Production Network Structure, Service Share, and Aggregate Volatility

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Abstract

This paper studies differences in production structures across countries and their implications for cross-country heterogeneity in GDP volatility. In particular, economies with more input-output connections—a denser network—are associated with less concentrated sales shares and lower volatility. The relationship between density and volatility is stronger in countries with a higher share of services in GDP (hereafter referred to as service share). To account for this evidence, I propose a generalized production network model in which denser economies display higher production complexity. If production is also specialized in industries that use labor and intermediates as substitute inputs, higher network density indeed lowers the concentration of sales shares and aggregate volatility. U.S. sectoral data suggest that the elasticity of substitution between labor and intermediates in service sectors is larger than one and larger than in non-service sectors. A calibrated model that then also matches each country's production network can quantitatively generate observed cross-country empirical patterns. Furthermore, in contrast to previous work, the model predicts that: i) sectoral shocks play only a modest role in accounting for the observed business cycle dynamics, especially in dense and service-oriented economies; and ii) production diversification does not always lower volatility.

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1 Introduction

The production process, such as the production of cell phones in Korea, entails complex connections between firms in different sectors of the economy. Samsung uses not only chips, plastic, and financial services, but also equipment, water, and gas. Every economy produces different goods and services, which then translate into a particular structure of input-output connections. What features of the production structure amplify or mitigate shocks? Do economies with more input-output connections display lower macroeconomic volatility? Finally, what does the relationship between volatility and input-output structure teach us about the role of sectoral shocks in explaining business cycle fluctuations?

In this paper, I use international data on input-output structures to investigate what features of the production network explain cross-country heterogeneity in sales shares and, therefore, aggregate volatility. I provide a generalized production network model that is able to qualitatively and quantitatively rationalize the facts. I then use the model to study the role of sectoral shocks in accounting for sectoral business cycle dynamics in the U.S.

From an empirical standpoint, this paper shows that production network density—the fraction of non-zero input-output connections—is a key driver of countries' sales shares concentration (as measured by the Herfindahl-Hirschman Index (HHI)) and GDP volatility. Specifically, in a panel of 48 OECD and non-OECD countries for the period 1984-2014, I document two facts: i) in countries with denser production networks, sales shares are less concentrated and GDP volatility is lower; and ii) the relationship between density and volatility is stronger in the group of high-service-share economies.

From a more theoretical standpoint, I generalize the canonical economy of Acemoglu et al. (2012) in two key ways to help explain the cross-country patterns I document. First, I allow for heterogeneous and non-unitary elasticities of substitution in production (flexibility). Second, I introduce a cost of complexity in producing with a different number of intermediates. I calibrate the model to match each country's input-output structure and estimate heterogeneous elasticities of substitution in production across sectors from U.S data. To the extent that U.S sectoral technologies also describe sectoral technologies across countries, I then use the model to perform several quantitative exercises.

The main results in this paper are as follows. First, the model economy suggests that a higher network density reduces the economy's HHI of sales shares and, therefore, volatility, as long as production technologies display high substitutability between intermediate inputs and labor. In denser economies, production is more complex, and high substitutability in production allows the use of more labor to overcome the complexity cost. The higher labor share reduces the effect of shocks along the production chain, which then reduces volatility. Using U.S sectoral data, I show that service industries have an elasticity of substitution between labor and intermediates that is larger than one and larger than in non-service sectors.

Therefore, through this mechanism, the model is also able to qualitatively replicate the fact that network density more evidently reduces volatility in high-service-share countries. Moreover, the calibrated model is quantitatively successful in replicating the observed empirical patterns. The model's implied relationship among density, service share, and volatility is of an order of magnitude similar to that observed in the data.

Second, in contrast to Foerster et al. (2011) or Atalay (2017), the model economy in this paper predicts only a mild role for sectoral shocks in explaining business cycle dynamics, especially in dense and service-oriented economies. While in my model Hulten (1978)'s theorem hold, implying that sectoral sales shares are sufficient statistics to determine GDP volatility, I show that network density and production flexibility play a crucial role in determining sectoral comovement and, therefore, in determining the exact sources of aggregate volatility, whether aggregate shocks or sectoral shocks. I calibrate economies with different elasticities and different costs of complexity parameters to match the same vector of sectoral sales. I shock each economy with the same series of sectoral productivity processes and show that, while an economy with low production flexibility and no role of network density delivers significant comovement out of sectoral shocks, in an economy with high production flexibility, in which network density plays a role, idiosyncratic shocks generate a median pairwise correlation of output growth close to zero, which is significantly smaller than the median pairwise correlation observed in the U.S (0.3) for the period 1998-2017.

Third, the results in this paper suggest that production diversification, as measured by the number of non-zero input-output connections, does not always reduce volatility. The model in this paper predicts that a dense production network specializing in industries with low production flexibility can be highly volatile. In a more diversified production structure, in which firms rely on intermediates from most other industries, low production elasticities also generate stronger sectoral links at the intensive margin, which then amplify the effect of sectoral shocks along the production chain. This result is of special interest to emerging economies, where discussions regarding diversifying the production structure are ongoing.

Several studies have highlighted the role of the domestic production structure in shaping aggregate fluctuations. Horvath (1998), Foerster et al. (2011), Acemoglu et al. (2012), Carvalho et al. (2016), Atalay (2017), and Baqaee and Farhi (2017) demonstrate the importance of production complementarities, whether they are input-output linkages and/or low substitutability between inputs. Moro (2012), Moro (2015), Koren and Tenreyro (2007), and Carvalho and Gabaix (2013) emphasize the role of sectoral composition. This paper is the first to provide an empirical and a theoretical link between production network density, service share, and volatility. Within this literature, this paper illustrates the importance of deviating from homogeneous and Cobb-Douglas sectoral technologies and the relevance of taking into account the production complexity of using more types of intermediates

In the literature of production diversification and volatility, di Giovanni and Levchenko

(2012) provide cross-country evidence and a theory to illustrate the role of international trade and country size in determining macroeconomic volatility. On the other hand, Koren and Tenreyro (2013) emphasize the role of production diversification via the use of more intermediate varieties. Different from the models in these studies, the model in this paper displays input-output linkages and heterogeneous sectoral elasticities in production.

A series of recent paper have studied the relationship between the structure of inter-sectoral linkages and macroeconomic outcomes such as a country's aggregate productivity and income level, as in Jones (2011), Bartelme and Gorodnichenko (2015), and Fadinger et al. (2015). The key difference between those studies and this paper is that the focus here is on business cycles instead of income level.

Finally, the relationship between production complexity—through higher network density—and the macroeconomy provided in this paper is related to those proposed by Costinot et al. (2013) and Jaimovich et al. (2017). The former study postulates that lengthier and more complex production processes require more inputs to produce a given level of output, while the latter documents that higher-quality products require a higher labor share.

2 Empirical facts

In this section, I study the empirical relationship between countries' production structure and the HHI of sales shares, as well as the empirical relationship between countries' production structure and macroeconomic volatility. I start by describing the data and then estimate cross-sectional and panel regressions while controlling for several country characteristics.

2.1 Data

The variables of interest are i) the HHI of sectoral sales shares and ii) the volatility of growth. I measure the HHI using OECD data on sectoral output for each country. I use countries' real GDP at constant national prices in millions of 2011 US$ dollars from the Penn World Tables 9.0 (rgdpna). To characterize the production structure, I use input-output matrix data from the OECD input-output database.[1] For each country, I have information for 33 sectors of the economy for the period 1995-2011. The sample retains the countries in the OECD database for which I have real GDP data since 1983, resulting in a final sample of 48 countries—25 with developed economies and 23 with emerging economies

I use the input-output data to generate three main production network measures: a measure of the extensive margin of connections (network density); a measure of the sectoral

[1] In particular, I collect the Input-Output Tables ISIC Rev. 3 available at `http://www.oecd.org/sti/ind/input-outputtables.htm`.

asymmetry in supplying intermediate inputs (outdegrees); and a measure of the asymmetry in the use of intermediates (indegrees).

To control for other country characteristics used by previous studies, I collect data from the IMF World Economic Outlook. These characteristics are the size of the country in the world economy and openness to trade (di Giovanni and Levchenko (2012)); the average growth of a country (Ramey and Ramey (1995)); the share of service sectors' gross output over total gross output (Moro (2015)); the size of the financial sector in the economy (Srivisal and Dabla-Norris (2013)), the size of the most volatile sectors (Koren and Tenreyro (2007)); and the average intermediate input share of the economy (Jones (2011)).[2]

2.2 Heterogeneity in input-output structure

Let Ω represent the input-output matrix of the economy. An element ω_{ij} is an input-output share that represents the importance of intermediate inputs shipped from sector i to sector's j total expenditure on intermediates. These input-output shares are not symmetric; it might be the case, for example, that sector i provides inputs to sector j, but sector j does not supply intermediates to sector i. The column i sum of Ω equals 1, as total intermediate expenses must be allocated to some (all) sector(s) of the economy. The row j sum represents how important sector j is as an intermediate input supplier to the economy as a share of other sectors' intermediate expenditures.

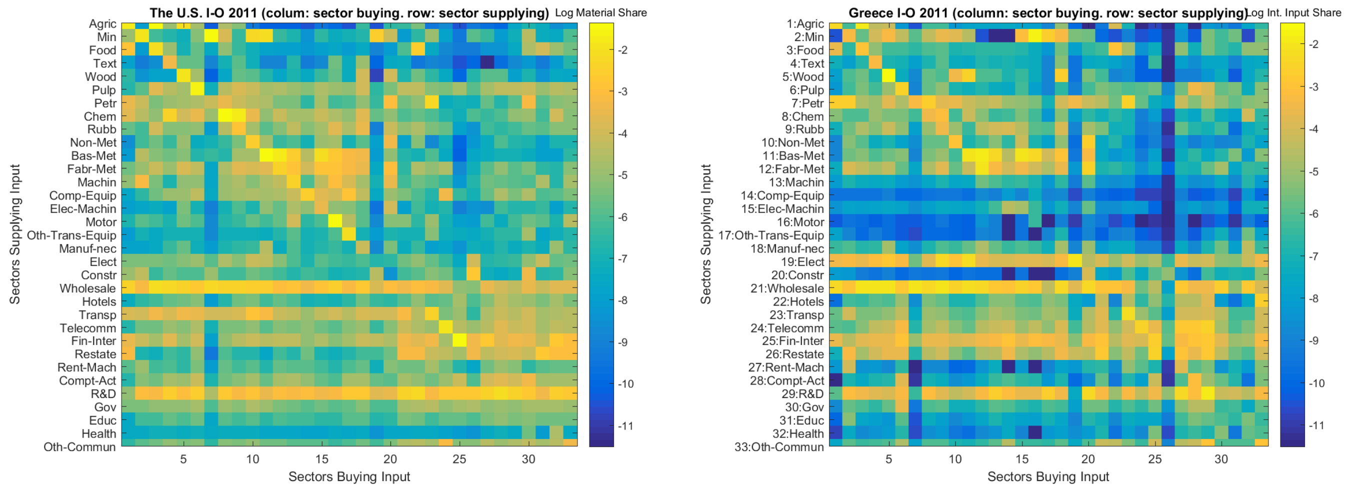
Figure 1 illustrates the input-output structures of the U.S and Greece. Each row represents an industry supplying intermediates of production, while each row represents industries using intermediates to produce final output. I report the logarithm of industries' intermediate input shares as a fraction of total output. Bright (yellow) colors represent high shares, while dark (blue) colors are small shares. The production network of the U.S. is such that most sectors are interconnected (panel a), while in Greece (panel b), sectors are more isolated. In particular, the U.S. has a higher number (264) of non-zero input-output shares than Greece, which is equivalent to setting seven entire column—out of 33—of the U.S. input-output matrix to be almost zero.[3]Regarding the intensity of connections, the U.S. and Greece have few sectors that supply intermediate inputs to all other sectors. This is expressed by few very bright rows of the input-output matrix in Figure 1.

The production network literature emphasizes the role of sectors' supplier (outdegree) importance (Horvath (1998) and Acemoglu et al. (2012)) as a network feature that amplifies and propagates sectoral shocks. Carvalho et al. (2016) underline the role of sectors'

[2]I also considered other country controls such as financial openness and rule of law. These controls do not alter any of the results presented later.

[3]Throughout the paper, and to avoid counting spurious connections due to data harmonization, I define non-zero input-output connections as shares of total output that are larger than a small threshold. The benchmark threshold is 0.0005. The results are robust to values of the threshold between 0.0005 and 0.005.

Figure 1
Production network of the U.S. and Greece



intermediate input shares (indegrees). On the other hand, the financial network literature emphasizes the role of production network density. Acemoglu et al. (2015b) find that networks with several active links (dense networks) can propagate large shocks more strongly but can also better mitigate small shocks. Thus, in my empirical analysis, I consider different features of the production network.[4] Specifically, I consider:

- Network density: ratio of positive input-output connections to total potential connections.
- Asymmetry in sectors' supplier importance (outdegree): I consider the skewness or variance of the sectoral row sum and sectoral dominance from Acemoglu et al. (2016).
- Asymmetry in sectors' use of intermediates (indegree): skewness or variance of sectoral column sum (intermediate input-share in production).

Figure 2, panel a depicts the histogram of the network density for the 48 countries in the sample using the input-output matrices in 2011.[5] There is substantial heterogeneity in the extent to which sectors interact in different countries. For example, emerging economies such as Brunei, Mexico, and Costa Rica have less than 60% of non-zero intermediate-input

[4]More details on the network measures can be found in Appendix B.

[5]In order to count positive input-output connections, I count input-output shares – as a fraction of total output – that are larger or equal to a threshold value between 0.05% and 0.5%. The threshold value aims to avoid counting potentially spurious connections due to data harmonization. Using the input-output matrices of previous years – and different small values for the threshold – yields very similar results (see robustness checks in Appendix A).

shares. In the developed world, Ireland, Denmark, and Greece have less than 70% of non-zero intermediate input shares. The opposite holds for China, Poland, Spain, and the U.S., where more than an 85% of the intermediate input shares are positive.

