Revenue and efficiency in pollution permit allocation mechanisms

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Abstract

The most contentious design issue within pollution markets is the choice of initial allocation mechanism. Within this debate, auctions have become the predominant method of initial permit allocation. Although auctions provide potential gains—such as revenue generation, allocative efficiency and clear price discovery—these benefits are rarely fully realized due to firms submitting non-truthful bids. We propose a mechanism that can improve on existing auctions. In our design the regulator determines the supply (up to an upper bound) once all bids have been submitted. This simple and applicable design incites truthful revelation of firms' private abatement costs, maximizes revenue, and allocates the permits efficiently. This design is relevant to all existing permit auctions including those in the European Union Emissions Trading Scheme (EU-ETS), Regional Greenhouse Gas Initiative (RGGI), and the California Cap-and-Trade Program.

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1 Introduction

Pollution markets are now a mainstream environmental policy. Although pollution market design has expanded at pace over the last number of decades, a continual debate exists over the design of the initial allocation mechanism: should permits be auctioned or freely allocated? In general, economists have favored auctioning due to the potential for revenue generation, allocative efficiency, and clear price discovery (e.g., Cramton and Kerr, 2002; Goeree et al., 2010; Lopomo et al., 2011). Indeed, these arguments are clearly visible in contemporary market design choice; auctioning is now the predominant method of permit initial allocation in the Regional Greenhouse Gas Initiative (RGGI), the European Union Emissions Trading Scheme (EU-ETS), and the California Cap-and-Trade Program.

With the increasing use of auctions in permit markets it has become clear, however, that the proposed benefits are often not fully realized. Indeed, the specific design and implementation of permit auctions may generate significant barriers for the generation of revenue, allocative efficiency, and price discovery. It is well known that the uniform-price format—used in the majority of permit auctions—is susceptible to 'demand reduction' where firms shade their bids in order to reduce the final price they pay for all permits (Ausubel et al., 2014).[1] The most direct consequence of demand reduction is that firms will submit non-truthful demand schedules (bids) within the auction. This then provides incorrect signals to the regulator as well as the market via an inaccurate equilibrium price. Thus current auctions may not be effective in true price discovery. It is also likely that firms shade their bids to varying degrees. Consequently, we may expect the auction to allocate permits inefficiently: permits may be allocated to firms that do not have the highest values. Demand reduction also has significant effects on the generation of revenue. If firms shade their bids, then it is quite clear revenue will not be maximized from such an auction. Given the potential issues with current initial allocation mechanisms, it is important then to consider if auctions can be (re-)designed to fully realize these potential benefits.

In this article we propose a simple mechanism for the initial allocation of permits. The key innovation within this proposed mechanism is that the regulator does not commit to a fixed permit supply. Instead, the regulator sets an upper bound of permit supply and once all bids have been collected, the regulator adjusts permit supply to maximize their revenue. In our model, we assume firms have private information over

[1] An alternative to a uniform-price auction is the pay-your-bid (discriminatory) auction, which is less commonly observed within pollution markets: the U.S. sulfur dioxide auctions being the only leading example (Lopomo et al., 2011)

their willingness to pay for permits. In particular, we allow each firm to have a private and diminishing marginal abatement cost function that determines their demand for permits up to a maximum (and heterogeneous) capacity for permits. Firms submit demand schedules using a sealed-bid auction, where the regulator aggregates all bids to determine the aggregate demand schedule. The regulator then selects the permit auction supply (up to a maximum bound) that maximizes revenue.

There are a number of merits of implementing this scheme. First, this simple process incites firms to submit their true demand curves (marginal abatement costs) to the regulator. If a firm were to submit a non-truthful demand schedule (i.e., they choose to demand reduce), the regulator—in order to maximize revenue—would reduce the permit supply, ensuring the clearing price does not decrease. Thus if firms submit untruthful (lower) demands they would end up receiving less units and recieving a lower payoff than if they truthfully reported their demand schedule. This means that the regulator can determine the true aggregate marginal abatement cost curve by using only the information provided by firms' submitted demands. Second, the revenue to the regulator is maximized. The collection of a consistent stream of revenue is now an important requirement in many schemes that can generate revenue-recycling effects.[2] Third, this process is allocatively efficient: as bids are truthful, the permits are allocated to buyers with the highest value and this provides improved price discovery for the secondary market.[3] Fourth, there is the potential for social welfare improvements using this auction. The regulator's upper bound of emissions is likely to be larger than the associated first-best level. Thus the mechanism proposed in this article has the ability to reduce emissions and generate social welfare improvements.

1.1 Related literature

This article proposes a new mechanism that could improve on existing auction designs. A relatively new literature exists on exploring the efficiency and cost effectiveness issues relating to permit auctions (e.g., Lopomo et al., 2011; Alvarez et al., 2017; Khezr and MacKenzie, 2018a,b). Alvarez et al. (2017) explores the cost effectiveness of permit auctions where bidders have linear and downward sloping abatement costs functions and shows that within the multiplicity of equilibria only one minimizes abatement costs. Khezr and MacKenzie (2018a) investigates permit auctions where the permits are

[2] See, for example, RGGI (2018) and European Commission (2017).

