An efficient and implementable auction for environmental rights

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Abstract

This article proposes a simple and efficient auction design to allocate environmental rights, such as tradable pollution permits. We show that if the auctioneer limits the number of bids that each buyer submits—coupled with a simple ex-post supply adjustment rule—then truthful bidding is obtained. Consequently, the uniform-price auction becomes efficient and revenue superior to conventional uniform-price auctions that are currently observed in pollution markets.

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1 Introduction

Auctions are now a common method to allocate environmental rights, which include pollution, water, and mineral rights. The very nature of environmental resources means that they are normally allocated in multiple units, which increases the complexity of designing an efficient allocation mechanism. Although multi-unit uniform-price auctions are a natural process to allocate these resources (and preferred by governments), there are significant concerns regarding the efficiency of these mechanisms (Wilson, 1979; Ausubel et al., 2014). Further, previous attempts to improve auction efficiency—for example, by designing Vickrey-Clarke-Groves (VCG) mechanisms—have never been used in large-scale practical applications due to complex payment rules that limit the adoption of these mechanisms. Thus it is currently unknown how to design a uniform-price auction that is both efficient and implementable in the vast array of environmental applications.

This article proposes an auction design that can lead to the efficient allocation of resources. A new rule is introduced within the uniform-price auction that can result in truthful bidding and revenue improvement. In particular, we limit the number of bids that each bidder can submit and introduce a marginal *ex-post* supply adjustment to achieve truthful bidding. These two implementable mechanisms require no extra informational requirements and are simpler than existing uniform-price auctions (such as those observed in current cap-and-trade markets).

The ideal outcome of the uniform-price auction is to achieve the Walrasian equilibrium. In other words, if each bidder submits their own true demand function, then the aggregate demand represents the true market demand. Using the pricing rule of the uniform-price auction therefore results in the same price as the Walrasian equilibrium. This is especially important in environmental auctions as the price associated with the Walrasian equilibrium may result in the correct valuation of the resource, which has important consequences for firms' emissions choices, activity on secondary markets, and investment decisions. Note, however, that the uniform-price auction usually fails to achieve the desirable Walrasian equilibrium. This is due to 'demand reduction' by buyers, which is the underreporting of their actual demand (Wilson, 1979; Back and Zender, 1993). In contrast, the proposed auction design in this article mitigates buyers' incentives for demand reduction and results in truthful bidding. Thus, if a regulator implements a similar auction to allocate environmental rights, then buyers with the highest realized valuations receive those rights, whenever the regulator decides to sell them.[1]

This article showcases a situation where buyers have private values for the environmental rights with asymmetric capacities. The basic model studies a case with buyers that have constant marginal values for the environmental rights and later extend it to a case with diminishing marginal values. We show, under both frameworks, it is possible to achieve efficiency if the auctioneer makes some small adjustments to the auction rules. There are two intuitive reasons for this efficiency result. First, each buyer's submitted bid is independent of the price they pay for the rights they win. Second, firms cannot manipulate their bids in order to win a proportion of their actual demand at lower prices. The first is addressed by the adjustment of the bidding

[1] Note that, following Armstrong (2000), we make a distinction between "weak" efficiency and "strong" efficiency: the difference being that, in the former, the seller can withhold rights and the efficiency focus is solely on the rights that are sold whereas, in the latter, all potential gains from trade between auctioneer and bidder need to be realized.

rules; either by limiting the number of bids or submitting linear bids. The second is addressed by ex-post supply adjustment. In particular, the supply adjustment is aimed at eliminating any remaining incentives for demand reduction at the margin.

Supply adjustment rules have previously been proposed to eliminate or reduce the demand reduction in uniform-price auctions (Back and Zender, 2001; LiCalzi and Pavan, 2005; McAdams, 2007; Damianov and Becker, 2010). Although these studies have shown the efficiency-enhancing properties of supply adjustments, they have done so in a relatively restrictive environment; namely, all previous studies assume buyers have a *common* value for the auctioned goods. In this article we allow each buyer to have their own private values for the environmental rights. It is more realistic—especially in an environmental context—that buyers have private values for the auctioned goods. For example this may represent firms' values of future expected prices on a secondary market or their individual abatement costs.