Figure 2
Cross-country distribution of network density and outdegrees

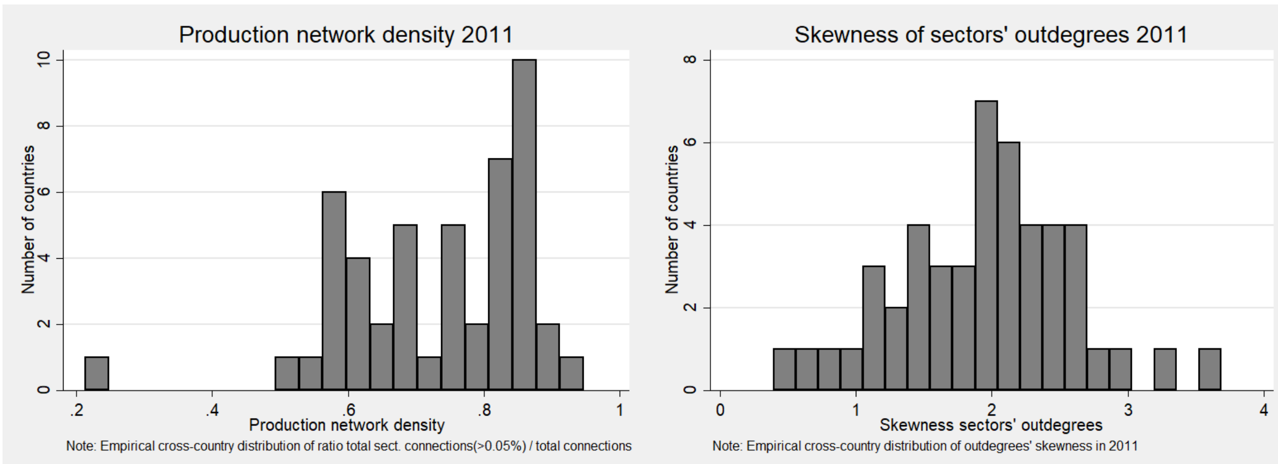


Figure 2, panel b depicts the skewness of sectors' outdegrees. The data show a global right tail in the distribution of sectoral outdegrees, evidenced by the fact that all countries have a positive skewness of outdegrees.[6] There is also significant cross-country heterogeneity in the asymmetry of input supplier importance. For instance, China has a relatively symmetric distribution of outdegrees, while in Luxembourg and Singapore two sectors are the main intermediate input providers of the economy.

Regarding the distribution of indegrees, the international evidence also shows substantial heterogeneity (Figure 9 in Appendix A). Countries such as Belgium and Italy have a negative skewness in the distribution indegrees, meaning a mass of sectors with intermediate input shares around the mean and few sectors with disproportionately small intermediate input shares. Other countries, such as Singapore and Bulgaria, have few sectors that use a disproportionately high intermediate input share in production (star input customers).

2.3 Sales shares and the production structure

I now estimate the conditional correlation between the HHI of sectoral sales and the production network measures described in Section 2.2, controlling for other determinants of sales shares used in the literature, such as the service share (Moro (2015)), the share of the

[6] Dungey and Volkov (2018) use the same OECD input-output data and document that the wholesale and R&D sectors are dominant in most countries in the sample.

country in world GDP, and the country's exposure to international trade (di Giovanni and Levchenko (2012)).

I estimate the following regression:

$$\log(HHI_T^k) = \mathbf{Z'_T}\beta + \mathbf{X'_T}\gamma + \epsilon_{T,k}, \tag{1}$$

where HHI_T^k represents the HHI of sectoral sales shares for country k in period T.[7] The vector $\mathbf{Z_T}$ contains the production network measures—density, indegrees, outdegrees—for every country k and period T. The vector $\mathbf{X_T}$ contains the additional cross-country controls used in the literature. I divide the sample into three sub-periods: 1984-1995, 1996-2005, and 2006-2014. Given that the OECD input-output data cover the period 1995-2011, I use the end of period HHI and production network measures. To be consistent, I use the end-of-period value of the control variables.

Table 1 illustrates the results of estimating Equation (1). There is a strong negative relationship between the production network density and the HHI of sectoral sales shares. To have an idea of the economic significance of the result, a one-standard-deviation increase in network density—e.g., an increase in density from 0.6 to 0.73—would decrease the HHI of India ($HHI = 0.23$) to the level of Germany ($HHI = 0.18$). The result is robust to controlling for production structure measures highlighted by Acemoglu et al. (2012), Carvalho et al. (2016), Jones (2011) and Moro (2015) (columns 2 and 3). Indeed, the production structure measures in column 3 account for a third of the observed heterogeneity in countries' HHI of sales shares. In columns 4 and 5, I show that the results are robust to controlling for trade openness and country size (di Giovanni and Levchenko (2012)), and robust to adding country fixed-effects and country-specific time trends.[8]

[7]For each country k and end of period T, I calculate $HH_T^k = \sqrt{\sum_{j=1}^{N} \left(\frac{s_{j,T}^k}{GDP_T^k}\right)^2}$, where $s_{j,T}^k$ are sector j's sales at time T in country k.

[8]The same results hold when adding additional controls, such as average growth rate, the size of the financial sector, and the size of the agriculture and mining sectors.

Table 1
Herfindahl, Density, and Service Share

VARIABLES	(1) $\log HHI_T^k$	(2) $\log HHI_T^k$	(3) $\log HHI_T^k$	(4) $\log HHI_T^k$	(5) $\log HHI_T^k$
log density	-0.894***	-0.828***	-1.039***	-0.829**	-0.776**
	(0.154)	(0.208)	(0.212)	(0.317)	(0.320)
log service		-0.306	-0.400*	-0.434**	-0.501***
		(0.275)	(0.227)	(0.195)	(0.184)
log av. int. share			0.912***	1.657***	1.570***
			(0.276)	(0.520)	(0.493)
outdegrees			0.100	0.003	-0.011
			(0.066)	(0.066)	(0.063)
indegrees			0.209***	0.060	0.066
			(0.080)	(0.066)	(0.063)
log trade/GDP				0.399***	0.269**
				(0.088)	(0.125)
log country size				-0.125	-0.075
				(0.095)	(0.103)
Observations	144	144	144	144	144
Adjusted R-squared	0.228	0.242	0.323	0.514	0.908
Country FE	No	No	No	Yes	No
Country Trend	No	No	No	No	Yes

Note: This table presents an OLS regression using the log Herfindahl index of sales shares as the dependent variable. The sample is divided into three sub-periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors (clustered at the country level) in parentheses.

2.4 Volatility and the production structure

The previous results illustrate a new feature of the production network affecting countries' sales shares: network density. In this section, I confirm that network density is also strongly related to countries' macroeconomic volatility. I also document an additional result that complements the findings in Table 1: the service share of the economy strengthens the relationship between density and volatility. I run the following regression:

$$\log(\sigma_T^k) = \bar{\mathbf{Z}}'_{\mathbf{T}}\beta + \bar{\mathbf{X}}'_{\mathbf{T}}\gamma + \bar{\epsilon}_{T,k}, \tag{2}$$

where σ_T^k is the standard deviation of real GDP growth for country k in period T. The vector $\bar{\mathbf{Z}}_\mathbf{T}$ contains the same production network measures considered in Equation (1), except that I add the interaction between the network density and the service share. The vector $\bar{\mathbf{X}}_\mathbf{T}$ expands the set of cross-country controls $\mathbf{X}_\mathbf{T}$ to include determinants of volatility previously used: the average growth of a country; the size of the financial sector in the economy; and the size of agriculture and mining sectors in the economy.[9]

Table 2 presents the results of estimating Equation (2). There is a strong negative relationship between the production network density and GDP growth volatility. A 10% increase in network density is associated with a 7.2% decrease in aggregate volatility. This result implies that a one-standard-deviation increase in network density (a 22% increase in density) decreases GDP volatility by between 0.2 to 0.5 percentage points, depending on the country's current volatility.[10] The network density by itself accounts for 8% ($R^2 = 0.08$) of the variation in cross-country volatility.

The result is robust to controlling for countries' service share (column 2). Interestingly, as shown in column 3, the density-volatility relationship interacts with the service share. In particular, in high-service-share countries, the network density has a stronger—i.e., more negative—effect on volatility. The network density and the service share alone account for a 25% of the observed variation in cross-country volatility. These results are robust to including other production network measures and to adding additional country controls (column 4). Columns 5 and 6 show that the results are also robust to adding countries fixed-effects and country-specific time trends, respectively.

When I include country-specific time trends (column 6), only the coefficient of the network density is statistically significant, at the 90% confidence level. As I will discuss later in the paper, once the HHI of sales shares is added as an additional regressor in column 6 (see Table 4), the relationship between network density/service share and volatility becomes more precise and stronger. This result is interesting, as it suggests that the input-output structure affects volatility beyond sales shares. The last section of the paper discusses this result.

In Appendix A, Tables 5-8, I provide several robustness checks. These rule out that the results in Table 2 are driven by the threshold to define network density (Tables 5 and 6), by outlier observations (Table 7), and by whether I construct network density using domestic input-output tables or total input-output tables, which include imported intermediates (Table 8).

Overall, the facts presented in Sections 2.3 and 2.4 contribute to understanding the

[9]In non-reported results, I also considered other country controls, such as financial openness and rule of law. These controls do not alter any of the results presented later.

[10]The sample average of GDP volatility is 2.87%. The 25th percentile of volatility is 1.75%, while the 75th percentile is 3.71%.

drivers of countries' HHI of sales shares and macroeconomic volatility. These facts also pose a challenge to existing multisector models with intersectoral linkages. The existing literature predicts no relationship between production network density and aggregate fluctuations (Dupor (1999), Acemoglu et al. (2012), Baqaee and Farhi (2017)). Nor do existing models predict either that the density-volatility relationship interacts with the service share of the economy. Therefore, the next section aims to give economic interpretation to the cross-country facts presented in this section.

Table 2
Volatility, Network Density and Service Share

VARIABLES	(1) $\log \sigma_T^k$	(2) $\log \sigma_T^k$	(3) $\log \sigma_T^k$	(4) $\log \sigma_T^k$	(5) $\log \sigma_T^k$	(6) $\log \sigma_T^k$
log density	-0.721***	-0.567**	-2.483***	-2.470***	-2.782***	-1.508*
	(0.246)	(0.277)	(0.422)	(0.515)	(0.903)	(0.894)
log service		-0.720***	-2.088***	-2.976***	-1.965*	0.370
		(0.230)	(0.374)	(0.405)	(1.149)	(1.081)
log density · log service			-3.032***	-3.746***	-4.058***	-1.613
			(0.559)	(0.496)	(1.041)	(1.089)
Observations	144	144	144	144	144	144
Adjusted R-squared	0.080	0.134	0.254	0.356	0.268	0.487
Controls	No	No	No	Yes	Yes	Yes
Country FE	No	No	No	No	Yes	No
Country Trend	No	No	No	No	No	Yes

Note: This table presents an OLS regression using real GDP growth as the dependent variable and the following set of time-varying country controls: trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, size of the financial sector, and the share of agriculture and mining sectors in gross output. The sample is divided into three sub-periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors (clustered at the country level) in parentheses.

3 The model economy

The model economy in this paper displays intersectoral linkages and general CES technologies, as in Atalay (2017) and Baqaee and Farhi (2017). Different from the aforementioned studies, and in order to match the empirical facts documented in Section 2, the model in this paper incorporates two elements of sectoral production technologies: i) higher flexibility in the production of service sectors; and ii) a *cost of complexity* in the bundle of intermediates.

Firms

There are N sectors, each of which has a continuum of homogeneous firms that behave competitively. The constant returns-to-scale technology of firms in sector j is:

$$Q_j = Z_j \Big[a_j^{\varrho_Q} L_j^{\frac{\epsilon_{Q,j}-1}{\epsilon_{Q,j}}} + (1-a_j)^{\varrho_Q} M_j^{\frac{\epsilon_{Q,j}-1}{\epsilon_{Q,j}}} \Big]^{\frac{\epsilon_{Q,j}}{\epsilon_{Q,j}-1}}, \tag{3}$$

where the intermediate input bundle is

$$M_j = \Big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M} M_{ij}^{\frac{\epsilon_{M_j}-1}{\epsilon_{M_j}}} \Big)^{\frac{\epsilon_{M_j}}{\epsilon_{M_j}-1}}. \tag{4}$$

The output of the representative firm in sector j is denoted by Q_j. Z_j is total factor productivity; L_j is labor; M_j is the intermediate input bundle of sector j, and M_{ij} is the amount of intermediates that sector j purchases from sector i. The parameter a_j represents how important labor is in the total value of production.

The element ω_{ij} reflects the importance of sector i as an input supplier to sector j, as a fraction of sector j's total intermediate input cost. Therefore, the square matrix Ω—of dimension N and typical element ω_{ij}—represents the input-output structure of the economy.

I assume constant returns to scale in the intermediate input bundle technology, $\sum_{i=1}^{N} \omega_{ij} = 1$. The sectoral elasticity of substitution between labor and intermediates is denoted by $\epsilon_{Q,j}$. The elasticity of substitution among material varieties is $\epsilon_{M,j}$. A key difference from Baqaee and Farhi (2017) and Atalay (2017) is that elasticities can be sector-specific and are also allowed to be larger than one. In the theoretical analysis, for tractability, I assume that elasticities are common across sectors. However, in the quantitative section I allow them to be different across industries.

Another difference with respect to previous studies is the treatment of ϱ_M. The parameter ϱ_M is what determines whether using more or fewer intermediate inputs in production affects the intermediate bundle productivity. Thus, the parameter ϱ_M governs what I call the *cost of complexity*. At the industry level, it is not that a firm will use different types of intermediate input varieties from the same industry (e.g., varieties of glass), but that it will use different type of goods (e.g., glass and metal). Therefore, adding different intermediates will make the production process more complex. Without increasing labor or capital to handle the new production process, all else equal, total output declines within a period.

This production technology can be seen as a reduced form for the production complexity in Costinot et al. (2013). The authors study a model in which more complex production entails lengthy production processes that require more operations to produce a given output. In this model, a *cost of complexity* in the bundle of intermediates exists as long as $\varrho_M < \frac{1}{\epsilon_M}$ ($\epsilon_M < 1$) or $\varrho_M > \frac{1}{\epsilon_M}$ ($\epsilon_M > 1$). The benchmark assumption of $\varrho_M = 1$ ensures the existence of a *cost of complexity*, when $\epsilon_M > 1$ and when $\epsilon_M < 1$.[11]

Households

The representative household maximizes utility

$$U(C,L) = \frac{C^{1-\varsigma}-1}{1-\varsigma} - \psi\frac{L^{1+\eta}}{1+\eta}, \tag{5}$$

subject to

$$wL + \sum_{j=1}^{N} \pi_j = P_c C \tag{6}$$

$$C = \sum_{j=1}^{N} \left(\beta_j^{\frac{1}{\epsilon_D}} C_j^{\frac{\epsilon_D - 1}{\epsilon_D}}\right)^{\frac{\epsilon_D}{\epsilon_D - 1}},$$

where C is the consumption bundle and L is total labor supply. $\varsigma \geq 0$ is, for a given wage, the income elasticity of labor supply, and $\eta > 0$ is the inverse Frish elasticity of labor supply. The parameter ψ is 0 for the tractable case of inelastic labor supply and is 1 when labor is elastically supplied. In the household budget constraint, π_j is profit from firms in sector j (in equilibrium $\pi_j = 0$ for all j); w is the wage rate; and P_c is the ideal price index of the consumption bundle C. The consumption shares β_j satisfy $\sum_{j=1}^{N} \beta_j = 1$. The elasticity of substitution between goods from different sectors is ϵ_D.