[3] This is increasingly important when the market is not perfectly competitive and thereby allocative efficiency is not assured (Dickson and MacKenzie, 2018).

initially consigned to the bidders. In this framework they show that the permit auction does not provide an accurate price signal. Further, Khezr and MacKenzie (2018b) analyzed permit auctions with allowance reserves, such as those used within the Regional Greenhouse Gas Initiative (RGGI). They show that in many cases the allowance reserve may not work as intended: the reserve of permits places upward pressure on the auction clearing price rather than reducing firms' costs. The current literature has focused on the weaknesses of existing auction design. While it is important to highlight the existing inefficiencies, what is currently missing is the design of a permit auction that avoids these potential weaknesses. This article fills this gap. We add to this literature by providing a new auction design. A key added value is the simplicity of the mechanism. In our approach, the regulator can use a conventional uniform-price auction with one modification: that they allow the permit supply to be determined after bids have been submitted. This change generates an allocatively efficient auction while simultaneously creating truthful bidding and the maximization of revenue.

Our auction design focuses on the *ex post* choice of permit supply. There exists a small but growing literature that investigates auctions with *ex post* supply adjustment (Back and Zender, 2001; LiCalzi and Pavan, 2005; McAdams, 2007; Damianov and Becker, 2010; Khezr and MacKenzie, 2018c). This literature has been able to explore the benefits of adjustable supply compared to a fixed supply. However, given the application of their models—mainly to treasury auctions—the focus has been on buyers that have constant and known marginal valuations.[4] While the assumption of a constant and common value for a pollution permit may make sense in a single auction, it is generally recognized that polluting firms have private information over their abatement costs and these costs may decrease in their emissions level. Thus to fully investigate a new permit auction design, we require a bespoke analysis that can incorporate firms' private information on their marginal abatement costs as well as the potential for decreasing marginal values over the use of permits. In this article, we allow for firms to have both asymmetric emission capacities and marginal abatement costs. Further, their capacities and slopes of their marginal abatement costs are assumed to be private information. Thus with this setup we advance the auction literature by highlighting the efficiency and revenue maximizing improvements with an *ex post* supply mechanism under more general conditions.

[4]Khezr and MacKenzie (2018c) investigates auction design when firms have constant marginal values that are private information but where firm capacities are common knowledge. Their supply adjustment focuses on ensuring all winning firms' capacities are fulfilled in the auction. This process is not possible in this article as we assume firms have private information about their emissions capacity (unconstrained level of emissions). Thus we are able to design a process that incites truthful bidding and maximizes revenue in a more complex informational environment.

A key outcome of this proposed approach is that firms have an incentive to truthfully reveal their marginal abatement cost structures (their true demand for permits). Dating back to Kwerel (1977), there has been a large literature that has attempted to create regulatory structures that incentivize truthful revelations of abatement costs (e.g., Dasgupta et al., 1980; Duggan and Roberts, 2002; Montero, 2008; Shrestha, 2017). For example, Montero (2008) creates a uniform-price auction that can incite truthful bidding if there exists firm-specific rebates (based on the damage they create) and an *ex post* permit supply choice for the regulator. Intuitively, having the rebate and *ex post* supply makes use of the Vickrey-Clarke-Groves (VCG) mechanism to incite truthful bidding. However, one drawback of the VCG mechanism in the presence of externalities is that it often requires substantial amounts of information for the mechanism to work efficiently.[5] For example, the regulator may need to know the true damage function of emissions to obtain the first-best outcome. In our approach we depart from using a VCG model by advancing existing auction designs to incite truthful bidding (and hence the truthful revelation of abatement costs without use of a VCG mechanism). In our mechanism we can ensure truthful bidding by only requiring the regulator to select the supply of permits *ex post* without any rebate. Thus we are able to generate truthful bids while simultaneously maximizing revenue for the regulator.

Our contribution is thus two-fold. First we advance the knowledge of auction design by extending the scope of *ex post* supply mechanisms. Thus we can consider such mechanisms within more general settings where firms' values are private information and decreasing in the number of units. Second, within these general settings we can now provide clear understanding as to how to improve existing permit auctions. Using such a mechanism may maximize revenue, efficiently allocate permits, and incite truthful revelation of abatement costs without the use of rebates or additional regulatory compliance structures.

The remainder of this article is structured as follows. In Section 2 the model is outlined and a benchmark case is derived using fixed permit supply. Subsection 2.3 outlines the variable permit supply aspect of the model. Section 3 compares the social welfare of the proposed mechanism. Section 4 provides some concluding remarks. All proofs are relegated to the Appendix.

[5]Also note that it would be impossible to use a VCG mechanism in a permit market auction without the use of rebates as the tendency is to use uniform-price auctions, where VCG mechanisms cannot work. For the rationale behind the use of uniform-price auctions for permits (rather than discriminatory auctions) see Lopomo et al. (2011).

2 The Model

2.1 Preliminaries

Consider a cap-and-trade market with n firms indexed by $i = \{1, ..., n\}$, where $n > 2$. Each firm i has a demand for permits given by

$$d_i(p) = k_i - \theta_i p, \tag{1}$$

where p is the price paid for all units, $\theta_i > 0$ is the type of firm i, and k_i is firm i's heterogeneous unconstrained level of emissions (their emissions capacity). We assume throughout that each firm's type θ_i and unconstrained level of emissions k_i are private information and thus only known to themselves. Also denote $D(p)$ as the aggregate demand so that $D(p) = \sum_{i=1}^{n} d_i(p)$.

