Pareto Efficiency, Inequality and Distribution Neutral Fiscal Policy - An Overview

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Abstract A structure of taxes and transfers that keep the income distribution unchanged even after positive or negative shocks to an economy, is referred as a Distribution Neutral Fiscal Policy. Marjit and Sarkar (2017) referred this as a Strong Pareto Superior (SPS) allocation which improves the standard Pareto criterion by keeping the degree of inequality, not the absolute level of income, intact. In this paper we show the existence of a SPS allocation in a general equilibrium framework, and we provide a brief survey of distribution neutral fiscal policies existing in the literature. We also provide an empirical illustration with Indian Human Development Survey data.

Keywords: Pareto Superiority; Strong Pareto Superiority; Inequality
JEL Classification: H20; H30; H23; D63

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1 Introduction

Marjit (2016) has argued that researchers must look for robust results in economic theory that hold across space and time. This is likely to bring economics closer to physical sciences. In this paper we review one such robust result also referred as the "*Distribution Neutral Fiscal Policy*" (DNFP). By DNFP we mean a structure of taxes and transfers that keep the income distribution unchanged even after positive or negative shocks to an economy. The idea of distribution neutrality evolved in different areas of economics in different forms. For example, in a classic article Datt and Ravallion (1992) uses this approach in the context of poverty decomposition analysis. Very recently Marjit and Sarkar (2017) generalize the notion of Pareto efficiency by focusing on Pareto efficient allocations that do not aggravates inequality. In this paper we discuss these results.

The Pareto ranking or Pareto efficiency is a topic economists are exposed to very early in their career. In particular the basic welfare comparison between two social situations starts with the ranking in terms of a principle Pareto had talked about in the nineteenth century. If we compare two social situations A and B, we say B is Pareto superior to A iff everyone is as well of in B as in A and at least one is strictly better off in situation B compared to A. This comparison is done in terms of utility or welfare levels individuals enjoy in A and B. In the theory of social welfare this has been a widely discussed topic with seminal contributions from De Scitovszky (1941), Samuelson (1958), Arrow (1963), Stiglitz (1987), Sen (1970) and recent treatments include Mandler (1999), Cornes and Sandler (2000) etc.

Pareto's principle provides a nice way to compare situations when some gain and some lose by considering whether transfer from gainers to losers can lead to a new distribution in B such that B turns out to be Pareto superior to A, the initial welfare distribution. It is obvious that if sum of utilities increases in B relative to A, then whatever be the actual distribution in B, a transfer mechanism will always exist such that transfer-induced redistribution will make B Pareto superior to A. The great example is how gains from international trade can be redistributed in favor of those who lose from trade such that everyone gains due to trade. Overall gains from trade lead to a higher level of welfare under ideal conditions and therefore one can show that under free trade eventually nobody may lose as gainers *'bribe'* the losers. But whatever it is Pareto ranking definitely does not address the inequality issue. There will be situations where B will be Pareto superior to A, but inequality in B can be much greater than A. The purpose of this survey paper is to extend the basic principle of Pareto's welfare ranking subjecting it to a stricter condition that keeps the degree of inequality intact between A and B after transfer from gainers to losers, but at the same time guaranteeing that everyone gains in the end.

In this paper we also provide the existence and uniqueness of SPS in a general equilibrium framework. Considering the case with only two agents we show that an SPS allocation would always exist on the contract curve i.e. in the set of Pareto efficient allocations. Further, we also show that this point is unique. We also provide an empirical illustration with Indian Human Development Survey data.

The paper is organized as follows. In Section 2 we present a brief discussion on the measurement of income inequality. In section 3 we present a discussion on Strong Pareto Superiority allocations. In the next section we discuss other Distribution Neutral Fiscal policies that exists in the literature. In section 5 we provide an empirical illustration with Indian data. The paper is concluded in section 6.

2 Inequality Measurement

In this section, we provide a discussion on the measurement of income inequality. We begin with some well-known axioms or postulates that any income inequality measure must satisfy. We then discuss some standard inequality measures proposed in the literature. We also provide a discussion on the Lorenz curve that stands as a basic tool for inequality ordering.

Inequality index is a scalar measure of interpersonal income differences within a given population. The class of inequality measures can be classified in two broad types, namely the relative and the absolute inequality measures. The relative inequality measures satisfy the scale invariance property, i.e., the inequality measure should remain unchanged if we multiply income of all individuals by a positive scalar. The absolute inequality measures are translation invariant, i.e., these inequality measures should remain unchanged if we change the origin by a positive scalar. has referred the relative and absolute the inequality measures as the rightists and leftists inequality measures, respectively.