This proposed auction design has direct application to pollution permit auctions (Lopomo et al., 2011). Within most cap-and-trade markets, quarterly uniform-price auctions are the predominant method of initial permit allocation. For example, the Regional Greenhouse Gas Initiative (RGGI), the European Union Emissions Trading Scheme (EU-ETS), and the California Cap-and-Trade Program run uniform-price auctions. While differences do exist in each of these auctions—such as the inclusion of Cost Containment Reserves (Khezr and MacKenzie, 2016) in RGGI and the process of consignment in California (Khezr and MacKenzie, 2018)—the basic uniform-price rules continue to apply, which may mean the existence of inefficiencies. The proposed auction design within this article shows that these schemes can be simplified and improved.

A substantive literature exists on investigating optimal (truthful inducing) regulatory design in environmental policy (e.g., Kwerel, 1977; Dasgupta et al., 1980; Duggan and Roberts, 2002; Montero, 2008; Shrestha, 2017). In this literature the goal of the regulator is to choose a regulatory design that maximizes social welfare. In these studies it is assumed that the regulator is certain about the damage function that is associated with the environmental policy but is uncertain about firms' private costs. The regulation, therefore, is designed to induce firms to act truthfully by submitting correct cost functions. In this vein, Montero (2008) proposes a uniform-price auction design that can induce truthful bidding. To achieve this, the auction must provide firm-specific revenue rebates as well as an endogenous supply of rights. The rebates are based on each firm's individual damage it exerts on others. Although the adjustment of auction supply is relatively straightforward, the use of auction revenue rebates may prove to be difficult to implement in existing auctions because the auctioneer requires knowledge over individual and aggregate damage functions for the first-best outcome to be realized. In this article we have a different initial set-up: we model an auction in a second-best scenario where the policy target is (quite realistically) set by a political process. Our objective, then, is somewhat different to this literature as we focus on how to design an efficient auction for a *given* policy level: one that improves revenue and incites truthful bidding.[2]

The proposed auction design has a number of comparative advantages. First, the proposed auction is administratively simpler in design as the auctioneer need only adjust supply downwards and restrict individual buyers to a single bid; both changes are implementable within

[2]Later in the article we provide a comparison between the proposed auction design and a conventional uniform-price auction to show the possibility of social welfare improvements.

current auction rules. Second, the auction design does not require buyers to receive revenue rebates. Previous proposals often use the revenue to incentivize truthful bidding. Yet in this new auction design the total revenue can be recycled by reducing distortionary taxation or investing in public goods. Third, efficiency within the auction is realized with minimal informational requirements for the auctioneer: knowledge of the cost and damage functions is not required for buyers to submit truthful bids.

The remainder of this article is organized as follows. Section 2 introduces the model and presents the results related to the single-bid uniform-price auction. Section 3 extends the model to allow for diminishing marginal values. Finally, Section 4 concludes the paper. All proofs are relegated to the appendix.

2 The basic model

Consider an auctioneer that sells up to $\bar{Q}$ identical units of an environmental right, such as pollution, water, or mineral rights. A set of potential buyers $\Theta = \{1, 2, \ldots, n\}$ exists where buyer $i \in \Theta$ has a marginal value v_i for all units up to a capacity λ_i.[3] A buyer's individual capacity is their maximum demand for the units and we focus on non-trivial cases where $\lambda \equiv \sum_1^n \lambda_i > \bar{Q}$.[4] Buyers' values are private information but their capacities are public information. For example, it is feasible that the capacities of firms are obtained from market and financial reports or accounting registries in secondary markets, while firms' values of the environmental rights are known only to themselves. Consequently, suppose marginal values are independently and identically distributed according to some known distribution function $F(.)$ on $[0, \bar{v}]$ with density $f < \infty$.[5] Also suppose the seller's reservation value for the rights is normalized to zero.

