The Strategic Industry Supply Curve[1]

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Abstract

In this paper we develop the concept of the strategic industry supply curve, representing the locus of Nash equilibrium outputs and prices arising from additive shocks to demand. We show that the standard analysis of partial equilibrium under perfect competition, including the graphical representation of supply and demand due to Marshall, can be extended to encompass imperfectly competitive markets, including monopoly, Cournot and Bertrand oligopoly and competition in linear supply schedules. We then derive a unified theory of cost pass-through and show that it satisfies the five principles of incidence set out by Weyl and Fabinger (2013).

Keywords: industry supply, cost pass-through, oligopoly.

JEL Code: D4, L1.

1 Introduction

Supply and demand curves, and associated concepts such as elasticities, have been central to partial equilibrium analysis since the 19th century. Supply and demand analysis provides a simple and elegant way of modelling the effects of shifts in consumer preferences, production costs and government interventions such as taxes. The graphical representation of the derivation of equilibrium prices and quantities as the intersection of demand and supply curves is an instantly recognizable iconic representation of economics.

Although commonly attributed to Marshall (1890), supply and demand curves were first presented by Cournot (1838), in the same volume that introduced his famous analysis of duopoly. The theoretical foundations of the demand curve were developed shortly afterwards by Dupuit (1844). Despite this overlap, Cournot and Dupuit worked in very different methodological frameworks, which Weyl (2017) distinguishes as 'reductionist' and 'price theory' respectively. Dupuit addressed institutional and historical factors as well as the purely economic determinants of equilibrium that were the focus of Cournot's analysis. Even more than Cournot's duopoly analysis, these early innovations were neglected, and the ideas were subsequently developed independently by a number of writers before being systematized by Marshall.[1]

Despite this early link with the theory of strategic behavior in imperfectly competitive markets, the supply–demand approach has been confined to the non-strategic case of competitive markets, where firms and consumers may be regarded as price takers. In this case, supply and demand quantities may be represented as functions of prices, and the associated curves are the graphs of those functions.

In the polar case of monopoly, the standard graphical analysis begins with the demand curve, which permits the derivation of the marginal revenue curve. Profit-maximizing output is determined by the intersection of the marginal revenue and marginal cost curves, and the associated price may then be read off the demand curve. In this standard analysis, there is no analog to the supply curve.

For the more general case of oligopoly, supply–demand analysis is rarely, if ever, used. To the extent that a graphical representation of equilibrium determination is employed, the standard approach is to represent the problem in terms of the reaction functions of the firms involved in a duopoly market, and thereby illustrate the Nash equilibrium solution. That is, in the terminology of Weyl (2017), theoretical analysis of the oligopoly problem is

[1] Ekelund and Hebert (1999) provides a detailed discussion of Marshall's predecessors in the development of the supply–demand diagram.

undertaken almost entirely within a reductionist framework.

The aim of the present note is to show how the tools of supply and demand analysis, fundamental to the price theory approach advocated by Weyl, may be extended to encompass strategic behavior. We examine the case of a market where producers are not price-takers, but face additive demand shocks, parametrized by a scalar shift variable. Firms compete in supply schedules, with Cournot and Bertrand competition as special cases, along with monopoly.[2]

We derive a strategic industry supply curve which maps out the (Nash) equilibrium price–quantity pairs associated with any given realization of the demand shock.[3] Given this setup, we may derive equilibrium supply elasticities, and show that the standard partial equilibrium analysis of cost and demand is applicable to the case of imperfect competition. In particular, in the linear case, we can apply the standard 'welfare triangle' analysis of consumer surplus and derive the deadweight loss from monopoly and oligopoly.

The standard methods of comparative statics are also applicable. We apply these methods to the analysis of 'cost pass-through'. We consider the 'five principles' proposed by Weyl and Fabinger (2013) and show that our approach permits a unified treatment of monopoly, oligopoly and competition.

2 The strategic industry supply curve

The central focus of this paper is on the implications of strategic behavior for firms' supply decisions. To tighten this focus and allow a simple and compact statement of results, we adopt very simple specifications for demand and cost: namely, a linear demand curve and constant marginal costs of production.

We assume that consumers do not behave strategically, so that the demand curve may be taken as exogenously given. Furthermore, as our focus is on constructing a strategic industry supply curve, we limit our analysis to a linear demand curve with additive shocks:

$$D(p, \varepsilon) = a - bp + \varepsilon, \tag{1}$$

where ε is a shock observed by firms before they make their strategic choices.

[2]Unlike Klemperer and Meyer (1989) we restrict attention to affine supply schedules with each strategy available to a firm represented by the value of a scalar shift parameter. By contrast with the Klemperer–Meyer result that any individually rational outcome can be derived as a Nash equilibrium for competition in supply schedules, our approach allows the derivation of a unique, symmetric Nash equilbrium.

[3]Busse (2012) independently developed, for the cases of monopoly and Cournot oligopoly, a similar concept, which she described as the 'equilibrium locus'.

It should be clear from our analysis below that the model can be extended to more general demand functions, but at the expense of more complex analysis to account for the curvature of the demand curve.[4] As we will show below, for constant marginal costs, and competition in linear supply schedules, the strategic industry supply curve is also linear, further simplifying the analysis.