The leftist inequality measures assigns a higher weight to the bottom of the income distribution. This can be illustrated as follows: consider the following income distribution $Y = \{1, 100\}$. Now following the relative inequality measure inequality should remain unchanged if we multiply the income of both individuals by a positive scalar. If we choose this scalar as 10, the new distribution becomes $Y^* = 10, 1000$. If we observe Y and Y^* closely, the poorer individual has gained only 9, whereas the richer individual has gained 900. The relative inequality remains unchanged despite the fact that absolute income differences between individuals increase. Notwithstanding this problem, the relative inequality index is widely used. This is because it is simple and because income inequality measures do not depend on the choice of the units. That is income inequality measured in dollars or pounds will exhibit same values.

We now discuss the following axioms that both absolute and relative inequality measures are expected to satisfy.We begin with the transfer axiom.

Transfer Axiom: A progressive transfer of income is defined as a transfer of income from a person to anyone who has a lower income so that the donor does not become poorer than the recipient. A regressive transfer is defined as a transfer from a person to anybody with a higher income, keeping their relative positions unchanged. This axiom requires that inequality should decline and increase as a result of progressive and regressive transfer, respectively. If the income distribution is ordered either in an ascending or in a descending order, then the transfers cannot change the rank orders of the individuals. Hence, these transfers are sometimes referred as rank preserving transfers.

Symmetry: This axiom requires that an income measure should not distinguish individuals by anything other than their incomes. This axiom is also referred as an anonymity axiom.

Normalization axiom: This axiom states that if for a society income of all individuals are same then there is no inequality. Eventually any inequality index that satisfies this axiom should take the value zero. This axiom always ensures that an inequality measure takes non-negative values.

Population Replication Invariance: This axiom states that inequality measurement should be invariant for replication of incomes. This implies that any inequality measure satisfying this axiom should be invariant between $Y = (2,3)$ and $Y^* = (2,3,2,3,2,3)$. This is because Y^* is obtained following three-fold replication of incomes of Y. Following this axiom, we can compare two income distributions with different population size. For example, consider $Y = (2,3)$ and $X = (1,2,3)$. If the inequality index is population invariant, then we can compare $Y^* = (2,3,2,3,2,3)$ and $X^* = (1,2,3,1,2,3)$.

A well known relative inequality measure that satisfies all the axioms listed above is the Gini coefficient. Variance is an absolute inequality measure that also satisfies all the axioms discussed so far.

Researchers often expressed interest on whether different inequality indices can rank alternative distributions of income in the same way. One may consider the Lorenz curve in order to address this issue. Before we address this issue let us formally define Lorenz curve. Assume that the income distribution $Y_t = (y_1, y_2, .., y_n)$ is designed in an ascending order. Lorenz curve represents the share of the total income enjoyed by the bottom p% of the population. The Lorenz curve is defined as the plot $L(Y_t, k/n) =$ against p where $p = k/n$. Note that 0% of the population enjoys 0% of the total income. Further, 100% of the population possesses the entire income. Hence, the curve starts from the south-west corner with coordinates (0,0) of the of unit square and terminates at the diametrically opposite north-east corner with coordinates (1,1).In the case of perfect equality, Lorenz curve coincides with the diagonal line of perfect equality, which is also referred to as the egalitarian line. This follows from the fact that if everybody has equal income then every p% of the population enjoys p% of the total income. In all other cases the curve will lie below the egalitarian line. If there is complete inequality, i.e., in a situation where only one person has positive income and all other persons have zero income, the curve will be L shaped. That is the curve will run through the horizontal axis until we reach the richest person where it will rise perpendicularly. In the context of Lorenz curve one important concept is that of Lorenz dominance. Any income distribution Yis said to be Lorenz dominant income distribution X if the Lorenz curve of Y lies strictly above X for at least one point and not below X at any of the point. In this context, we can also refer that inequality in Yis less than that of X, for all relative inequality measures that satisfies Symmetry, Transfer, and Population Replication Invariance. These class of measures is also referred as Lorenz consistent inequality measures.