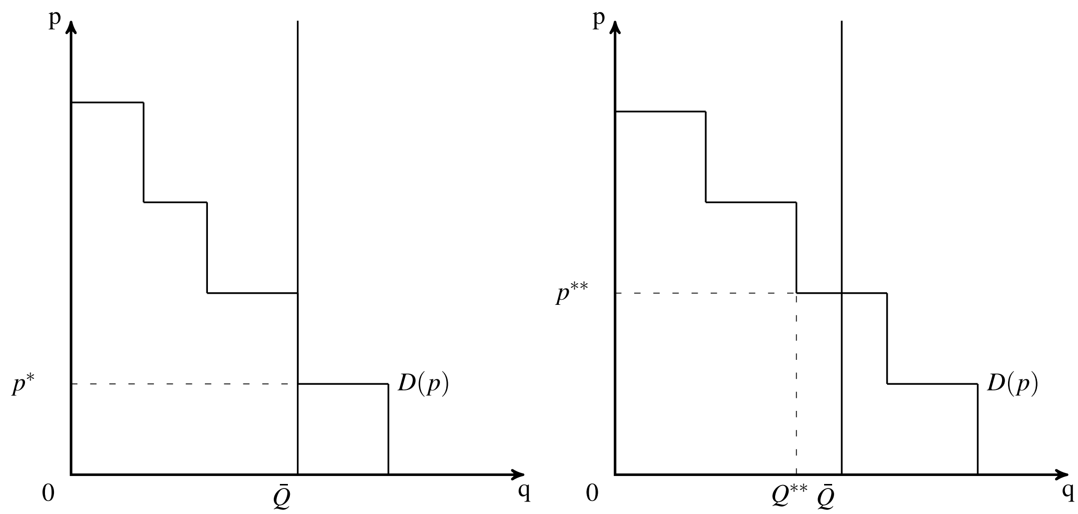
The selling method is a modified version of a sealed-bid uniform-price auction. In particular, the auctioneer only allows bidders to submit a *single* sealed-bid for all units they demand. The auctioneer sorts the submitted bids from the highest to the lowest value, which generates the aggregate demand as a step function. The allocation rule is defined as follows. At the beginning of the game the auctioneer announces $\bar{Q}$ as the maximum quantity available for sale. However, the auctioneer determines the exact amount of supply after the bids are submitted. The final quantity supplied by the seller is equal to the largest amount less than or equal to $\bar{Q}$ such that the whole capacity of successful buyers is satisfied. That is, if the lowest-valued (successful) buyer cannot obtain their full capacity then the auctioneer reduces supply until the next lowest-valued buyers can satisfy their full capacity. To observe this consider Figure

[3]In Section 3 we extend the model to allow for diminishing marginal values and provide similar results. We initially focus on constant values as there exists a number of justifications within an environmental setting. In the short run—when technologies are fixed—bidders' values of the auction units can be interpreted as their expected value of selling the right on a secondary market or their marginal cost of using their abatement technology, which would be constant for the short-run capacity limits within this auction.

[4]This capacity (or maximum demand) can be interpreted as buyers' unregulated choice of environmental actions associated with the single auction, such as their pre-regulation level of pollution or their capacity based on their current desirable level of production. Typically these environmental rights (especially pollution permits) are based on vintages (normally expiring after a few years), so buyers' capacities are based on their vintage-specific actions within this short regulatory timespan.

[5]In this model values are independent from each other. Depending on the application, one may consider a case with interdependent values. The results are robust for a situation with interdependent values.

1. In Figure 1 (a) the aggregate demand is not flat at $\bar{Q}$, therefore all buyers to the left of $\bar{Q}$ receive quantities equal to their capacities: $\bar{Q}$ is the determined auction supply. In Figure 1 (b), however, the aggregate demand is flat at $\bar{Q}$, meaning that the lowest successful buyer(s) cannot satisfy their full capacity. The auctioneer then reduces supply such that all buyers to the left of $\bar{Q}$, except those at the flat would receive quantities equal to their capacities and the buyers at the flat receive nothing. In this case the final quantity supplied by the seller, denoted by Q^{**}, is less than $\bar{Q}$.



(a) Not flat

(b) Flat

Figure 1: Supply adjustment and auction clearing price.

To specify the payment rule, denote b_i as the bid submitted by bidder i which determines the maximum willingness to pay for λ_i units. The aggregate demand is therefore the sum of individual bids, and can be represented by the following step function $D(p)$, where p is the price per environmental right:

$$D(p) = \sum_{i=1}^{n} \lambda_i|_{[b_i \geq p]}. \tag{1}$$

The auction clearing price, which is the price for all quantities won, is set by the highest losing bid. The formal expression of the auction clearing price is,

$$p^*(\bar{Q}, \lambda) = \begin{cases} \max\{p|D(p) > \bar{Q}\} & \text{if not flat,} \\ \{p|D(p) = \bar{Q}\} & \text{if flat.} \end{cases} \tag{2}$$

The first expression of (2) is equivalent to the highest losing bid when the aggregate demand is not flat at $\bar{Q}$ (Figure 1 (a)). The second expression is for the case where the aggregate demand is flat at $\bar{Q}$. In this instance, the price is set at the flat (Figure 1 (b)) .

2.1 Results

We now show how the auction design performs with respect to efficiency and how this design compares to conventional uniform-price auctions. Given the new auction design, the next proposition shows that bidders would act truthfully given the current setting.

