Firm Size Distribution, Production Efficiency, and Returns to Scale: A Stochastic Frontier Approach *

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Abstract

This paper empirically investigates the relationship between firm size, production efficiency, and returns to scale. Using a recently developed stochastic frontier approach and data from Vietnam, our analysis shows that across all of the sectors we consider, production efficiency is most variable among middle-sized firms, with these firms across all sectors tending to have the lowest production efficiency. While most firms across different sized groups show constant returns-to-scale technologies, our analysis using Spearman coefficients shows that there is a significant difference in technologies and this difference varies substantially across size groups in all sectors. Furthermore, we show that the least-efficient size also differs across sectors. Although our analysis is a snapshot of the Vietnamese manufacturing industry, the diverse production efficiencies in the middle-sized groups can be thought of as a risk that small-sized firms would face in expanding their business.

Key Words: Firm size distribution, Missing middle, Productivity, Efficiency, Stochastic frontier.

JEL Classification Numbers: D21, D22, L25.

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1 Introduction

The missing-middle phenomenon, as first documented by Liedholm and Mead (1987), refers to the empirical feature that most employment in developing countries is located in either small- or large-sized firms. Tybout (2000) also finds that there is a large spike in the size distribution for small-sized firms, but that this drops off quickly among middle-sized firms in the poorest countries, and thus argues that strong business regulation could be a reason behind the existence of too many small firms[1].

Considering that the missing middle is more evident in developing nations, we envisage that there are some reasons for its existence that particularly apply to developing countries, but not so much to developed countries. In search of these reasons, we employ firm-level data from Vietnam and examine whether there is any difference in production efficiency across different firm sizes. For this purpose, we estimate each firm's production efficiency and returns to scale (RTS) in particular manufacturing sectors. Using a recently developed stochastic frontier approach and data from Vietnam, our analysis shows that across all of the sectors we consider, production efficiency is most variable among middle-sized firms, with these same firms across all sectors tending to have the lowest production efficiency. While most firms across different size groups show constant returns-to-scale technologies, our analysis using Spearman coefficients shows that there is a significant difference in technologies and this difference varies substantially with size groups across sectors. Furthermore, we show that the least-efficient size also differs across sectors. This suggests the necessity of sector-by-sector analysis when considering the underlying mechanism for the missing-middle phenomenon.

We also find that there are many firms whose efficiency is low; in fact, the efficiency of some firms is less than 70% of that of the most efficient firms, across the different firm sizes in all sectors. Almost half a century ago, Leibenstein (1966) introduced the concept of *X-efficiency*, with the argument that this inefficiency, unlike allocative efficiency, is mainly the result of incentive misalignment in the workplace. Leibenstein (1966) reviews several empirical studies and shows that allocative inefficiency, such as that caused by a tax, could account for only very small losses in total production whereas improving management or motivational efficiency in a workplace could increase production

[1]See also Little et al. (1987) for South Korea and Steel and Webster (1992) for Ghana. The presence of the missing-middle phenomenon and our definition have been intensively discussed in the literature (see Hsieh and Olken, 2014; Tybout, 2014a,b).

by approximately 50%. Although it is generally difficult to separate these inefficiencies empirically, our results suggest that there are many inefficient firms of different sizes in each sector and that this feature is more prominent in middle-sized firms across the different sectors.

Hsieh and Olken (2014) consider the possibility that regulatory obstacles generate the missing-middle phenomenon, but find no evidence of discontinuities in the firm size distribution. In contrast, Tybout (2014b) claims that the missing middle is at least partly policy induced and states that when policies are imperfectly enforced, producers tend to prefer to remain small in order to avoid taxes and costly regulations. One difference of our approach from the previous literature is that rather than looking at taxes or regulations themselves, we study production efficiency as a manifestation of the business environment in developing countries. Certainly, business environments vary across countries. However, while it is difficult to measure precisely the health of the business environment or how well managed the firms are, production efficiency can yield evidence about the management of an organization. We provide empirical evidence that the use of resources in developing countries might be less than efficient and that this inefficiency is more prominent in middle-sized firms.

We begin with a test of the firm size distribution against bimodal and unimodal distributions. The bimodality statistics for the entire sample of the manufacturing industry that we examine imply the existence of bimodality in Vietnam. We then study the relationship between production efficiency and firm size for various sectors of the manufacturing industry by using the stochastic frontier approach. Our first contribution is to show that production efficiency is most diverged in the group of middle-sized firms. We obtain this same finding using a nonparametric approach, which we also present in this paper. Like our companion paper, Pham and Takayama (2015), which provides evidence that the likelihood of paying bribes increases with firm size, this paper provides evidence that some middle-sized firms are disadvantaged in production. Further, our paper extends the analyses in Pham and Takayama (2015) by using a stochastic frontier approach, which allows us to consider the relationship between returns to scale, technological differences for each firm, and firm sizes. As mentioned, most firms show constant returns-to-scale technologies, although our detailed analysis of Spearman coefficients across different size groups shows that firms' technologies are significantly different.

Overall, our analysis shows that the middle-sized group is a 'mixed bag' in the sense that firms with very high and very low production efficiencies coexist in this size group. Meanwhile, there is a significant heterogeneity in firms' technologies. Although our analysis is a snapshot of the Vietnamese manufacturing industry, the diverse production efficiencies in the middle-sized groups can be thought of as a risk that small-sized firms would face in expanding their businesses.

In recent years, many methods have been developed to estimate production efficiency using the frontier approach.[2] Comparing average products across different inputs is a complicated task. As pointed out by Ray (2004), measures of a firm's productivity relying on a single input disregarding other inputs may fail to reflect total factor productivity. Taking this into account, we use the frontier approach.[3]

Aigner et al. (1977) and Meeusen and van den Broeck (1977) propose a stochastic production frontier approach. This approach uses a parametric estimation method, where the form of the production function is known and can be estimated statistically. The advantages of this approach are then that hypotheses can be tested with statistical rigor, and that we can apply some known functional form between outputs and inputs. Using this approach, we can simultaneously estimate technical efficiency and returns to scale. Several studies have already measured the performance of manufacturing sectors using this method (see Caves and Barton, 1990; Greene and Mayes, 1991). However, both these studies assumed that firms share the same technology.

One of the challenges in the stochastic frontier literature is the treatment of heterogeneity and analyzing its effects on efficiency estimations.[4] For this reason, a random coefficient stochastic frontier model has been developed to incorporate heterogeneous effects. For example, Tsionas (2002) uses Gibbs sampling to estimate the random coefficient stochastic frontier model for US electric utility companies in 1970, and provides evidence that the random coefficient model is a better fit than the fixed coefficient model. Elsewhere, using a similar method Diaz and Sanchez (2008) shows that compared with larger-sized firms, small and middle-sized firms in Spain tend to be more efficient.

As pointed out by Feng and Zhang (2014), not considering the technological heterogeneity of an

[2] Production frontiers can be estimated using parametric estimates (so-called deterministic or stochastic frontier analysis) or calculated using nonparametric approaches (such as the free-disposal hull). For details, see Kalirajan and Shand (1994).

[3] Some other studies, including the following, estimate total factor productivity (TFP). Johannes (2005) provides evidence on the differences in the evolution of firm size and the productivity distribution across nine sub-Saharan African countries and developed countries, including the US. Banerjee and Duflo (2005) provide a survey and analyze what accounts for lower TFP, such as access to technology or human capital, in poor countries (which they call "macro puzzles").

[4] The omission of heterogeneity in stochastic frontier models has been well documented to produce biased efficiency estimations (see Greene, 2008; Kumbhakar and Lovell, 2000, for complete reviews). Empirical studies have also shown relevant implications of heterogeneity in the estimation of both efficiency levels and rankings (see Bos et al., 2009; Greene and Mayes, 2004, for examples in the banking and health sector).

individual firm even within a single sector can cause bias in measuring returns to scale. Consolidating data from multiple sectors may further distort the estimation of returns to scale. It is particularly important to study this measure at the sectoral level, as the underlying technology in each sector would not be much different in comparison with different sectors. To the best of our knowledge, we know of no other studies that use a random stochastic frontier analysis to consider firms' heterogenous technologies and estimate production efficiencies across different firm sizes in various sectors of a developing country in the context of the missing-middle phenomenon. In particular, this is because analysis of the underlying mechanisms for the observed firm size distribution is less developed, perhaps because of the lack of detailed data.[5] The dataset we use is the Enterprise Census conducted by the General Statistics Office of Vietnam (GSO) containing information on all registered firms in Vietnam. Our data include capital, labor, and intermediate materials as inputs, and goods measured in monetary terms as the output. Table 1 details the seven sectors[6] we examine in this paper.