3 Strong Pareto Superiority

Pareto superiority (PS) is defined as the situation where no one loses from the initial to the final period but at least one individual gains. However, PS allocation may aggravate inequality. We thus introduce "*Strong Pareto Superiority*" (SPS). By SPS we mean a situation where the utility of all the individuals increases and the inequality (either absolute or relative) also remains same, compared to that of the initial distribution. In order to derive such a SPS allocation we first assume that their exists a social planner who taxes a subgroup of the population and distributes the collected tax to the rest of the population. Note that as we move through this paper we show that in general the SPS allocations and their associated tax transfer vector will be different for the relative and absolute inequality measures. From here onwards we refer the SPS allocation's that preserves relative and absolute inequality as RSPS and ASPS, respectively.

In order to illustrate the existence of SPS we consider a two-period economy with only two individuals observed in both the period. Let the distribution in the first period be denoted by the vector $Y_0 = (y_1, y_2)$ and that of the second period as $Y_1 = (g_1 y_1, g_2 y_2)$, where g_1 and g_2 denotes the growth rate of first and second individual respectively. Let $g_1 y_1 > g_2 y_2$. In order to keep the degree of relative inequality we must tax the first individual such that the following condition is satisfied:

$$\frac{y_1}{y_2} = \frac{g_1 y_1 - T}{g_2 y_2 + T} \tag{1}$$

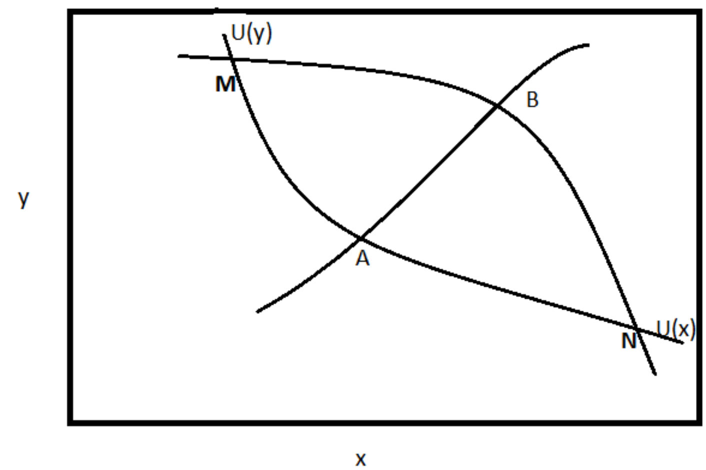
We can solve T from the above equation in the following fashion

$$T = \frac{y_1 y_2 (y_1 - y_2)}{g_1 y_1 + g_2 y_2} \tag{2}$$

Note that the income profiles $y = (y_1, y_2)$ and $\hat{y} = (\hat{y}_1 = g_1 y_1 - T, \hat{y}_2 = g_2 y_2 + T)$ have same level of inequality, following any measure that is relative in nature. Further, $\hat{y}_1 > y_1$ and $\hat{y}_2 > y_2$, holds if the society enjoys positive growth (i.e., $g_1 y_1 + g_2 y_2 > y_1 + y_2$). Hence $\hat{y} = (\hat{y}_1, \hat{y}_2)$ qualifies as a relative-SPS allocation. Similarly we can design the absolute-SPS allocation, where T has to be solved from the following absolute inequality preserving condition: $y_1 - y_2 = (g_1 y_1 - T) - (g_2 y_2 - T)$. Note that following the results from Marjit and Sarkar (2017); Marjit et al. (2018); Gupta et al. (2018) it can be easily established that SPS always exists and is unique. The existence of an SPS can also follow if we do the same exercise in the utility space. We use the well known box-diagram (figure 3) to highlight the welfare implication of the result.

Along the contract curve AB all points are Pareto superior to M. All such points are non-comparable in the sense of Pareto. This is because one can move from one point to the other (within AB) only by redistribution. Thus one is made better off and the other is worse off. However, our discussion so far ensures that among the non-comparable Pareto points on AB there exists a unique point which is distribution neutral. The formal proof is provided in the appendix.

Fig. 1 Edgeworth Box Digram reflecting existence and uniqueness of SPS



So far our discussion has been limited only two the case where the number of individuals is 2. Consider the number of individuals being n. Let $Y_0 = (y_1, y_2, .., y_n)$ be the initial income distribution. $Y_1 = (g_1.y_1, g_2.y_2, .., g_n.y_n)$ as the final income distribution. Thus g_i ($g_i \in \Re$) denotes the growth rate of income of the i^{th} individual $\forall i \in \{1, 2, .., n\}$. The SPS allocation that have same level of relative inequality as in Y_0 is given by

$$RSPS = (y_1^{RSPS}, y_2^{RSPS}, .., y_n^{RSPS}) \tag{3}$$

where $y_i^{RSPS} = y_i.\left(\frac{\sum_{i=1}^{n} g_i.y_i}{\sum_{i=1}^{n} y_i}\right)$.