In the case of competitive markets, graphical analysis using the supply curve has the following desirable features. First, and most importantly, the equilibrium price and quantity are given by the intersection of the demand and supply curves. Second, we can undertake comparative static analysis both with respect to shifts in the demand curve and with respect to cost shocks. In the latter case, we can use the concept of 'cost pass-through' to describe the equilibrium incidence of cost shocks.

In the general case of imperfect competition, the strategic choices of firms will depend on the anticipated responses of other firms as well as on market demand. Hence, there is no uniquely defined relationship between market prices and the quantity supplied by individual firms or by the industry as a whole.

Nevertheless, a form of the supply curve arises naturally when we consider the response to shifts in the demand curve, characterized by the shock (ε). For each value of ε, the (Nash) equilibrium strategic choices of firms determine a market equilibrium, that is a price–quantity pair on the demand curve at which the market clears. The locus of such points will be a one-dimensional manifold, upward-sloping in price–quantity space, that is, a (strategic industry) supply curve. The analysis is particularly simple in the case where the firms' strategy spaces consist of a family of firm supply curves, with a single strategic shift parameter, and particularly when both the demand parameter and the strategic supply parameter represent additive shifts.

We assume that firms $n = 1, ..., N, N \geq 1$, choose strategies in linear supply schedules

$$S_n = \alpha_n + \beta p \tag{A2}$$

where α_n is the strategic variable for firm n and β is an exogenously given slope parameter, interpreted as representing the competitiveness of the market. Cournot competition is represented by $\beta = 0$. The opposite polar case of Bertrand competition is approached as $\beta \to \infty$.

Firms seek to maximize profit

$$\pi_n = (p - c) S_n$$

[4] For a comprehensive treatment of the implications of non-linear demand see Weyl and Fabinger (2013).

where c is the constant marginal cost of production.[5]

From the viewpoint of any given firm i, the strategic choices of the other firms $n \neq i$, along with the market demand curve and the realization of the demand shock ε, determine the residual demand curve faced by that firm. Nash equilibrium requires that firm i chooses its optimal price–quantity pair from the residual demand curve, holding the strategic choices of the other firms, $\alpha_n, n \neq i$ (and the exogenously given[6] $\bar{\beta}_n, n \neq i$) constant.

It does not matter, however, whether firm i conceives of its own choice as picking the strategic variable α_i, the associated quantity or price, or some other variable such as the markup on marginal cost. All that matters is that the decision variable should uniquely characterize the profit-maximizing price–quantity pair on the residual demand curve for firm i.

The residual demand facing firm i, given the realized value of ε, is

$$\begin{aligned} q_i(\varepsilon) &= a - bp(\varepsilon) - \sum_{n \neq i} (\alpha_n + \beta p(\varepsilon)) + \varepsilon \qquad (2) \\ &= a - \sum_{n \neq i} \alpha_n - (b + (N-1)\beta) p(\varepsilon) + \varepsilon, \end{aligned}$$

and firm i can be regarded as a monopolist facing this demand schedule.[7]

We can rearrange the expression above as:

$$p(\varepsilon) = \theta_i(\varepsilon) - \gamma_i q_i(\varepsilon)$$

where

$$\begin{aligned} \theta_i(\varepsilon) &= \frac{a - \sum_{n \neq i} \alpha_n + \varepsilon}{b + (N-1)\beta} \qquad (3) \\ \gamma_i &= \frac{1}{b + (N-1)\beta}. \end{aligned}$$

The same analysis is applicable to the case of monopoly, where $N = 1$, and the expressions above become:

$$\begin{aligned} \theta(\varepsilon) &= \frac{a + \varepsilon}{b} \qquad (4) \\ \gamma &= \frac{1}{b}. \end{aligned}$$

[5]The case of a general convex cost function is straightforward but complicates the statement of the results. This case is addressed in an Appendix available, from the authors.

[6]In this paper, we have adopted the simplifying assumption that all firms have the same β_n. This assumption can be relaxed. The important point is that, from the perspective of firm i, it is the set of strategies available to other firms $n \neq i$ that determine the residual demand curve facing i. Whatever the strategy space for i, any point on the residual demand curve may be selected as an equilibrium outcome.

[7]For the special case of monopoly, (2) is simply the demand curve.

In general, we can write firm i's profit as:

$$p(\varepsilon) q_i(\varepsilon) - cq_i(\varepsilon) = \theta_i(\varepsilon) q_i(\varepsilon) - \gamma_i q_i^2(\varepsilon) - cq_i(\varepsilon).$$

The FOC on $q_i(\varepsilon)$ becomes

$$p(\varepsilon) - \gamma_i q_i(\varepsilon) - c = 0$$

$$q_i(\varepsilon) = \frac{(p(\varepsilon) - c)}{\gamma_i}. \tag{5}$$

Since each firm i acts as a monopolist facing the residual demand curve, we may also consider any given firm as setting p, with the first order condition

$$p(\varepsilon) = \frac{\theta_i(\varepsilon) + c}{2}. \tag{6}$$

Given the linearity of demand, and of the supply schedules available to firms, the game has a unique solution, which is symmetric. In this symmetric equilibrium, for given ε, all firms produce the same output $q(\varepsilon)$. Hence, θ and γ are the same for all firms

$$\begin{aligned} \theta(\varepsilon) &= \frac{a - (N-1)\alpha + \varepsilon}{b + (N-1)\beta} \qquad (7) \\ \gamma &= \frac{1}{b + (N-1)\beta} \end{aligned}$$

and we obtain

$$q(\varepsilon) = \frac{p(\varepsilon) - c}{\gamma},$$

or, summing across firms,

$$Q(\varepsilon) = N\left(\frac{p(\varepsilon) - c}{\gamma}\right). \tag{8}$$

Equation 8 represents the equilibrium price and quantity for a given value of the additive shock ε. In terms of the price theory approach described by Weyl (2017), equation (8) represents a description sufficient for the class of phenomena under consideration.