Proposition 1. *It is a weakly dominant strategy for each bidder to bid truthfully, that is, to bid equal to their marginal values.*

An essential point in the result of Proposition 1 is that a buyer's bid is independent of the price they pay.[6] This is the result of both the payment rule and the supply adjustment. The payment rule guarantees that a bidder never pays a price equal to their own bid. Also as a result of the supply adjustment, a bidder cannot manipulate their bid to win a proportion of the total capacity at a lower price. Recall that without the supply adjustment truthful bidding may not be the optimal outcome. When there is no *ex-post* supply adjustment, bidders could win some of their capacities at the flat section of the demand schedule. In that case, there are clear incentives for demand reduction in order to reduce the final price for a proportion of the total capacity.

The next proposition compares the performance of the proposed auction design with the standard uniform-price auction.

Proposition 2. *The uniform-price auction with single bids is weakly efficient and revenue superior to the uniform-price auction with multiple bids for selling similar quantities.*

Proposition 2 compares the expected revenue of the two mechanisms for the sale of similar quantities.[7] Since in the standard uniform-price auction, demand reduction is optimal for bidders, the aggregate submitted demand is also lower than the true aggregate demand. While in the single-bid case, as a result of truthful bidding, the aggregate demand represents the true market demand. Another important result of Proposition 2 is the efficiency of the single-bid uniform-price auction. When the seller allocates these environmental units, they will be allocated to the buyers with the highest value. This is denoted "weak efficiency" as the auctioneer may decide to retain some environmental units even if there were bidders with positive value for them.

The new auction design can be compared to previous proposals for auctioning environmental rights. For example consider the approach of Montero (2008), which ensures truthful bidding by allowing the auctioneer to endogenously select the number of rights and rebate the auction revenue to firms based on their individual damage. A key difference in this new approach is that auction revenue is not used to incite truthful bidding. Under the Montero (2008) approach, the use of rebates develops the uniform-price auction into a mechanism akin to a discriminatory auction, where firms now have different net costs from the auction (thus ensuring truthful bidding). In this new approach presented here, truthful bidding can be induced without using

[6] The result in Proposition 1 can also be extended to values that are interdependent, similar to the model in (Ausubel et al., 2014).

[7] Clearly, a standard-uniform price auction for the sale of $\bar{Q}$ units could result in higher revenues than a single-bid uniform-price auction with $Q^{**} < \bar{Q}$ units. Thus to have a fair comparison, we compare both auctions for the sale of similar units.

auction revenue. This allows the revenue to be recycled in the form of cuts in distortionary taxes, investment in public goods, and so on. Thus using this proposed auction mechanism allows one of the key advantages of revenue-raising instruments—the revenue recycling effect—to be realized. In many proposals that use the auction revenue to incite truthful bidding, a key issue is ensuring the mechanism balances the budget, but with this new design this is no longer a concern. This approach is also simple to administer. In previous approaches, information is required about firm-specific damages to society. Yet no such information is required to incite truthful bidding. The auction rules are designed to ensure firms can only submit one bid and the total supply of environmental rights can be adjusted after bids have been submitted.

Throughout this part of the analysis we allowed buyers to have a constant marginal value for a given capacity (we provide the case of diminishing marginal values in Section 3). Constant values are realistic within the majority of environmental contexts, including pollution markets (e.g., Khezr and MacKenzie, 2018). For environmental rights there exist a number of justifications. First, as environmental rights are often tradable, the marginal valuation can be interpreted as firms' (differing) estimates of the expected future price on the secondary market that they could obtain for selling a right. Second, the marginal value can be interpreted as firms' marginal pollution abatement costs. As presented within the model we allow each buyer to have a capacity constraint within one auction. This can be interpreted as firms' marginal abatement costs for a single technology class. This fits well with estimates of real-world abatement cost functions that show firms have long-run convex abatement costs where constant marginal abatement costs exist for specific technology classes but there exist discontinuities between technology classes (e.g., Mckinsey and Company, 2016).

2.2 Social welfare implications

Throughout the analysis our focus has been on auction efficiency: to allocate auctioned units to those with the highest value. Thus our context is within a second-best world where the maximum externality target $\bar{Q}$ is not chosen to maximize social welfare but simply (and realistically) a political decision. Yet, even in this environment, implications for social welfare can be obtained. Consider a welfare maximizing level of Q denoted by Q^*, where the marginal damage of the environmental rights is equal to the true demand for rights (marginal cost) in Figure 2. It is likely that any real-world policy target $\bar{Q}$ is larger than the level required within a first-best world (given the nature of negative externalities). For example, this is the case in most current cap-and-trade markets. In this framework the auctioneer has the ability to reduce the level of units and—simply assuming increasing marginal damages over Q—will result in improvements in social welfare due to a reduction in the policy target away from $\bar{Q}$. From Figure 2, the social welfare gain from this auction is the hatched area under the marginal damage function (MD) and above the demand schedule. Thus social welfare improvements can be obtained without obtaining additional information about the damages associated with $\bar{Q}$.