Note that if there is growth in the economy (i.e., $\sum_{i=1}^{n} g_i.y_i > \sum_{i=1}^{n} y_i$), then every individual is better off in the distribution RSPS and relative inequality also remains the same. The tax transfers vector following which one can derive the RSPS allocation form the final distribution is given by the following vector:

$$t_{RSPS} = (t_1, t_2, .., t_n) \tag{4}$$

where $t_i = g_i.y_i - RSPS_i$. Note that if $t_i > 0$ then the individual has to pay tax. One the other hand if $t_i < 0$ then he enjoys transfer.

One can also preserve absolute inequality and make every one better off considering the following distribution.

$$ASPS = (y_1^{ASPS}, y_2^{ASPS}, .., y_n^{ASPS}) \tag{5}$$

where $y_i^{ASPS} = y_i + \left(\sum_{i=1}^{n} g_i . y_i - \sum_{i=1}^{n} y_i \right)$.

$$t_{RSPS} = (\tilde{t}_1, \tilde{t}_2, .., \tilde{t}_n) \tag{6}$$

where $\tilde{t}_i = g_i . y_i - ASPS_i$. The individuals pays tax (receives transfer) if $\tilde{t}_i > 0$ ($\tilde{t}_i < 0$).

4 Related Literature

The notion of distribution neutral fiscal policies exists in different forms in the literature. In this section we provide a discussion on some of the papers in this direction. We begin with a discussion of the poverty decomposition analysis introduced in Datt and Ravallion (1992). The authors introduced a methodology that decomposes poverty changes into growth and redistribution components.[1] The methodology is applicable only if the poverty measure is a function of poverty line (z) mean income (μ_t) and the underlying Lorenz curve (L_t). That is the poverty measure can be written as $P_t = P(z/\mu_t, L_t)$. Here t denotes the time point. Note that the level of poverty may change due to a change in the mean income μ_t relative to the poverty line, or due to a change in relative inequalities i.e. L_t. Let L_r denotes the Lorenz curve at any given reference frame.[2] Following Datt and Ravallion, a change in poverty from time t to t+n can be decomposed as

$$P_{t+n} - P_t = G(t, t+n; r) - D(t, t+n; r) + R(t, t+n; r) \tag{7}$$

where $G(t, t+n; r)$, $D(t, t+n; r)$ and $R(t, t+n; r)$ denotes the growth component, redistribution component and the residual, respectively.

The growth component (i.e., G()) refers to the the change in poverty due to a change in the mean while holding the Lorenz curve constant at the reference frame (i.e., L_r). Mathematically this can be written as:

$$G(t, t+n; r) = P(z/\mu_{t+n}, L_r) - P(z/\mu_t, L_r) \tag{8}$$

Note that if the Lorenz curve of the initial distribution is set to be the reference frame (i.e., $L_r = L_t$) then the component $G(t, t+n; r)$ can also be interpreted as the change of poverty from a SPS allocation.

The redistribution component is the change in poverty due to a change in the Lorenz curve while keeping the mean income constant at μ_r.

$$D(t, t+n; r) = P(z/\mu_r, L_{t+n}) - P(z/\mu_r, L_t) \tag{9}$$

[1] Readers interested in this topics are referred to Jain and Tendulkar (1990); Kakwani and Subbarao (1990); Kakwani (2000); Shorrocks (2013).

[2] This can also be interpreted as the desired level of inequality the policy maker is anticipating.

Poverty reduction is an important fiscal policy, and the total reduction of poverty following a distribution that do not aggravates inequality might also be an important tool for policy analysis. In the next section we shed some further lights on this issue using data from India. Note that presence of the residual term is often considered as a major criticism of the Datt and Ravallion poverty decomposition methodology. This problem has been addressed in Kakwani (2000); Shorrocks (2013).