This description may be represented in graphical terms by taking the locus of solutions $(p(\varepsilon), Q(\varepsilon))$ as ε varies over its range. In a standard competitive model, with additive demand shocks, this locus of solutions would trace out the supply curve. In the more general strategic setting proposed here, we therefore refer to the locus of equilibrium solutions as the *strategic industry supply curve*:

$$S(p) = N\frac{(p-c)}{\gamma}. \tag{9}$$

By substituting the value of γ into (9), we obtain the following result:

Proposition 1 *Given constant marginal costs and linear demand, the industry supply curve is*

$$S(p; N, \beta) = N(p-c)(b + (N-1)\beta), \tag{10}$$

which is linear with intercept c and slope $\frac{1}{N(b+(N-1)\beta)}$ in the $p \times S(p; N, \beta)$ axis.

For the monopoly case, the slope is $\frac{1}{b}$. For the symmetric oligopoly case, the slope ranges between $\frac{1}{Nb}$ for Cournot ($\beta = 0$) and 0 in the limit as $\beta \to \infty$ (Bertrand/perfect competition, where $p \to c$).

We also have:

Corollary 1 *For the symmetric case with constant marginal costs, $S'(p) = N(b + (N-1)\beta) > \sum \beta_n = N\beta$.*

In particular, for the case of Cournot oligopoly, the strategic industry supply curve is strictly upward sloping even though the firms' equilibrium supply schedules are all vertical. This reflects the fact that the strategic industry supply curve is derived from a locus of equilibria, one for each value of ε.

Menezes and Quiggin (2012) observe that an increase in the number of competitors N will have similar effects on equilibrium market outcomes as an increase in the competitiveness of the market (higher β). The concept of the strategic industry supply curve enables us to sharpen this point. Consider as a benchmark the Cournot case, where $S'(p) = Nb$. Now, for any $2 \leq M < N$, let $\beta(M) = \frac{(N-M)b}{M(M-1)} > 0$. Then, for all p

$$S(p; N, 0) = S(p; M, \beta(M)).$$

More generally, for any initial β and $M < N$, we can find $\beta(M)$ such that, for all p,

$$S(p; N, \beta) = S(p; M, \beta(M)).$$

Remark 1 *The linear strategic industry supply curve (9) , obtained under constant marginal cost, is equivalent to a competitive industry supply curve with appropriately defined quadratic costs. For example, consider the cost function* $c(Q) = \gamma Q + \xi Q^2$*, so that, under competition,* $p = \gamma + 2\xi Q$*. This is equivalent to (10) with* $\gamma = c$ *and* $\xi = \frac{1}{2N(b+(N-1)\beta)}$.

We can now turn to the determination of the equilibrium. For given ε, (9) coincides with the equilibrium price and quantity and, therefore, it can be used to determine the Nash equilibrium outcome:

$$p(\varepsilon) = c + \frac{a - bc + \varepsilon}{\left(\frac{N}{\gamma} + b\right)} \tag{11}$$

$$= c + \frac{\gamma(a - bc + \varepsilon)}{N + b\gamma} \tag{12}$$

$$Q(\varepsilon) = \frac{N}{\gamma}\frac{a - bc + \varepsilon}{\left(\frac{N}{\gamma} + b\right)} = \frac{N(a - bc + \varepsilon)}{N + b\gamma}.$$

In the monopoly case, we obtain

$$p^M(\varepsilon) = c + \frac{a - bc + \varepsilon}{2b}$$
$$Q^M(\varepsilon) = \frac{a - bc + \varepsilon}{2}.$$

For the symmetric oligopoly case, we denote the equilibrium price–quantity pair associated with slope parameter β and shock ε by $\left(p^\beta(\varepsilon), Q^\beta(\varepsilon)\right)$.

For Cournot, we set $\beta = 0$ in (7), to obtain $\gamma = \frac{1}{b}$ and

$$p^0(\varepsilon) = c + \frac{1}{N+1}\frac{a - bc + \varepsilon}{b}$$
$$Q^0(\varepsilon) = \frac{N}{N+1}(a - bc + \varepsilon),$$

which reduces to the familiar $p^0 = \frac{1}{N+1}$, $Q^0 = \frac{N}{N+1}$ for zero cost, $a = 1$ and $\varepsilon = 0$.