Definition 1 *(Competitive Equilibrium) A decentralized competitive equilibrium is a set of prices* $\{w, (P_j, P_j^M)_{j=1}^N\}$ *and allocations* $\{C, L, (C_j, Q_j, L_j)_j^N\}$, $\{(M_{ij})_{ij}^N\}$ *such that, for a given vector of sectoral productivity shocks* $\{Z_j\}_{j=1}^N$ *and prices:*

- *the representative consumer maximizes utility* (5) *subject to the budget constraint* (6)*;*
- *firms maximize profits; and*
- *the goods and labor markets clear:*

$$Q_j = C_j + \sum_{i=1}^{N} M_{ji},$$

[11]Note that multisector (or multifirm) models using similar CES technologies—$\epsilon_M \neq 1$ and $\varrho_M = 1$—are Horvath (2000), Koren and Tenreyro (2013), Baqaee and Farhi (2017), and Acemoglu and Azar (2017). Other studies that also assume the existence of *cost of complexity* ($\varrho_M = 1$), but assume $\epsilon_M = 1$ and $\epsilon_Q = 1$, are Acemoglu et al. (2012) and Foerster et al. (2011). The previous studies do not focus on understanding the role of network density in such an environment.

$$L = \sum_{j=1}^{N} L_j.$$

Notation

Let β, Z, and a be the $N \times 1$ vector of consumption shares, the $N \times 1$ vector of sectoral productivities, and the $N \times 1$ vector containing the importance of labor in each sector's technology, respectively. An expression $e \circ f$, where e and f are vectors of the same dimension, should be interpreted as an element-by-element multiplication (Hadamard product). An expression e^f should be interpreted as an element-wise exponent. Finally, an expression $g \circ h$, where g is a vector and h is an N by N matrix should be interpreted as the multiplication of the element i of vector g with every element in row i of matrix h (same as diagonal(g) $\times$ h).

3.1 The network amplification

I study the general model's implications for the relationship between the network structure—the intensive and extensive margin of connections—and aggregate GDP. For tractability, I assume that sectoral elasticities are common across sectors.[12] The next proposition summarizes the main implications of the model.

Proposition 1 *Assume that $\epsilon_Q = \epsilon_M$, $\varrho_Q = \varrho_M$, $\epsilon_D = 1$, $\varsigma = 1$, $\psi = 0$, and a labor endowment $\bar{L} = 1$.*[13] *The GDP elasticity to a shock in sector j is:*

$$\frac{\partial \log GDP}{\partial \log Z_j} = \frac{P_j Q_j}{GDP} = \left(\beta' [I - \xi^{1-\epsilon_Q} \circ Z^{(\epsilon_Q - 1)} \circ \tilde{\Gamma}']^{-1}\right)_j,$$

where $\Gamma = (1-a)' \circ \Omega$, $\tilde{\Gamma} = \Gamma^{\varrho_Q \epsilon_Q}$, *and* $\xi_i = \dfrac{\left([I - Z'^{(\epsilon_Q - 1)} \circ \tilde{\Gamma}]^{-1} (Z^{\epsilon_Q - 1} \circ \tilde{a})\right)_j}{\left([I - Z'^{\epsilon_Q - 1} \circ \tilde{\Gamma}]^{-1} (Z^{\epsilon_Q - 1} \circ \tilde{a})\right)_i}$, *and* $\tilde{a}_j = a_j^{\varrho_Q \epsilon_Q}$.

Proof: See Appendix C.

The results in Proposition 1 generalize the network centrality in Acemoglu et al. (2012) by introducing non-unitary elasticities and a cost of complexity in the bundle of intermediates. As in Hulten (1978), the aggregate effect of a sectoral shock depends on the sector's sales-to-GDP ratio. Similarly, equilibrium sales shares depend on how interconnected sector j is to the rest of the economy (Acemoglu et al. (2012)). The network centrality of a sector is given by the Leontief inverse elements, which depend on sectors' input-output connections, as well as on sectoral elasticities and the cost of complexity parameter.

[12]This is, $\epsilon_{Q,j} = \epsilon_Q$ and $\epsilon_{M,j} = \epsilon_M$ for all j. In the general calibration of the model I allow $\epsilon_{Q,j} \neq \epsilon_{M,j}$.

[13]This parametrization of the utility is convenient to obtain closed form solutions. In the simulations of the next section I show that the main results still hold with an upward sloping labor supply

When $\epsilon_Q = \epsilon_M = \varrho_M = \varrho_Q = 1$, the network centrality reduces to $\beta'[I - \Gamma']^{-1}_j$, as in Acemoglu et al. (2012). The same statistic describes the effect of sectoral shocks when $\epsilon_Q \neq 1$, $\varrho_Q = \varrho_M = \frac{1}{\epsilon_Q} = \frac{1}{\epsilon_M}$, and the steady state productivity is $Z_j = 1$ for all j. In these cases, as Dupor (1999) and Acemoglu et al. (2012) show, the network density is irrelevant, and the model is not able to rationalize the facts in Section 2. I now show that the generalized model does predict a role for network density.

3.1.1 The role of network density

It is instructive to study the implications of Proposition 1 with two common networks that differ only in terms of the network density: the *sparse* network, in which each sector uses (provides) intermediate inputs only from (to) one other sector; and the *dense* network, in which every sector is connected to every other sector in the same intensity.

$$\Omega^{sparse} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \Omega^{dense} = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \end{bmatrix}.$$

In both economies, labor and intermediates have the same importance in the sectoral production, implying that $a_j^{sparse} = a_j^{dense} = a$ for all j. Also, to highlight the role of the cost of complexity in the presence of non-unitary elasticities of substitution, I assume that $\varrho_M = 1$. The GDP elasticity in this case is:

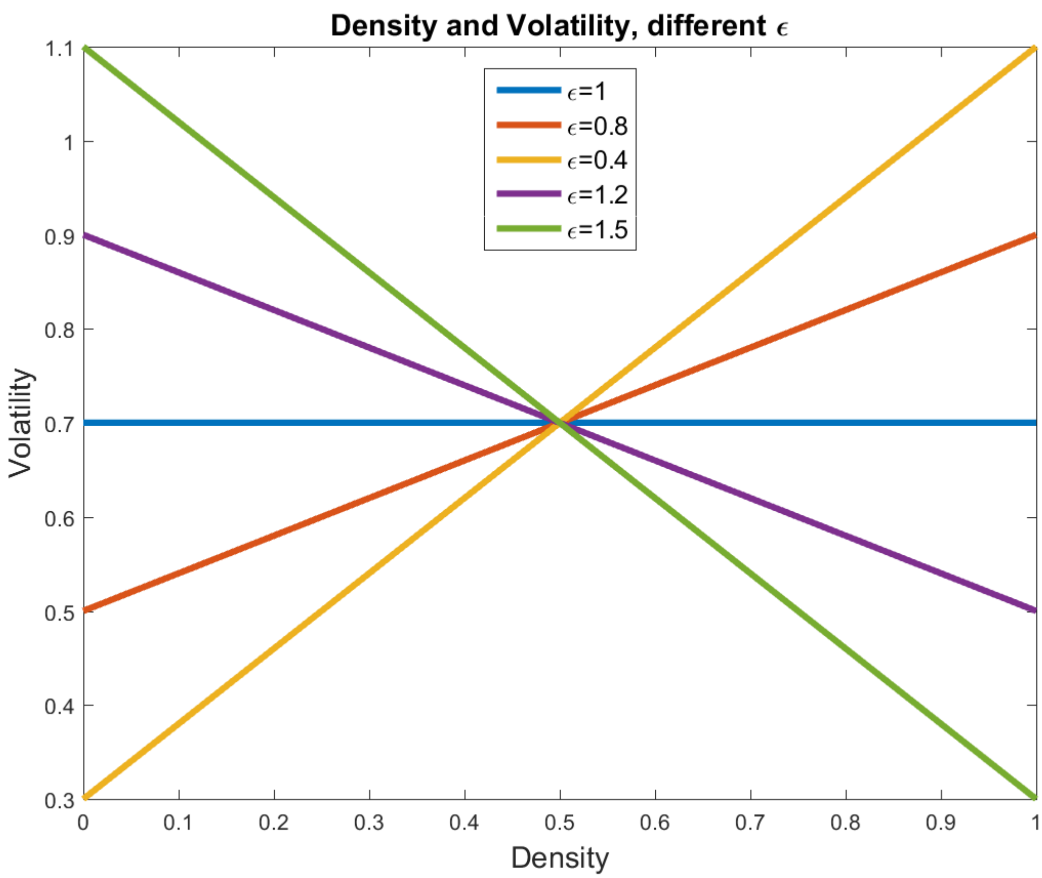
$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \Big(\beta'[I - Z^{\epsilon_Q - 1} \circ (1-a)^{\epsilon_Q} \circ \Omega^{'}]^{-1}\Big)_j = \Big(\frac{P_j Q_j}{GDP}\Big)^{sparse}$$

$$\frac{\partial \log GDP^{dense}}{\partial \log Z_j} = \Big(\beta'[I - N^{1-\epsilon_Q} Z^{\epsilon_Q - 1} \circ (1-a)^{\epsilon_Q} \circ \Omega^{'}]^{-1}\Big)_j = \Big(\frac{P_j Q_j}{GDP}\Big)^{dense}$$

A first result is that when $\epsilon_Q = 1$, the network density is irrelevant. As in Dupor (1999) and Acemoglu et al. (2012), even with an extensive margin of connections that is somewhat different in these two networks, the two economies are equally stable. The empirically relevant case for this paper arises when $\epsilon_Q \neq 1$, and a cost of complexity exists ($\varrho_M = 1$). With non-unitary elasticities of substitution, and a cost of complexity in the bundle of intermediates, the dense network displays an extra *diversification* term $N^{1-\epsilon_Q}$. In particular, when $\epsilon_Q < 1$, shocks have larger effects in the dense network, and the opposite applies when $\epsilon_Q > 1$.

Before formally studying the implications for GDP volatility, in Figure 3, I report numerical simulations to illustrate the role of network density as a function of production flexibility. I assume that $N = 6$ and, for a given value of β, a, I simulate series of sectoral productivity for networks with different densities and elasticities ϵ_Q. In this simulation, and also in the cross-country calibration of the next section, I demonstrate that the key elasticity is ϵ_Q rather than ϵ_M. I set $\epsilon_M = 1$, $\varrho_M = 1$ and vary the value of ϵ_Q. When $\epsilon_Q > 1$, a denser production network displays lower GDP volatility, and this effect is stronger—i.e., a steeper line—the larger is the elasticity. The opposite occurs when $\epsilon_Q < 1$. A denser network is more volatile, and the effect is stronger the smaller the elasticity. With Cobb-Douglas production technologies, the network density is irrelevant for volatility.

Figure 3
Density, Flexibility and Volatility



Thus, the general network statistic in Proposition 1 is useful for understanding not only the cross-country correlation between density and volatility, but also why the density-volatility relationship interacts with the service share. In particular, if service sectors had higher flexibility in production, the model would explain why the density-volatility relationship is stronger in countries with a higher service share. In the next section, using U.S data, I show that, indeed, service industries have substantially higher flexibility in production.

I now formally study the relationship among the production network structure, the HHI of sectoral sales shares, and GDP volatility, emphasizing the role of network density. Assume that sectoral productivity follows a random walk:

$$\log Z_{jt} = \log Z_{jt-1} + \kappa_{jt} \tag{7}$$

where the productivity shock κ is normally distributed with mean zero and standard deviation σ_j.

Proposition 2 *Define σ_{GDP} the volatility of the log change of real GDP. Assume that $\epsilon_Q = \epsilon_M$, $\varrho_Q = \varrho_M$, $\epsilon_D = 1$, $\varsigma = 1$, $\psi = 0$, and a labor endowment $\bar{L} = 1$. Assume that sectoral productivity shocks κ_{jt} are independent and have a volatility of σ_j. Then, up to a first order, the volatility of the log change of GDP is:*

$$\sigma_{GDP} = \sqrt{\sum_{j=1}^{N} \Big(\frac{P_j Q_j}{GDP}\Big)^2 \sigma_j^2} = \sqrt{\sum_{j=1}^{N} \Big(\beta'[I - \xi^{1-\epsilon_Q} \circ Z^{\epsilon_Q - 1} \circ \tilde{\Gamma}']^{-1}\Big)_j^2 \sigma_j^2}.$$

If we further assume that $\sigma_j = \sigma$, $a = a_j$, $\varrho_M = 1$, and $\beta = \beta_j$ for all j, the dense and the sparse networks have the following aggregate volatility:

- $\sigma_{GDP}^{dense} = \frac{\sigma\sqrt{\beta}}{(1 - N^{1-\epsilon_Q}(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})} > \sigma_{GDP}^{sparse} = \frac{\sigma\sqrt{\beta}}{(1-(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})}$ *if* $\epsilon_Q < 1$

- $\sigma_{GDP}^{dense} = \frac{\sigma\sqrt{\beta}}{(1 - N^{1-\epsilon_Q}(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})} < \sigma_{GDP}^{sparse} = \frac{\sigma\sqrt{\beta}}{(1-(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})}$ *if* $\epsilon_Q > 1$

Proof: See Appendix C.

Proposition 2 generalizes the results in Acemoglu et al. (2012) to account for the roles of density and elasticity. GDP volatility is determined by the HHI index of sectoral sales shares, which, in turn, depends on the production network structure of the economy. To better illustrate the effect of network density on volatility, Proposition 2 also reports the implied volatility of the dense and sparse networks. These two networks have perfectly symmetric distributions of sectoral sales, but, as shown in Proposition 2, they can display rather different aggregate volatilities from idiosyncratic productivity shocks, depending on the degree of complementarity between labor and intermediates. When $\epsilon_Q \neq 1$, the details of how sectors are connected—in particular, the density of connections—play an important role in shaping the aggregate effect of shocks. Large complementarities in production imply a dense network that is much more volatile than the sparse network.

It is important to note that, while the result in Proposition 2 holds for independent productivity shocks, the same holds for correlated shocks. More generally, the relevant of network density is not restricted to a world with purely idiosyncratic shocks.

3.2 The mechanism

The results in Propositions 1 and 2 are a direct implication of the interaction among non-unitary elasticities of substitution, network density, and the cost of complexity in the bundle of intermediates. In the mix of intermediate inputs, there is a trade off between producing with several intermediate types, with each type having a small share ω_{ij}, versus using few intermediate types, with each type representing a large share of the total intermediate bundle.

