\beta][x^i P(X;\alpha) - C(x^i)] + \beta v(x^i P(X;\alpha)).$$

Their marginal payoff is therefore given by

$$\phi(x^i, X^{-i}; \alpha) \equiv \frac{\partial U}{\partial x^i} = [1-\beta][P + x^i P_X - C'] + \beta v'[P + x^i P_X]. \tag{1}$$

Note that

$$\phi_{x^i} = [1-\beta+\beta v'][2P_X + x^i P_{XX}] + \beta v''[P + x^i P_X]^2 - [1-\beta]C'' < 0$$

under our assumptions, and therefore a decision maker's best response, ignoring for the moment the non-negative profit condition, is given by the reaction function $\hat{x}^i(X^{-i}) = \max\{0, x^i\}$, where x^i is the solution to the first-order condition

$$\phi(x^i, X^{-i}; \alpha) \equiv [1-\beta][P + x^i P_X - C'] + \beta v'[P + x^i P_X] = 0.$$

Note that if $\beta = 0$, the problem becomes one of equating marginal revenue, $P + x^i P_X$, to marginal cost, C', as in a standard profit-maximization context. If $\beta = 1$, the problem collapses to $v'[P + x^i P_X] = 0$ the solution to which (since $v' > 0$) necessitates equating marginal revenue to zero, as would be expected because in this case the payoff function is $v(r)$, which is a monotonic transformation of revenue, maximized where revenue is maximized. In intermediate cases where $\beta \in (0, 1)$ the decision-maker cares about both profit and revenue. Note that in this case marginal revenue must be positive at the chosen level of output, for if $P + x^i P_X \leq 0$ equality with zero in the first-order condition can never be achieved, and by the same reasoning it must be the case that marginal revenue is less than marginal cost.

Decision makers are subject to the constraint that profit is positive. As such, their reaction function is actually given by the minimum of that defined above, and $\bar{x}^i(X^{-i})$, which is the output at which profit is zero. We suppose throughout that at equilibrium the positive profit constraint is not binding so that when we consider a shock to the environment decision-makers can respond following the incentives they face in our setup. If the firm is operating at their constraint they could not increase their output further.

The slope of the reaction function with respect to the action of any other player $j \neq i$ is given by

$$\frac{\partial \hat{x}^i(X^{-i})}{\partial x^j} = \frac{\partial \hat{x}^i(X^{-i})}{\partial X^{-i}} = -\frac{\phi_{X^{-i}}}{\phi_{x^i}},$$

where $\phi_{x^i} < 0$ and

$$\phi_{X^{-i}} = [1 - \beta + \beta v'][P_X + x^i P_{XX}] + \beta v'' x^i P_X [P + x^i P_X].$$

The sign of this is not determined by the current assumptions as while the first term is negative, marginal revenue in the last square brackets of the second term is positive making the second term positive (since $v'' < 0$ and $P_X < 0$).

To establish uniqueness of the equilibrium we require that the joint reaction function map is a contraction. A sufficient condition is that $\sum_{j \neq i} \frac{\partial \hat{x}^i(X^{-i})}{\partial x^j} < 1$ (see, for example, Vives (1999)), which requires $[n-1]\frac{\mathrm{d}\hat{x}^i(X^{-i})}{\mathrm{d}X^{-i}} < 1$, and therefore that $\phi_{x^i} + [n-1]\phi_{X^{-i}} < 0$. As such, uniqueness of equilibrium requires

$$[1 - \beta + \beta v'][2P_X + x^i P_{XX}] + \beta v''[P + x^i P_X]^2 - [1 - \beta]C'' + \\ [n-1][[1 - \beta + \beta v'][P_X + x^i P_{XX}] + \beta v'' x^i P_X [P + x^i P_X]] < 0.$$

This can be reduced to

$$[1 - \beta][P_X + n[P_X + x^i P_{XX}] - C''] + \beta[v'[P_X + n[P_X + x^i P_{XX}]] + v''[P + x^i P_X][P + n x^i P_X]] < 0.$$

The first and second terms in this expression are negative by assumption. The third term requires more scrutiny. We know that on the reaction function $P + x^i P_X > 0$, so if $P + n x^i P_X > 0$ as well then the third term is negative. However, if $P + n x^i P_X < 0$ then a sufficient condition to ensure the whole expression is negative is that $P + x^i P_X < -\frac{v'}{v''}\frac{\frac{\partial}{\partial x^i}[P + n x^i P_X]}{P + n x^i P_X}$, which we assume to hold at least in equilibrium, for then uniqueness of equilibrium is guaranteed.