Poverty decomposition analysis discussed is also a widely used in the pro-poor growth literature. The notion "*pro poor growth*" may be defined in two different senses. In general growth is said to be pro poor in an absolute sense, if it raises income of the poor and consequently poverty reduces (See Kraay, 2006) . Following Kakwani and Pernia (2000), growth is labeled as "pro-poor" in a relative sense, if it raises the incomes of poor proportionately more than that of the non poor. For further references readers interested are referred to Osmani (2005); Ravallion and Chen (2003); Son (2004); Nssah (2005); Duclos (2009).

5 Empirical Illustration

In this section we provide an application of distribution neutral fiscal policies. We show how poverty reduction which is viewed as an important fiscal policy is effected because of presence of inequality. Or in other words we are interested in the additional poverty reduction if the final distribution being replaced by the SPS allocation. This can also be viewed as an application of the Datt and Ravallion methodology that has been discussed earlier.

For this exercise we use the India Human Development Survey (IHDS) which is a nationally representative, multi-topic survey of households in India. The survey consists of two rounds. The first round was completed in 2004-05. The second round was conducted in 2011-12. This is an unbalanced panel data. That is, in the second round most households in 2004-05 were interviewed again. From here onwards we denote IHDS data for 2004-05 and 2011-12 as IHDS1 and IHDS2, respectively. Throughout this exercise, we consider IHDS1 as the initial time point and IHDS2 as the final time point. We use monthly per-capita expenditure as a proxy of income.[3]

Before we move through the formal application of the distributional fiscal policies we discuss how poverty and inequality has changed from 2004-05 to 2011-12. Throughout this exercise we focus our analysis on three mutually exclusive and exhaustive subgroups of India, namely rural, urban and the slum areas. Following Table 5 it is readily observable that poverty has decreed substantially from period 2004-05 to 2011-12. However, following the same table it is also readily observable that inequality figures has also increased substantially. Clearly following the discus-

[3] Note that per-capita income data is available in this survey. However, income data have standard problems in the sense that people often misreports their income. Furthermore, the poverty line in India is usually constructed using the per-capita consumption figures. Thus using such figures for income data might be incomplete.

sions in the previous sections the poverty reduction would have been more, had the degree of inequality remaining same as that of the initial distribution.

In Table 1 we present the actual poverty reduction. That is we simply compute the difference between poverty 2004-05 to that of 2011-12. In the same table we also compute the SPS poverty reduction. This is the difference between the poverty figures of the initial distribution and SPS distribution. It is readily observable that the absolute value of the SPS poverty reduction rates is much higher than that of the actual rates. For example, the net poverty reduction for rural India is 18.8% where as the same for an SPS distribution is 21.6%. Thus moving from the final to the SPS distribution would have reduced additional 3% poverty. Similar result also holds for the rest of the cases.

Table 1 Summary Statistics

	2004-05			**2011-12**		
	Rural	Urban	Slum	Rural	Urban	Slum
Real and Nominal Mean						
Mean	694.23	1792.67	1159.49	2881.88	828.02	2019.67
Inequality						
Gini	0.36	0.38	0.37	0.39	0.32	0.33
Atkinson (a=0.5)	0.11	0.12	0.12	0.13	0.08	0.09
Generalized Entropy (e=0)	0.22	0.24	0.23	0.25	0.16	0.18
Generalized Entropy (e=1)	0.25	0.29	0.26	0.31	0.17	0.21
Generalized Entropy (e=-1)	0.24	0.27	0.27	0.29	0.18	0.20
Poverty Rates						
Head Count Ratio	0.37	0.18	0.24	0.10	0.38	0.18
Poverty Gap	0.10	0.04	0.06	0.02	0.10	0.04
Poverty Reduction						
Head Count Ratio						
Net Poverty Reduction				-0.188	-0.140	-0.207
SPS Poverty Reduction				-0.216	-0.151	-0.225
Poverty Gap						
Net Poverty Reduction				-0.062	-0.039	-0.061
SPS Poverty Reduction				-0.070	-0.043	-0.065
Squared Poverty Gap						
Net Poverty Reduction				-0.026	-0.015	-0.025
SPS Poverty Reduction				-0.030	-0.016	-0.027

Notes

[1] *Author's calculations based on monthly per capita expenditure from IHDS data.*

[2] *Poverty figures are obtained considering poverty line as suggested by the Tendulkar Committee report for 2004-05. The poverty line for 2011-12 is obtained by inflating the 2004-05 line using CPIAL and CPIIW for rural and urban India respectively. Thus the poverty line for rural and urban India for 20111-12 is* $\frac{611}{340} \times 477 = 803$ *and* $\frac{195}{112} \times 579 = 1007$ *(ignoring the decimals), respectively. Real Mean is also obtained following the same procedure.*