Note that this demand function can be interpreted as the marginal abatement cost function of each firm. In particular, rearranging (1), firm i's marginal cost function is $c_i^{'}(q_i) = \bar{v} - \frac{1}{\theta_i} q_i$, where $q_i > 0$ is their level of emissions, the slope of the marginal abatement cost function is determined by the parameter $\theta_i \equiv \frac{k_i}{\bar{v}} > 0$ where $\bar{v} > 0$ is the marginal cost with zero emissions, $c_i(0) = \bar{v}$ (i.e., their value of a permit with zero emissions). This then means that firms' marginal abatement cost functions are linear and private information with a common vertical intercept. The sum of firms' unconstrained emissions—the aggregate unregulated emissions level—is denoted by $K = \sum_{i=1}^{n} k_i$. The regulator has an emissions target given by $\bar{Q} > 0$ and we assume throughout that this policy is a second-best target derived through a political process.

Assumption 1. $\frac{K}{2} \leq \bar{Q} < K$.

Assumption 1 avoids trivialities by ensuring that the target is indeed binding so that firms' aggregate unconstrained emissions are larger than the regulator's emissions target as well as the policy not becoming prohibitively restrictive, something that appears to be realistic for most pollution regulation.[6] If $\frac{K}{2} > \bar{Q}$ then the analysis reduces to a conventional permit auction.

Denote Q^{**} as the welfare maximizing emissions level, where the true aggregate permit demand (aggregate costs of abatement) equals the marginal damages of emissions.

[6]Exceptions, of course, do exist. Most notable was the over supply of pollution permits within the EU-ETS (Kollenberg and Taschini, 2016; Perino and Willner, 2016). This problem was tackled by the European Commission introducing the Market Stability Reserve (MSR) to ensure an appropriate level of liquidity within the market.

Assumption 2. $Q^{**} < \bar{Q}$.

Assumption 2 ensures the welfare maximizing level of emissions to be lower than the regulator's upper bound of permits. Again, in most existing permit markets, this is indeed the case as permit supply is not first-best and usually much higher.

Our initial allocation mechanism for permits is a sealed-bid uniform-price auction where each firm submits a demand schedule as a sealed bid for the permits they require. We denote p^e as the clearing price derived from the intersection of the true aggregate demand and permit supply $\bar{Q}$ within the auction.

2.2 Benchmark case: an auction with fixed supply

Before outlining a new auction design, it is important to provide a clear benchmark of the conventional permit auction with the associated equilibrium characteristics. Consider then a case where the regulator sells all $\bar{Q}$ permits in a uniform-price auction. The clearing price from the auction is given by

$$p^* = \max\left\{p | D(p) \geq \bar{Q}\right\}. \tag{2}$$

It is well-known that when the supply is fixed every undominated equilibrium of the uniform-price auction is inefficient (Krishna, 2002, Chp. 13). This inefficiency is the result of the so-called *demand reduction*. In particular, buyers submit demand schedules that are below their actual demand function. Intuitively, given the clearing price is the price paid for *all* units there is an incentive for firms to bid lower than their maximum willingness to pay for at least some units in order to lower the auction clearing price. As a result, the aggregate demand function is strictly below the true aggregate demand. The following proposition summarizes the known results related to the equilibrium behavior in uniform-price auction with fixed supply.

Proposition 1. *Any undominated equilibrium of the uniform-price auction with fixed supply has the following properties.*

1. *All firms submit demand schedules that are strictly below their actual demand.*

2. *The aggregate demand is strictly below the true aggregate demand.*

3. *The regulator's revenue is not maximized.*

4. At least some units are not allocated to buyers with the highest values for them.

To elicit the result of the above proposition suppose $D^{\dagger}(p)$ is a representative submitted aggregate demand of the uniform-price auction with a fixed supply and $p^{\dagger}$ is the corresponding clearing price. Given Proposition 1, $D^{\dagger}(p)$ must be lower than $D(p)$ and therefore $p^{\dagger}$ is lower than p^{e}, as shown in Figure 1. Due to the existence of demand reduction the fixed supply of permits is now sold at a price lower that the competitive equilibrium price and, consequently, the revenue generated is lower (and not maximized). Thus there has been an information rent transfer from the regulator to the regulated firms.

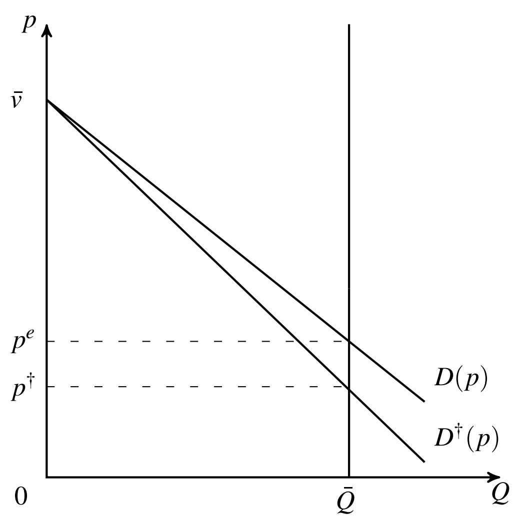

Figure 1: Fixed permit supply with the associated demand reduction.

Another problem with demand reduction is the impact on allocative efficiency. Given that all firms have incentives to bid lower for at least some units they demand, the resulting permit allocation—which is the result of their reduced demand—is not necessarily to those with the highest willingness to pay. In other words, firms are willing to lose some units in order to pay a lower price for other units they win (Krishna, 2002, Chp. 13).