In a symmetric Nash equilibrium in which $x^{i*} = x^*$ for all $i = 1, ..., n$ equilibrium supply

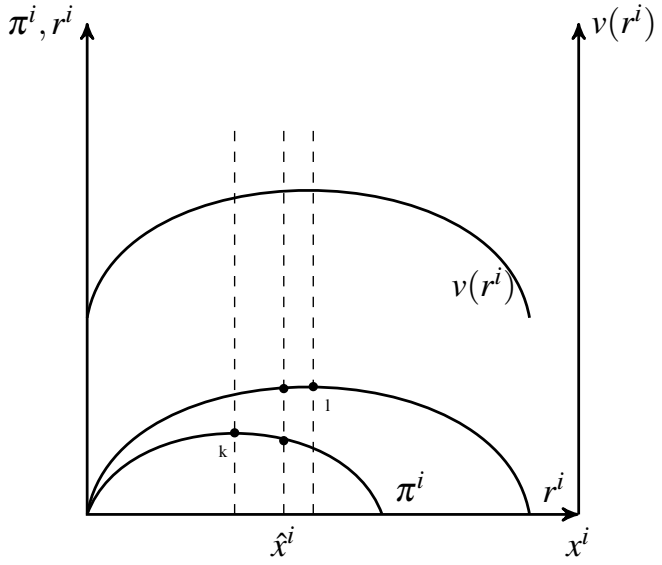


Figure 1: The firm's functions for profit π^i, revenue r^i, and evaluation of revenue $v(r^i)$. Profit and revenue are measured on the left-hand axis, while the evaluation of revenue is measured on the right-hand axis.

is implicitly defined by[4]

$$\phi(x^*, [n-1]x^*; \alpha) \equiv [1-\beta][P(nx^*;\alpha) + x^* P_X(nx^*;\alpha) - C'(x^*)] + \beta v'(x^*(P(nx^*;\alpha)))[P(nx^*;\alpha) + x^* P_X(nx^*;\alpha)] = 0. \quad (2)$$

Figure 1 illustrates both profit (π^i) and revenue functions (r^i), as well as the decision maker's value of revenue ($v(r^i)$). Having assumed that $P_X + x^i P_{XX} < 0$, both the revenue and profit functions are concave. When $\beta = 0$, the Nash equilibrium will correspond to the Cournot equilibrium in which each firm maximizes profit by producing output where its marginal revenue is equal to its marginal cost, as observed in Figure 1 at point k. When $\beta = 1$, the Nash equilibrium will involve each firm maximizing revenue, which, for graphical purposes, we have assumed to be downward sloping for large enough levels of output. In that instance, firms will produce at point l, where the firm's marginal revenue is zero, which by extension also implies that the marginal valuation of revenue is zero. Observe that since both revenue and profit functions are concave, and at the profit-maximizing level of output marginal revenue is positive, this necessitates a higher level of output under revenue maximization than under profit maximization. For intermediate cases where $\beta \in (0,1)$, the decision maker will produce output until their weighted valuation of marginal profits and

[4]Note that $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*} = \phi_{x^i} + [n-1]\phi_{X^{-i}}$, which is negative as this is exactly the condition that imposes the joint reaction function is a contraction.

revenues is zero, as shown as output $\hat{x}^i$ in Figure 1, which will lie somewhere between the profit maximizing level of output and the revenue maximizing level of output.