[3] *Poverty differences:*
Net Poverty Reduction=(poverty rate in 2011-12)-(poverty rate in 2004-05).
SPS Poverty Reduction= (poverty rate for the SPS distribution)-(poverty rate in 2004-05)

6 Conclusion

In this paper, we provide a survey on distributional neutral fiscal policy and related literature highlighting the ideas of our new research in this area. We introduce the notion of "*Strongly Pareto Superior*" allocation or SPS. In order to focus on inequality-neutral or distribution neutral Pareto superior allocation we first discuss SPS allocation introduced in Marjit and Sarkar (2017) which guarantees higher individual welfare keeping the degree of inequality same as before. Also Marjit et al. (2018) provide an analysis in the context of international trade.

It is possible to generate a SPS allocation by taxing a subgroup of population and redistributing the collected tax to the rest of the population. The only condition required is that there should be growth in the society. The construction of SPS is different when relative and absolute inequality is preserved. The SPS allocation preserving the relative inequality is obtained by redistribution of the aggregate gains among the individuals proportional to their utilities of the initial distribution. On the other hand, the SPS allocation which preserves absolute inequality is obtained by equally distributing the aggregate gains among all the individuals. SPS is a general condition and whenever there is growth in the society one can generate both relative and absolute SPS uniquely. This paper also provides the existence and uniqueness of SPS in a general equilibrium framework. Considering the case with only two agents we show that SPS allocation is a unique point in the edgeworth box and also lies in the locus of the contract curve. Major contributions of this approach is that it provides a new interpretation of Pareto criterion, an interpretation that should have been done long ago. We propose that Pareto criterion IS also about inequality or distribution, a point totally missed and misinterpreted so far[4]. Additionally we rigorously prove that a unique Pareto efficient allocation is DIFFERENT from all non-comparable Pareto allocations on the well known contract curve.

We also provide an empirical illustration on distribution neutral fiscal policies. We show that poverty reduction following a SPS distribution is actually 3% higher than that of the actual poverty reduction. The exercise is somewhat similar to the poverty decomposition analysis introduced in Datt and Ravallion (1992).

The paper is consistent with the concern raised in Marjit (2016) regarding robust results in economics. Independent of the country or context we focus, we can always find out a Pareto efficient allocation that will keep the distribution intact. Our idea convincingly proves the point that Pareto criterion could be refined to include distribution and would have wider applications in the areas of growth, trade and public policy.

[4] An Wikipedia entry argues that "*It would be incorrect to treat Pareto efficiency as equivalent to societal optimization, as the latter is a normative concept that is a matter of interpretation that typically would account for the consequence of degrees of inequality of distribution.*" (https://en.wikipedia.org/wiki/Pareto_efficiency).

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7 Mathematical Appendix

Let $\{W^i\}$: endowment vectors $i = 1, 2, .., n; \sum\limits_i W^i = W$.

Utility functions, real valued, assumed to be continuous, increasing, strictly quasiconcave : $\{\bar{U}^i\}$ The consumption possibility set is $\mathfrak{R}^n_+$ the non-negative orthant. The set of feasible allocation vectors $\mathfrak{F} = \{\{Y^i\} : Y^i \geq 0, \sum_i Y^i \leq W\}$; $\{W^i\} \in \mathfrak{F}$; $\mathfrak{U} = \{\{U^i\} : \exists\{Y^i\} \in \mathfrak{F}, U^i(Y^i) = \bar{U}^i \forall i\}$ is the set of feasible utilities. Notice $\{U^i(W^i)\} \in \mathfrak{U}$.

Statement 1: $\mathfrak{U}$ is a nonempty compact subset of $\mathfrak{R}^n_+$
Proof: The above follows given the continuity of U^i over a compact set $\mathfrak{F}$.

Let $U^i(W^i) = \hat{U}^i \forall i$; there is a $P^* = (P_1^*, P_2^*, .., P_n^*)$ which is a competitive equilibrium i.e., X^{i*} solves the problem $maxU^i(.)$ subject to $P^*x \leq P^*.W^i \forall i$ and $\sum_i X^{i*} = W$. Let $U^i(X^{i*}) = U^{i*}$.