Our work also relates to the literature on estimating returns to scale. By using a similar method, Feng and Zhang (2014) investigate the returns to scale of large banks in the US over the period 1997–2010 by considering technological heterogeneity and show that most bank production functions exhibit constant returns to scale. Some other works estimate returns to scale for an entire industry. For instance, Basu and Fernald (1997) employ US data to estimate returns to scale and find that a typical industry appears to have constant or slightly decreasing returns to scale. Diewert et al. (2011) find a similar result using Japanese data over the period 1964 to 1988 (see also Miller and Noulas,

[5]Recently, there has been growing interest in the relationship between the firm size distribution and productivity in developed countries. For example, Leung et al. (2008) investigate differences between the US and Canada in the distribution of employment over firm size, and confirm that a larger average-sized firm supports higher productivity at both the plant and firm levels. Crosato et al. (2009) investigate data from a micro-survey in Italy, and using nonparametric estimates, reveal that firms in the concave part of a Zipf plot of the Italian firm distribution overwhelmingly experience increasing returns to scale, while firms in the linear part are mainly characterized by constant returns to scale. Elsewhere, Diaz and Sanchez (2008) employ a stochastic frontier model to investigate small- and medium-sized manufacturing enterprises in Spain and find that these firms tend to be less efficient than their larger peers.

[6]These sector codes are based on International Standard Industrial Classification (ISIC) codes. There are 14 sectors with more than 200 observations each year, for which we can ensure the robustness of the empirical results. Then we choose five sectors among them, using the convergence criterion for the Markov Chain Monte Carlo (MCMC) procedure in our model of translog production functions. For the other nine sectors, we have used a Cobb–Douglas production function, which provides a better fit for these nine sectors. A summary of the results for the other nine sectors is available in Section 6.2 and the details are available upon request from the authors.

1996; Doraszelski, 2004). Our paper is one of the first to apply a random coefficient model to estimate returns to scale for each firm using the stochastic frontier approach to data from a developing country. We intensively study inefficiencies and returns to scale across different firm sizes at the sectoral level.

The remainder of the paper is organized as follows. Section 2 describes our dataset. Section 3 provides some data statistics about the size distribution of Vietnamese firms. Section 4 describes our methodology for estimating production efficiency and returns to scale. Section 5 presents the main results. Section 6 further examines the main results. The final section concludes.

Table 1: ISIC Codes and Industry Description

ISIC	Industry Description	ISIC	Industry Description
18	Wearing apparel etc.	20	Wood and wood products
26	Nonmetallic mineral products	28	Fabricated metal products
36	Furniture		

2 Data Description

An enterprise census has been conducted annually in Vietnam since 2000, where an enterprise is defined as "an economic unit that independently keeps a business account and acquires its own legal status". In this paper, we focus on the nine-year period from 2000 to 2008. We exclude recent years (from 2009 to the present) to separate the size distribution from the effects of the global financial crisis and certain size-biased policies such as a 30% reduction in corporate income tax for qualifying entities implemented in the fourth quarter of 2008 and all of 2009[7] or the tax reduction for small and middle-sized firms involved in labor-intensive production and processing activities under Decree 60/2011. We also exclude inconsistent data from our sample, such as twice-recorded observations for the same firm in the same year, firms with negative or zero revenue values, and firms with an implausibly large number of employees. We primarily present the results for five industries that have at least 200 annual observations and for which translog specifications of the production functions provide the best fit. In our main analysis, we have chosen firms that have data for at least four years.

In our analysis, the size of a firm is related to a head-count of the number of workers in the firm at the end of the fiscal year. Labor is measured as the total income of employees in a firm. This includes total wages and other employee labor-related costs such as social security, insurance, and

[7]See Circular No. 03/2009/TT-BTC published in January 2009 by Vietnam's Ministry of Finance.

other benefits. Intermediate materials include costs such as fuel and the value of other materials. Capital is measured as assets used in production.

We should note that at the firm level, prices and quantities may not be well measured, and therefore revenue, rather than gross output or cost, is normally used. Hsieh and Klenow (2009) discuss some of the problems: if products are differentiated, revenue-based productivity measures may mainly reflect market distortions, mark-ups, and adjustment costs. Basu and Fernald (1997) provide evidence that elasticities of labor and capital in revenue estimations are downward biased. Klette and Griliches (1996) report that changes in sector prices are substantially diversified and correlated with changes in labor and capital. Ideally, physical output should be used to eliminate the price bias that arises if output is constructed by deflating firm revenues by an industry-level price index. Owing to data constraints, we use the deflated revenue as a proxy for physical output. Moreover, because our main interest is the relative ranking rather than the level of firm efficiencies, we expect that the estimations relying on revenue data do not markedly affect our discussion.[8]

All variable values are adjusted for inflation. The annual inflation data are from the World Bank. Detailed descriptions of the variables can be found in Table 7.

3 Preliminary Statistics: The Missing Middle

3.1 The Bimodality Coefficient

We now present some statistics and provide a more detailed analysis of the firm size distribution. First, we conduct the bimodality coefficient test. This test examines the relationship between the bimodality, skewness, and kurtosis of the distribution. The bimodality coefficient (BC) is $\frac{m_0^2+1}{m_1+\frac{3(n-1)^2}{(n-2)(n-3)}}$, where m_0 is skewness and m_1 is the excess kurtosis. In words, BC is proportional to the division of squared skewness by uncorrected kurtosis. This is consistent with the logic that the bimodal distribution tends to have low kurtosis and/or asymmetric characteristics.

The BC statistics range from 0 to 1. The maximum value of 1 is obtained if the population has only two distinct values. Heavy-tailed distributions will have small values of BC, regardless of the number of modes (see Knapp, 2007). A BC that exceeds 0.55 (the value of a uniform population)

[8]Strategies for dealing with revenue data range from calibrated modeling Hsieh and Klenow (see 2009) to econometric estimators that take a stand on the nature of consumer preferences (see Loecker et al., 2015; Loecker and Warzynski, 2012; Loecker, 2011; Katayama et al., 2009; Klette and Griliches, 1996).

Table 2: The Dip Statistics and Bimodality Coefficients for Each Sector

Sec	18	20	26	28	36
Dip	0.016	0.025	0.014	0.030	0.022
p-value	0 0	0	0	0	0
BC	0.62	0.68	0.37	0.66	0.69

Note - BC indicates the Bimodality Coefficients.

suggests bimodality. [9] As shown, all BC statistics are larger than 0.55, which indicates the existence of bimodality. The BC test, however, is criticized because of its sensitivity to the skewness of the distribution. Thus, we also include the results of the Dip test, which is considered to be more robust (see Freeman and Dale, 2013).

3.2 The Dip Test

Based on the fact that a truly bimodal distribution should have a large dip between the two modes, Hartigan and Hartigan (1985) develops the dip test to distinguish between unimodality and bimodality. In particular, the test examines the departure of the observed density function from the unimodal one which is assumed to have a single inflection point between the convex and concave segments Freeman and Dale (2013). Therefore, Dip statistics tend to be robust to skewness as they test the observed function against the presence of a single rejection point. By using the Dip test, we can study the null hypothesis H_0 that the sample distribution has a unimodal density, against the alternative hypothesis H_1 that the sample distribution has more than one mode. Hartigan and Hartigan (1985) define the Dip statistics as follows. Let $\rho(F, G) = \sup_x |F(x) - G(x)|$ for any bounded functions F, G. Define $\rho(F, \mathscr{A}) = \inf_{G \in \mathscr{A}} \rho(F, G)$ for any class $\mathscr{A}$ of bounded functions. Let $\mathscr{U}$ be the class of unimodal distribution functions. The Dip statistics of a distribution are defined by $D(F) = \rho(F, \mathscr{U})$. According to the results of the Dip test, the null hypothesis is rejected at the 5% significance level for all years, which indicates the presence of bimodality or multimodality.[10]

Table 2 presents the results for the bimodality coefficients and the Dip tests. The BCs indicate that the five sectors' firm size distributions are bimodal. This is consistent with the results of the Dip tests.[11]

[9] For more details, see p. 1258 in SAS Institute Inc. (2008).

[10] A more detailed interpretation of this statistic is found in Freeman and Dale (2013).