As more importance is placed on revenue (i.e., β increases), ceteris paribus, equilibrium output increases from the Cournot level to the revenue-maximizing level. This can be deduced from implicit differentiation of (2):

$$\frac{\mathrm{d}x^*}{\mathrm{d}\beta} = -\frac{v'[P + x^* P_x] - [P + x^* P_X - C'(\cdot)]}{\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*}}. \tag{3}$$

Since $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*} < 0$ and in equilibrium marginal revenue must be positive—but at the same time less than marginal cost—this term is positive.

4.2 Exogenous demand shocks

We now turn to understand how the equilibrium supply of decision makers responds to a change in market conditions. More specifically, we are interested in the effect of an exogenous demand shock on the equilibrium. Implicit differentiation of (2) gives

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} = -\frac{\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}\alpha}}{\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*}}.$$

Since we have deduced that $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*} < 0$, the sign of $\frac{\mathrm{d}x^*}{\mathrm{d}\alpha}$ will be equal to the sign of $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}\alpha}$. Now

$$\begin{aligned}\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}\alpha} &= [1-\beta][P_\alpha + x^* P_{X\alpha}] + \beta\left[v'' x^* P_\alpha [P + x^* P_X] + v'[P_\alpha + x^* P_{X\alpha}]\right] \\ &= P_\alpha\left[[1-\beta][1+\epsilon_{P_\alpha x}] + \beta[v'' x^* P[1+\epsilon_{P,X}] + v'[1+\epsilon_{P_\alpha x}]]\right],\end{aligned}$$

where $\epsilon_{P,X} \equiv \frac{x^* P_X}{P}$ is the elasticity of the inverse demand to quantities (i.e., the inverse of the price-elasticity of the demand), and $\epsilon_{P_\alpha x} \equiv \frac{x^* P_{X\alpha}}{P_\alpha}$ is the x-elasticity of the sensitivity of the demand to α.

We begin by analyzing the two polar cases; namely, the pure profit-maximizing case ($\beta = 0$) and the pure revenue-maximizing case ($\beta = 1$). In the pure profit-maximizing case where $\beta = 0$, the effect of the shock comes solely through the effect on marginal revenue since $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}\alpha} = P_\alpha + x^* P_{X\alpha}$. Following the shock, marginal revenue increases because the price increases due to the shift in demand, but because the slope of demand might also change the effect of expanding supply on the price of existing units that are being sold may be positive or negative. We assume that the overall effect is positive so following the demand shock marginal revenue increases (i.e., $P_\alpha + x^* P_{X\alpha} > 0$). As such, and without surprise, the equilibrium supply of pure profit maximising firms will increase following a positive demand shock.

The same is true if firms are purely revenue maximising. To see this note that if $\beta = 1$ then $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}\alpha} = [P + x^*P_X]v''x^*P_\alpha + v'[P_\alpha + x^*P_{X\alpha}] = v'[P_\alpha + x^*P_{X\alpha}] > 0$, since $P + x^*P_X = 0$ in equilibrium. The effect of a positive demand shock on a firm's output decision is therefore dictated by the exact same forces that exist in the profit-maximizing case; namely, the change in the marginal revenue $P_\alpha + x^*P_{X\alpha}$. We can thus state the following result:

Proposition 1. *Pure profit-maximizing firms and pure revenue-maximizing firms increase their output with a positive demand shock.*

This finding is quite intuitive. A revenue-maximizing decision maker will increase its firm's production until the marginal revenue becomes zero, since this will maximize their utility irrespective of the shape of their utility function. Any increase in the marginal revenue will thus incentivize a firm to increase its output. A profit-maximizing decision maker sets its firm's marginal revenue equal to the marginal cost. Since the demand shock leaves the marginal cost unaffected, but increases the firm's marginal revenue, this incentivizes the manager to increase output.