Several previous studies assume that $\varrho_M = \varrho_Q = \frac{1}{\epsilon_M} = \frac{1}{\epsilon_Q}$. In this case, producing with three types of intermediate inputs ($i = 1,2,3$)—where each one is $M_{ij} = \frac{1}{3}$ and has weights $\omega_{ij} = \frac{1}{3}$—versus producing with one input ($i = k$)—where $M_{kj} = 1$ and $\omega_{kj} = 1$—yields exactly the same output. On the other hand, when $\varrho_M < \frac{1}{\epsilon_M}$ ($\epsilon_M < 1$) or $\varrho_M > \frac{1}{\epsilon_M}$ ($\epsilon_M > 1$), there is a *cost of complexity* in which the complex bundle, all else equal, produces less output. The benchmark assumption of $\varrho_M = \varrho_Q = 1$ ensures the existence of the *cost of complexity*, when $\epsilon_Q > 1$ and when $\epsilon_Q < 1$.[14] This specification of production technologies shares the idea in Costinot et al. (2013). Producing with more intermediates requires more operations or tasks in order to produce a given level of output.

Therefore, a denser production network is also a network that produces more complex goods. When firms use the bundle of intermediates and labor as gross substitutes ($\epsilon_Q > 1$), firms in a denser network are able to use more labor to compensate for the more complex production process. Hence, a denser network displays smaller sectors with a lower intermediate input share, which then mitigates the effect of shocks along the production chain. When firms have low production flexibility ($\epsilon_Q < 1$), in equilibrium, a denser network displays larger intermediate shares and larger sectors, which then amplify the effect of sectoral shocks and increase volatility.

4 Quantitative assessment

In this section, I calibrate the model using each country's input-output matrix. In addition, I use U.S sectoral data to estimate sectoral elasticities of substitution. The goal is to study the quantitative predictions of the model regarding the empirical cross-country correlations among density, service share, and volatility documented in Section 2. Note that the goal here is not to disentangle the role of sectoral shocks versus aggregate shocks in generating the empirical correlations between the production network structure and volatility. I address this concern for the U.S economy in the next section.

[14] That is not to say that adding more varieties necessarily reduces productivity. If we change the example to have $M_{ij} = 1$ ($\forall$ i and j) in the complex bundle, which is a standard assumption in the literature on gains from input variety (Koren and Tenreyro (2013)), the more diversified bundle will produce more, as long as $\varrho_M > \frac{1}{\epsilon_M}$ ($\epsilon_M > 1$).

Estimation of elasticities

A key parameter for studying the role of network density is the elasticity of substitution between labor and intermediates. To estimate the elasticities, I follow Atalay (2017) and use the model's implied cost minimization condition for inputs. As in Miranda-Pinto and Young (2019), I allow for the elasticities to differ across sectors. However, instead of estimating elasticities for every industry—and guided by the empirical facts in Section 2—I separate service industries from non-service industries. Another reason to group sectors is that the industry classification used to estimate elasticities in the U.S is different from the industry classification in the OECD input-output tables. The empirical counterpart of the firms' cost minimization condition is:

$$\Delta \log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right) = \gamma_{ij} + \phi_t + \alpha\Delta\log\left(\frac{P^M_{jt}}{P_{it}}\right) + \beta\Delta\log\left(\frac{P_{jt}}{P^M_{jt}}\right) + \nu_{ijt}, \tag{8}$$

where γ_{ij} and ϕ_t are buyer-seller and time fixed effects, respectively. Buyer-seller fixed effects aim to control for unobserved time-invariant intermediate-input trade partner relationships. The error term is denoted by ν_{ijt}. Estimating Equation (8) via OLS would yield biased coefficients due to endogeneity problems. Sectoral productivities (Z_j) are part of the error term ν_{ijt} and are correlated with sectoral prices (P_j, P^M_j).

Therefore, I use military spending as instruments for sectoral prices (see Atalay (2017) and Miranda-Pinto and Young (2019) for more details). Since the instruments are relevant to the United States, I assume that sectoral technologies in the U.S are similar to sectoral technologies elsewhere. I use BEA annual input-output data on sectoral prices and intermediate shares. The data contain 66 non-government sectors of the economy and cover the period 1997-2014.[15] Table 9 in Appendix A (from Miranda-Pinto and Young (2019)) contains detailed information on each sector's input shares and sales shares. Service sectors are sectors 5, 6, 27-66; other sectors are sectors 1-4, 7, and 24, while the rest are manufacturing industries.

Tables 10-12 in the Appendix report the estimated elasticities for three groups of sectors: manufacturing, service, and others. Other sectors include the agriculture, forestry, oil and gas, construction, and petroleum industries. Columns 1 and 2 present the OLS results with fixed effects. Columns 3 and 4 use military spending instruments, while columns 5 and 6 use military spending and lags of sectoral prices as instruments. Given that the weak instruments test is rejected in column 4 (for all sectors), I use the results from column 4 as the benchmark elasticity estimates.

The main take away from the estimation of sectoral elasticities is that service sectors

[15] For each sector, I keep the top 25 intermediate goods' supplier sectors. The results are similar when using the top 20 or 30 suppliers.

have higher elasticity of substitution than other sectors, including manufacturing sectors. In particular, service sectors consistently display $\epsilon_Q > 1$ ($\epsilon_Q \approx 6$), while the evidence on ϵ_M suggests that $\epsilon_M \geq 1$. For manufacturing sectors, the hypothesis of Cobb-Douglas production functions ($\epsilon_Q = \epsilon_M = 1$) cannot be rejected, while for other sectors, the evidence suggests that $\epsilon_Q > 1$ ($\epsilon_Q \approx 2$) and $\epsilon_M \approx 0$.

Calibration

I set sectoral elasticities for each country based on the U.S estimates. To highlight the role played by ϵ_Q, and also motivated by the sectoral estimates of ϵ_M for the manufacturing and service sectors, I set $\epsilon_M = 1 \ \forall \ j$ and k. To highlight the role of the service share, through production flexibility, I let ϵ_Q differ across sectors and be larger than one for service sectors. The benchmark calibration considers $\epsilon_Q = 1$ for non-service sectors and $\epsilon_Q \in [3, 6]$ for service sectors.

The process for sectoral productivity follows a random walk as in Equation (7), in which shocks κ_{jt} are independent and normally distributed with zero mean and variance σ_j^2. I calibrate the variance for sectoral productivity following Horvath (2000), who uses the Jorgenson dataset to estimate sectoral productivities for the U.S. sectors at a level of disaggregation of 36 sectors. The estimates apply to annual productivity; thus, they represent a good benchmark for my analysis, as the empirical results in Section 2 are at an annual frequency and at a level of disaggregation of 33 sectors. The persistence of sectoral productivities estimated in Horvath (2000) is high and does not differ much across sectors. However, the author finds important differences in the volatility of sectoral shocks σ_j across sectors, especially between manufacturing and services. The benchmark calibration assumes that $\sigma = 0.04$ for manufacturing sectors and $\sigma = 0.02$ for service sectors.[16] I assume the following parameters for households' utility $\varsigma = 1$, $\psi = 1$, and $\eta = 0$, implying that $U(C, L) = \log C - L$.

The countries' intermediates input shares ω_{ij} are matched to the observed shares in year 2005 using the OECD input-output tables at a level of disaggregation of 33 sectors. The consumption shares β_j are calibrated to be the observed consumption shares in the 2005 input-output tables for each country.[17] The importance of labor in sectoral production (a_j) is matched to be the value added (labor + capital) shares of output in 2005. Therefore, I set $a_j = 1 - \sum_{i=1}^{N} \frac{P_i M_{ij}}{P_j Q_j}$.

[16] Koren and Tenreyro (2007) find similar results for the volatility of sectoral shocks. Service sectors tend to be less volatile than manufacturing sectors.

[17] The assumption of unitary elasticity between intermediate varieties and consumption goods implies that ω_{ij} and β_j are exactly the corresponding observed shares in the data.

Model's simulated regression

I simulate series of sectoral productivities and aggregate GDP for every country in the sample. I then re-estimate the cross-sectional relationship between the model's implied GDP volatility, input-output structure, and the service share. In particular, I simulate series of size $T (= 50$ years) for each economy $S (= 500)$ times. With the implied path for real GDP, I calculate each country's times series of real GDP growth. I then calculate the standard deviation of growth for each economy and simulation. The countries' average volatility of GDP growth over the S simulations is the dependent variable.

The results of estimating Equation (2) using the model's implied volatility are in Table 3. I control for outdegrees , indegrees, average intermediate input share, and network density separately, as in the empirical section. The model is able to deliver the observed empirical patterns. When $\epsilon_Q = 6$ for service sectors, we observe that denser economies are less volatile and this is strengthened by the service share (column 2). The model's implied density-volatility relationship is of an order of magnitude similar to that in the data (column 1). The model's implied coefficient for density and the interaction between density and service share in column 2 are three quarters as big as the observed cross-sectional coefficients in column 1.

The third column of the table presents the model's implied regression with a relatively lower flexibility in service sectors ($\epsilon_Q = 3$). Similar results hold; however, due to the relatively lower production flexibility of service sectors, the coefficients are smaller. In column 4, I assume that service and non-service sectors have the same TFP volatility ($\sigma = 0.04$). Although, similar results hold for the density-volatility relationship, the service-volatility relationship weakens.

Finally, in column 5, I report the model's implied regression coefficients for Cobb-Douglas production technologies. As Proposition 2 predicts, under Cobb-Douglas technologies the model is unable to account for the international facts.

Table 3
Volatility, Network Density and Service Share Model

VARIABLES	(1) Data	(2) $\epsilon_Q^s = 6; \epsilon_Q^{ns} = 1$ $\sigma_s < \sigma_{ns}$	(3) $\epsilon_Q^s = 3; \epsilon_Q^{ns} = 1$ $\sigma_s < \sigma_{ns}$	(4) $\epsilon_Q^s = 3; \epsilon_Q^{ns} = 1$ $\sigma_s = \sigma_{ns}$	(5) $\epsilon_Q^s = 1; \epsilon_Q^{ns} = 1$ $\sigma_s = \sigma_{ns}$
log density	-1.988***	-1.610***	-1.520***	-0.874**	0.94
	(0.71)	(0.568)	(0.50)	(0.43)	(1.32)
log service	-2.924***	-1.03***	-1.11***	-0.294	0.802
	(0.50)	(0.349)	(0.31)	(0.214)	(0.48)
log density · log service	-3.07***	-2.41**	-2.34***	-1.265*	1.57
	(0.91)	(1.06)	(0.93)	(0.70)	(1.33)

Note: Columns 2-5 of this table present an OLS regression using the model's implied volatility of GDP growth as the dependent variable and the following set of time-varying country controls: outdegrees, indegrees, and average intermediate input share. The empirical OLS regression, column (1), also controls for countries' trade openness, country size, average growth, financial sector size, and size of mining and agriculture sectors. Robust standard errors in parentheses.

5 Discussion of implications

In this final section, I discuss three important implications that arise from this paper. First, I discuss the model's quantitative implications for the role of sectoral shocks. Second, I consider the fact that more domestic sectoral connections can lead to higher volatility. Third, I discuss the deviation from Hulten (1978) implied by the international evidence.

5.1 Sectoral shocks and comovements

I illustrate an important quantitative implication of the model in this paper: sectoral shocks are not capable of generating the observed business cycle dynamics. This result is specially true for dense and service-oriented economies. While sectoral sales shares (Hulten (1978)) determine GDP volatility, the details of the input-output structure are crucial to determining sectoral comovement and identifying the exact sources of aggregate volatility, whether sectoral shocks are propagated through input-output connections or through aggregate shocks affecting all firms equally.

Suppose that two economies, Economy 1 and Economy 2, display the same vector of sectoral sales shares. Economy 1 is an island economy, where firms produce using only labor input. On the other hand, Economy 2 displays input-output connections, meaning that firms use labor and intermediate inputs in production. As both economies have the same vector of sales shares, shocking both economies with the same sequence of idiosyncratic firm-level shocks will generate the same level of macroeconomic volatility. Nevertheless, Economy 1 will display negligible comovement across firms, while Economy 2 will display significant firm-level comovement due to input-output linkages. Therefore, if the data show that both economies also display high firm-level comovement, it must be the case that, in Economy 1 aggregate shocks play a much more important role than in Economy 2. This example addresses the core question of this section: can sectoral shocks explain the observed business cycle dynamics?

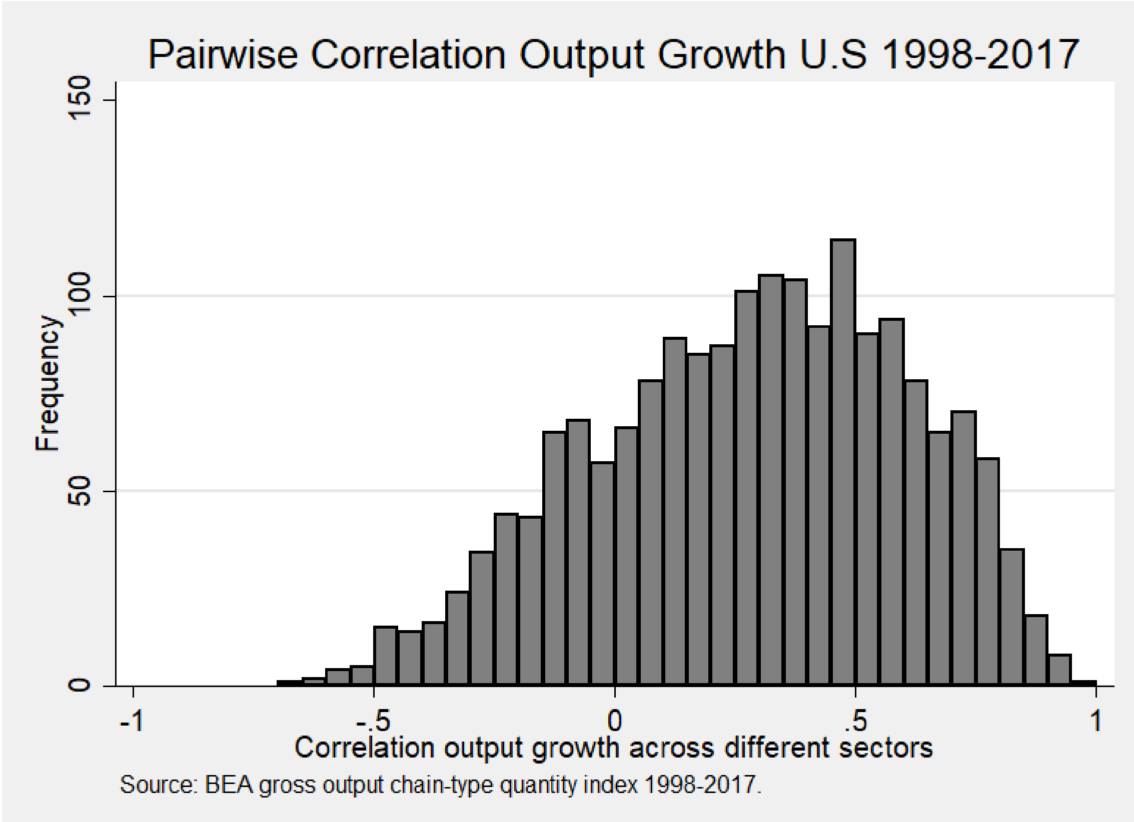
The quantitative exercise I perform shares the spirit of Foerster et al. (2011). However, to clearly study my model's implications for sectoral comovements, rather than backing out its implied series of sectoral shocks, I shock each of the models with the same sequence of independent productivity processes. One advantage of this procedure is that one can analyze more clearly how sectoral shocks are propagated and amplified through input-output connections, and, therefore, it is easily to evaluate whether sectoral shocks are capable of generating the observed fluctuations in sectoral output. Another advantage of this experiment is that it does not require approximating the model solution via loglinearization but, instead, provides the exact solution of the model.