[11] Pham and Takayama (2017) expands the set of sectors to 14 sectors, and presents the results of the bimodality coefficients and the Dip tests including the other nine sectors.

Table 3: The Dip Statistics and Bimodality Coefficients for Each Sector and Each Year

Bimodality Coefficients

Sec.	18	20	26	28	36
2000	0.69	0.70	0.64	0.72	0.69
2001	0.64	0.74	0.67	0.72	0.78
2002	0.63	0.71	0.68	0.74	0.75
2003	0.59	0.72	0.66	0.75	0.66
2004	0.59	0.73	0.66	0.78	0.74
2005	0.67	0.76	0.67	0.71	0.70
2006	0.67	0.66	0.65	0.69	0.75
2007	0.64	0.58	0.66	0.69	0.72
2008	0.66	0.71	0.80	0.72	0.71

The Dip Statistics (p-value)

Sec.	18	20	26	28	36
2000	0.021	0.024	0.025	0.020	0.021
	(0.30)	(0.01)	(0.00)	(0.19)	(0.29)
2001	0.016	0.023	0.016	0.026	0.016
	(0.42)	(0.02)	(0.06)	(0.00)	(0.43)
2002	0.018	0.021	0.016	0.29	0.018
	(0.14)	(0.02)	(0.05)	(0.00)	(0.14)
2003	0.016	0.017	0.017	0.025	0.016
	(0.10)	(0.03)	(0.02)	(0.00)	(0.10)
2004	0.016	0.021	0.014	0.027	0.016
	(0.05)	(0.00)	(0.07)	(0.00)	(0.06)
2005	0.019	0.021	0.015	0.022	0.019
	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)
2006	0.026	0.031	0.017	0.057	0.026
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
2007	0.023	0.026	0.014	0.033	0.022
	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)
2008	0.034	0.030	0.018	0.035	0.034
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: Variables inside parentheses are the p-values. The p-value is calculated by comparing the Dip statistics obtained from the data, and the one from random resamplings by using STATA.

We have also computed the BCs for each sector by year as shown in Table 3. Because the number of observations in each year for each sector decreases, the results fluctuate. In the yearly results, the BCs tend to increase, and even for sectors 26, the BCs are higher than 0.65. Perhaps sector 26 is the interesting case. Although it shows the lowest score for any year, sector 26's score for each year is higher than 0.65. We further analyzed sector 26's histograms of firm size distribution for each year and found that small firms grew noticeably in this sector year by year. According to Anh et al. (2014), the nonmetallic mineral products industry is one of the "sunrise" industries in Vietnam. This industry's ranking in the share of total output has increased from eighth (in 2001) to second in 2007, and continues to be the second highest after Food products.

Finally, we present the Dip test results for each sector by year in Table 3. According to the results of the Dip test, in more recent years, the null hypothesis is rejected at the 5% significance level for all sectors, which indicates the presence of bimodality or multimodality, although between 2000 and 2004, we cannot reject the null hypothesis for some sectors.

4 Estimation Methodology

The stochastic frontier method is selected because it allows for statistical noise resulting from events outside the firm's control as well as other types of specification error and the omitted variable problem.

Moreover, the random coefficient model is chosen. The advantage of our method is that it allows us to capture the unobserved technological heterogeneity across different firms in an industry. By allowing the coefficient to vary across firms, we can estimate the returns to scale for each firm in the industry. In particular, we allow the coefficient of the standard fixed coefficient stochastic frontier model to vary by decomposing the firm-specific coefficients into two parts: the mean vector and the random vector. The random vector is drawn from a multivariate normal distribution of mean zero. The fixed coefficient model can be obtained easily by excluding the random components.

In general, the advantage of our random coefficient model over the fixed version is that it provides a better approximation of the underlying heterogeneity of technologies across different firms and thus allows us to estimate the returns to scale of each firm more accurately. With this method, we can obtain, for each firm, a separate production frontier and thus estimate the returns to scale of each firm according to its own frontier. This method is especially useful when there exists a high degree of heterogeneity across observations.

We employ both the fixed and random coefficient methods to estimate the efficiency of production. Our main analysis employs a stochastic frontier (SF) model using panel data. In this section, we briefly detail the Bayesian procedure for estimating the random coefficient SF model. We adopt the methodology detailed in O'Donnell and Coelli (2005) and Feng and Zhang (2014). The Bayesian approach is preferable over the maximum likelihood estimation because the first method does not require the specification of an explicit distribution for the inefficiency term. At the same time, the estimation can utilize prior information to update the posterior estimation. We estimate the following translog production function:

$$\begin{aligned}\ln y \quad &= \quad a_0 + \textstyle\sum_n^3 b_n \ln x_n + \frac{1}{2}\sum_n^3 \sum_j^3 b_{nj} \ln x_n \times \ln x_j \\ &\quad +\sigma_t t + \tfrac{1}{2}\sigma_{tt} t^2 + \textstyle\sum_n^3 \rho_n t \ln x_n + u + v.\end{aligned} \tag{1}$$

The translog function allows interactions between inputs and is widely used in the literature. We compare the deviance information criterion (DIC) between that from the Cobb–Douglas analysis and that from the translog function analysis. Because that in the translog analysis shows better fit, we mainly report the results from the translog analysis.

Let $\ln y = q_{it}$ for some i, t, and we rewrite (1) as:

$$q_{it} = \mathbf{x}_{it}\boldsymbol{\beta}_i + u_{it} + v_{it}, \tag{2}$$

where $\boldsymbol{\beta}_i$ follows a multivariance normal distribution: $\boldsymbol{\beta}_i \sim i.i.d. N(\bar{\boldsymbol{\beta}}, \boldsymbol{\Omega})$. Also, $v_{it} \sim i.i.d. N(0, \sigma_v^2)$ and $u_{it} \sim i.i.d. \exp(\lambda^{-1})$.

Decomposing $\boldsymbol{\beta}_i$ into its mean vector of $\bar{\boldsymbol{\beta}}$ and a random vector $\boldsymbol{\delta}_i$ yields

$$\boldsymbol{\beta}_i = \bar{\boldsymbol{\beta}} + \boldsymbol{\delta}_i, \tag{3}$$

where $\boldsymbol{\delta}_i \sim i.i.d. N(\mathbf{0}, \boldsymbol{\Omega})$.

Given these specifications, we can rewrite the stochastic frontier model as

$$q_{it} = \mathbf{z}_{it}'\bar{\boldsymbol{\beta}} + \mathbf{z}_{it}'\boldsymbol{\delta}_i + u_{it} + v_{it}, \tag{4}$$

where v_{it} and u_{it} are defined as above. When $\boldsymbol{\delta}_i = \mathbf{0}_{s\times 1}$, the random coefficient translog SF model reduces to the fixed coefficient SF model. Now, we briefly describe the Bayesian procedure for estimating the random coefficient SF model. We use a flat prior for $\bar{\boldsymbol{\beta}}$, as

$$p(\bar{\boldsymbol{\beta}}) \propto 1, \tag{5}$$

and the following distribution for $h_v \equiv \frac{1}{\sigma_v^2}$,

$$p(h_v) \propto \frac{1}{h_v}. \tag{6}$$

The prior for u_{it} can be written as

$$p(u_{it}|\lambda^{-1}) = f_{Gamma}(u_{it}|1, \lambda^{-1}). \tag{7}$$

To obtain the prior, we use:

$$p(\lambda^{-1}) = f_{Gamma}(\lambda^{-1}|1, -\ln \tau^*). \tag{8}$$

We use the median values from our nonparametric analysis, which we discuss in the next section, as the prior for each sector. For priors of $\boldsymbol{\delta}_i$ and $\boldsymbol{\Omega}$, we follow the following:

$$\boldsymbol{\delta}_i \sim i.i.d.N(\mathbf{0}, \boldsymbol{\Omega}) \text{ and } \boldsymbol{\Omega} \propto IW(\mathbf{1}, \boldsymbol{\Omega}) \tag{9}$$

where $IW(1, \boldsymbol{\Omega})$ is an inverse Wishart density with 1 degree of freedom and scale matrix $\boldsymbol{\Omega} = 10^{-6} I_s$.