The most interesting scenerio, however, emerges for the more general intermediate cases where $\beta \in (0,1)$. In such instances we obtain

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} \gtreqless 0 \Leftrightarrow [1 - \beta + \beta v'][1 + \epsilon_{P_\alpha x}] \gtreqless -\beta v''x^*P[1 + \epsilon_{P,X}]. \tag{4}$$

While the left-hand side of this expression is positive as a result of our assumption that $P_\alpha + x^*P_{X\alpha} > 0$, the right-hand side will be strictly positive for strictly concave utility functions, thereby implying that it is feasible to witness the counter-intuitive situation summarized in the following proposition:

Proposition 2. *A decision maker that has non-linear evaluation of revenue and maximizes a combination of profit and revenue may reduce their firm's output following a positive demand shock.*

To grasp the essence of this result, we focus on the right-hand side of the inequality in (4). That term captures how the slope of the marginal revenue-related utility generated by an extra unit of output is affected by an increase in demand. In other words, since the marginal revenue-related utility generated by an extra unit of output is the product of the decision maker's marginal utility of total revenue, v', and the marginal revenue, $P + x^*P_X = 1 + \epsilon_{P,X}$, a demand shock will impact total revenue, and thereby this (weighted) marginal utility of total revenue. If $v'' < 0$, then a positive demand shock will reduce the (weighted) marginal utility of total revenue. The intensity of the drop in this marginal valuation will be dictated by the degree of concavity of the utility function, but equally by the level of total revenue which is

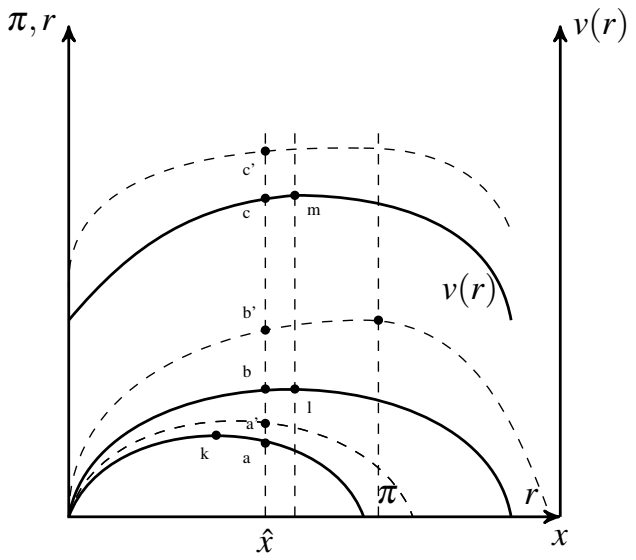


Figure 2: A positive demand shock results in the dashed expansion of profit, revenue, and the evaluation of revenue. Output will decrease if the decrease in the slope at point a is offset by the decrease in the product of the slopes at points b and c. This occurs if the evaluation of revenue $v(r)$ is sufficiently concave.

directly impacted by changes in demand. It thus follows that if the utility function's concavity is sufficiently strong then it is possible that $dx^*/d\alpha < 0$ and so the response of decision makers to a change in market conditions will break the conventional wisdom.

To observe the intuition of this result, Figure 2 illustrates a decision maker's choice; dashed curves indicate post-demand expansion functions. In our most general case, the decision maker has a weighted preference for profit and revenue maximization. Denote the optimum output as $\hat{x}$. At this point the (weighted) slope of the profit function at point a must be equal to the (weighted and product of) slopes of the revenue function at point b and the value function at point c.

When demand expands both revenue and marginal revenue increase. Thus the profit function and the slope of the profit function increase, so at point a' the slope is less negative; the slope of the revenue function increases at point b'; and the slope of the value function decreases at point c'. Thus the slope at point a will decrease and the product of b and c can increase or decrease. If it decreases a lot, then at the equilibrium we could have the first-order condition becoming negative, so the optimum needs to move in the direction that increases the slopes of b and c and makes the slope at point a less negative, that is, have a lower $\hat{x}$. The rate at which the product of the slopes b and c decrease depends on the concavity of the value function $v(r)$.