The Bertrand case is obtained by taking the limit of (7) as $\beta \to \infty$, yielding $\gamma \to 0$ and

$$p^\infty(\varepsilon) = c$$
$$Q^\infty(\varepsilon) = a - bc + \varepsilon.$$

The elasticity of industry supply with respect to price is simply

$$\epsilon_S = \frac{p}{p-c}.$$

This expression does not contain β explicitly, but the equilibrium price p depends on β. As would be expected, ϵ_S approaches infinity for the Bertrand case $\beta \to \infty$. For Cournot, $\epsilon_S = 1 + \frac{Nbc}{a-bp+\varepsilon}$. In particular, for the case of zero costs, ϵ_S takes values in the range $[1, \infty)$.

Our approach to constructing the strategic industry supply curve is illustrated, for the case of a symmetric Cournot duopoly with constant marginal costs, in Figures 1 and 2 below.

Figure 1 shows how the Cournot equilibrium quantity was obtained for three values of ε. For the case of linear demand (1), firm i's reaction function, $i = 1, 2$, $i \neq j$, is given by:

$$\begin{aligned} q_i &= \frac{p-c}{\gamma} \\ &\quad \frac{a - q_j + \varepsilon - bc}{2}. \end{aligned}$$

Figure 2 shows the derivation of the strategic industry supply curve, which is obtained by tracing the equilibrium price–quantity supplied pairs as ε varies over its range.

In Figure 3 below, we show how the standard first-year supply–demand graphical approach can be extended to the analysis of symmetric oligopoly and to the case of monopoly, drawn below for constant marginal cost c.

As it is clear from Figure 3, the strategic industry supply curve is an equilibrium concept in the sense that it is derived from the firms' profit maximization for each realization of demand.

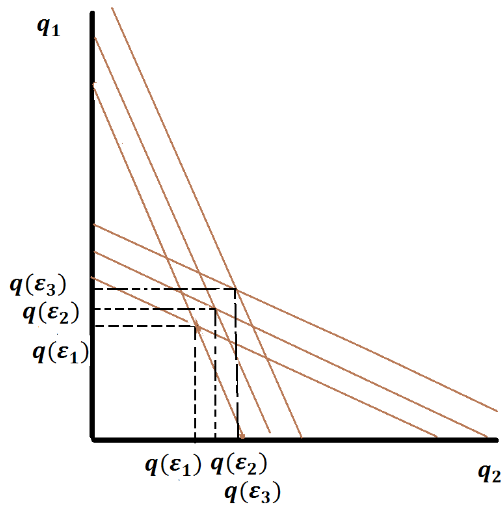
3 Welfare

The construction of a strategic industry supply curve also allows us to undertake the standard graphical analysis of welfare using a supply–demand diagram. This is illustrated in Figure 4 below for the case of linear demand (1) and constant marginal costs c. Figure 4 depicts consumer surplus (CS), producer surplus (PS), total surplus (TS) and deadweight loss (DWL) for given values of β and ε.

Under our construct, the direct demand (1) is parametrized by the realization of ε. Consumer surplus (CS) is given by:

$$CS(\varepsilon) = \frac{1}{2}(p_0(\varepsilon) - p)D(p, \varepsilon),$$

Figure 1: Reaction Curves and the Strategic Industry Supply Curve



where $p_0(\varepsilon)$ is the price for which $D(p,\varepsilon) = 0$. Expressing CS in terms of p, we have

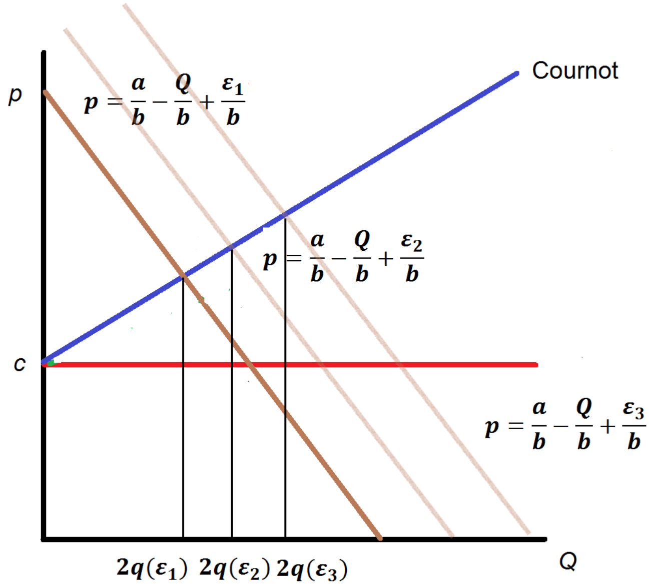
$$\begin{aligned} p_o &= \frac{a+\varepsilon}{b} \\ CS(\varepsilon) &= \frac{1}{2}\left(\frac{a+\varepsilon}{b} - p\right)(a+\varepsilon-bp) \\ &= \frac{1}{2b}(a+\varepsilon-bp)^2 \\ &= \frac{(D(p,\varepsilon))^2}{2b}. \end{aligned}$$

In Figure 4, consumer surplus is represented by the area of the triangle ABF. The maximum value of consumer surplus is ACE, arising as $\beta \to \infty$, $p \to c$.