We can also compare the proposed auction design to a conventional uniform-price auction. Figure 3 is similar to Figure 2 but now we have introduced the aggregate demand from a conventional uniform-price auction denoted by the red line. It is clear that under a conventional uniform-price auction demand reduction occurs and a lower clearing price is established at $\hat{p}$. Note that in terms of social welfare each unit is now being bought below the clearing price

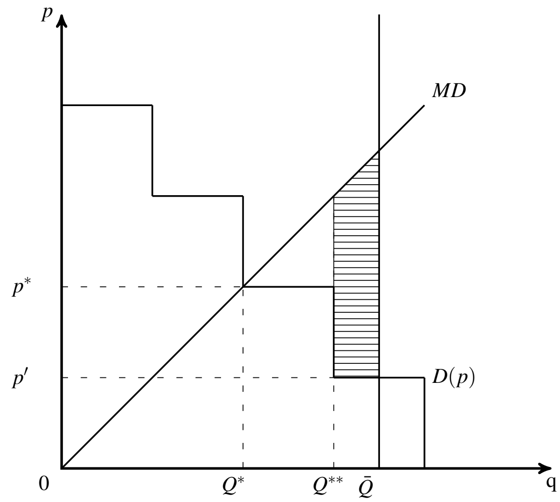


Figure 2: Impact on social welfare. A reduction in auction supply will increase social welfare.

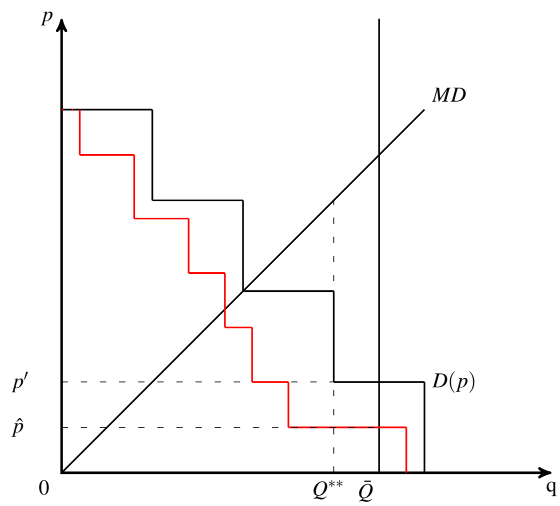


Figure 3: Comparison of single- and multiple-bid uniform-price auctions.

derived in this new auction design, p'. Thus relative to a conventional uniform-price auction, the proposed auction design can generate social welfare improvements.

3 Diminishing marginal values

In this section we extend the original model to a case where buyers have diminishing marginal values for environmental rights. Similar to the above analysis, suppose buyer i's maximum willingness to pay for an environmental right is v_i, where v_i is private information. We now suppose, similar to Ausubel et al. (2014), that buyers' values diminish at a rate equal to ρ, which is common for all buyers.

The new auction rules for the diminishing marginal values setting are as follows. Each buyer i submits a linear demand schedule $d_i(v_i, q_i)$, where q_i is the amount of demand at each prices. The auctioneer sorts these demands from the highest to the lowest and calculates the aggregate demand schedule. The auctioneer then allocates the units based on the aggregate demand function from the highest price to the lowest until $\bar{Q}$ is reached. Let us first discuss the shape of the aggregate demand. The aggregate demand starts from the highest submitted value, and diminishes at a rate equal to ρ until it reaches the second highest submitted value. At this point, there exists a kink in the aggregate demand where it diminishes at a rate equal to $\frac{\rho}{2}$ until the next kink, which is the third highest submitted value, and so on.

The supply adjustment is as follows. Every bidder to the left of $\bar{Q}$ receive their demands as long as the aggregate demand diminishes at a rate equal to $\frac{\rho}{k}$ after kink k. Otherwise, the supply reduces to the first kink k' that satisfies the right diminishing rate, that is, $\frac{\rho}{k'}$.