I choose to calibrate the model to the U.S economy. The U.S provides the perfect example, as it produces 75% of total output in flexible-service industries and displays 87% of non-zero sectoral connections. Figure 4 presents the histogram of pairwise correlations of sectoral output growth among 61 U.S non-government industries for the period 1998-2017. Sectoral output displays significant comovement. The median pairwise correlation is 0.3, while the 25th percentile and the 75th percentile are 0.04 and 0.53, respectively. Does sectoral comovement arise from sectoral shocks being propagated through input-output linkages or through to aggregate shocks?

I study the model's ability to deliver a distribution of pairwise correlation of output growth similar to the one observed in Figure 4. To highlight the relevance of the network, even in the presence of Hulten (1978)'s theorem, I calibrate different versions of the model—different values of ϵ_Q and ϱ_M—to match a given vector of sectoral sales. In addition, to preserve mathematical tractability and to obtain the exact solution of the model, I assume that $\epsilon_Q = \epsilon_M$. I also assume that ϵ_Q is common across sectors. Assuming common elasticities allows me to directly compare my results with those of the previous literature. Sectoral sales in the model are described by:

$$P \circ Q = \beta' [I - \xi^{1-\epsilon_Q} \circ Z^{\epsilon_Q - 1} \circ \tilde{\Gamma}']^{-1},$$

Figure 4
Sectoral Comovements U.S



where ξ and $\tilde{\Gamma}$ are defined in Proposition 1. Given a set of structural parameters $(\Omega, \beta, \epsilon_Q, \varrho_Q)$ and steady-state vector of sectoral productivity $\tilde{Z}$, I can choose a to match a given vector of sectoral sales shares. Then, I use the fact that

$$\log P \circ Q = \log P + \log Q,$$

and

$$P^{1-\epsilon_Q} = [I - Z^{\epsilon_Q - 1} \circ ((1-a) \circ \Omega)^{\varrho_Q \epsilon_Q}]^{-1} (Z^{\epsilon_Q - 1} \circ a^{\varrho_Q \epsilon_Q}),$$

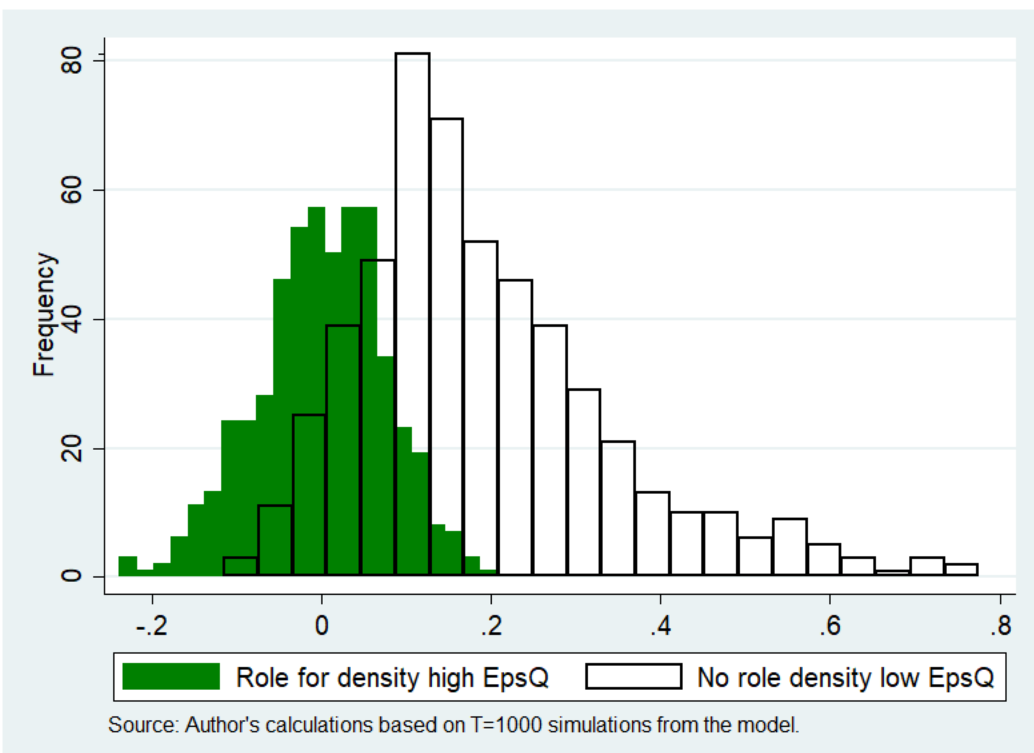
to obtain the model's implied vector for sectoral output (Q):

$$\log Q = \log[I - (\varrho^{1-\epsilon_Q} \circ Z^{\epsilon_Q - 1} \circ (1-a)) \circ \Omega]^{-1} \beta - \frac{1}{1-\epsilon_Q} \log\Big([I - Z^{\epsilon_Q - 1} \circ (((1-a) \circ \Omega')]^{-1} (Z^{\epsilon_Q - 1} \circ a) \Big). \tag{9}$$

Sectoral output depends on structural parameters $(\Omega, \beta, \epsilon_Q, \varrho_Q, a)$ and the realizations of sectoral productivities. From Equation (9), it is clear that sectoral output will strongly comove if sectoral shocks $\{Z_j\}_{t=1}^{T}$ are highly correlated. Sectoral output could also comove in the presence of idiosyncratic shocks due to complementarities in production, whether input-output connections (Foerster et al. (2011)) and/or low flexibility in the substitution of inputs (Atalay (2017)).

I assume that sectoral technologies obey equation (7). Shocks $\{\kappa_{jt}\}_{t=1}^{T}$ are independent and normally distributed with mean zero and standard deviation of 0.02. Figure 5 plots the model's implied pairwise correlation of sectoral output growth from i) a model with high elasticity ($\epsilon_Q = \epsilon_M = 3$) and a role for network density ($\varrho_Q = \varrho_M = 1$); and ii) a model with low production elasticity ($\epsilon_Q = \epsilon_M = 0.6$) and no role for network density ($\varrho_M = 1/\epsilon_M$). The model with high elasticity ($\epsilon_Q = 3$) and a role for network density displays a median pairwise correlation of 0.01, while the low-production-flexibility model displays a median pairwise correlation of 0.17. As expected, in the absence of aggregate shocks, both models—especially the model in this paper—fall short of matching the observed median pairwise correlation of 0.3 in the U.S.

Figure 5
Model's implied sectoral comovements U.S



Figures 6 and 7 decompose the total effect in Figure 5 into the role of network density and the role of production flexibility. In Figure 6, two economies with the same production flexibility ($\epsilon_Q = 3$), but different roles of network density, yield very different sectoral co-movement. As expected for a dense production network economy like the U.S.'s, the model with high elasticity and a role for network density displays less sectoral comovement than a model with high flexibility but that shuts down the role of network density.

Figure 6
Model's implied sectoral comovements U.S: role of network density

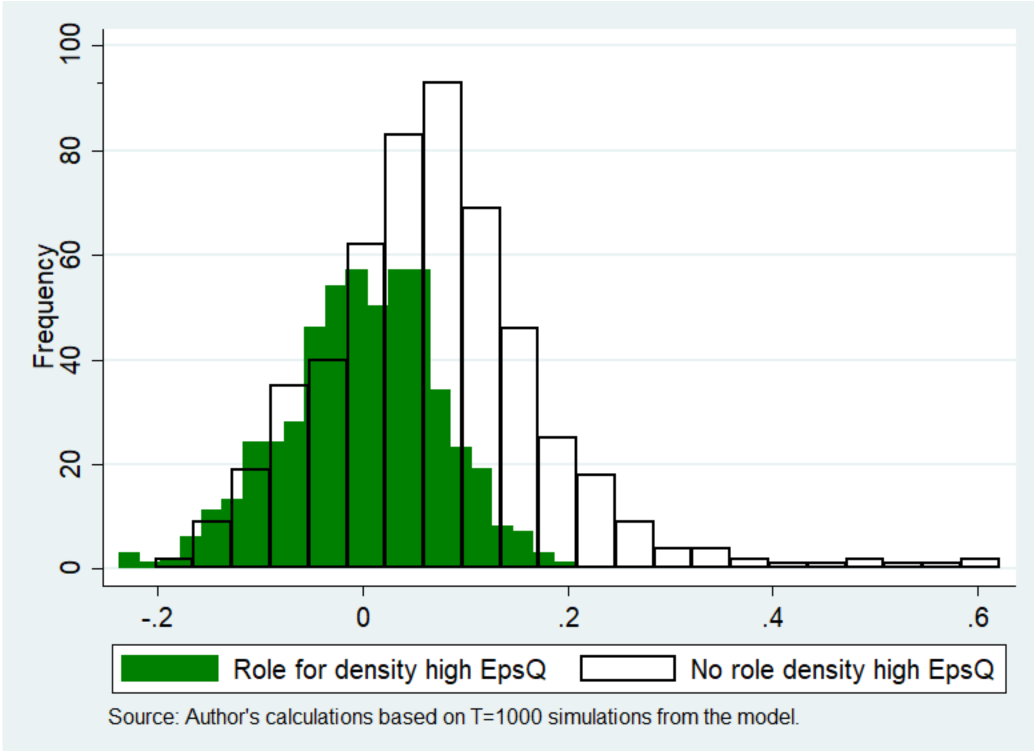


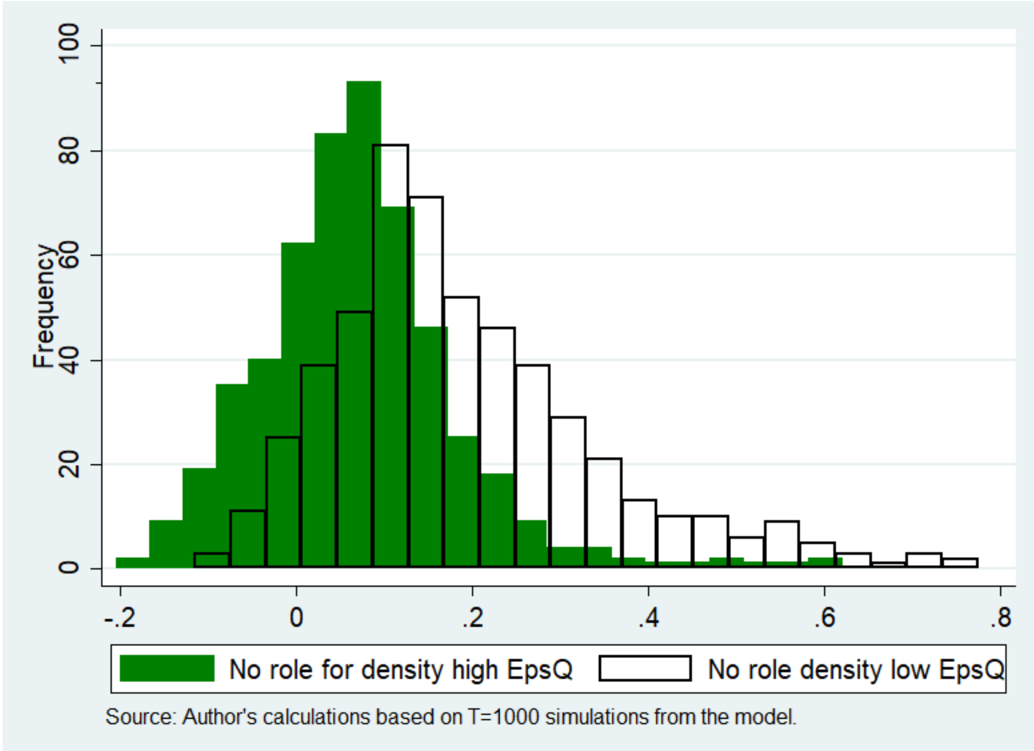
Figure 7 describes the role of production flexibility for two economies with no role for network density. As Atalay (2017) emphasizes, an economy with low flexibility in production displays higher sectoral comovement out of idiosyncratic sectoral shocks.

In a nutshell, the results in Figures 5-7 predict that in a model with high production flexibility and a role for network density, consistent with the international facts in Section 2, sectoral shocks—propagated through input-output linkages—are not able to reproduce the observed business cycle dynamics in the U.S displayed in Figure 4. Therefore, aggregate shocks seem to play the more predominant role in driving business cycle fluctuations in a dense and service-oriented economy like that of the U.S.