Producer surplus (PS) can be calculated from (9):

$$\begin{aligned} PS(\varepsilon) &= (p(\varepsilon)-c)S(p(\varepsilon)) = \frac{S(p(\varepsilon))^2}{N(b+(N-1)\beta)} \\ &= \frac{N(p(\varepsilon)-c)^2}{\gamma}. \end{aligned}$$

Figure 2: Strategic Industry Supply Curve (Cournot)

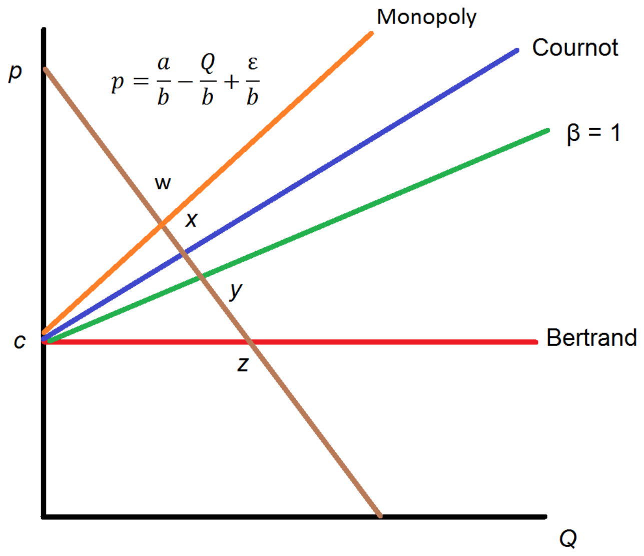


Note that producer surplus is not equal to the area of the triangle BEF between the price and the supply curve in Figure 4.[8] Rather, in the case of constant marginal cost examined here, producer surplus for given ε is equal to the rectangle BDEF with height $(p(\varepsilon)-c)$ and base $Q(\varepsilon)$. That is, producer surplus in the case of a linear strategic supply curve and constant marginal is twice the surplus that arises in a competitive market with the same supply curve resulting from increasing marginal cost.

Remark 2 *As noted in remark 1, the strategic supply curve derived here is equivalent to a competitive supply curve with appropriately defined quadratic costs. In this case, the producer surplus would be the triangle BEF while the complementary triangle BDE would represent costs incurred by producers. Thus, imperfect competition is analogous to a case where producers engage in 'cost-padding' and recoup both the resulting producer surplus and the spurious costs.*

[8]We are indebted to Glen Weyl for this observation.

Figure 3: Industry Strategic Supply Curve: Constant Marginal Cost



The total surplus (TS) associated with ε can be written as:

$$\begin{aligned} TS(\varepsilon) &= CS(\varepsilon) + PS(\varepsilon) \\ &= \frac{Q(\varepsilon)^2}{N\gamma} + \frac{Q(\varepsilon)^2}{2b} \end{aligned} \tag{13}$$

where $Q(\varepsilon)$ is the equilibrium output given by (11):

$$Q(\varepsilon) = \frac{(a - bc + \varepsilon)\, N}{N + b\gamma} \tag{14}$$

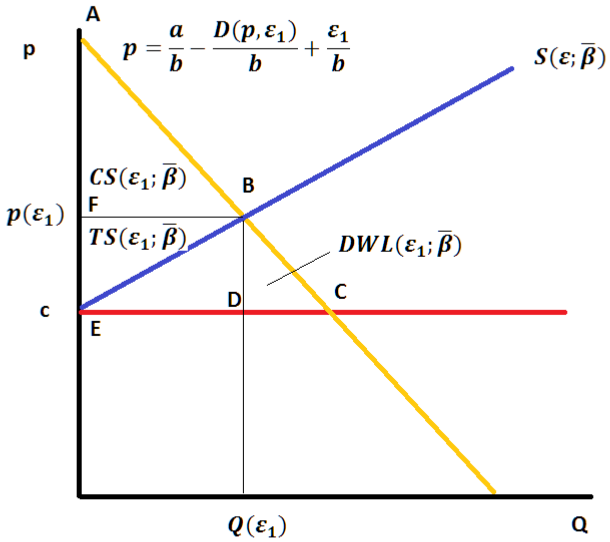
and is represented by the area ABDEF.

The deadweight loss (DWL), relative to the Bertrand equilibrium $p = c$, is:

$$\begin{aligned} DWL(\varepsilon) &= \frac{1}{2}(p(\varepsilon) - c)(D(c, \varepsilon) - Q(\varepsilon)) \\ &= \frac{1}{2}\frac{\gamma Q(\varepsilon)}{N}(a - bc + \varepsilon - Q(\varepsilon)) \end{aligned} \tag{15}$$

where $D(c, \varepsilon)$ represents the quantity demanded, given ε, when $p = c$.

Figure 4: Welfare analysis with the strategic industry supply curve



Substituting (14) into (13) and (15) and simplifying yields:

$$TS(\varepsilon) = \frac{N\left(a - bc + \varepsilon\right)^2 \left(2b + N\gamma\right)}{2b\gamma\left(N + b\gamma\right)^2} \tag{16}$$

and

$$DWL\left(\varepsilon\right) = \frac{b\gamma^2\left(a - bc + \varepsilon\right)^2}{2\left(N + b\gamma\right)^2}. \tag{17}$$

As the calculations above hold for any value of ε, we can derive the following result. The proof is omitted as it follows from inspection of (16) and (13):

Proposition 2 *For the case of linear demand and constant marginal cost, consumer surplus and total surplus increase with β, while producer surplus and deadweight loss decrease with β.*

4 Cost Pass-through

The problem of cost pass-through is a special case of the comparative statics of Marshallian partial equilibrium analysis. The analysis begins with a market equilibrium disturbed by a shock to suppliers' input prices or technology, which may be represented as an increase of Δc in unit costs. The problem is to determine the resulting change in the equilibrium price Δp, and, more particularly, the ratio $\rho = \frac{\Delta p}{\Delta c}$, which measures the proportion of the cost increase passed through to consumers. Although input prices and technology are subject to constant change, the term 'pass-through' is most commonly used in contexts where the change in equilibrium prices is seen to be of policy concern.