To see the importance of the utility function's concavity, consider instead a linear utility

function as has been assumed in the related literature (e.g., Sklivas, 1985; Vickers, 1985; Fershtman and Judd, 1987). Positive demand shocks will then leave the slope of the marginal revenue-related utility generated by an extra unit of output unchanged. In other words, the valuation of the total revenue will only be impacted through the effect the demand shock has on the firm's marginal revenue. Therefore, since both marginal profits and the marginal valuation of revenues are impacted in the same way; namely, through an increase in the firm's marginal revenue, the firm's output will always increase with α.

An interesting question pertains to the effect on equilibrium profits where there is a shock to the market when decision makers are motivated by revenue concerns. In general, the effect of a change in α can be decomposed into the direct effect on profit, and the indirect effect that comes through the change in activity as a result. Thus:

$$\frac{d\pi}{d\alpha} = \frac{\partial \pi}{\partial \alpha} + \frac{\partial \pi}{\partial x}\frac{\partial x^*}{\partial \alpha} \tag{5}$$

Note that since $\pi = xP(X, \alpha) - C(x)$, $\partial \pi / \partial \alpha = xP_\alpha$. When $\beta = 0$ so the decision-maker is motivated by profit we can use the envelope theorem to deduce that $\partial \pi / \partial x = 0$ and therefore:

$$\left.\frac{\mathrm{d}\pi}{\mathrm{d}\alpha}\right|_{\beta=0} = xP_\alpha > 0. \tag{6}$$

When $\beta > 0$, the decision maker's payoff function is $U = [1-\beta]\pi + \beta v(r)$, and so the first-order condition requires $[1-\beta]\frac{\partial \pi}{\partial x} + \beta \frac{\partial v(r)}{\partial x} = 0$. This implies $\frac{\partial \pi}{\partial x} = -\frac{\beta}{1-\beta}\frac{\partial v(r)}{\partial x} < 0$ in equilibrium, since $\partial v(r)/\partial x > 0$. Moreover $x^*|_{\beta>0} > x^*|_{\beta=0}$. As such, so long as $P_{x\alpha} > 0$, $x^* P_\alpha|_{\beta>0} > x^* P_\alpha|_{\beta=0}$. Inspection of (5) therefore allows us to conclude that when a decision maker is motivated by revenue and as a result of a positive shock reduces supply ($\frac{\partial x^*}{\partial \alpha} < 0$), not only does profit increase, but it increases by more than would be the case if the firm were operated by a profit-maximizing decision maker.

5 Worked example

To illustrate the above results consider the following example, in which demand is taken to be isoelastic, taking the form

$$P(X; \alpha) = \frac{\alpha}{X}, \tag{7}$$

the benefits from revenue are assumed to given by

$$v^i(r^i) = \delta r^i - \frac{\gamma}{2}[r^i]^2, \tag{8}$$

with $\delta, \gamma > 0$, and production costs are assumed to be given by cx^i with $c > 0$.

It is clear to show that revenue is $r^i = \frac{x^i \alpha}{X}$ with $MR = \frac{\alpha X^{-i}}{X^2} = \frac{\alpha[n-1]}{n^2 x}$ when symmetry is imposed. For the symmetric profit maximizing condition $MR = MC$ we have $x^* = \frac{\alpha[n-1]}{n^2 c}$ and for the symmetric revenue maximization output $MR = 0$ implying $x^* \to \infty$. We now turn to the intermediate cases where $\beta \in (0,1)$. The first-order condition is:

$$U' = \left[1 - \beta + \beta \left[\delta - \frac{\gamma \alpha x^i}{X}\right]\right] \left[\frac{\alpha}{X} - \frac{x^i \alpha}{X^2}\right] - [1-\beta]c = 0 \tag{9}$$

and under a symmetric equilibrium this results in:

$$x^* = \frac{n-1}{n^2[1-\beta]c} \alpha \left[1 - \beta + \beta \left[\delta - \frac{\gamma \alpha}{n}\right]\right]. \tag{10}$$