Let K be the total number of firms in the dataset and T_i be the set of time periods for which firm i's sample is taken in the dataset. Let N be the total number of observations in the dataset. The likelihood function is

$$\begin{aligned} \mathcal{L}(q|\bar{\boldsymbol{\beta}}, h_v, u, \lambda^{-1}, \boldsymbol{\delta}, \boldsymbol{\Omega}) &= \prod_{i=1}^{K} \prod_{t \in T_i} \left[\sqrt{\frac{h_v}{2\pi}} \exp\left(q_{it} - \mathbf{z}'_{it}\bar{\boldsymbol{\beta}} - \mathbf{z}_{it}\boldsymbol{\delta}_i - u_{it}\right)^2 \right] \\ &\propto (h_v)^{\frac{N}{2}} \exp\left[-\frac{h_v}{2}\mathbf{v}'\mathbf{v}\right], \end{aligned} \tag{10}$$

where $\mathbf{v}$ is a $N \times 1$ vector such that $\mathbf{v} = q_{it} - \mathbf{z}'_{it}\bar{\boldsymbol{\beta}} - \mathbf{z}_{it}\boldsymbol{\delta}_i - u_{it}$.

By Bayes' Theorem, using the likelihood function (10) and the prior distributions (5) to (9), we obtain the posterior joint density function

$$\begin{aligned} f(\bar{\boldsymbol{\beta}}, h_v, \mathbf{u}, \lambda^{-1}, \boldsymbol{\delta}, \boldsymbol{\Omega}|\mathbf{q}) &\propto h_v^{N/2-1} \exp\left[-\frac{h_v}{2}\mathbf{v}'\mathbf{v}\right] \prod_{i=1}^{K} \left[|\boldsymbol{\Omega}|^{-1/2} \exp\left(-\frac{1}{2}\boldsymbol{\delta}'_i\boldsymbol{\Omega}^{-1}\boldsymbol{\delta}_i\right)\right] \\ &\times \prod_{i=1}^{K} \prod_{t \in T_i} \left[\lambda^{-1} \exp(-\lambda^{-1}u_{it}) \exp(\lambda^{-1} \ln \tau^*) \times |\boldsymbol{\Omega}|^{\frac{t+s+1}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1})\right]\right]. \end{aligned} \tag{11}$$

Table 4: Priors for MCMC

Sector	18	20	26	28	36
Prior	0.29	0.25	0.22	0.28	0.27

Then, the system of the full conditional posterior distribution is:

$$\begin{aligned}
p(\bar{\boldsymbol{\beta}}|\mathbf{q}, h_v, \mathbf{u}, \lambda^{-1}, \boldsymbol{\Delta}, \boldsymbol{\Omega}) &\propto f_{Normal}(\bar{\boldsymbol{\beta}}|\bar{\boldsymbol{b}}, h_v^{-1}, (\mathbf{zz}')^{-1}) \\
p(h_v|\mathbf{q}, \bar{\boldsymbol{\beta}}, \mathbf{u}, \lambda^{-1}, \boldsymbol{\Delta}, \boldsymbol{\Omega}) &\propto f_{Gamma}(h_v|\tfrac{N}{2}, \tfrac{1}{2}\mathbf{v}'\mathbf{v}) \\
p(y_{it}|\mathbf{q}, \bar{\boldsymbol{\beta}}, h_v, \lambda^{-1}, \boldsymbol{\Delta}, \boldsymbol{\Omega}) &\propto f_{Norma}(u_{it}|q_{it} - \mathbf{z}_{it}'\bar{\boldsymbol{\beta}} - \mathbf{z}_{it}'\boldsymbol{\delta}_i - (h_v\lambda)^{-1}, h_v^{-1}) \cdot I(u_{it} \geq 0) \\
p(\lambda^{-1}|\mathbf{q}, \bar{\boldsymbol{\beta}}, h_v, \mathbf{u}, \boldsymbol{\Delta}, \boldsymbol{\Omega}) &\propto f_{Gamma}(\lambda^{-1}|N+1, \mathbf{u}'\boldsymbol{\iota}_N - \ln\tau^*) \\
p(\delta_i|\mathbf{q}, \bar{\boldsymbol{\beta}}, h_v, \mathbf{u}, \lambda^{-1}, \boldsymbol{\Omega}) &\propto f_{Normal}(\boldsymbol{\delta}_i|\bar{\boldsymbol{\delta}}, (\boldsymbol{\Omega}^{-1} + h_v\mathbf{z}\mathbf{z}_i')^{-1}) \\
p(\boldsymbol{\Omega}|\mathbf{q}, \bar{\boldsymbol{\beta}}, h_v, \mathbf{u}, \lambda^{-1}, \boldsymbol{\Delta}) &\propto IW(r+K, \boldsymbol{\Phi} + \boldsymbol{\Delta}\boldsymbol{\Delta}').
\end{aligned}$$

The efficiency score is given by $\exp(-u_{it})$ for each i, t. We then compute the returns to scale. To do so, we take the first derivative of $\ln y$ with respect to each $\ln x$. This gives us the measure of RTS. Now,

$$\frac{\mathrm{d}\ln y}{\mathrm{d}\ln x_i} = b_i + \frac{1}{2}b_{ni}\ln x_n + \rho_i t, \tag{12}$$

and RTS is given by

$$RTS_i = \sum_i^3 \frac{\mathrm{d}\ln y}{\mathrm{d}\ln x_i}. \tag{13}$$

To estimate the model, we use the Gibbs sampling algorithm, which draws from the joint posterior density by sampling from a series of conditional posteriors. Once draws from the joint distribution have been obtained, any posterior feature of interest can be calculated. When estimating the model for each sector, we discard the first 10,000 as a *burn-in* and then repeat the procedure 390,000 times for each sector.[12] All the results are based on the last 200,000 times after ensuring convergence.

As mentioned, we use the mean values of the estimates in the nonparametric analysis or *free-disposal hull*, described in Section 6.4, as the priors for the MCMC. The following table summarizes these.

Finally, we compute RTS by using (13) and categorize firms into three groups, namely, Increasing Returns to Scale (IRS), Decreasing Returns to Scale (DRS), and Constant Returns to Scale (CRS) by

[12]We have chosen this by checking convergence using a scale reduction factor (SRF).

examining whether the 95% credible intervals are strictly greater than 1.0 (IRS), strictly less than 1.0 (DRS), or include 1.0 (CRS). These definitions are also adopted in Orea (2002), or Feng and Zhang (2014).[13]

4.1 Spearman Test

Spearman rank correlation is used to test the association between two ranked variables without requiring knowledge of the joint probability distribution of the variables. For a sample of size n, let $\mathrm{rg}(X_i)$ denote the ranking of firm i's RTS using the fixed-effect estimation, and $\mathrm{rg}(Y_i)$ denote the ranking of firm i's RTS using the random-effect estimation. Then the Spearman correlation coefficient ρ_s can be computed using the formula:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)},$$

where $d_i = \mathrm{rg}(X_i) - \mathrm{rg}(Y_i)$, is the difference between the two ranks of each observation and n is the number of observations.

A Spearman correlation of zero indicates that there is no tendency for a variable to either increase or decrease when the other one increases. The Spearman correlation increases in magnitude as the two variables become closer to being perfect monotone functions of each other.

5 The Main Results

To start, Tables 9 and 10 present our estimated results. We also present the scale reduction factor (SRF) and simulation inefficiency factor (SIF) for each coefficient, and the deviance information criterion (DIC) for the model. The first two factors help us check the convergence of MCMC, while the last factor helps us choose which production function is more appropriate for our analysis. For brevity, we only present the estimation results for Sectors 18 and 36. We have made sure of the convergence by using SRF and SIF across all sectors and a similar result holds. The estimation results for the other sectors are available upon request. As shown in the tables, SRF and SIF show good convergence in our results across sectors. Furthermore, we also check convergence by visually checking the trace, mean, and density plots (see Figure 1).

Table 2 presents the estimated results for the efficiency scores.[14] Figure 2 presents a scatterplot of the efficiency scores computed using the SF method for some of our sample industries for the entire

[13] For a more detailed discussion, see Caves et al. (1982) or Ch. 5 in Färe and Grosskopf (1994).

[14] The figures for the other sectors are available upon request.

nine-year period, and Figure 3 is a scatterplot of averaged efficiency scores computed using the SF method for each size. Figure 5 presents a scatterplot of RTS computed using the SF method for each size. Figure 4 is a scatterplot of efficiency scores computed using the FDH method. In this figure, we also insert the curve that shows the function representing the moving averages of the efficiency scores for every 0.1 interval of $log(size)$.