Proposition 3. *It is weakly dominant strategy for each buyer i to submit their true demand.*

The intuition behind Proposition 3 is that being untruthful results in a punishment by the supply adjustment. A buyer may influence the price that other buyers pay by submitting an untruthful demand, but cannot reduce the price they pay for their own units by reducing their own demand schedule. The weakly dominant strategy provides several more desirable features for the proposed mechanism, which are presented in the next proposition.

Proposition 4. *The mechanism is fully efficient and revenue superior to all undominated equilibria of the uniform-price auction.*

Proposition 4 indicates that, in the case of diminishing marginal values, the proposed mechanism with supply adjustment would allocate all the $\bar{Q}$ units to those with the highest value. Therefore, it results in strong efficiency. This result is noteworthy because almost all of the existing literature suggests that most variants of the uniform-price auction are inefficient. Also note that when buyers have diminishing marginal values, it is usually more difficult for the regulator to identify demand reduction. In the current mechanism this is not a problem as the simple supply adjustment acts to prevent buyers from demand reduction. Note, however, that in equilibrium the mechanism achieves full efficiency but the supply adjustment process is not used.

The diminishing marginal value setup extends the previous results of this article to address situations where buyers have downward sloping demands rather than flat demands. Although there are reasonable grounds to assume that buyers in environmental markets may have constant marginal values for a single auction (as discussed in the previous section), the extension to the case of diminishing marginal values helps to highlight the robustness of this auction design and the potential applicability of this framework to other areas of interest.

4 Conclusions

This article proposes an auction that results in truthful bidding for environmental rights. The proposed auction design has two distinct features (i) the auctioneer allows bidders to submit only one bid for their full demand of environmental rights and (ii) we allow a simple *ex-post* supply adjustment rule, where the aggregate supply of rights can be reduced after bids have been submitted. Using these two features we show truthful bidding occurs. In comparison to previous proposals for environmental auctions, this proposed mechanism is administratively simple, requires no knowledge of firms' marginal costs or marginal damages, and does not use any of the generated revenue as firm rebates—the use of rebates is often proposed to incite truthful bidding. Thus this mechanism is able to efficiently allocated environmental rights, and therefore generate maximum revenue that can be used in revenue recycling policies.

A natural application of this auction is to tradable permit auctions. In all current permit auctions, the uniform-price design is chosen to generate a single equilibrium price that can assist the development of the secondary market. Yet this design comes at a cost: these uniform-price auctions are known to be inefficient as bidders normally shade their bids with a resulting lower equilibrium auction price (Khezr and MacKenzie, 2018). This newly proposed auction can generate an efficient permit auction that will generate the Walrasian equilibrium.

Although tradable permit auctions are a key application, this auction design can be further applied to other environmental (multi-unit) resources (such as rights for water, conservation, and so on) as well as non-environmental rights such as treasury bills.

Appendix

Proof of Proposition 1. Suppose bidder i with value v_i submits a bid $b_i < v_i$ and the price at which all units are sold is p. We claim that bidding $b_i = v_i$ weakly dominates $b_i < v_i$. If $p > v_i$ in both situations the payoff are zero. If $b_i < p < v_i$, then bidding b_i guarantees zero payoff while bidding v_i would result in positive payoff. Finally, if $p < b_i < v_i$ both cases would result in similar payoff. Also p is independent of b_i, that is, demand reduction cannot influence the final price. This is due to the payment rule and the supply adjustment. Therefore, bidding v_i weakly dominates bidding $b_i < v_i$.

Now suppose bidder i submits a bid equal to $b'_i > v_i$. We show this is also weakly dominated by a bid equal to her value. If $v_i < b'_i < p$, then both cases would result in zero payoff. If $v_i < p < b'_i$, then b'_i wins but results in negative payoff, since $v_i < p$. In this case bidding equal to v_i results in zero payoff. Finally, if $p < v_i < b'_i$ both bids would result in similar payoffs. □

Proof of Proposition 2. The efficiency part is a straightforward result of truthful bidding. Since the mechanism allocates the objects to those with the highest values, then it is efficient. However, since there are instances that the seller may sell less than $\bar{Q}$, then it is *weakly efficient*.