Given that the conventional permit auction may incite demand reduction, a loss in generated revenue, and a lowering of allocative efficiency, we now propose a mechanism that attempts to address these issues.

2.3 An auction with variable supply

Let us now consider the case where the regulator can choose the exact amount of permit supply after the bids have been submitted. The structure of the auction is as follows. First, firms submit linear demand schedules to the auctioneer (regulator). Given our setting, this is equivalent to submitting a capacity k_i. Then the regulator aggregates the submitted demand schedules and selects a quantity Q^* within the bounds $Q^* \in [0, \bar{Q}]$. In particular, the regulator chooses Q^* that maximizes the revenue based on the submitted demands.[7] Given the regulator's choice of permit supply Q^*, and given the aggregate demand submitted by firms, all the units demanded up to Q^* will be allocated to firms with highest submitted demands and they will pay a uniform price equal to p^*. The payoff of each buyer is the area under their demand function up to the number of units they win minus the amount of money they pay. So if firm i with a demand function $d_i(p) = k_i - \theta_i p$ submits a demand equal to $d_i'(p) = k_i' - \theta_i' p$, their payoff becomes

$$\pi_i(p^*, q_i^*, k_i, k_i') = \int_0^{q_i^*(k_i')} p_i(x)dx - p^*(k_i')q_i^*(k_i'), \tag{3}$$

where $p(x)$ is the inverse of the demand function $d_i(.)$.

As mentioned, the payoff firm i receives is the area under their true demand up to the allocated units q_i^*. However, the submitted demand affects the aggregate demand and consequently the price experienced by all firms. This would also affect the allocated units to firm i, q_i^*, and therefore this quantity is also a function of k_i'. The next proposition shows that the variable supply scheme eliminates any incentives for demand reduction.

Proposition 2. *The unique equilibrium of the variable supply mechanism has the following properties.*

1. *All firms submit their true demand functions.*
2. *The aggregate demand represents the true market demand.*
3. *The seller's revenue is maximized.*
4. *Units are allocated to buyers with the highest values.*

Given the equilibrium of Proposition 2, define $D^*(p)$ as the equilibrium aggregate demand schedule

[7] This is similar to the supply adjustment first proposed in Back and Zender (2001).

submitted by firms. As suggested by the above result $D^*(p)$ represents the true demand of all firms. Also denote p_m^* as the equilibrium clearing price for the variable supple scheme.

An important consequence of this auction design is that there exists an incentive to submit truthful demand schedules (that is to truthfully reveal their marginal abatement costs). To see this consider Figure 2. Along with the true demand schedule $D(p)$ we have also drawn a non-truthful aggregate demand $D^{\dagger}(p)$. If firms submitted truthful demand schedules then the equilibrium supply would be Q^* with a corresponding clearing price p^*. Now if some firms submit demands that are lower than their actual demand, the auction mechanism would decrease the supply to $Q^{\dagger}$. This not only decreases the allocated units to those firms but also ensures the equilibrium price never decreases. As a result, firms' payoffs would decrease with such actions. Therefore, the supply reduction acts as a valid punishment and makes the mechanism incentive compatible or truth-telling.

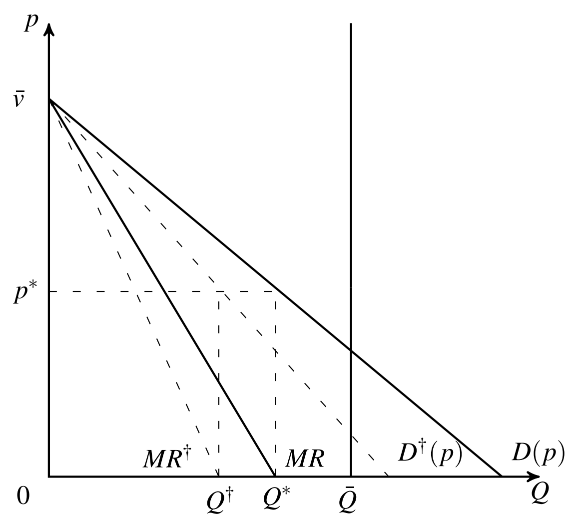

Figure 2: Truth revelation within variable permit supply.

As a consequence of truthful bidding it follows that the auction is allocatively efficient. Firms that have the highest value for permits will receive them. Moreover, as the regulator selects permit supply to maximize revenue, it is assured that revenue is maximized as firms truthfully submit their demand schedules.

3 Social welfare implications

In the previous section we highlighted the potential benefits of a variable supply in permit auction design. The focus was on revenue maximization and allocative efficiency. Yet a regulator may also have other objectives associated with, for example, improving social welfare. In this section we investigate the consequences for social welfare with this new auction design. To begin let us denote the marginal damage function of emissions by $S(q) = \alpha + \beta q$, with $\alpha, \beta \geq 0$. The intersection of the marginal damage function and the true aggregate demand gives us the optimal level of permit quantity and price, which we denote as Q^{**} and the price as p^{**}, respectively.

Given the marginal damage function and the aggregate demand, the optimal level of permit quantities becomes

$$Q^{**} = \frac{\bar{v} - \alpha}{\beta + \frac{\bar{v}}{K}}, \tag{4}$$

and the optimal price becomes

$$p^{**} = \frac{K\beta + \alpha}{1 + \frac{K\beta}{\bar{v}}}. \tag{5}$$

Although our main focus will be on comparing second-best outcomes, it is useful to consider when the variable permit supply design is socially efficient.