Our analysis shows that across all the sectors we study, production efficiencies are most variable in the middle-sized firm group, and that the firms with the lowest production efficiency are middle-sized firms in all of the sectors. This result is observed in both Figures 2 and 4. As shown in Figure 3, the average scores vary most for the middle-sized firm group. We should mention that in Figure 4, these scores are measured relative to best practice for each firm's similar production scale, while we statistically decompose the difference from the best practice into two parts, error and efficiency, in Figure 2. As Figure 4 does not include this decomposition, it shows some separation of low scores and high scores, even within similarly sized firm groups. Even so, the tendency that the middle-sized firm group has the most inefficient scores still holds using both methods. Furthermore, we show that the least-efficient size differs by sector, which indicates the necessity of sector-by-sector analysis when examining the underlying mechanisms of the missing-middle phenomenon.

We further study what portion of the lower efficiency group of each sector belongs to the middle-sized group. We consider observations whose efficiency score is lower than 0.6. In most sectors, the proportion of middle-sized firms in the low-efficiency group is higher than the ratio of the middle-sized firms relative to all observations. For example, (although this varies significantly across sectors) in Sector 36, 36% of the low-efficiency group has between 51 and 150 employees, while the ratio of this size group to the whole sample is 17%. Except for the two sectors whose two ratios are similar, the former is higher by a few percentage points on average than the latter. This implies that when randomly drawing an observation from a very low-efficiency score group, there is some tendency to draw a middle-sized firm observation, while the likelihood of drawing a middle-sized firm observation from the pool of high-efficiency scores is as likely as drawing one from the whole sample.

Our analysis of RTS shows that the majority of most size groups in all sectors have constant returns-to-scale technologies. As discussed at the end of Section 3.1, we use 95% credible intervals to categorize returns to scale and a sample firm for which the interval contains 1 is classified as a group with CRS. In Table 11, we group the samples for each sector by the number of employees in five groups, where groups 1 to 5 respectively include firms with $1--10$, $11--50$, $51--100$,

Table 5: % Difference in Spearman Coefficient for Each Group

Sector	G1	G2	G3	G4	G5	Total
Sector 18	0.4891	0.0898	0.2458	0.3297	0.1632	0.1810
Sector 20	0.3616	0.2858	0.3372	0.3671	0.3646	0.3298
Sector 26	0.0072	0.2592	0.1464	0.1235	0.1477	0.1795
Sector 28	0.1306	0.1470	0.2032	0.4130	0.2818	0.2053
Sector 36	-0.2463	0.2781	0.1573	0.1509	0.1008	0.1352

$101 - -200$, and 201 or more employees.

The differences in technologies are more striking in Table 5. This table presents how different each group's Spearman coefficient is, compared with the one for the whole sector. In this analysis, G1 or G2 tend to show lower coefficients than for the overall sample or other groups. This shows that these groups tend to have more heterogenous technologies among them. Moreover, our results show that there is a significant heterogeneity in firms' technologies. This also justifies our methodology of using a random coefficient stochastic frontier approach, which considers firms' technological heterogeneity.

In summary, our analysis so far indicates that the middle-sized group is a 'mixed bag' where firms with very high and very low production efficiencies coexist. This shows a possible risk for a small-sized firm's expanding the business. Meanwhile, there is a significant heterogeneity in firms' technologies across different size groups. The heterogeneity in technologies may make a small-sized firm to clearly predict how their expansion of size would result in, because even if some firm succeeds in expanding, the firm's technologies are different. We would not know how the heterogeneity of technologies affect the future productivity of the firm. Although our analysis is a snapshot of the Vietnamese manufacturing industry, the diverse production efficiencies in the middle-sized groups can be thought of as a risk that small-sized firms would face in expanding their businesses.

6 Further Examination of the Results

6.1 Further Robustness Checks

Furthermore, to check the robustness of the findings in this analysis, we also measure the efficiency of production using a nonparametric method, which imposes no or only very limited assumptions on the data. In addition, we also conduct the same analysis by using Cobb–Douglas functions and by using the data of firms that have more than two observations. Finally, we have also conducted the same analysis using a half-normal distribution function instead of an exponential distribution for the efficiency terms.

The following results still hold in these analyses of different settings.

- The efficiency is most diverse in the middle-sized firms across sectors.
- The lowest efficiency firms belong to the middle-sized groups.
- The majority of firms show constant returns-to-scale technologies.

6.2 A Cobb–Douglas Production Function

As we mentioned in Footnote 6, we started with 14 sectors that have more than 200 observations in each year, so that we can ensure the robustness of the empirical results. Based on the convergence criterion for the MCMC procedure, we have chosen 5 sectors in Table 1 for the translog production functions analyses as in the previous section and we also conduct similar analyses using Cobb–Douglas production functions for the other 9 sectors, which are found in Table 6. The above-mentioned results still hold.

Table 6: ISIC Codes and Industry Description

ISIC	Industry Description	ISIC	Industry Description
14	Other mining and quarrying	15	Food products and beverages
17	Textiles	19	Tanning and dressing leather
21	Paper and paper products	22	Publishing, printing, etc.
24	Chemical products, etc.	25	Rubber and plastic products
29	Machinery and equipment, etc.		

6.3 Locations and Ownership of Firms

In this section, we group firms based on their locations and ownership types and study whether our findings are still robust. The results suggest are findings are robust. The groups of locations we consider consist of nine groups: northern midland and mountainous area, Red River delta, north central coast, south central coast, central highlands, southeast area, Mekong River delta, Hanoi, and

Ho Chi Minh City. The ownership types we consider are the following three types: state-owned firms, private firms, foreign firms.

As presented in Figures 6 and 7, for the middle-sized groups, the efficiency scores are relatively diversified across firms and regions, and different ownership types still remain. Table 12 presents the means and variances of the efficiency scores for each size group in Sector 36. It varies across ownership types, although the middle-sized groups tend to have higher variances. We further study whether a higher variance is caused by a larger number of firms. Although it also depends on the sector, in the groups of state-owned firms, across sectors, a size group with a higher number of firms tends to have a higher variance, while in the groups of foreign firms, a size group with a higher number of firms tends to have a smaller variance.

6.4 Nonparametric Estimation: FDH

Pham and Takayama (2015) measures the efficiency of production using a nonparametric method called *free-disposal hull* (hereafter, FDH). The results in their paper support our findings. Here, we briefly discuss their method and summarize the results so that we can see how robust our findings are across different methodologies of estimating efficiency scores.

Suppose there are L industrial sectors in the economy. Take a sector $l \in \{1, \cdots, L\}$ where inputs $x \in \mathbb{R}^3_+$ are used to produce an output $y \in \mathbb{R}_+$. Let N^l denote the number of firms in Sector l of the dataset. Firm i's input and output combination is called a *productive unit*, denoted by (x_i, y_i) with $x_i \in \mathbb{R}^3_+$ and $y_i \in \mathbb{R}_+$. The efficiency score for a productive unit (x_i, y_i) is defined by:

$$E(x_i, y_i) = \inf_{\theta}\{\theta : (\theta x_i, y_i) \in \Psi^l\}, \tag{14}$$

where the production set is given by: for some $\lambda = \{\lambda^i\}$ with $\sum_{i=1}^{n} \lambda^i = 1$ and $\lambda^i \in \mathbb{R}_+$ for every i,

$$\Psi^l = \{(x, y) : x \geq \sum_{i=1}^{N^l} \lambda^i x_i, y \leq \sum_{i=1}^{N^l} \lambda^i y_i\}. \tag{15}$$

The efficiency score is the distance between the observed quantity of inputs and outputs, and the quantity of inputs and outputs at the production possibility frontier, which is the best possible outcome for a firm in its cluster (industry). The efficiency score[15] lies between 0 and 1, and represents

[15]This is called the "input-oriented" efficiency score. There is an alternative in the form of the "output-oriented" efficiency score. In this analysis, we use the input-oriented efficiency score because the results do not change substantially.

the minimal proportional reduction of all inputs while maintaining the same output level within the production set. Solving Problem (14) requires constructing the production set. Hence, by using the data, and through linear programming, we need to calculate λs so that the assumptions required by each method are satisfied. The FDH approach does not require the convexity of the production set.

Table 4 provides the distributions of the efficiency scores through this method.[16] As we see, the middle-sized firms are a 'mixed bag'. Furthermore, there are many middle-sized firms with the lowest efficiency scores.