The problem of cost pass-through was recently examined by Weyl and Fabinger (2013), who draw on a long tradition of work on tax incidence, going back to Dupuit (1844), Jenkin (1871-72) and Marshall (1890). Like Weyl and Fabinger, we extend the standard analysis of incidence under competition to the case of imperfectly competitive markets. Although our representation of the problem is different, it is equivalent to that of Weyl and Fabinger in some cases, most notably that of a monopolist facing linear demand. Our results for that case coincide with theirs, as we shall show.

For the case of symmetric oligopoly, there are subtle differences. These arise from the fact that Weyl and Fabinger focus on a conjectural variations model derived from the work of Bresnahan (1981). By contrast, our analysis begins with a Nash equilibrium in affine supply schedules, as derived above. We explore some of the similarities and distinctions below.

The following proposition can be derived directly from (11):

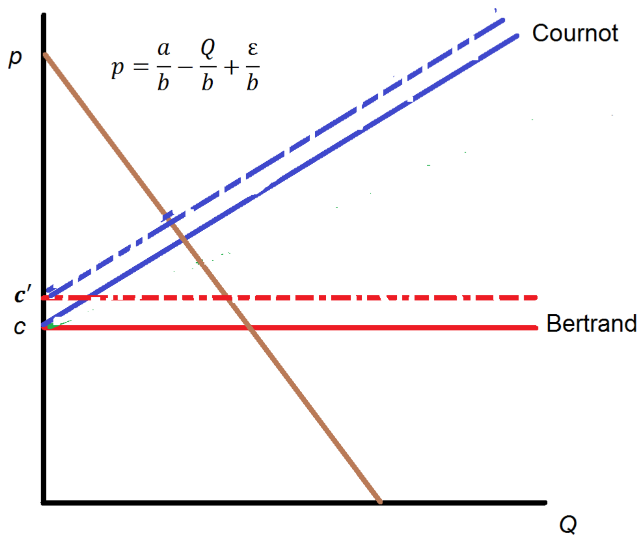
Proposition 3 *Cost pass-through for symmetric oligopoly with constant marginal cost is given by:*

$$\rho = \frac{Nb + N(N-1)\beta}{(N+1)b + N(N-1)\beta}. \tag{18}$$

From (18), we can recover the standard pass-through expression for Cournot models with linear demand and constant marginal cost ($\rho = \frac{N}{N+1}$). For Bertrand (when $\beta \to \infty$) cost pass-through is equal to 1, as for perfect competition. As observed above, an increase in the number of competitors N has the same effect as an appropriately chosen increase in β. In particular, for any fixed β, as $N \to \infty$, $\rho \to 1$. The minimum value of ρ is $\rho = \frac{1}{2}$, attained in the monopoly case $N = 1$.

The Bertrand and Cournot examples are shown in Figure 5 below.

Figure 5: Cost Pass Through for Cournot and Bertrand



The next proposition relates (18) to the relevant elasticities, namely that of demand and of the strategic industry supply curve, extending the standard analysis of cost pass-through to cover monopoly and (symmetric) oligopoly.

Proposition 4 *Cost pass-through is given by*

$$\rho = \frac{\epsilon_S}{\epsilon_D + \epsilon_S},$$

where ϵ_D denotes the price elasticity of demand and ϵ_S the price elasticity of the strategic industry supply curve.

Proof. The expression for ϵ_D can be derived as follows

$$\epsilon_D = -\frac{\partial D(p,\varepsilon)}{\partial p}\frac{p}{D(p,\varepsilon)} = \frac{bp}{a+\varepsilon-bp}.$$

Substituting the expression for (11) yields:

$$\epsilon_D = \frac{b\left[c\left[(N+1)\,b + N\,(N-1)\,\beta\right] + a - bc + \varepsilon\right]}{(a-bc+\varepsilon)\left[(N+1)\,b + N\,(N-1)\,\beta - bc\right]}.$$

Similarly, replacing (11) into the expression for $\epsilon_S = \frac{p}{p-c}$ yields:

$$= \frac{c\left[(N+1)\,b + N\,(N-1)\,\beta\right] + a - bc + \varepsilon}{a - bc + \varepsilon}.$$

It follows, by simple algebra, that

$$\frac{\epsilon_S}{\epsilon_D + \epsilon_S} = 1 - \frac{b}{\left[(N+1)\,b + N\,(N-1)\,\beta\right]} = \rho.$$

■

For the special case of monopoly, we have $\rho = 1/2$. For Cournot, $\rho = \frac{N}{N+1}$. For Bertrand, $\rho = 1$.

4.1 Incidence

Now consider the case when a cost increase arises from the imposition of a tax. In this case, we are interested in the tax burden, that is, the ratio of the loss in producer and consumer surplus to the revenue raised by the tax.