Proposition 3. *The variable supply mechanism results in a socially efficient outcome if and only if*

$$2\alpha + K\beta = \bar{v}. \tag{6}$$

The condition required for the first-best outcome to be realized is relatively straightforward for the regulator, which involves the characteristics of the marginal damage function, the aggregate unconstrained level of emissions, as well as the maximum willingness to pay for a permit. Note that there can clearly be a misalignment between revenue maximization and the first-best outcome. An interesting point to note is that if the regulator decided to select permit supply to maximize social welfare, then truthful bidding would not occur. Recall from Proposition 2 that truthful bidding is a consequence of the regulator's choice to maximize revenue and the associated implicit punishment mechanism with supply adjustment.

So far we have assumed that marginal damages are increasing in Q. This makes sense if we think of the auction as the allocation of rights for a large level of emissions (perhaps, for example, for the full environmental problem). Yet most auctions are organized quarterly, where the level of emissions is sufficiently small relative to the total environmental problem, which thus means that marginal damages can be approximated with a constant function. Indeed this can accurately reflect the case of CO_2 pollution permits, where the estimated marginal damages are flat (See, for example, Newell and Pizer, 2003). The following corollary shows the condition for the first-best outcome to occur with constant marginal damages.

Corollary 1. *With a flat marginal damage function the condition in Proposition* 3 *becomes*

$$2\alpha = \bar{v}. \tag{7}$$

Intuitively, the first-best outcome occurs when the maximum willingness to pay for a permit is twice the size of the marginal damage. For pollution permits, such as those for CO_2 allowances, this provides a very quick method to observe the extent to which this variable permit supply is socially optimal.

For the fixed permit supply auction, the socially optimal outcome occurs simply when the target is set equal to the optimal level.

Proposition 4. *When the target $\bar{Q}$ is not set at the socially optimal level Q^{**}, the fixed supply mechanism would result in socially inefficient outcomes. If the target $\bar{Q}$ is exactly equal to Q^{**}, even though the mechanism achieves the social optimal level of pollution, there is still a transfer of information rents from the regulator to polluters.*

Proposition 4 suggests that when the target $\bar{Q}$ is set above the social optimal level of emissions, then the fixed supply mechanism would always result in a deadweight loss to society. Also given the existence of asymmetric information in this market, it is impossible for the regulator to realize the optimal level of emissions from an *ex-ante* perspective and set the target accordingly. However, even if it was possible to set the target equal to Q^{**}, because of the strategic behavior of firms which results in the demand reduction, there exists an information rent transferred from regulator to firms of the size $(p^e - p^*)\bar{Q}$. Contrast this with the variable supply design that would eliminate such informational rents.

While it is interesting to consider the first-best outcomes, it is more relevant to compare these auction designs in the context of a second-best environment, where the regulator knows neither firms' abatement costs nor marginal damages associated with the emissions.

3.1 Social welfare comparisons

Let us begin by determining the deadweight losses associated with a sub-optimal target. Now for any submitted aggregate demand schedule $D(p)$ the following expression defines the deadweight loss for the fixed supply auction

$$\psi(q) = \int_{Q^{**}}^{\bar{Q}} (S(q) - P(q))dq, \tag{8}$$

where $P(q)$ is the inverse of $D(p)$. This can be observed within Figure 3 by the triangle ade. Due to the existence of demand reduction the lower submitted demand will generate a lower clearing price than p^e and, as explained previously, this will generate an information rent to firms.

Next we define the deadweight loss for the variable supply mechanism as follows

$$\Delta(q) = \int_{Q^{*}}^{Q^{**}} (P(q) - S(q))dq. \tag{9}$$

Note two cases occur, either (i) $Q^* > Q^{**}$ with loss $|\Delta(q)|$ or (ii) $Q^* < Q^{**}$ with the resulting loss $\Delta(q)$. Let us begin with considering case (i). In Figure 3 (a) case (i) is represented by the triangle abc. It is quite clear that triangle abc is strictly smaller than ade. Therefore it is straightforward to conclude that the variable supply mechanism results in higher social welfare than the fixed supply mechanism.

Let us now consider the case where $Q^* < Q^{**}$. Figure 3 (b) shows the case where the deadweight loss from the variable supply is given by abc and the loss from fixed supply is given by ade. From Figure 3 (b) it is now ambiguous whether the fixed or variable permit supply results in an equilibrium closest to the first-best equilibrium. It is, however, clear from Figure 3 (b) that the variable permit supply is likely to be closest to first-best as Q^* increases. The following proposition provides the exact conditions for this to hold.