7 Concluding Remarks

In this paper, we focused on the relationship between firm size and production efficiency using Vietnamese data. Using a recently developed stochastic frontier approach and data from Vietnam, our analysis has shown that across all the sectors we studied, production efficiencies are most variable among middle-sized firms, most large-sized firms tend to exhibit constant returns-to-scale technologies, and middle-sized firms tend to have the lowest production efficiency. Furthermore, we have shown that the least efficient size differs across sectors, which indicates the necessity of sector-by-sector analyses when considering the underlying mechanism for the missing-middle phenomenon, which refers to the feature that most employment in developing countries is located in either small- or large-sized firms.[17] Moreover, the heterogeneity of firm size at the bottom of the U-shaped curve across industries indicates the importance of sector-level analysis in investigating the missing middle. Our analysis also contributes in this regard.

Needless to say, one might question whether our findings from a transition economy—namely, Vietnam—provide a sufficient basis for generalization, particularly because much of the missing middle literature concerns Africa. Business environments and their features vary substantially from country to country. It would then be interesting to see if we observe a similar feature in a different country.

Several interesting future research paths follow from our analysis. A next step is to conduct a similar analysis using data from developed countries to measure production efficiency and to com-

[16] In this paper, we use the R-package *FEAR* (see Wilson, 2008) to compute the scores.

[17] In the literature, there has been an increasing interest in the relationship between firm size and other characteristics, such as innovation and market structure (see Acs and Audretsch, 1987), growth and productivity (see Bentzen et al., 2011), and job creation (see Dalton et al., 2011). However, few empirical studies consider the relationship between firm size and the level of efficiency, particularly in a developing country context. Also, in preceding works, large firms are found to be the most efficient (see Angelini and Generate, 2008; Leung et al., 2008).

pare the results for Vietnam. If we still observe lower productivity for middle-sized firms, we could then consider the cause. If we only observe this U-shaped pattern in developing countries, we could examine the reasons that this pattern exists in developing countries, but not developed countries.

Finally, it would be very interesting and important to study the causes of these differences in productivity across different firm sizes in developing countries. OECD Publishing (2013) reports two main reasons for the differences in productivity across different firm sizes: (i) firm size matters for productivity, and (ii) structural differences in the industrial composition of economies influence the relative performance of large and small firms across countries. First, in most countries, there is the possibility for improving production efficiency by increasing firm size. Larger firms are on average more productive than smaller ones, and this generally holds for all industries. Second, in countries with large industrial sectors and relatively low per capita income, large firms are, on average, 2–3 times as productive as small firms. However, in countries with large service sectors and relatively high income per capita, small firms are often more productive than large firms. It would be interesting to investigate whether these factors are also important in developing countries.

List of Tables and Figures

Table 7: Summary of Enterprise Census Data Statistics (Firm Size)

		Size			Capital			Material		
Sector	# Observations	Mean	Median	St. Dev.	Mean	Median	St. Dev.	Mean	Median	St. Dev.
Sector 18	3181	674.59	330.00	1030.94	6770.87	2199.87	49571.76	21614.99	6380.57	49571.76
Sector 20	3261	110.72	35.00	228.37	881.06	194.01	19669.44	6385.77	1219.79	19669.44
Sector 26	5543	190.69	80.00	353.33	2297.49	535.73	219330.93	38622.39	2525.31	219330.93
Sector 28	4041	103.66	33.00	219.02	1379.89	299.78	48581.66	17672.86	2554.10	48581.66
Sector 36	2846	341.42	132.00	634.71	3830.20	1108.38	50102.10	21685.72	5418.01	50102.10

[1] Size measured using the number of employees.

[2] Capital and Material are listed in the home currency (inflation adjusted).

[3] St. Dev. is standard deviation.

Table 8: Summary of Enterprise Census Data Statistics (Number of Observations and Firms)

Year	Total	Sec 18	Sec 20	Sec 26	Sec 28	Sec 36
2000	1513	267	259	561	250	176
2001	2092	357	373	688	399	275
2002	2061	353	364	670	401	273
2003	2078	345	367	684	401	281
2004	1972	341	337	639	386	269
2005	2292	380	391	577	551	393
2006	2292	379	391	577	551	394
2007	2290	380	390	575	551	394
2008	2282	379	389	572	551	391
# Total	18872	3181	3261	5543	4041	2846
# Firms		743	773	1277	962	679

Note: Figures except for the last row are the number of observations in each sector for each industry in each year. The last row shows the total number of firms in each sector.

Table 9: Sector 18

	a_0	b_1	b_2	b_3	b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}	σ_t	σ_{tt}
	-0.127	0.678	0.210	0.145	0.165	-0.107	-0.029	0.120	-0.007	0.027	0.023	-0.003	-0.010	0.005	-0.006
95 % CI	-0.845	0.395	-0.129	0.039	-0.101	-0.331	-0.139	-0.146	-0.101	-0.017	-0.200	-0.041	-0.062	-0.029	-0.025
	0.594	0.958	0.549	0.251	0.430	0.117	0.085	0.391	0.085	0.070	0.252	0.036	0.041	0.040	0.013
SRF*	1.021	1.067	1.022	1.006	1.023	1.000	1.001	1.004	1.000	1.000	1.017	1.002	1.051	1.009	1.007
SIF†	34.764	37.138	34.235	34.069	36.460	34.045	33.211	30.809	28.696	27.142	27.735	22.919	35.215	33.410	33.911
DIC‡	-1459.12														

Table 10: Sector 36

	a_0	b_1	b_2	b_3	b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}	σ_t	σ_{tt}
	-0.100	0.580	0.348	0.082	0.148	-0.121	-0.015	0.141	-0.025	0.039	0.040	-0.004	0.009	-0.016	0.002
95 % CI	-0.814	0.036	-0.293	-0.224	0.032	-0.254	-0.096	-0.045	-0.085	-0.020	-0.207	-0.052	-0.047	-0.095	-0.032
	0.605	1.129	0.973	0.387	0.255	0.015	0.066	0.323	0.037	0.100	0.297	0.044	0.064	0.065	0.036
SRF*	1.005	1.001	1.007	1.013	1.018	1.056	1.006	1.048	1.002	1.029	1.002	1.001	1.014	1.019	1.003
SIF†	32.583	41.173	36.548	30.901	32.577	32.058	32.540	31.786	31.029	27.396	27.276	25.596	38.132	33.630	29.479
DIC‡	-2346.54														

Note: $*$ SRF is *Scale Reduction Factor*; † SIF is *simulation inefficiency factor*; ‡ DIC is *deviance information criterion*.