5.2 Production diversification and volatility

Another implication of the model in this paper is that when an economy specializes in industries with low production flexibility, a more diversified production structure—denser production network—amplifies the effect of sectoral shocks along the production chain. This result is especially relevant for emerging countries, in which discussion about the need to diversify the production structure is ongoing. The possibility that production diversification leads to higher volatility stands in contrast to Koren and Tenreyro (2013)'s model, in which the use of more intermediates in production is always associated with lower macroeconomic

Figure 7
Model's implied sectoral comovements U.S: role of flexibility

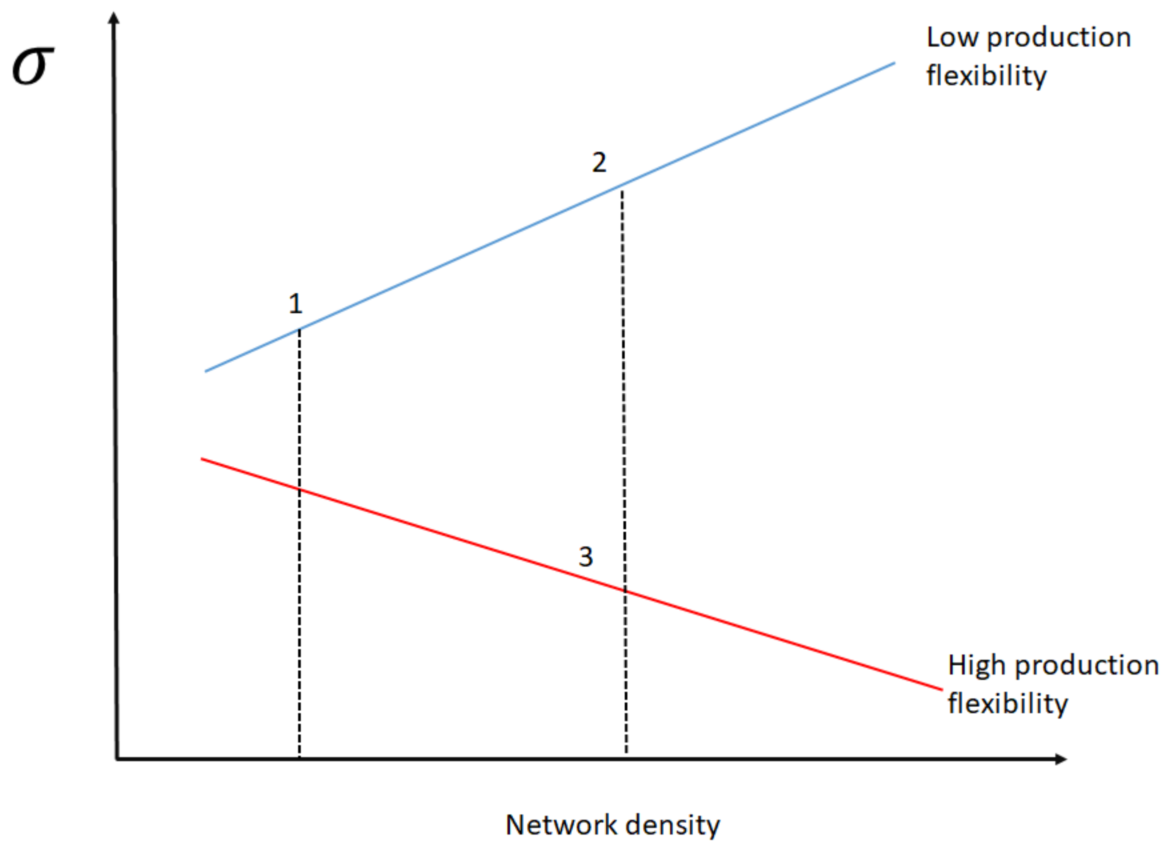


volatility.

The fact that more intermediates in production—i.e., diversification—reduce macroeconomic volatility in Koren and Tenreyro (2013) is due to the law of large numbers with a growing number of industries (N). The mechanism in this paper is rather different. In particular, the number of sectors N is fixed, and input-output connections are explicitly modeled. In this environment, provided that a cost of complexity in the bundle of intermediates exists—in the spirit of Costinot et al. (2013)—more-diversified economies with low elasticity of substitution between labor and intermediates will amplify the effect of sectoral shocks along the production chain.

Figure 8 illustrates the predictions from the model. The vertical axis represents GDP growth volatility, while the horizontal axis corresponds to network density. The upward-sloping curve corresponds to the density-volatility relationship in an economy with low production flexibility, while the downward-sloping curve represents the density-volatility relationship for an economy with high production flexibility. Suppose that point 1 represents the initial situation of an economy with low network density and high production share in low-flexibility industries. Points 2 and 3 are two potential situations in the same economy ten years later, with a higher network density. If the economy continues to produce in low-flexibility sectors, a higher network density increases macroeconomic volatility (point 2). However, if the economy's production structure shifts towards production in more-flexible sectors, the increased diversification manifests in an important decline in macroeconomic volatility (point 3).

Figure 8
Diversification and Volatility



Poland and Turkey provide good anecdotal evidence. From 1995 to 2005, Turkey's network density increased from 0.65 to 0.85, while its service share decreased from 0.54 to 0.49. At the same time, Turkey's volatility increased from 3.7% in 1984-1995 to 4.9% in 1996-2005. On the other hand, from 1995 to 2005, Poland's network density increased from 0.8 to 0.9, while its service share increased from 0.55 to 0.61. Poland's volatility declined from 5.6% (1984-1995) to 1.5% (1996-2005).

5.3 Deviations from Hulten (1978)

In this final section, I discuss another implication of the international evidence documented in section 2. In the model presented in this paper, the input-output structure affects macroeconomic volatility only through its effect on sales shares. However, the facts documented in Section 2.4, on the relationship between the input-output structure and volatility, hold even after controlling for countries' HHI of sales shares (see Table 4). Therefore, the data suggest that the input-output structure has a role beyond sales shares in explaining aggregate volatility.

A potential explanation for the results in Table 4 is the existence of sectoral distortions in the use of inputs. Bigio and La'O (2016) show how sectoral distortions are able to break the one-to-one mapping between sectoral sales shares and the input-output structure. Miranda-Pinto and Young (2019) provide evidence for the U.S showing that sectoral distortions played an important role during the Great Recession. Interestingly, in an environment with production networks and sectoral distortions, there are network pecuniary externalities that can be corrected, via input subsidies, to increase GDP (Miranda-Pinto (2018)). These models, however, do not display a role for network density in affecting macroeconomic volatility. Therefore, I leave the results in Table 4 as a puzzle for future research.

Table 4
Volatility, Network Density, Service Share, Herfindahl

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	$\log \sigma_T^k$	$\log \sigma_T^k$	$\log \sigma_T^k$	$\log \sigma_T^k$	$\log \sigma_T^k$	$\log \sigma_T^k$
log density	-0.721***	-0.567**	-2.483***	-2.377***	-3.549***	-2.092**
	(0.246)	(0.277)	(0.422)	(0.531)	(0.979)	(0.935)
log service		-0.720***	-2.088***	-2.905***	-2.595**	-0.208
		(0.230)	(0.374)	(0.436)	(1.093)	(1.066)
log density · log service			-3.032***	-3.726***	-4.090***	-1.820*
			(0.559)	(0.504)	(0.970)	(0.988)
log HHI				0.087	-0.778	-0.493
				(0.105)	(0.563)	(0.408)
Observations	144	144	144	144	144	144
Adjusted R-squared	0.080	0.134	0.254	0.353	0.296	0.491
Controls	No	No	No	Yes	Yes	Yes
Country FE	No	No	No	No	Yes	No
Country Trend	No	No	No	No	No	Yes

Note: This table presents an OLS regression using real GDP growth as the dependent variable and the following set of time-varying country controls: trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, sectoral HHI, size of the financial sector, and the share of agriculture and mining sectors (two most volatile sectors). The sample is divided into three sub-periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors clustered at the country level in parentheses.

6 Conclusion

This paper provides cross-country evidence on the importance of input-output structure in accounting for the heterogeneity in cross-country HHI of sales shares and macroeconomic volatility. I show that higher production network density—the fraction of non-zero input-output connections—is associated with lower HHI and lower GDP volatility. The density-volatility relationship interacts with the service share of the economy and accounts for a sizable fraction of the heterogeneity in cross-country volatility in the data.

I build a multisector model with general CES technologies that can account for these facts. I find that the production network density has a differential effect on volatility depending on the flexibility in production—in particular, the elasticity of substitution between intermediates and labor. The existence of a *cost of complexity* in producing with more intermediates implies that a denser production network mitigates shocks and reduces volatility when labor and intermediates are substitutes. However, when these inputs are complements, a denser network amplifies shocks and displays higher volatility.

Using U.S data, I confirm that service sectors have an elasticity between labor and intermediates that is larger than one and larger than in non-service sectors. Therefore, calibrating the model for each country, I show that, similar to the empirical results, the regressions using data simulated from the model yield a negative relationship between density and volatility. This relationship is also stronger the larger is the fraction of flexible-service industries in the economy.

The model in this paper has important implications for the role of sectoral shocks in driving short-run macroeconomic fluctuations. A quantitative exercise for the U.S, one of the most dense and service oriented economies in the world, predicts that sectoral shocks play only a modest role in driving the observed fluctuations in sectoral output growth.

References

Acemoglu, D., Akcigit, U., and Kerr, W. (2015a). *Networks and the Macroeconomy: An Empirical Exploration*, pages 276–335. University of Chicago Press.

Acemoglu, D. and Azar, P. D. (2017). Endogenous production networks. Working Paper 24116, National Bureau of Economic Research.

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015b). Systemic Risk and Stability in Financial Networks. *American Economic Review*, 105(2):564–608.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2016). Networks, shocks, and systemic risk. *In The Oxford Handbook of the Economics of Networks*.

Atalay, E. (2017). How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4):254–80.

Baqaee, D. R. and Farhi, E. (2017). The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. Working Paper 23145, National Bureau of Economic Research.

Bartelme, D. and Gorodnichenko, Y. (2015). Linkages and economic development. Working Paper 21251, National Bureau of Economic Research.

Bigio, S. and La'O, J. (2016). Financial frictions in production networks. Working Paper 22212, National Bureau of Economic Research.

Carvalho, V. and Gabaix, X. (2013). The Great Diversification and Its Undoing. *American Economic Review*, 103(5):1697–1727.

Carvalho, V. M., Nirei, M., Saito, Y. U., and Alireza Tahbaz-Salehi, A. (2016). Supply Chain Disruptions: Evidence from the Great East Japan Earthquake. Cambridge Working Papers in Economics 1670, Faculty of Economics, University of Cambridge.

Costinot, A., Vogel, J., and Wang, S. (2013). An Elementary Theory of Global Supply Chains. *Review of Economic Studies*, 80(1):109–144.

di Giovanni, J. and Levchenko, A. A. (2012). Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy*, 120(6):1083–1132.

Dungey, M. and Volkov, V. (2018). Rd and wholesale trade are critical to the economy: Identifying dominant sectors from economic networks. *Economics Letters*, 162:81 – 85.

Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics*, 43(2):391–409.

Fadinger, H., Ghiglino, C., and Teteryatnikova, M. (2015). Income Differences and Input-Output Structure. Vienna Economics Papers 1510, University of Vienna, Department of Economics.

Foerster, Sarte, and Watson (2011). Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production. *Journal of Political Economy*, 119(1):1 – 38.

Horvath, M. (1998). Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks. *Review of Economic Dynamics*, 1(4):781–808.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *Review of Economic Studies*, 45(3):511–518.

Jaimovich, N., Rebelo, S., and Wong, A. (2017). Trading Down and the Business Cycle. *Working paper.*

Jones, C. I. (2011). Intermediate Goods and Weak Links in the Theory of Economic Development. *American Economic Journal: Macroeconomics*, 3(2):1–28.

Koren, M. and Tenreyro, S. (2007). Volatility and Development. *The Quarterly Journal of Economics*, 122(1):243–287.

Koren, M. and Tenreyro, S. (2013). Technological diversification. *American Economic Review*, 103(1):378–414.

Luo, S. (2015). Propagation of Financial Shocks in an Input-Output Economy with Trade and Financial Linkages of Firms. Manuscript, Columbia University.

Miranda-Pinto, J. (2018). A note on optimal sectoral policies in production networks. *Economics Letters*, 172:152 – 156.

Miranda-Pinto, J. and Young, E. (2019). Flexibility and frictions in multisector models. Manuscript, University of Virginia.

Moro, A. (2012). The Structural Transformation Between Manufacturing and Services and the Decline in the US GDP Volatility. *Review of Economic Dynamics*, 15(3):402–415.

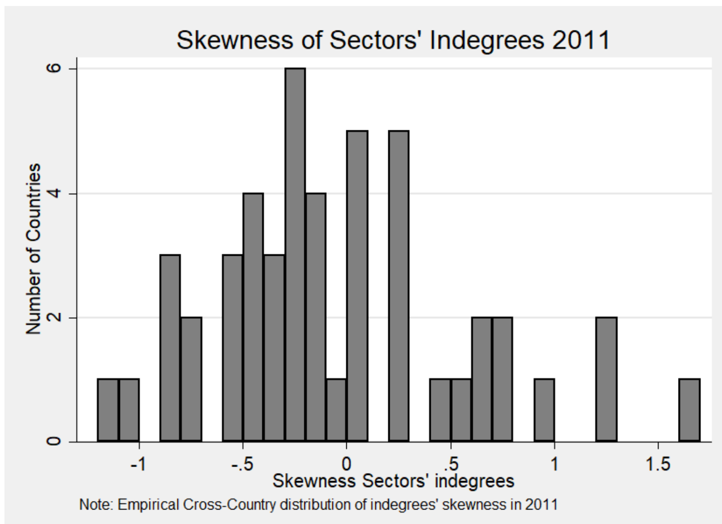
Moro, A. (2015). Structural Change, Growth, and Volatility. *American Economic Journal: Macroeconomics*, 7(3):259–94.

Ramey, G. and Ramey, V. A. (1995). Cross-Country Evidence on the Link between Volatility and Growth. *American Economic Review*, 85(5):1138–1151.

Srivisal, N. and Dabla-Norris, E. (2013). Revisiting the Link Between Finance and Macroeconomic Volatility . *IMF working paper*, (13:29).

Appendix A: figures and tables

Figure 9
Distribution of in-degrees Skewness of 48 countries in 2011



Robustness checks cross-country regressions

Density threshold

Table 5
Robustness: Threshold 0.1%

VARIABLES	(1) $\log \sigma_k^T$	(2) $\log \sigma_k^T$	(3) $\log \sigma_k^T$	(4) $\log \sigma_k^T$	(5) $\log \sigma_k^T$
log density2	-0.668***	-0.542**	-2.151***	-2.176***	-3.672***
	(0.224)	(0.256)	(0.392)	(0.544)	(1.044)
log service		-0.751***	-2.317***	-3.239***	-3.035***
		(0.230)	(0.424)	(0.491)	(1.128)
log density2 · log service			-2.633***	-3.411***	-3.929***
			(0.530)	(0.553)	(0.889)
Observations	144	144	144	144	144
Adjusted R^2	0.074	0.135	0.238	0.342	0.307
Controls	No	No	No	Yes	Yes
Country FE	No	No	No	No	Yes
R^2	0.0804	0.147	0.254	0.397	0.365

Note: This table presents an OLS regression using real GDP growth as dependent variable and the following set of time-varying country controls: trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, sectoral Herfindahl index, size of the financial sector, and the share of agriculture and mining sectors (two most volatile sectors). The sample is divided into three sub periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors clustered at the country level in parentheses.

Table 6
Robustness: Threshold 0.5%

VARIABLES	(1) $\log \sigma_k^T$	(2) $\log \sigma_k^T$	(3) $\log \sigma_k^T$	(4) $\log \sigma_k^T$	(5) $\log \sigma_k^T$
log density3	-0.596***	-0.563**	-1.644***	-2.243***	-4.325***
	(0.190)	(0.224)	(0.388)	(0.657)	(0.971)
log service		-0.848***	-3.194***	-4.546***	-5.125***
		(0.239)	(0.735)	(0.808)	(1.308)
log density3 $\cdot$ log service			-1.951***	-2.978***	-4.003***
			(0.567)	(0.732)	(0.727)
Observations	144	144	144	144	144
Adjusted R^2	0.063	0.146	0.213	0.329	0.348
Controls	No	No	No	Yes	Yes
Country FE	No	No	No	No	Yes
R^2	0.0700	0.158	0.230	0.386	0.403

Note: This table presents an OLS regression using real GDP growth as dependent variable and the following set of time-varying country controls: trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, sectoral Herfindahl index, size of the financial sector, and the share of agriculture and mining sectors (two most volatile sectors). The sample is divided into three sub periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors clustered at the country level in parentheses.