Proposition 5. *If $Q^* < Q^{**}$, then the variable supply mechanism results in higher social surplus compared to the fixed supply as long as*

$$\frac{2(\bar{v} - \alpha)}{\beta + \frac{\bar{v}}{K}} - \frac{K}{2} < \bar{Q}. \tag{10}$$

As can be observed in Proposition 5 the variable supply design results in higher social welfare when

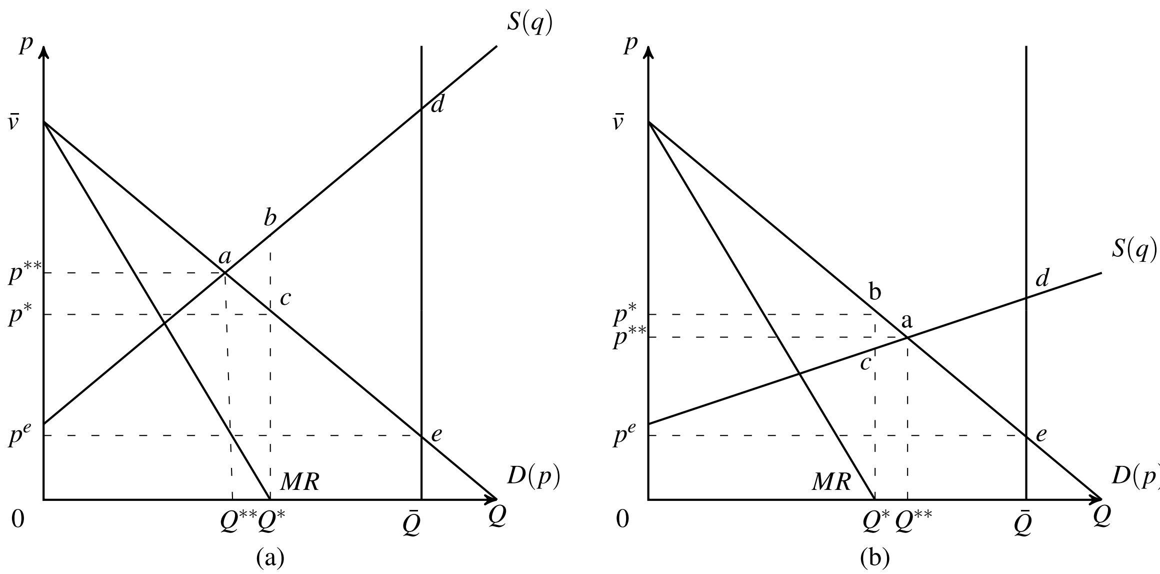

Figure 3: Respective social losses from variable and fixed supply.

the marginal damages are sufficiently steep (larger β) (or a large α) as well as when the regulator's fixed target is sufficiently large.

Thus using an auction with an *ex post* permit supply choice can incite truthful bidding, maximize revenue and, in many cases, generate an equilibrium closer to the welfare-maximizing policy target. Recall that this can be done by the regulator with only minimal information. In particular, the regulator can achieve this outcome without knowing the marginal damages, firms' unconstrained emissions, or firms' marginal abatement costs.

4 Concluding remarks

The purpose of this article is to design a mechanism that can allocate pollution permits efficiently while simultaneously maximizing the generated revenue. Current auction designs use a uniform-price format, which ensures firms pay the same price for each permit. Although this is the preferred method to allocate permits at present, it is however, susceptible to 'demand reduction', where firms shade their bids in order to reduce the price they pay for all permits won. Demand reduction is an important issue: as firms submit bids that are untruthful this results in allocative inefficiency, a loss in generated revenue, as well as having inaccurate price discovery for the secondary market.

To solve these problems we propose a new design of the uniform auction. In particular, instead of allocating a fixed supply of permits—as is done in all existing auctions—we allow the regulator to select the supply of permits (from an upper bound of permits) after all bids have been submitted. The regulator selects the number of permits that will maximize revenue from the submitted bids. This simple mechanism has the benefit of inciting truthful bids and, therefore, revealing firms' private marginal abatement costs. If a firm decides to shade their bids then, in order to maximize revenue, the regulator would reduce supply while ensuring the clearing price was non-decreasing. This would mean firms receive less permits and, consequently, have a lower payoff. Firms thus always have an incentive to reveal their true permit demands. Due to the truthful bidding of firms, the auction then becomes allocatively efficient with the maximum revenue generated.

Previous attempts to incite truthful revelations of firms' marginal abatement costs require mechanisms that are often informationally demanding. The main innovation of this design is the minimal information requirements to incite truthful behavior. We investigate this mechanism in a second-best world, where the regulator knows neither firms' marginal abatement costs nor the marginal damages associated with the emissions. Even so, we provide conditions where the proposed auction design may generate an equilibrium closer to the first-best outcome compared to conventional auction designs.

This auction design has direct relevance to current auctions in the Regional Greenhouse Gas Initiative (RGGI), the European Union Emissions Trading Scheme (EU-ETS), and the California Cap-and-Trade Program, and can be implemented with only minor modifications to existing formats.

Appendix

Proof of Proposition 1

Proof. Suppose all firms except firm i submit their true demand function. We first want to show it is not optimal for firm i to do so. To see this note that firm i's payoff is

$$\pi_i(p^*, q_i^*(k_i)) = \int_0^{q_i^*(k_i)} p_i(x)dx - p^*(k_i)q_i^*(k_i). \tag{11}$$

where $p_i(x)$ is the inverse of the demand function $d_i(.)$. Differentiating the payoff with respect to k_i gives

$$\frac{\partial \pi_i(p^*, q_i^*, k_i)}{\partial k_i} = \frac{dq_i^*}{dk_i} p_i(q_i^*) - p^*(k_i)\frac{dq_i^*}{dk_i} - \frac{dp^*}{dk_i} q_i^*(k_i). \tag{12}$$

Note that when firm i acts truthful, the first two terms become equal and cancel. What remains is the third term. It is easy to check that the price is increasing in k_i and therefore the last term makes the derivative negative. Therefore by reducing k_i firm i can increase their payoff.