Consider the case when a tax t is imposed. In the linear case considered here, local and global analysis will coincide. However, for notational convenience we will focus on derivatives evaluated at $t = 0$.

We have, for producer surplus,

$$\begin{aligned}
\frac{\partial PS(\varepsilon)}{\partial t} &= \frac{2N}{\gamma}(p(\varepsilon) - c)\frac{\partial(p(\varepsilon) - c)}{\partial t} \\
&= (\rho - 1)\,\frac{2N}{\gamma}(p(\varepsilon) - c) \\
&= (\rho - 1)\,Q\,(\varepsilon) \\
&= 2\,(\rho - 1)\,\frac{\partial R}{\partial t}
\end{aligned}$$

where $R = tQ\,(\varepsilon)$ is tax revenue.

For consumer surplus,

$$CS = \frac{1}{2b}\,(a + \varepsilon - bp)^2\,.$$

So, in equilibrium.

$$\begin{aligned}
\frac{\partial CS}{\partial t} &= -\,(a + \varepsilon - bp)\,\frac{\partial p}{\partial t} \\
&= -\rho Q \\
&= -\rho\frac{\partial R}{\partial t} \\
&= \frac{\rho}{2\,(1-\rho)}\frac{\partial PS(\varepsilon)}{\partial t}.
\end{aligned}$$

The total burden of the tax is given by

$$\begin{aligned}\left|\frac{\partial PS(\varepsilon)}{\partial t}+\frac{\partial CS}{\partial t}\right| &= (2-\rho)\frac{\partial R}{\partial t} \\ &\geq \frac{\partial R}{\partial t},\end{aligned}$$

where equality holds only for the Bertrand case $\rho = 1$.

For the Bertrand case, we have

$$\begin{aligned}\frac{\partial PS(\varepsilon)}{\partial t} &= 2(\rho-1)\frac{\partial R}{\partial t} = 0 \\ \frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{\partial R}{\partial t}\end{aligned}$$

That is. the burden of a small tax is entirely borne by consumers, and there is no deadweight loss. For monopoly, $\rho = \frac{1}{2}$, and we have

$$\begin{aligned}\frac{\partial PS(\varepsilon)}{\partial t} &= -\frac{\partial R}{\partial t} \\ \frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{1}{2}\frac{\partial R}{\partial t}.\end{aligned}$$

Thus, we obtain the well known result that the full tax revenue is paid by the monopolist, and an additional burden is borne by consumers. In the linear case considered here, this additional burden is equal to half of the revenue raised by the tax.

For Cournot $\rho = \frac{N}{N+1}$, we have

$$\begin{aligned}\frac{\partial PS(\varepsilon)}{\partial t} &= -\frac{2}{N+1}\frac{\partial R}{\partial t} \\ \frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{N}{N+1}\frac{\partial R}{\partial t} \\ \left|\frac{\partial PS(\varepsilon)}{\partial t}+\frac{\partial CS(\varepsilon)}{\partial t}\right| &= \frac{N+2}{N+1}\frac{\partial R}{\partial t} \\ I &= \frac{N}{2}\end{aligned}$$

where I (incidence) is the ratio of the burden borne by consumers to the burden borne by producers.

For the general oligopoly case, with $\beta < \infty$ and $N > 1$, we have

$$\begin{aligned}
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{Nb + N(N-1)\beta}{(N+1)b + N(N-1)\beta}\frac{\partial R}{\partial t} \\
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{2b}{(N+1)b + N(N-1)\beta}\frac{\partial R}{\partial t} \\
\left|\frac{\partial PS(\varepsilon)}{\partial t} + \frac{\partial CS(\varepsilon)}{\partial t}\right| &= \frac{(N+2)b + N(N-1)\beta}{(N+1)b + N(N-1)\beta}\frac{\partial R}{\partial t} \\
I &= \frac{Nb + N(N-1)\beta}{2b}.
\end{aligned}$$

Once again, the total burden exceeds revenue and is shared between producers and consumers. Producers bear less than the full burden of the tax.

To sum up, the concept of the strategic industry supply curve yields a unified analysis of cost pass-through, encompassing the cases of monopoly, symmetric oligopoly and competition (interpreted either as oligopoly with Bertrand competition or as competition between large numbers of firms).

5 The five principles

Weyl and Fabinger (2013) analyze the problem of cost pass-through, drawing on the literature on tax incidence. Their analysis is organized around five principles, drawing on the analysis of tax incidence under perfect competition. These principles are extended, with appropriate modifications, to the cases of monopoly and oligopoly.

To analyze pass-through, Weyl and Fabinger introduce the elasticity of the inverse marginal cost curve, and of the inverse marginal surplus curve. Weyl and Fabinger denote the elasticity of the inverse marginal cost curve by ϵ_s, but we have used this notation for the elasticity of the industry supply curve. We will therefore denote the elasticity of the inverse marginal cost curve by ϵ_c. Because we focus on linear demand, the elasticity of the inverse marginal surplus curve is identically equal to 1, and we will make this substitution throughout in stating the Weyl–Fabinger principles.