To prove the revenue superiority of the single-bid uniform-price auction, we first claim that if buyers are allowed to submit multiple bids then it is optimal for them to shade their bids, that is, they bid lower than their marginal values for at least some of the units they demand. Imagine bidders are allowed to submit different bids for every unit they demand. So the sequence of these bids for each bidder represents the demand schedule of the bidder. Let us focus on a bidder, say bidder i, and assume that they only demand two units, that is, $\lambda_i = 2$. The value for both units is equal to v_i. We are going to show if all other bidders submit truthful bids equal to their values, then it is not the best response for bidder i to do so, as long as they are allowed to submit two bids. Denote $b_i^1(v_i)$ as the bid for the first unit. We first show that it is a weakly dominant strategy for the bidder to bid $b_i^1(v_i) = v_i$. Suppose they bids $\beta_i^1(v_i) > v_i$ instead of $b_i^1(v_i) = v_i$. In the case that the auction clearing price p is lower than v_i the payoff remains the same, because they win the object and pay p. But if $\beta_i^1(v_i) > p > v_i$ then they receive a negative payoff while bidding $b_i^1(v_i) = v_i$ and result in zero payoff. If they bid lower than their value, that is, $\beta_i^1(v_i) < v_i$ then again it is weakly dominated by bidding $b_i^1(v_i) = v_i$. This is because in the case that $\beta_i^1(v_i) < p < v_i$ the payoff is zero while bidding bidding $b_i^1(v_i) = v_i$ results in a positive payoff. If $p < \beta_i^1(v_i)$ both bids results in the same payoff. So we can conclude that bidding $b_i^1(v_i) = v_i$ for the first unit is a weakly dominant strategy.

Denote $b_i^2(v_i)$ as the bid for the second unit. Also define $\{F_{(1)}, ..., F_{(n)}\}$, as the distribution of order statistics of values $\{v_{(1)} \geq v_{(2)}, ..., \geq v_{(n)}\}$. So with a probability equal to $F_{(m)}(v_i)$, bidder i has the mth highest value among other bidders. Now if buyer i has the mth highest value, since the capacities of other bidders are asymmetric, there are two possibilities. One, is that the sum of the capacities of those bidders with higher values is less than Q^{**}, the equilibrium level of auction supply after the adjustment. In this case bidder i is pivotal, or in other words, has a chance to win the object. Two, is the case that the sum of the capacities of those bidders with higher values is more than Q^{**}. In this case, bidder i is not pivotal and even her first bid cannot win. For each order $m \in \{1, ..., n\}$ of i's value, denote δ_m as the proportion of cases they are pivotal in all possible cases and $(1 - \delta_m)$ as the proportion of cases they are not pivotal, where

$0 \leq \delta_m \leq 1$. Also it is easy to check that $\delta_1 \geq \delta_2, ..., \geq \delta_n$, that is, for higher orders the bidder is pivotal in less cases.

Let us focus on one particular order of bidders excluding bidder i, where $n' < n$ bidders have the highest values such that the sum of their capacities is larger or equal to Q^{**}. Also when we exclude the n'th highest bid the sum of capacities becomes lower than Q^{**} and bidder i becomes pivotal. Denote v_1 and v_2 as the $n'-1$th and n'th highest values. Also define $F_{(n'-1)}(.)$ and $F_{(n')}(.)$ as their marginal distribution with densities $f_{(n'-1)}$ and $f_{(n')}$. The expected payoff of bidder i when they submit $b_i^1(v_i)$ and $b_i^2(v_i)$ is,

$$\begin{aligned}\pi_i(b_i^1, b_i^2) = & F_{(n'-1)}(b_i^2)(2v_i) - 2\int_0^{b_i^2} v_1 f_{(n'-1)}(v_1)dv_1 + (F_{(n')}(b_i^1) - F_{(n'-1)}(b_i^2))v_i \\ & - (F_{(n')}(b_i^2) - F_{(n'-1)}(b_i^2))b_i^2 - \int_{b_i^2}^{b_i^1} v_2 f_{(n')}(v_2)dv_2.\end{aligned} \tag{3}$$

Differentiate the above expected payoff with respect to the second bid. After some cancellation we have,

$$\frac{\partial \pi_i(b_i^1, b_i^2)}{\partial b_i^2} = v_i f_{(n'-1)}(b_i^2) - b_i^2 f_{(n'-1)}(b_i^2) - (F_{(n')}(b_i^2) - F_{(n'-1)}(b_i^2)). \tag{4}$$

Finally when $b_i^2 = v_i$ we have,

$$\frac{\partial \pi_i(b_i^1, b_i^2)}{\partial b_i^2} = -(F_{(n')}(b_i^2) - F_{(n'-1)}(b_i^2)) < 0. \tag{5}$$

Therefore, when bidder i is pivotal, bidding equal to the marginal value is not optimal for the second unit. It is possible to extend this analysis to a case with $\lambda_i > 2$. In this case, we can show that it is not optimal to bid equal to v_i for at least some proportion of the total capacity λ_i. In particular, suppose bidder i still submits two bids, one for some proportion of the whole capacity and the other for the remaining. With the same argument as above we can show that it is not optimal to bid equal to v_i for the second part of its capacity. However, it is possible that given the capacity of buyer i we need to exclude more than one bidder to make bidder i pivotal. The only thing that changes is the marginal probability while the rest of the analysis remains the same.