This argument is true for all firms and therefore, submitted demand schedules are strictly below their actual demands. This concludes part 1. Part 2 is the corollary of part 1, that is, when all firms submit lower demands the aggregate demand is strictly below the true aggregate demand. This is enough to conclude that the regulator's payoff is always below the maximum payoff. Finally, when firms submit lower demands, they lose some units in order to pay lower price for other units they win. Given that the overall allocated units are still $\bar{Q}$ these units are allocated to some other firms with lower willingness to pay. This is essential in order to reduce the equilibrium price. Thus at least some of the units are not allocated to those with highest values for them.

□

Proof of Proposition 2

Proof. We start by proving the existence of the equilibrium. First, suppose all buyers except buyer i submit their true demands. If firm i submits the true demand, then based on the supply adjustment,

$MR(Q) = 0$ gives us the Q^* and the aggregate demand gives p^*. Firm i's payoff is then

$$\pi_i(p^*, q_i^*(k_i)) = \int_0^{q_i^*(k_i)} p_i(x)dx - p^*(k_i)q_i^*(k_i). \tag{13}$$

The regulator's profit function for any quantity Q is, $\Pi(Q) = p(Q)Q$. So the supply is given by,

$$p'(Q^*)Q^* + p(Q^*) = 0, \tag{14}$$

which gives $Q^* = \frac{K}{2}$.

Now suppose that firm i submits a demand with $k_i' < k_i$, then the aggregate demand becomes steeper and shifts to the left. Thus the supply adjustment rule results in an equilibrium quantity $Q' < Q^*$ with a price $p' \geq p^*$. Also note that since the submitted demand is below the previous demand (the true demand), the final allocated quantity to firm i, which is based on the submitted demand, reduces. To see the effect on the payoff, differentiate Equation (3) with respect to k_i',

$$\frac{\partial \pi_i(p^*, q_i^*, k_i, k_i')}{\partial k_i'} = \frac{dq_i^*}{dk_i'} p_i(q_i^*) - p^*(k_i') \frac{dq_i^*}{dk_i'} - \frac{dp^*}{dk_i'} q_i^*(k_i'). \tag{15}$$

The first two terms are equal when $k_i' = k_i$. Also the first term is larger than the second term in absolute value when $k_i' < k_i$. Given the current supply adjustment the last term becomes zero. So we can conclude that $k_i' < k_i$ is dominated by truthful bidding.

Note that the first term is smaller than the second term in absolute value when $k_i' > k_i$. This also proves that $k_i' > k_i$, is dominated by $k_i' = k_i$. That is, if firm i submits a demand with $k_i' > k_i$, then the aggregate demand will become flatter and the equilibrium price p' will be lower or equal to p^*. However, at any price $p' > p^*$ firm i's allocated units is above its true demand. So they make losses for the extra units and their overall payoff decreases. This concludes the proof for claim number 1. Claim number 2 is a corollary of claim 1. In particular, when all firms submit their true demand then the aggregate demand is the true market demand. Claim number 3 is the result of the supply adjustment and the truthful bidding together. So given that the regulator essentially maximized the revenue based on the true aggregate demand, this mechanism is revenue maximizing. Claim number 4 is also the result of truthful bidding.

Now we prove the uniqueness of this equilibrium by contradiction. Suppose there exists another equilibrium strategy with demand schedules $d^{\dagger}(q_i) < d^{*}(q_i)$ which is untruthful. So it must be the case that at least some buyers submit demand schedules below their actual demand. Focus on one of those buyers, say buyer i. If she increases her demand to her actual demand, she would not pay a higher price and would receive more units at a price not larger than her willingness to pay, which is a contradiction as such equilibria cannot exist.

□

Proof of Proposition 3

Proof. As we know from the previous section, the equilibrium quantity of the variable supply mechanism is $Q^{*} = \frac{K}{2}$. Given that the equilibrium of the variable supply is truth-telling, it will result in a socially efficient outcome if the quantity of supply is equal to Q^{**}. This happens when the marginal damage function intersect the demand at Q^{*}. The quantity at the intersection is given by,

$$\alpha + \beta Q = \bar{v} - \frac{\bar{v}}{K} Q. \tag{16}$$

This given us Q^{**}. So if we set $Q^{**} = Q^{*} = \frac{K}{2}$, we get $2\alpha + K\beta = \bar{v}$. The reverse argument is also true, that is, if $2\alpha + K\beta = \bar{v}$ then the variable supply mechanism results in a socially efficient quantity. □

Proof of Corollary 1

Proof. Set $\beta = 0$ in (6) to get $2\alpha = \bar{v}$. □

Proof of Proposition 4

Proof. First note that for any $\bar{Q} > Q^{**}$, the fixed supply mechanism results in social inefficiencies equal to the area of the triangle between $S(q)$, $p(q)$, Q^{**} and $\bar{Q}$. Now suppose the regulator sets $\bar{Q} = Q^{**}$. In this case define $D(p)$ as the true aggregate demand function and $D^{\dagger}(p)$ as the submitted one. From Proposition 1 we know that $D^{\dagger}(p)$ in any equilibria is strictly below $D(p)$. Denote p^{e} as the intersection of $D(p)$ and $\bar{Q}$. Thus the equilibrium price $p^{\dagger}$, which is the result of submitted demand schedules, is strictly below p^{e}. Now at any point $\bar{Q}$ we know that the lowest maximum willingness to pay is p^{e}. Also note that if $\bar{Q} = Q^{**}$ then the intersection of $S(q)$ and $D(p)$ is also at p^{e}. This means that at this point the minimum price that needs to be charged to overcome the cost of marginal damage is p^e. Since $p^\dagger < p^e$, although the overall surplus does not change there will continue to be a transfer from the society to polluters equal to $(p^e - p^\dagger)Q^{**}$.