In the case of symmetric oligopoly with a homogenous product, the competitiveness of the market is represented by a parameter R, which varies between 0 (for Cournot) and -1 (for Bertrand). Weyl and Fabinger use the derived variable $\theta = \frac{1+R}{N}$, which varies between 0 (Bertrand) and $\frac{1}{N}$ (Cournot). A straightforward manipulation shows that, translating to the terms of our model, we have $\theta = \frac{1}{N+N\beta(N-1)}$. Thus, our model provides an explicit game-theoretic foundation for the derivation of the parameters R and θ.

The idea of the strategic industry supply curve allows for a more unified treatment of the Weyl–Fabinger principles, with a single statement of the principles applicable to competition, monopoly and oligopoly. As in the argument above, we will confine attention to the case of linear demand and constant marginal costs, so that the only variation arises from the competitiveness or otherwise of the market structure.

We now consider the Weyl–Fabinger principles in turn:

Principle of incidence 1 (Economic versus physical incidence)

The physical incidence of taxes is neutral in the sense that a tax levied on consumers, or a unit parallel downward shift in consumer inverse demand, causes nominal prices to consumers to fall by $1 - \rho$.

This principle of neutrality is fundamental. The same principle underlies the crucial observation that from the viewpoint of any individual producer, a shock to residual demand is identical whether it arises from a shock to market demand or from the (equilibrium) supply of other producers. Weyl and Fabinger (2013) attribute this insight to Jeremy Bulow.

Principle of incidence 2 (Split of tax burden)

(i) Under competition, the total burden of the infinitesimal tax beginning from zero tax is equal to the tax revenue and is shared between consumers and producers.

(ii) Under monopoly, the total burden of the tax is more than fully shared by consumers and producers. While the monopolist fully pays the tax out of her welfare, consumers also bear an excess burden.

(iii) Under homogenous products oligopoly, the total burden of the tax is more than fully shared by consumers and producers. Producers bear less than the full burden of the tax.

As shown above, Principle 2 is satisfied by our model.

Principle of incidence 3 (Local incidence formula)

The ratio of the tax borne by consumers to that borne by producers, the incidence, I, equals:

(i) $\frac{\rho}{1-\rho}$ in the case of perfect competition;

(ii) ρ in the case of monopoly; and

(iii) $\frac{\rho}{1-(1-\theta)\rho}$ in the case of oligopoly.

Our results coincide with those of Weyl and Fabinger in all cases.

Principle of incidence 4 (Pass– through**)**

The pass-through rate is:

(i) $\rho = \frac{1}{1+\epsilon_D/\epsilon_C}$ in the case of perfect competition;

(ii) $\rho = \frac{1}{2+(\epsilon_D-1)/\epsilon_C}$in the case of monopoly; and

(iii) $\rho = \frac{1}{1+\theta+\frac{\theta}{\epsilon_\theta}+(\epsilon_D-\theta)/\epsilon_C}$ in the case of oligopoly.

For the special case of constant marginal costs, ϵ_C is infinite, so we obtain $\rho = 1$ for competition and $\rho = \frac{1}{2}$ for monopoly. For the case of linear supply schedules, β and therefore also θ are constant. Hence ϵ_θ is also infinite, so $\rho = \frac{1}{1+\theta}$. Substituting $\theta = \frac{1}{N+N\beta(N-1)}$ into this expression, we obtain $\rho = \frac{N+[(N+1)+N(N-1)\beta]}{[(N+1)+N(N-1)\beta]}$, which coincides with our result for the case $b = 1$.

Finally, we have

Principle of incidence 5 (Global incidence)

Weyl and Fabinger derive global incidence as a weighted average of the pass-through rate which is, in general, variable. For the case of linear demand, considered here, the pass-through rate is constant, and therefore global incidence is the same as local incidence. The extension of our analysis to the case of non-linear demand can be undertaken using the tools provided by Weyl and Fabinger.

6 Concluding comments

We have shown that, using the concept of the strategic industry supply curve, the standard analysis of partial equilibrium under perfect competition, including the graphical representation of supply and demand due to Marshall, can be extended to encompass imperfectly competitive markets. The class of market structures encompasses monopoly and competition, as well as an entire class of oligopoly models represented by competition in linear supply schedules, with Cournot and Bertrand as polar cases. For the oligopoly case, the results show the interaction between the number of firms N and the competitiveness of the market structure, characterized by the parameter β.

Furthermore, this representation of supply allows for a unified treatment of comparative static problems such as cost pass-through, which have previously required separate treatments for competition, monopoly and oligopoly. In particular, we provide both a game-theoretic foundation for and a simple derivation of the Weyl–Fabinger principles of incidence. The tools used here could be applied to a wide range of problems, such as the analysis of mergers. Similarly, we can extend the diagrammatic tools of welfare analysis, such as the representation of deadweight losses as welfare triangles.

The analysis here has focused on the case of linear demand and constant marginal cost, to allow for a simple statement of results that illuminates the crucial aspects of the problem, and to permit a simple graphical representation. But the concept of the strategic industry supply curve is valid under more general conditions.

We have not addressed issues of estimation. However, by observing market outcomes in the presence of demand shocks, it should be possible to use standard techniques to estimate the strategic industry supply curve. Combined with information about technology and input costs, the slope of the estimated industry supply curve would provide information about the competitiveness of the market, measured by the slope of the supply curves available as strategies to individual firms.

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