Figure 1: Trace Plots, Density Plots and Mean Plot of Parameters for Sector 36

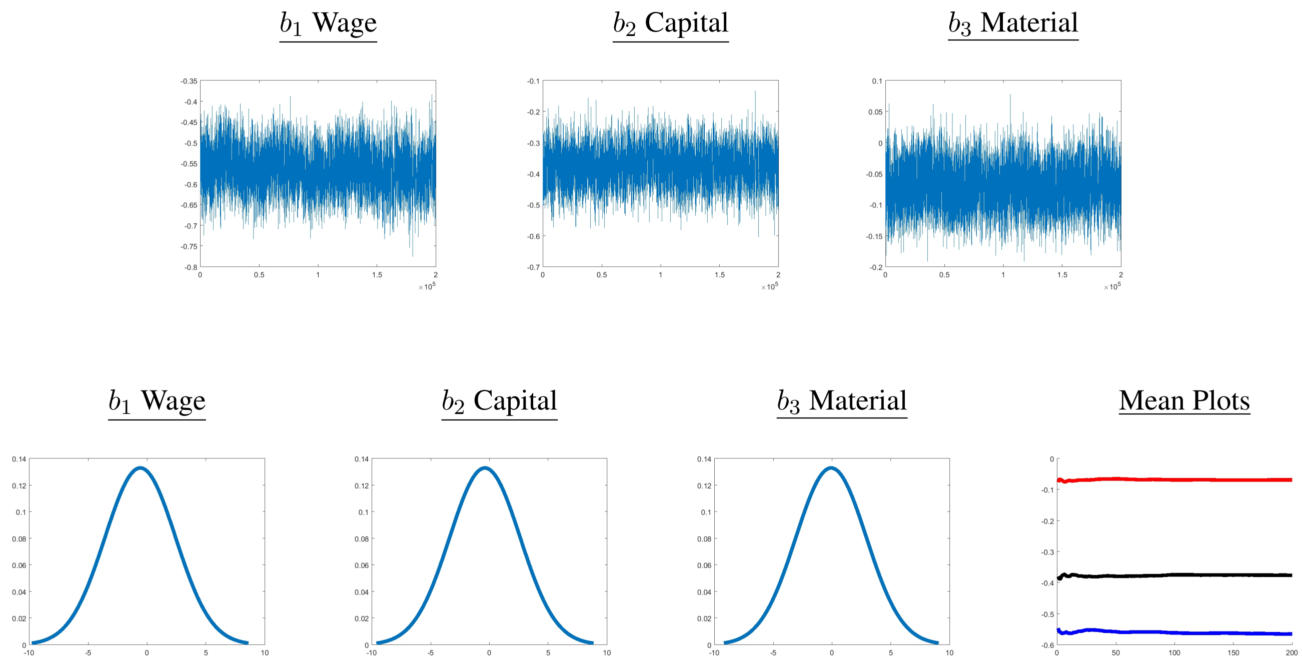


Figure 2: Distribution of SF Efficiency Scores across Different Firm Sizes

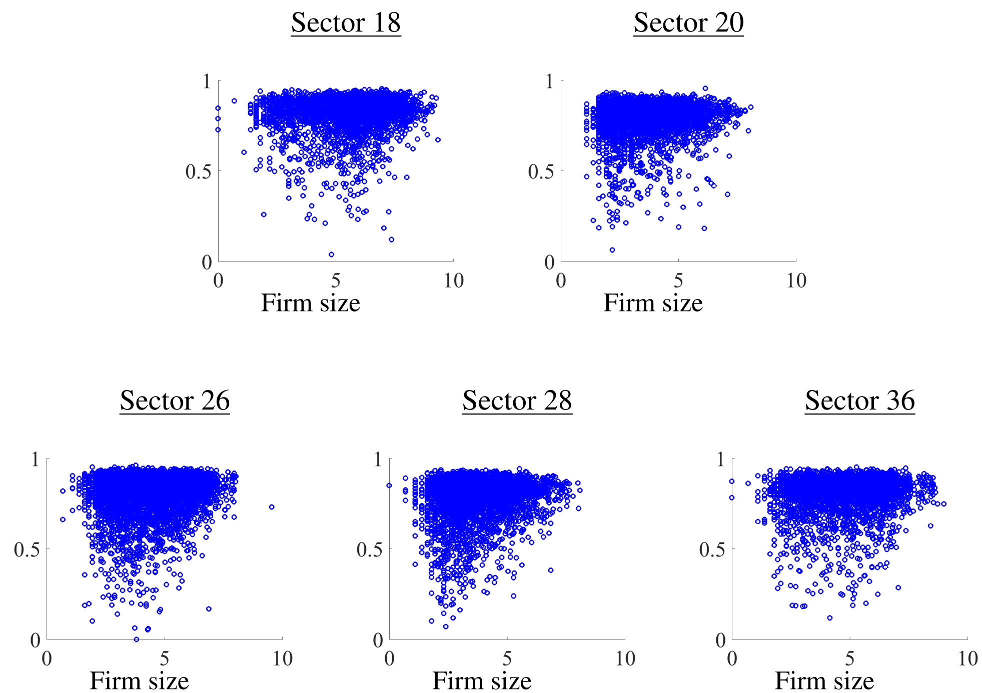


Figure 3: Distribution of Average Efficiency Scores across Different Firm Sizes

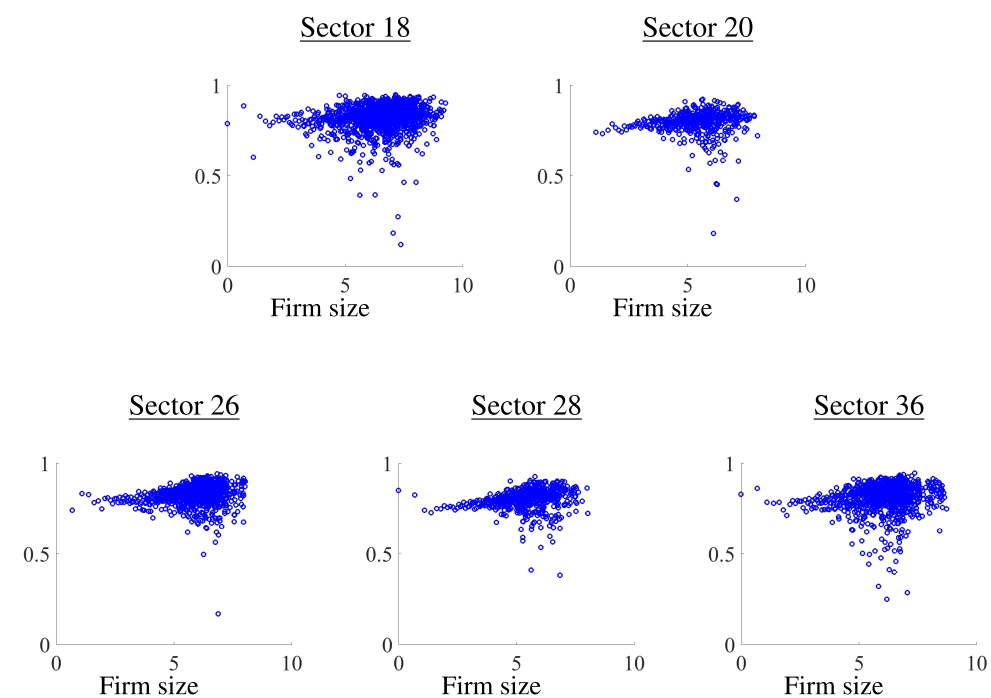
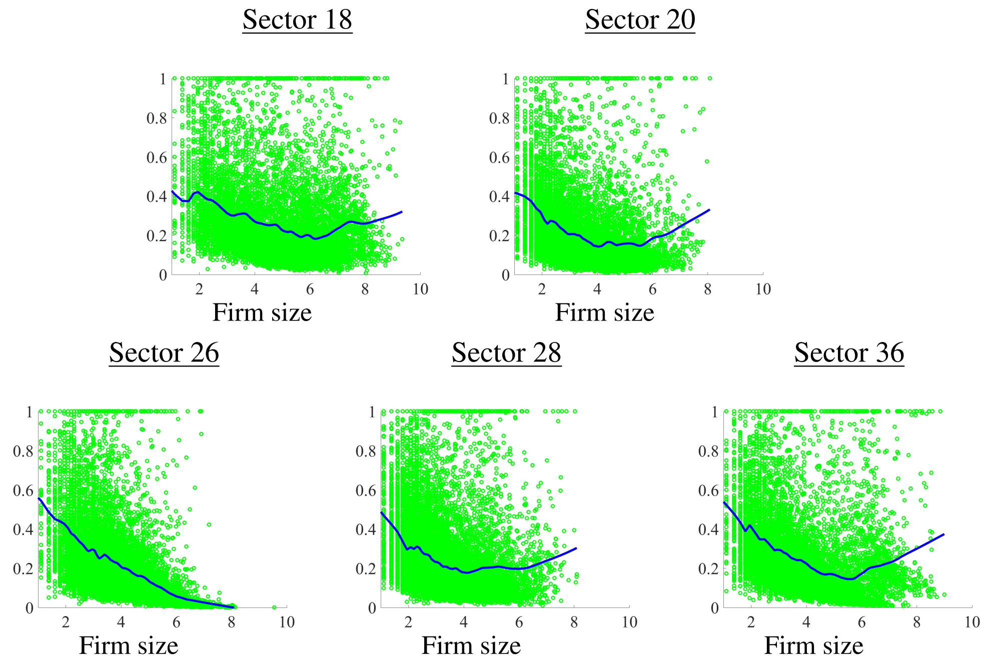


Figure 4: Distribution of FDH Efficiency Scores across Different Firm Sizes



Note: The curve represents the smooth trend line of the efficiency scores for every 0.1 interval of $log(size)$. We use Matlab's `smooth` command. This inserts a smooth trend line using the local regression method.