□

Proof of Proposition 5

Proof. When $Q^* < Q^{**}$, the deadweight loss for the variable supply mechanism is given by $\Delta(q)$ as in Equation (9). Also note that the deadweight loss for the fixed supply auction is,

$$\Delta^\dagger(q) = \int_{Q^{**}}^{\bar{Q}} (S(q) - p(q))dq \tag{17}$$

The sufficient condition for higher social surplus is, $\Delta(q) > \Delta^\dagger(q)$. Thus we must have,

$$\int_{Q^*}^{Q^{**}} (P(q) - S(q))dq > \int_{Q^{**}}^{\bar{Q}} (S(q) - p(q))dq.$$

The above is satisfies as long as $Q^{**} - Q^* < \bar{Q} - Q^{**}$. Substituting for Q^* and Q^{**} gives the condition in (10).

□

References

Alvarez, F., Mazón, C. and André, F. J. (2017). 'Assigning pollution permits: are uniform auctions efficient?', *Economic Theory*.

Ausubel, L. M., Cramton, P., Pycia, M., Rostek, M. and Weretka, M. (2014). 'Demand reduction and inefficiency in multi-unit auctions', *The Review of Economic Studies*, **81**, pp. 1366–1400.

Back, K. and Zender, J. F. (2001). 'Auctions of divisible goods with endogenous supply', *Economics Letters*, **73**, pp. 29–34.

Cramton, P. and Kerr, S. (2002). 'Tradeable carbon permit auctions: How and why to auction not grandfather', *Energy Policy*, **30**, pp. 333 – 345.

Damianov, D. S. and Becker, J. G. (2010). 'Auctions with variable supply: Uniform price versus discriminatory', *European Economic Review*, **54**, pp. 571–593.

Dasgupta, P., Hammond, P. and Maskin, E. (1980). 'On imperfect information and optimal pollution control', *The Review of Economic Studies*, **47**, pp. 857–860.

Dickson, A. and MacKenzie, I. A. (2018). 'Strategic trade in pollution permits', *Journal of Environmental Economics and Management*, **87**, pp. 94 – 113.

Duggan, J. and Roberts, J. (2002). 'Implementing the efficient allocation of pollution', *The American Economic Review*, **92**, pp. 1070–1078.

European Commission (2017). 'Analysis of the use of auction revenues by the member states',Technical report, European Commission.

Goeree, J. K., Palmer, K., Holt, C. A., Shobe, W. and Burtraw, D. (2010). 'An experimental study of auctions versus grandfathering to assign pollution permits', *Journal of the European Economic Association*, **8**, pp. 514–525.

Khezr, P. and MacKenzie, I. A. (2018a). 'Consignment auctions', *Journal of Environmental Economics and Management*, **87**, pp. 42 – 51.

Khezr, P. and MacKenzie, I. A. (2018b). 'Permit market auctions with allowance reserves', *International Journal of Industrial Organization*, **61**, pp. 283 – 306.

Khezr, P. and MacKenzie, I. A. (2018c). 'An efficient and implementable auction for environmental rights', Discussion Papers Series 587, School of Economics, University of Queensland, Australia.

Kollenberg, S. and Taschini, L. (2016). 'Emissions trading systems with cap adjustments', *Journal of Environmental Economics and Management*, **80**, pp. 20 – 36.

Krishna, V. (2002). *Auction Theory*, San Diego: Academic Press.

Kwerel, E. (1977). 'To tell the truth: Imperfect information and optimal pollution control', *The Review of Economic Studies*, **44**, pp. 595–601.

LiCalzi, M. and Pavan, A. (2005). 'Tilting the supply schedule to enhance competition in uniform-price auctions', *European Economic Review*, **49**, pp. 227–250.

Lopomo, G., Marx, L. M., McAdams, D. and Murray, B. (2011). 'Carbon allowance auction design: An assessment of options for the United States', *Review of Environmental Economics and Policy*, **5**, pp. 25–43.

McAdams, D. (2007). 'Adjustable supply in uniform price auctions: Non-commitment as a strategic tool', *Economics Letters*, **95**, pp. 48–53.

Montero, J.-P. (2008). 'A simple auction mechanism for the optimal allocation of the commons', *The American Economic Review*, **98**, pp. 496–518.

Newell, R. G. and Pizer, W. A. (2003). 'Regulating stock externalities under uncertainty', *Journal of Environmental Economics and Management*, **45**, pp. 416 – 432.

Perino, G. and Willner, M. (2016). 'Procrastinating reform: The impact of the market stability reserve on the eu ets', *Journal of Environmental Economics and Management*, **80**, pp. 37 – 52.

RGGI (2018). 'The investment of RGGI proceeds in 2016'. available at: https://www.rggi.org/sites/default/files/Uploads/Proceeds/RGGI_Proceeds_Report_2016.pdf.

Shrestha, R. K. (2017). 'Menus of price-quantity contracts for inducing the truth in environmental regulation', *Journal of Environmental Economics and Management*, **83**, pp. 1 – 7.