We can now identify the conditions where the firm either expands or contracts output as product demand changes:

$$\frac{dx^*}{d\alpha} = \frac{n-1}{n^2[1-\beta]c} \left[1 - \beta + \beta\delta - \frac{2\alpha\beta\gamma}{n}\right], \tag{11}$$

and therefore

$$\frac{dx^*}{d\alpha} \lessgtr 0 \Leftrightarrow \gamma \gtrless \frac{n[1-\beta+\beta\delta]}{2\alpha\beta}. \tag{12}$$

This confirms our findings within the general model in Section 3; namely, demand expansion can result in a firm reducing output when the decision maker has a sufficiently concave benefit function over the firm's revenue. In other words, for a sufficiently large γ, firm output can fall when demand increases.[5]

6 Concluding remarks

A conventional assumption within the industrial organization literature is that firms maximize profits. Although this is a reasonable assumption from the point of view of the firm owners, many important strategic decisions are taken by agents such as managers, who hold complementary objectives, such as maximizing sales, market share, or performance relative to competitors. To investigate the consequences of these alternative objectives, current approaches often assume a linear evaluation of these goals. Yet there is no *a priori* rationale to assume such a restrictive setting and, indeed, a general approach may capture more of the

[5]The existence of a γ conducive to decreasing output is guaranteed. There are four conditions imposed on the structure of the game: we require positive profit $r - cx > 0$; marginal valuation of revenue being positive $\delta - \gamma r > 0$; output being positive $x > 0$; and finally $\frac{dx^*}{d\alpha} < 0$. If $v' > 0$ then is must hold that $x^* > 0$ since $\frac{\delta n}{\alpha} < \frac{n[1-\beta+\beta\delta]}{\beta\alpha}$. We also know that $\frac{\delta n}{\alpha} > \frac{n}{\alpha\beta}\left[\frac{(n-1)\beta\delta-(1-\beta)}{n-1}\right]$, which implies that there exists a range of γ values such that $v' > 0$ and $\pi^* > 0$. Lastly, there exists a γ that satisfies $v' > 0$ and $\frac{dx^*}{d\alpha} < 0$ if $\frac{\delta n}{\alpha} > \frac{n(1-\beta+\beta\delta)}{2\beta\alpha}$, which reduces to $\delta > \frac{1-\beta}{\beta}$.

realistic settings with which decision makers are faced, such as incentives to increase their power, status, prestige or empire building.

This article investigates an oligopoly framework in which decision makers' objectives are a weighted sum of profit and utility over revenue. Our major innovation has been to assume non-linear preferences over revenue; namely, we allow the decision maker to obtain (concave) utility over revenue. This means that our model includes cases where incremental gains in revenue are more valuable when revenue is scarce compared to when it is abundant. The non-linearity in preferences has important consequences for the functioning of the market. If the decision maker is *solely* either profit maximizing or revenue maximizing, a negative demand shock reduces marginal revenue and output will fall. Yet we find that if the decision maker's objective is a weighted average of profit and the utility derived from revenue, then an exogenous negative demand shock may result in *increases* in output, both at the individual and aggregate level. Intuitively, a contraction in market demand will (i) reduce marginal profits but (ii) increase the marginal utility of revenue. If the latter effect is stronger than the former effect then demand shocks can result in an increase in output. This occurs when the decision maker's utility over revenue is sufficiently concave. Specularly, positive demand shocks may result in reductions in individual and aggregate output. The existing literature is nested within our framework by assuming specific conditions on the utility function, i.e. by imposing a constant marginal valuation of revenue to the firm's manager. Our more general approach can therefore provide a broader understanding of output choices in the presence of demand shocks.

Appendix: Heterogeneous firms

In the main body of this article we assume homogeneous firms for ease of exposition, but in this appendix we extend the analysis to consider heterogeneous firms, where there is common knowledge of firms' characteristics.