Outliers countries

Table 7
Robustness: drops countries with smallest and largest average network density (Brunei and Poland, respectively)

VARIABLES	(1) $\log \sigma_k^T$	(2) $\log \sigma_k^T$	(3) $\log \sigma_k^T$	(4) $\log \sigma_k^T$
log density	-0.942***	-2.296***	-1.781***	-3.941*
	(0.219)	(0.582)	(0.623)	(2.029)
log service	-0.919***	-1.998***	-2.793***	-3.007*
	(0.219)	(0.544)	(0.453)	(1.529)
log density · log service		-2.781**	-2.845***	-5.114*
		(1.148)	(0.868)	(3.005)
Observations	138	138	138	138
Adjusted R-squared	0.199	0.233	0.332	0.222
Controls	No	No	Yes	Yes
Country FE	No	No	No	Yes

Note: This table presents an OLS regression using real GDP growth as dependent variable and the following set of time-varying country controls: trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, sectoral Herfindahl index, size of the financial sector, and the share of agriculture and mining sectors (two most volatile sectors). The sample is divided into three sub periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Total (Domestic+imports) input-output measures

Table 8
Robustness: Total input-output

VARIABLES	(1) $\log \sigma_k^T$	(2) $\log \sigma_k^T$	(3) $\log \sigma_k^T$	(4) $\log \sigma_k^T$	(5) $\log \sigma_k^T$
log density tot	-1.146***	-0.814**	-3.860***	-4.348***	-4.002***
	(0.351)	(0.395)	(0.713)	(0.784)	(1.271)
log service		-0.636***	-2.084***	-3.059***	-2.408*
		(0.240)	(0.412)	(0.604)	(1.251)
log density tot · log service			-4.921***	-6.477***	-5.110***
			(0.957)	(1.251)	(1.668)
Observations	144	144	144	144	144
Adjusted R^2	0.088	0.124	0.226	0.342	0.343
Controls	No	No	No	Yes	Yes
Country FE	No	No	No	No	Yes

Note: This table presents an OLS regression using real GDP growth as dependent variable and the following set of time-varying country controls: country-specific time dummies, trade to GDP ratio, average growth rate, share of country in world GDP, skewness outdegrees, skewness of indegrees, average intermediate input share, sectoral Herfindahl index, size of the financial sector, and the share of agriculture and mining sectors (two most volatile sectors). The sample is divided into three sub periods: 1984-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 9
U.S. Sectors 2014 (BEA) Miranda-Pinto and Young (2019)

Sector Number	Iocode	Sector Name	Capital	Labor	Intermediates	Sales Share
1	111CA	Farms	34%	7%	59%	1.41%
2	113FF	Forestry, fishing, and related activities	30%	42%	28%	0.17%
3	211	Oil and gas extraction	61%	9%	30%	1.39%
4	212	Mining, except oil and gas	48%	14%	38%	0.42%
5	213	Support activities for mining	26%	41%	33%	0.34%
6	22	Utilities	49%	18%	33%	1.35%
7	23	Construction	20%	35%	45%	3.89%
8	321	Wood products	10%	20%	71%	0.32%
9	327	Nonmetallic mineral products	18%	22%	60%	0.38%
10	331	Primary metals	10%	11%	79%	0.91%
11	332	Fabricated metal products	13%	25%	61%	1.22%
12	333	Machinery	14%	23%	63%	1.31%
13	334	Computer and electronic products	35%	34%	31%	1.25%
14	335	Electrical equipment, appliances, and components	16%	27%	57%	0.41%
15	3361MV	Motor vehicles, bodies and trailers, and parts	13%	11%	76%	1.92%
16	3364OT	Other transportation equipment	15%	22%	64%	1.12%
17	337	Furniture and related products	9%	26%	65%	0.23%
18	339	Miscellaneous manufacturing	19%	29%	52%	0.54%
19	311FT	Food and beverage and tobacco products	15%	10%	75%	3.13%
20	313TT	Textile mills and textile product mills	9%	22%	69%	0.18%
21	315AL	Apparel and leather and allied products	6%	21%	72%	0.13%
22	322	Paper products	13%	15%	71%	0.63%
23	323	Printing and related support activities	14%	30%	55%	0.28%
24	324	Petroleum and coal products	19%	2%	79%	2.64%
25	325	Chemical products	33%	12%	56%	2.62%
26	326	Plastics and rubber products	14%	18%	68%	0.75%
27	42	Wholesale trade	35%	31%	34%	5.09%
28	441	Motor vehicle and parts dealers	31%	41%	28%	0.81%
29	445	Food and beverage stores	28%	39%	32%	0.72%
30	452	General merchandise stores	26%	39%	35%	0.72%
31	4A0	Other retail	29%	31%	39%	2.76%
32	481	Air transportation	21%	23%	55%	0.61%
33	482	Rail transportation	27%	25%	48%	0.29%
34	483	Water transportation	18%	11%	71%	0.20%
35	484	Truck transportation	15%	26%	59%	1.07%
36	485	Transit and ground passenger transportation	25%	32%	42%	0.18%
37	486	Pipeline transportation	57%	19%	23%	0.11%
38	487OS	Other transportation and support activities	19%	33%	48%	0.70%
39	493	Warehousing and storage	15%	42%	43%	0.29%
40	511	Publishing industries, except internet	31%	32%	36%	1.07%
41	512	Motion picture and sound recording industries	54%	21%	25%	0.49%
42	513	Broadcasting and telecommunications	36%	14%	50%	2.65%
43	514	Data processing, internet pub., and other inf. servi	19%	24%	57%	0.67%
44	521CI	Federal Reserve banks, credit interm., and rel. act.	38%	32%	31%	2.28%
45	523	Securities, commodity contracts, and investments	4%	47%	49%	1.55%
46	524	Insurance carriers and related activities	26%	28%	47%	2.73%
47	525	Funds, trusts, and other financial vehicles	26%	1%	73%	0.49%
48	HS	Housing Services	90%	1%	9%	5.88%
49	ORE	Other Real Estate	33%	8%	59%	3.09%
50	532RL	Rental and leasing services and lessors of int. asse	46%	10%	44%	1.10%
51	5411	Legal services	33%	39%	28%	0.99%
52	5415	Computer systems design and related services	10%	60%	29%	1.14%
53	5412OP	Miscellaneous professional, scientific, and tech. S	17%	42%	42%	4.00%
54	55	Management of companies and enterprises	8%	48%	44%	1.93%
55	561	Administrative and support services	18%	47%	35%	2.41%
56	562	Waste management and remediation services	19%	28%	53%	0.30%
57	61	Educational services	7%	54%	40%	1.03%
58	621	Ambulatory health care services	13%	50%	37%	3.01%
59	622	Hospitals	6%	46%	49%	2.45%
60	623	Nursing and residential care facilities	7%	53%	39%	0.72%
61	624	Social assistance	9%	55%	36%	0.55%
62	711AS	Performing arts, spectator sports, museums	28%	32%	40%	0.50%
63	713	Amusements, gambling, and recreation industries	24%	33%	43%	0.45%
64	721	Accommodation	30%	32%	38%	0.73%
65	722	Food services and drinking places	16%	36%	48%	2.15%
66	81	Other services, except government	17%	43%	41%	2.07%
67	GFGD	Federal general government (defense)	26%	38%	36%	2.02%

Table 10
Estimated Elasticities Manufacturing

VARIABLES	(1) FE	(2) FE	(3) IV Mil.	(4) IV Mil.	(5) IV all	(6) IV all
$\epsilon_M - 1$	-0.73***	-0.74***	1.79	2.75	0.52	-0.01
	(0.00)	(0.00)	(0.44)	(0.45)	(0.43)	(0.99)
$\epsilon_Q - 1$	-0.74***	-0.59***	2.23	0.12	0.72	-0.06
	(0.00)	(0.00)	(0.18)	(0.97)	(0.22)	(0.95)
Observations	7,200	7,200	7,200	7,200	6,300	6,300
Number of partner	450	450	450	450	450	450
Year FE	No	Yes	No	Yes	No	Yes
F Kleibergen-Paap			0.83	0.49	5.69	5.92
P-value Hansen test			0.36	0.39	0.01	0.48

Note: P-value in parentheses. Stock-Yogo test critical value 10%: 13.43.

Table 11
Estimated Elasticities Service

VARIABLES	(1) FE	(2) FE	(3) IV Mil.	(4) IV Mil.	(5) IV all	(6) IV all
$\epsilon_M - 1$	-0.30***	-0.32***	2.34	5.46***	1.91	-0.68
	(0.00)	(0.00)	(0.30)	(0.00)	(0.16)	(0.30)
$\epsilon_Q - 1$	-0.48***	-0.20*	4.37*	5.01**	3.76**	-0.85
	(0.00)	(0.07)	(0.06)	(0.01)	(0.03)	(0.49)
Observations	16,800	16,800	16,800	16,800	14,700	14,700
Number of partner	1,050	1,050	1,050	1,050	1,050	1,050
Year FE	No	Yes	No	Yes	No	Yes
F Kleibergen-Paap			4.80	8.89	3.99	7.45
P-value Hansen test			0.38	0.69	0.26	0.00

Note: P-value in parentheses. Stock-Yogo test critical value 10%: 13.43.

Table 12
Estimated Elasticities Others

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	FE	FE	IV Mil.	IV Mil.	IV all	IV all
$\epsilon_M - 1$	-1.10***	-1.21***	-3.23***	-4.24***	-2.06***	-2.71***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$\epsilon_Q - 1$	0.96***	1.18***	0.79	1.52**	1.23***	1.99***
	(0.00)	(0.00)	(0.20)	(0.02)	(0.01)	(0.00)
Observations	2,398	2,398	2,398	2,398	2,098	2,098
Number of partner	150	150	150	150	150	150
Year FE	No	Yes	No	Yes	No	Yes
F Kleibergen-Paap			114.17	95.04	340.38	116.82
P-value Hansen test			0.66	0.19	0.00	0.00

Note: P-value in parentheses. Stock-Yogo test critical value 10%: 13.43.

Appendix B: production network measures

Production network density

The network density measure (Acemoglu et al. (2015b)) counts the number of active edges over the maximum possible number of edges in the network.[18] Therefore, a production network with N sectors is dense if a large number of intermediate-input shares are positive over the maximum number of sectoral connections $N(N-1)$.[19] A production network is sparse if few intermediate-input shares are positive. The standard measure in network theory to describe the density of linkages is the following. Let $T = \sum_i^N \sum_{j=1}^N 1[\gamma_{ij} \geq 0]$, the number input-output shares (as a share of total output) above zero. Then, the density measure is defined as:

$$\text{density} = \frac{T - N}{N(N-1)},$$

[18]The term density is widely used in the literature of networks. In the literature of multisector models, Horvath (1998) used the term density when studying how having more zero rows in the input-output would alter the law of large numbers in the propagation of sectoral shocks. The author really meant outdegrees asymmetry.

[19]I suppress the diagonal of the input-output to focus in the connectivity among different sectors. In fact, in most countries firms of the same sector also purchase and sell intermediates to firms in the same sector.

where a network with density of 1 is a fully dense network while a density 0 implies a fully sparse network where firms in a sector only trade with one other sector. Due to measurement errors and the fact that input-output tables are harmonized to be comparable across countries, I count the input-output connections that are larger than a small threshold $\hat{\gamma}$. I consider a range of values between 0.05% and 0.5%.

Intensity of sectoral connections

The literature on production networks (Horvath (1998) and Acemoglu et al. (2012)) puts emphasis on the distribution of sectors' intermediate input supplier importance in shaping the way sectoral productivity shocks are propagated and amplified. The intermediate input supplier intensity in total sales is given by the weighted outdegree d_j^{out}:

$$d_j^{out} = \sum_{i=1}^{N} (1 - a_i)\omega_{ji},$$

where a_i is the importance of labor in production of sector i and ω_{ji} is the cost share of intermediates from sector j in the total cost of intermediates of sector i. I use the skewness of sectoral d_j^{out} as a measure of outdegree dominance in a given country. An alternative way to account for the asymmetry in sectors importance is by using the sectoral dominance statistic in Acemoglu et al. (2016). The authors define the influence of a sector by:

$$\nu_j = \sum_{i=1}^{N} \beta_i L_{ji},$$

where β_j is the share of households consumption in sector j goods and L_{ji} is the (j, i) element of the Leontief inverse matrix. The j_{th} element of the Leontief inverse can be expressed as:

$$L_j = (I - (1-a)' \circ \Omega')_j^{-1} = \sum_{k=1}^{\infty} \Big((1-a)' \circ \Omega\Big)_j^{\prime k},$$

and it contains the direct and indirect effects of a shock to sector j. The value of k represents the higher order network effects. When $k = 1$, a shock to sector j has a direct effect on the sectors that use sector j's output as input. When k is bigger than 1, the shock to sector j affects the sectors that are indirectly connected to sector j by being directly connected to sector j's customers.[20] The sectoral dominance index is defined as

$$\text{dominance} = \frac{\nu_{max}\sqrt{N}}{(\sum_{i=1}^{N} \nu_i^2)^{1/2}},$$

and captures the existence of sectors that are extremely important in the network.

[20] In non-reported results, I also consider moments (variance or skewness) of sectors' first order outdegrees (row sum of the input-output elements). The results are unaffected.

Finally, I also consider the intensity in which a sectors use intermediate inputs. As noted by Acemoglu et al. (2015a) and Luo (2015), government spending shocks and financial shocks propagate mainly from intermediate input user sectors to intermediate input provider sectors. Similarly, Carvalho et al. (2016) find that when firms use intermediate inputs and labor as substitute inputs, a productivity shock to sector j not only affects the users of sector j's output but it also affects the demand of firms in sectors that supply goods to firms in sector j.