What remains is to show that the expected revenue from single-bid uniform-price is at least as large as the expected revenue of a standard uniform-price auction. Since we have already shown that in the single-bid case bidders submit truthful bids, the aggregate demand function submitted by bidders in a single-bid case is above the aggregate demand submitted by bidders in a standard case given the same quantity Q^{**}. Thus it must be the case that the equilibrium price for the single-bid case is larger than the one for multiple bids when selling the same quantity Q^{**} and the seller's revenue which is price times quantity is larger for the single-bid uniform-price auction. □

Proof of Proposition 3. To show that bidding truthfully is a weakly dominant strategy we start by a representative buyer and show that this buyer cannot benefit from demand reduction. Suppose buyer i bids its true demand schedule, that is, $p = v_i - \frac{v_i}{\lambda_i} q_i$. First, calculate the aggregate demand without the consideration of buyer i. Define c_{-i} as the value of the buyer who sets the last kink before $\bar{Q}$ when excluding buyer i in the aggregate demand.

Case 1: If $v_i > c_{-i}$, then buyer i wins some units, say q_i, and pays a price $p < v_i$. Thus, the payoff for buyer i is,

$$\pi_t = \frac{(v_i - p)q_i}{2}. \tag{6}$$

If $v_i < c_{-i}$, then two outcomes are possible. The kink at v_i on the aggregate demand could be either to the left of $\bar{Q}$ or to the right and the last one before $\bar{Q}$.

Case 2: When the kink is at the left of $\bar{Q}$, but lower than c_{-i}, they win q_i' units and pay a price equal to p' with a positive payoff equal to,

$$\pi_{t'} = \frac{(v_i - p')q_i'}{2}. \tag{7}$$

Case 3: The last case where the kink is at the right of $\bar{Q}$, buyer i receives zero payoff.

Now, suppose buyer i submits a value $z_i < v_i$ in its demand schedule then in Case 3 they obviously receive zero payoff again. But now in Case 1 and 2 they would also receive a zero payoff due to the supply adjustment. To see this, suppose the kink at z_i is the kth highest kink. The slope of the aggregate demand after this kink is,

$$slope = \frac{\frac{\rho}{(k-1)}\rho'}{\frac{\rho}{(k-1)} + \rho'}, \tag{8}$$

where $\rho' < \rho$ is the slope of the buyer i's demand function with z_i. The above slope is less than $\frac{\rho}{k}$ because,

$$slope = \frac{\frac{\rho}{(k-1)}\rho'}{\frac{\rho}{(k-1)} + \rho'} < \frac{\rho}{k} \Leftrightarrow \frac{\rho'}{(k-1)} < \frac{\rho + (k-1)\rho'}{k(k-1)} \Leftrightarrow k\rho' < \rho + (k-1)\rho'. \tag{9}$$

The last inequality is true because $\rho' < \rho$. According to the supply adjustment, when the slope of aggregate demand is less than $\frac{\rho}{k}$ after kink k, the supply would reduce to kink k and buyer i receives zero units.

Thus it is a weakly dominant strategy for each buyer to bid their true demand.

□

Proof of Proposition 4. Given the result of Proposition 3, if all buyers except buyer i play their weakly dominant strategy, then there is no incentive for buyer i to deviate from this equilibrium. So submitting the true value is a dominant strategy equilibrium of this game. The efficiency part is straightforward because the mechanism allocates the objects to those with the highest values, since in equilibrium the aggregate demand represents the true demand of each buyer. Also, since the supply adjustment does not take place in equilibrium, all quantities available would be allocated to buyers. Thus the mechanism is fully efficient.

To show the revenue superiority, we rely on the existing results for the uniform-price auction that suggests every equilibria of the uniform-price auction is inefficient (Ausubel et al., 2014). Therefore, the aggregate demand submitted in a standard uniform-price auction is always lower than the one for the current mechanism. As a result the auction clearing price is lower for the standard uniform-price and the revenue of the auction, which is $p\bar{Q}$, is also lower for the standard uniform-price compared to the current mechanism.

□

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