Define $\mathfrak{U}^{\mathfrak{P}} = \{\{\bar{U}^i\} : \{\bar{U}^i\} \in \mathfrak{U}, \nexists\{U^i\} \in \mathfrak{U}$ such that $, U^i > \bar{U}^i \forall i\}$: Pareto Frontier. The First Fundamental Theorem assures us that $\{U^{i*}\} \in \mathfrak{U}^{\mathfrak{P}}$.

Define $\bar{\mathfrak{U}} = \{\{U^i\} : U^i \in \mathfrak{U}$ *and* $U^i \geq U^i(W^i) \forall i$; we shall be concerned with the set $\mathfrak{V} = \bar{\mathfrak{U}} \cap \mathfrak{U}^{\mathfrak{P}}$. Notice that $U^{i*} \in \mathfrak{V}$.

In case $\{U^i(W^i)\} \in \mathfrak{V}$ there is nothing more to prove; since then the initial endowment distribution is Pareto Optimal and it must be the case that $X^{i*} = W^i \forall i$. Excluding this case, we proceed as follows:

Define $U(\theta) = \theta\{U^i(W^i)\}, \theta > 0$; for $\theta = 1$, $U(1) = \{U^i(W^i)\} \in \bar{\mathfrak{U}}$, since $\bar{\mathfrak{U}}$ is a closed subset (weak inequalities in the definition) of a compact set $\mathfrak{U}$, it too is compact and consequently $sup_{\{U(\theta)\} \in \bar{\mathfrak{U}}} \theta = \hat{\theta}$ is well defined. Note that the supremum of a bounded non-empty set of real numbers always exist; nonempty, because $\theta = 1$ is a possibility and bounded because $\bar{\mathfrak{U}}$ is compact.

Statement 2: Properties of $\hat{\theta}$: $\theta > \hat{\theta}$ implies $U(\theta) \notin \bar{\mathfrak{U}}$ further $\theta \leq \hat{\theta}$ implies $U(\theta) \in \bar{\mathfrak{U}}$.

Proof: The first part follows from the definition of $\hat{\theta}$. The second property can be broken into two parts. Consider $U(\hat{\theta})$ first; by definition, there is a sequence $\theta^s \to \hat{\theta}$ such that $U(\theta^s) \in \bar{\mathfrak{U}}$; continuity of $U(.)$ and compactness of $\bar{\mathfrak{U}}$ completes the assertion that $U(\hat{\theta}) \in \bar{\mathfrak{U}}$; hence there is $\{\hat{Y}^i\} \in \mathfrak{F}$ such that $U^i(\hat{Y}^i) = \hat{\theta} U^i(W^i) \forall i$.

Consider next $\bar{\theta} < \hat{\theta}$; from the property of the supremum, there is $\tilde{\theta} > \bar{\theta}$ such that $U(\tilde{\theta}) \in \mathfrak{U}$ i.e., there is $\tilde{Y}^i$ such that $\{\tilde{Y}\} \in \mathfrak{F}$ and $U^i(\tilde{Y}^i) = \tilde{\theta} U^i(W^i) \forall i$. Now there must be a scalar $1 > \alpha_i \geq 0$ such that $U^i(\alpha_i \tilde{Y}^i) = \bar{\theta}.U^i(W^i)$ since by shrinking the scalar α_i we can make the left hand side go to zero (U is increasing), whereas for $\alpha_i = 1$, the left hand side is greater; also then we can claim $\{\alpha_i \tilde{Y}^i\} \in \mathfrak{F}$ since

$$\alpha_i \tilde{Y}^i \geq 0$$

and

$$\sum_i \alpha_i \tilde{Y}^i \leq max_i \alpha_i \sum_i \tilde{Y}^i \leq W$$

Q.E.D.

Statement 3 $U(\hat{\theta}) \in \mathfrak{V}$

Proof: Since, the initial configuration has been taken to be not Optimal, $\hat{\theta} > 1$ and hence $U(\hat{\theta}) \in \bar{\mathfrak{U}}$. Remains to show that $U(\hat{\theta}) \in \mathfrak{U}^{\mathfrak{P}}$. If not, it must be possible to increase θ and still remain within $\mathfrak{U}$: a contradiction. Hence the claim. **Q.E.D.**

Thus one may conclude that $\{\hat{Y}^i\}$ is a Pareto optimal configuration with the property that $U^i(\hat{Y}^i) = \hat{\theta} U^i(W^i) \forall i$. And since $\hat{\theta}$ is uniquely determined, so is the configuration $\{\hat{Y}^i\}$.