Figure 5: Distribution of RTS across Different Sizes

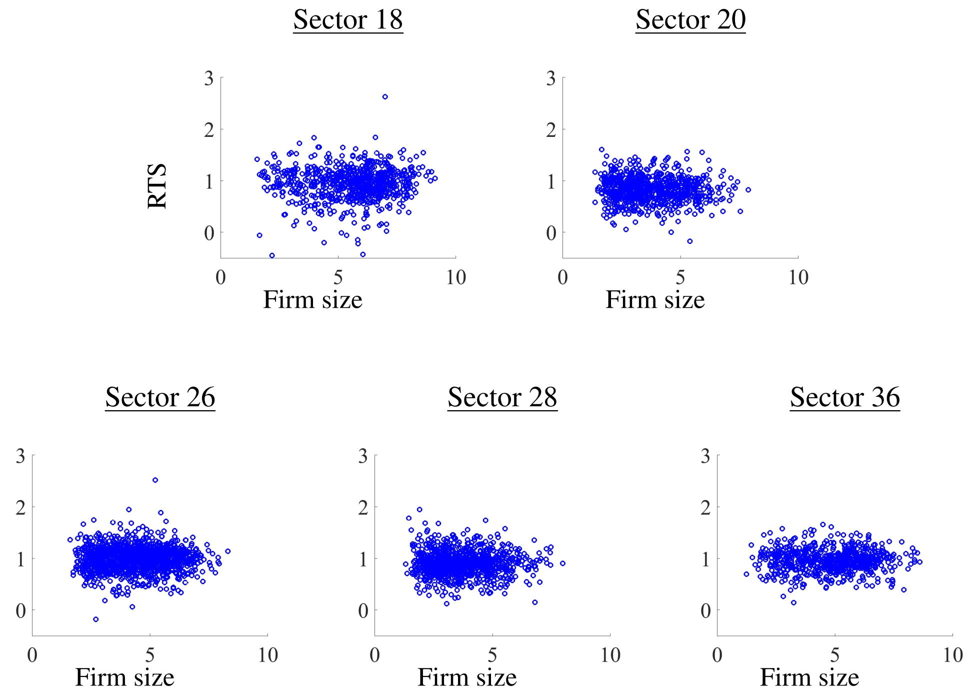


Table 11: Distribution of RTS across Different Firm Sizes

(%)	IRS	DRS	CRS
G1	0.00	0.00	100.00
G2	2.02	3.03	94.95
G3	0.00	2.99	97.01
G4	0.00	8.89	91.11
G5	2.37	4.53	93.10

Sector 18

(%)	IRS	DRS	CRS
G1	1.67	16.67	81.67
G2	1.22	10.09	88.69
G3	2.59	9.48	87.93
G4	2.00	16.00	82.00
G5	0.91	14.55	84.55

Sector 20

(%)	IRS	DRS	CRS
G1	3.45	3.45	93.10
G2	2.30	1.84	95.85
G3	4.15	3.23	92.63
G4	3.27	0.47	96.26
G5	2.26	2.54	95.20

Sector 26

(%)	IRS	DRS	CRS
G1	3.36	6.72	89.92
G2	1.52	5.84	92.64
G3	3.38	8.78	87.84
G4	1.65	4.13	94.21
G5	2.68	8.93	88.39

Sector 28

(%)	IRS	DRS	CRS
G1	3.03	3.03	93.94
G2	3.35	5.59	91.06
G3	5.41	4.05	90.54
G4	6.73	6.73	86.54
G5	3.11	5.19	91.70

Sector 36

Note: G1, G2, G3, G4, and G5 indicate firms with $1--10$, $11--50$, $51--100$, $101--200$, *and* 201 *or more employees, respectively.*

Figure 6: Efficiency and Ownership in Sector 36

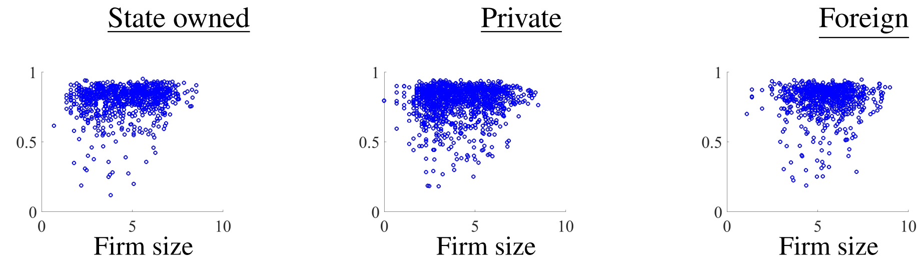
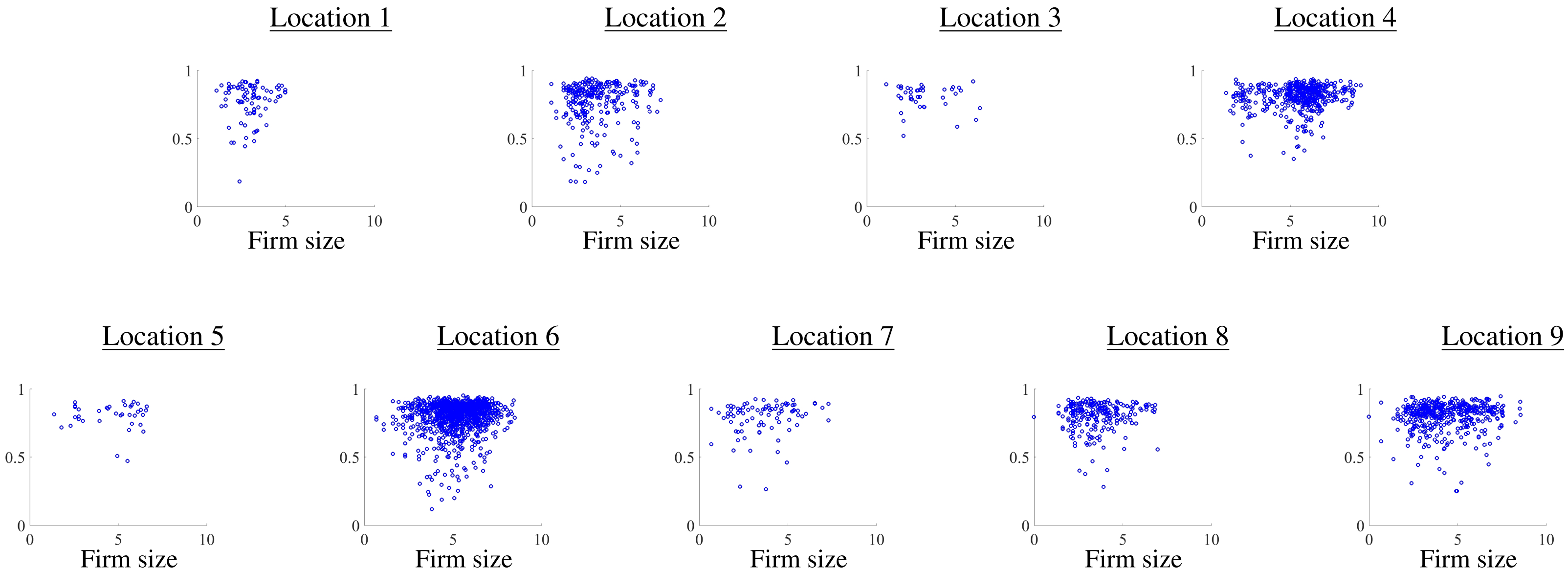


Table 12: Mean and Variance of Efficiency Scores across Different Firm Sizes

	All Samples			State-Owned			Private			Foreign		
	Mean	Variance	Samples	Mean	Variance	Samples	Mean	Variance	Samples	Mean	Variance	Samples
G1	0.7738	0.0153	233	0.7791	0.0150	71	0.7768	0.0145	156	0.8064	0.0050	6
G2	0.7844	0.0168	741	0.7809	0.0180	237	0.7848	0.0158	421	0.7882	0.0214	83
G3	0.7973	0.0133	336	0.8055	0.0073	99	0.7917	0.0140	139	0.7956	0.0177	98
G4	0.7992	0.0147	420	0.8017	0.0140	108	0.7988	0.0144	157	0.7970	0.0162	155
G5	0.8103	0.0094	1059	0.8194	0.0074	329	0.7991	0.0110	308	0.8123	0.0098	422
Total	0.7970	0.0133	2789	0.8013	0.0121	844	0.7901	0.0140	1181	0.8044	0.0133	764

Note: G1, G2, G3, G4, and G5 indicate firms with $1--10$, $11--50$, $51--100$, $101--200$, *and* 201 *or more employees, respectively.*

Figure 7: Efficiency and Locations in Sector 36



Note: Location 1 - northern midland and mountainous area; Location 2 - Red River delta; Location 3 - north central coast; Location 4 - south central coast; Location 5 - central highlands; Location 6 - southeast area; Location 7 - Mekong River delta; Location 8 - Hanoi; Location 9 - Ho Chi Minh City.

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