The payoff function of firm i takes the form

$$U^i = [1-\beta^i][x^i P(X;\alpha) - C^i(x^i)] + \beta^i v^i(x^i P(X;\alpha)).$$

The marginal payoff is therefore

$$\phi^i(x^i, X^{-i};\alpha) \equiv \frac{\partial U^i}{\partial x^i} = [1-\beta^i][P + x^i P_X - C^{i\prime}] + \beta^i v^{i\prime}[P + x^i P_X].$$

Following the approach of the aggregative games literature, we now define, for each firm, a *share function* which is derived from the first-order condition in which X^{-i} is replaced with $X - x^i$ and x^i is replaced with $\sigma^i X$, with σ^i denoting firm i's share of total supply. Firm i's share function, when multiplied by X, will give the supply of firm i consistent with a Nash equilibrium in which the aggregate supply of all firms is X. Share functions are defined only when their value is no larger than 1, and take the form $s^i(X;\alpha) = \max\{0, \sigma^i\}$ where σ^i is such that

$$\begin{aligned}\phi^i(\sigma^i X, X[1-\sigma^i];\alpha) = 0 \Rightarrow \\ l^i(\sigma^i, X;\alpha) \equiv [1-\beta^i][P + \sigma^i X P_X - C^{i\prime}] + \beta^i v^{i\prime}[P + \sigma^i X P_X] = 0.\end{aligned}$$

Share functions identify individually consistent behavior. Nash equilibrium requires consistency at the aggregate level. It is a matter of definition-chasing to recognise that there is a Nash equilibrium with aggregate supply $X > 0$ if and only if

$$\sum_{i=1}^{n} X s^i(X;\alpha) = X \Leftrightarrow S(X;\alpha) \equiv \sum_{i=1}^{n} s^i(X;\alpha) = 1.$$

We now want to determine the properties of individual, and consequently aggregate, share functions. By the implicit function theorem,

$$s^i_X = -\frac{l^i_X}{l^i_{\sigma^i}}.$$

Now, bearing in mind that $v^i = v^i(\sigma^i X P(X;\alpha))$,

$$l^i_{\sigma^i} = \beta^i v^{i\prime\prime} X P[P + \sigma^i X P_X] + [1 - \beta^i + \beta^i v^{i\prime}] X P_X - [1-\beta^i] X C^{i\prime\prime} < 0$$

as each term is negative by assumption, and

$$\begin{aligned}l^i_X &= \beta^i v^{i\prime\prime}\sigma^i[P + XP_X][P + \sigma^i X P_X] + [1 - \beta^i + \beta^i v^{i\prime}][P_X + \sigma^i[P_X + XP_{XX}]] - [1-\beta^i]\sigma^i C^{i\prime\prime} \\ &< \beta^i v^{i\prime\prime}\sigma^i[P + XP_X][P + \sigma^i X P_X] + [1 - \beta^i + \beta^i v^{i\prime}]\sigma^i[2P_X + XP_{XX}] - [1-\beta^i]\sigma^i C^{i\prime\prime} < 0\end{aligned}$$

as the second and third terms are negative by assumption, and the discussion on the additional assumption we require for uniqueness on p7 implies the first term is negative. This allows us to conclude that individual share functions are strictly decreasing in $X > 0$.

To derive the other properties of share functions, it is useful to envisage $l^i(\sigma^i, X; \alpha)$ plotted as a (strictly decreasing) function of σ^i. First, note that

$$l^i(0, X; \alpha) = [1 - \beta^i][P - C^{i'}(0)] + \beta^i v^{i'} P \text{ and}$$
$$l^i(1, X; \alpha) = [1 - \beta^i + v^{i'}\beta^i][P + XP_X] - [1 - \beta^i]C^{i'}(X).$$