The intermediate input intensity in total sales is given by the weighted indegree d_j^{in}:

$$d_j^{in} = \sum_{i=1}^{N} \frac{P_{ij} M_{ij}}{P_j Q_j}.$$

I consider, for each country k, the average indegree as well as the asymmetry in sectors indegrees.[21]

Appendix C: mathematical appendix

The model's general solution

To ease notation define $\rho_{Q_j} = \frac{\epsilon_{Q,j}-1}{\epsilon_{Q,j}}$, $\epsilon_{Q,j} = \frac{1}{1-\rho_{Q,j}}$, $\rho_{M_j} = \frac{\epsilon_{M,j}-1}{\epsilon_{M,j}}$, $\epsilon_{M,j} = \frac{1}{1-\rho_{M,j}}$. Also, let $b_j = 1 - a_j$ the importance of materials in production. The firms first order conditions with respect to inputs are:

$$L_j : P_j Z^{\rho} a_j^{\varrho_{Q_j}} \left(\frac{Q_j}{L_j}\right)^{1-\rho_{Q,j}} = w$$

$$M_j : P_j Z^{\rho} b_j^{\varrho_{Q_j}} \left(\frac{Q_j}{M_j}\right)^{1-\rho_{Q,j}} = P_j^M$$

$$M_{ji} : P_i Z_i^{\rho_Q} Q_i^{1-\rho_Q} (1-a_j)^{\varrho_Q} M_i^{\rho_Q - \rho_M} M_{ji}^{\rho_M - 1} \omega_{ji}^{\varrho_M} = P_j$$

From the household FONC I have:

$$P_j^{\epsilon_D} C_j = \beta_j P_c^{\epsilon_D} C.$$

The price index for consumption goods is obtained from minimizing the households'

[21] To account for star intermediate input users I consider the sample skewness of sectoral in-degrees in country k. The sample skewness of a realization of $\{x\}_{i=1}^N$ is given by

$$S = \frac{\frac{1}{n}\sum (x_i - \bar{x})^3}{\frac{1}{n-1}\left(\sum (x_i - \bar{x})^2\right)^{3/2}}$$

expenditures $\sum_{j=1}^{N} P_j C_j$ subject to $C = \left(\sum_{j=1}^{N} \beta_j^{1/\epsilon_D} C_j^{\frac{\epsilon_D - 1}{\epsilon_D}}\right)^{\frac{\epsilon_D}{\epsilon_D - 1}}$, I obtain:

$$P_c = \Big(\sum_{j=1}^{N} \beta_j P_j^{1-\epsilon_D}\Big)^{\frac{1}{1-\epsilon_D}}.$$

Similarly, firms minimize the cost of the bundle of intermediates $\sum_{i=1}^{N} P_i M_{ij}$ subject to $M_j = \left(\sum_{i=1}^{N} \omega_{ij}^{\varrho_{M,j}} M_{ij}^{\rho_{M,j}}\right)^{1/\rho_{M,j}}$. In competitive markets, I obtain:

$$P_j^M = \Big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_{M,j}\epsilon_{M,j}} P_i^{1-\epsilon_{M,j}}\Big)^{\frac{1}{1-\epsilon_{M,j}}}.$$

Assuming common sectoral elasticities, the production function of firms in sector j can be expressed as:

$$Z_j^{-\rho_Q} = a_j^{\varrho_Q}\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} + b_j^{\varrho_Q}\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q},$$

which combined with the FONC gives:

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j^{\varrho_Q \epsilon_Q} w^{1-\epsilon_Q} + Z_j^{\epsilon_Q - 1} b^{\varrho_Q \epsilon_Q}\Big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M \epsilon_M} P_i^{1-\epsilon_M}\Big)^{\frac{1-\epsilon_Q}{1-\epsilon_M}},$$

assuming $\epsilon_Q = \epsilon_M$ we have, in matrices, the solution for prices:

$$P^{1-\epsilon_Q} = [I - Z^{\epsilon_Q - 1} \circ ((1-a)\circ\Omega)^{\varrho_Q \epsilon_Q}]^{-1}(Z^{\epsilon_Q - 1} \circ a^{\varrho_Q \epsilon_Q}), \tag{10}$$

where $\tilde{\Gamma} = ((1-a)\circ\Omega)^{\varrho_Q \epsilon_Q}$ and $\tilde{a} = a^{\varrho_Q \epsilon_Q}$.

Proposition 1

Hulten (1978) shows that, up to a first order, sector j's sales to GDP ratio describes the aggregate effect of a productivity shock to sector j. Baqaee and Farhi (2017) show how this result also extends to models with general CES technologies (*theorem 2.1*). I also demonstrate that Hulten (1978)'s theorem applies for the model in this paper.

I proceed to find sectoral sales in this economy, as a function of the network structure. From the market clearing conditions of sectoral output I have:

$$Q_j = C_j + \sum_{i}^{N} M_{ji},$$

multiplying each size of the equation by P_j we have $P_j Q_j = P_j C_j + \sum_i^N P_j M_{ji}$. From the household optimal consumption share for each good and provided we assume $\epsilon_D = 1$ we have that $P_j C_j = \beta_j P_c C$. From the firms FONC for M_{ij} we have:

$$P_i Z_i^{\rho_Q} Q_i^{1-\rho_Q}(1-a_j)^{\varrho_Q} M_i^{\rho_Q-\rho_M} M_{ji}^{\rho_M-1}\omega_{ji}^{\varrho_M} = P_j,$$

which under the simplification assumption of $\epsilon_Q = \epsilon_M$ becomes:

$$P_j M_{ji} = \Big(\frac{P_j}{P_i}\Big)^{1-\epsilon_Q} Z_i^{\epsilon_Q-1}(1-a_j)^{\varrho_Q\epsilon_Q}\omega_{ji}^{\varrho_M\epsilon_Q} P_i Q_i.$$

Replacing these equations into the goods market clearing condition and defining sectoral sales as S_j, we obtain the following expression

$$S_j = \beta_j P_c C + P_j^{\epsilon_Q-1}\sum_i^N P_i^{\epsilon_Q-1} Z_i^{\epsilon_Q-1}(1-a_i)^{\varrho_Q\epsilon_Q}\omega_{ji}^{\varrho_M\epsilon_Q} S_i.$$

We have already solved for prices in Equation (10). Given thar $P_c C$ is nominal GDP in this economy, we can express the vector of sales to GDP ratio ($s_j = \frac{S_j}{GDP}$) as:

$$s = \beta'[I - \xi^{1-\epsilon_Q}\circ Z^{\epsilon_Q-1}\circ\tilde{\Gamma}']^{-1}, \tag{11}$$

where $\tilde{\Gamma} = ((1-a)'\circ\Omega)^{\varrho_Q\epsilon_Q}$

The term ξ is one when $a_j = a$ and $Z = Z_j$ for all industries, as in Carvalho et al. (2016). This is, in every sector, the importance of intermediates is the same. Conditional on this, different structures of Ω still yield, in steady state, $P = P_j$ for all j.

To complete the proposition, recall that, up to a first order, real GDP in this economy is

$$\log C \approx \sum_{j=1}^N (\log Z_j - \log\bar{Z})\cdot s_j. \tag{12}$$

Later, I show that this indeed corresponds to the first order Taylor approximation of GDP in the economies for which I have simple closed form solution for equilibrium prices.

Therefore, combining (11) and (12), the aggregate effect of a sectoral shock in sector j is

$$\frac{\partial\log C}{\partial\log Z_j} = \Big(\beta'[I - \xi^{1-\epsilon_Q}\circ Z^{\epsilon_Q-1}\circ\tilde{\Gamma}']^{-1}\Big)_j,$$

which proves Proposition 1.

Applying Proposition 1 for the sparse network yields

$$\frac{\partial\log C^{sparse}}{\partial\log Z_j} = \Big(\beta'[I - (1-a)^{\epsilon_Q}\circ Z^{\epsilon_Q-1}\circ\Omega']^{-1}\Big)_j,$$

and the following for the dense network

$$\frac{\partial\log C^{dense}}{\partial\log Z_j} = \Big(\beta'[I - N^{1-\epsilon_Q}\circ(1-a)^{\epsilon_Q} Z^{\epsilon_Q-1}\circ\Omega']^{-1}\Big)_j,$$

where $\frac{\partial \log C^{dense}}{\partial \log Z_j} > \frac{\partial \log C^{sparse}}{\partial \log Z_j}$ if $\epsilon_Q < 1$ and $\frac{\partial \log C^{dense}}{\partial \log Z_j} < \frac{\partial \log C^{sparse}}{\partial \log Z_j}$ if $\epsilon_Q > 1$

Proposition 1 special case

I have derived the general case using Hulten (1978). I now show for the simple network structures discussed in the paper, the dense and the sparse, that I obtain the same result using the solutions for prices. In this case, I also highlight the role of ϵ_Q by assuming that $\epsilon_D = 1 = \epsilon_M = 1$. Real GDP in this economy is

$$GDP = \prod_{j=1}^{N} \Big(\frac{\beta_j}{P_j}\Big)^{\beta_j}.$$

Using the solution for prices (when $a = a_j$ and $Z = Z_j$ for all j) we can now compare the dense and the sparse network

$$GDP^{sparse} = \prod_{j=1}^{N} \Big(\beta_j \Big(\frac{a^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}}{1 - (1-a)^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}}\Big)^{\frac{1}{\epsilon_Q - 1}}\Big)^{\beta_j}$$

$$GDP^{dense} = \prod_{j=1}^{N} \Big(\beta_j \Big(\frac{a^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}}{1 - N^{1-\epsilon_Q}(1-a)^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}}\Big)^{\frac{1}{\epsilon_Q - 1}}\Big)^{\beta_j}$$

$$\log GDP^{sparse} = \sum_{j=1}^{N} \beta_j \Big(\log \beta_j + \frac{1}{\epsilon_Q - 1} [\log(a^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}) - \log(1 - (1-a)^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}])\Big)$$

$$\log GDP^{dense} = \sum_{j=1}^{N} \beta_j \Big(\log \beta_j + \frac{1}{\epsilon_Q - 1} [\log(a^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1}) - \log(1 - N^{1-\epsilon_Q}(1-a)^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1})]\Big),$$

therefore

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \frac{\beta_j}{1 - (1-a)^{\varrho_Q \epsilon_Q} Z^{\epsilon_Q - 1}}$$

$$\frac{\partial \log GDP^{dense}}{\partial \log Z_j} = \frac{\beta_j}{1 - N^{1-\epsilon_Q}(1-a)^{\varrho_Q \epsilon_Q} Z^{\epsilon_Q - 1}}.$$

It is direct from equation (11) that this GDP elasticity is exactly the sales to GDP ratio in the dense and sparse. In particular, $s_j = \frac{P_j Q_j}{PC} = \frac{\beta_j}{1-(1-a)^{\varrho_Q \epsilon_Q} Z^{\epsilon_Q - 1}}$ in the sparse network and $s_j = \frac{P_j Q_j}{PC} = \frac{\beta_j}{1-N^{1-\epsilon_Q}(1-a)^{\varrho_Q \epsilon_Q} Z^{\epsilon_Q - 1}}$ in the dense.

Therefore, when $\epsilon_Q = 1$, $N^{1-\epsilon_Q} = 1$, I have

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \frac{\partial \log GDP^{dense}}{\partial \log Z_j},$$

whenever $\epsilon_Q > 1$, $N^{1-\epsilon_Q} < 1$, I obtain

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} > \frac{\partial \log GDP^{dense}}{\partial \log Z_j},$$

while when $\epsilon_Q < 1$, $N^{1-\epsilon_Q} > 1$, I have

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} < \frac{\partial \log GDP^{dense}}{\partial \log Z_j},$$

To obtain the volatility of GDP growth, I assume that sectoral productivity shocks follow a random walk

$$\log Z_{jt} = \log Z_{jt-1} + \kappa_{jt},$$

where κ_{jt} is normally distributed with mean zero and standard deviation σ_j. Therefore, log GDP changes is

$$\Delta \log GDP_t \approx \sum_{j=1}^{N} \kappa_{jt} \cdot s_j.$$

Therefore, the variance of the log change of GDP is

$$\sqrt{Var(\Delta \log GDP)} = \sqrt{\sum_{j=1}^{N} \left(\beta'[I - \xi^{1-\epsilon_Q} \circ Z^{\epsilon_Q - 1} \circ \tilde{\Gamma}']^{-1}\right)_j^2 \sigma^j}.$$

Assuming common sectoral volatilities and consumption shares, in the dense network, where $s_j = \frac{\beta}{1 - N^{1-\epsilon_Q}(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1}}$, we have

$$\sqrt{Var(\Delta \log GDP^{dense})} = \frac{\sigma\sqrt{\beta}}{(1 - N^{1-\epsilon_Q}(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})}.$$

and in the sparse, where $s_j = \frac{\beta}{1-(1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1}}$, implies

$$\sqrt{Var(\Delta \log GDP^{sparse})} = \frac{\sigma\sqrt{\beta}}{(1 - (1-a)^{\epsilon_Q} Z^{\epsilon_Q - 1})},$$

Volatility for the special networks

Using the solution for prices, I check that aggregate GDP can be approximated according to Hulten (1978) as in Proposition 1.

$$\log GDP = \sum_{j=1}^{N} \beta_j \log\left(\beta_j \left(\frac{a^{\varrho_Q \epsilon_Q} Z_j^{\epsilon - 1}}{1 - (1-a)^{\varrho_Q \epsilon_Q} Z_j^{\epsilon_Q - 1} \tilde{\xi}}\right)^{\frac{1}{\epsilon_Q - 1}}\right),$$

where $\tilde{\xi} = 1$ in the sparse network and $\tilde{\xi} = N^{1-\epsilon_Q}$ in the dense network. Taking a first order Taylor approximation of GDP around the steady state productivity $\log \bar{Z}$ we obtain the desired result

$$\log GDP \approx \sum_{j=1}^{N}(\log Z_j - \log \bar{Z}) \cdot \frac{\beta_j}{1-(1-a)^{\varrho_Q \epsilon_Q} \bar{Z}_j^{\epsilon_Q - 1} \tilde{\xi}} = \sum_{j=1}^{N}(\log Z_j - \log \bar{Z}) \cdot s_j$$

Model's simulations

With the assumptions from section 4, real GDP in this economy is $\log GDP = -\log P_c$. In addition, as $\epsilon_D = 1$ we have $P_c = \prod_{j=1}^{N}\left(\frac{\beta_j}{P_j}\right)^{\beta_j}$. Assuming $\varrho_M = \varrho_Q = \epsilon_M = 1$, equation (6) becomes:

$$P_j^{1-\epsilon_{Q_j}} = Z_j^{\epsilon_{Q_j}-1}\Big(a_j^{\epsilon_{Q_j}} w^{1-\epsilon_{Q_j}} + (1-a_j)^{\epsilon_{Q_j}}\Big(\prod_{i=1}^{N}\Big(\frac{P_i}{\omega_{ij}}\Big)^{\omega_{ij}}\Big)^{1-\epsilon_{Q_j}}\Big)$$

which in matrices becomes

$$(1-\epsilon_Q) \circ \log P = (\epsilon_Q - 1) \circ \log Z + \log\Big(a^{\epsilon_Q} + (1-a)^{\epsilon_Q} \circ exp[(1-\epsilon_Q) \circ (\Omega' \log P - diag(\Omega' \log \Omega))]\Big).$$

For a given path for $\left(\{Z_j\}_{j=1}^{N}\right)_{t=1}^{T}$ and the calibrated values of a, Ω, and ϵ_Q, we can solve the non-linear system of equations for prices. Then, we find real GDP as:

$$\log GDP = -\log P_c = \sum_{j=1}^{N} \beta_j \log\Big(\frac{\beta_j}{P_j}\Big).$$