Let us define $\underline{X}^i$ by $l^i(1, \underline{X}^i; \alpha) = 1$, i.e., the value of X where the share function is just equal to 1. Such an $\underline{X}^i$ exists by virtue of the intermediate value theorem, noting that

$$l^i(1, 0; \alpha) = [1 - \beta^i + v^{i'}(0)\beta^i]P(0) - [1 - \beta^i]C^{i'}(0) > 0 \text{ and}$$
$$\lim_{X\to\infty} l^i(1, X; \alpha) = [1 - \beta^i + \lim_{X\to\infty} v^{i'}(XP)\beta^i] \lim_{X\to\infty}[P + XP_X] - [1 - \beta^i] \lim_{X\to\infty} C^{i'}(X) < 0,$$

where the second line follows from marginal revenue in the large-X limit being either zero or negative. For $X < \underline{X}^i$ the share function would take a value that exceeds 1 and is therefore undefined. For $X \geq \underline{X}^i$ the share function, as previously noted, is strictly decreasing in X. As X increases without bound, note that $\lim_{X\to\infty} l^i(0, X; \alpha) \leq 0$ since $\lim_{X\to\infty} P(X) = 0$. If this is strictly negative then there is an $\bar{X}^i < \infty$, defined such that $l^i(0, \bar{X}^i; \alpha) = 0$ in which case the share function takes the value zero for all $X \geq \bar{X}^i$. If it is equal to zero then $s^i(X; \alpha) \to 0$ as $X \to \infty$.

To summarize, individual share functions are undefined for $X < \underline{X}^i$, take the value of 1 at $X = \underline{X}^i$, and then strictly decrease in X to take the value of zero at $\bar{X}^i < \infty$ if such a value exists, or converge to zero as X increases without bound.

The aggregate share function $S(X; \alpha)$ is defined only for $X \geq \max_i\{\underline{X}^i\}$ where it takes a value that exceeds 1. It then inherits the property of individual share functions that it is strictly decreasing in X and is either equal to zero for some large-enough X, or converges to zero as $X \to \infty$. Since it is continuous, the intermediate value theorem therefore allows us to conclude that there is a single value of X where $S(X; \alpha) = 1$, and therefore a unique Nash equilibrium.

When we consider a shock to demand which changes α, because share functions are strictly decreasing in X we can understand the effect of this from the effect of α on share functions. Formally, if we denote by X^* the equilibrium aggregate supply which satisfied $S(X^*; \alpha) = 1$, then

$$\frac{\mathrm{d}X^*}{\mathrm{d}\alpha} = -\frac{\sum_{i=1}^n s^i_\alpha}{\sum_{i=1}^n s^i_X}$$

the sign of which, since we know $s^i_X < 0$, depends on the sign of the s^i_α. Using the implicit function theorem,

$$s^i_\alpha = -\frac{l^i_\alpha}{l^i_{\sigma^i}},$$

the sign of which (since $l^i_{\sigma^i} < 0$) depends on the sign of l^i_α. Now,

$$\begin{aligned} l^i_\alpha &= [1 - \beta^i + \beta^i v^{i\prime}][P_\alpha + \sigma^i X P_{X\alpha}] + \beta^i v^{i\prime\prime} x^i P_\alpha [P + \sigma^i X P_X] \\ &= P_\alpha [[1 - \beta^i + \beta^i v^{i\prime}][1 + \epsilon_{P_\alpha x^i}] + \beta^i v^{i\prime\prime} x^i P [1 + \epsilon_{P,X}]] \end{aligned}$$

after some simplification, where as previously defined, $\epsilon_{P,X} \equiv \frac{x^{i*} P_X}{P}$ and $\epsilon_{P_\alpha x^i} \equiv \frac{x^{i*} P_{X\alpha}}{P_\alpha}$. As such, exactly the same conditions that govern whether a positive shock to demand results in a contraction in supply in the model with homogeneous firms apply to the case of heterogeneous firms if the sign of this effect is the same for all heterogeneous firms.

Notice that if the sign of s^i_α is negative for all firms then the aggregate supply decreases, but it could also be the case that $s^i_\alpha < 0$ only for a subset of firms and yet aggregate supply still declines so long as the aggregation of the effects makes $\frac{\mathrm{d}X^*}{\mathrm{d}\alpha} < 0$. Notice also that, even if $s^i_\alpha < 0$ for all firms, the individual supply of every firm does not necessarily decrease, but any increases are offset by sufficiently large reductions in supply by other firms so that the aggregate supply declines.

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