Asking Price Mechanism with Dynamic Arrivals

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Abstract

This paper studies a popular selling mechanism relevant to the Australian housing market in which the seller of a property posts an asking price to attract potential buyers for further negotiations. The game is studied in a dynamic setting with the possibility of more than one potential buyer arriving at each period. The game is designed such that in the event when only one buyer arrives, the seller engages in negotiation with that buyer and when two or more buyers arrive, the seller runs an auction with a reserve price. We show the conditions under which this mechanism can extract more expected payoffs to the seller comparing to a uniform price selling or an standard auction. The selling mechanism we study is applicable to many real world markets, specially the housing market of Australia and adds to the theoretical literature by explaining why sometimes sale prices are higher than the asking prices. We also explain the role of the asking price in the relevant markets. Other small variations of this mechanism are also studied for the purposes of comparison.

Keywords: Asking Price, Bargaining, Dynamic Selling, Housing Market, Reservation Price.

JEL Classification: D44, D83, R32.

1 Introduction

There are several markets in which the seller of an indivisible object posts a price to attract potential buyers to make offers. In these markets, sometimes the posted price acts as a commitment device from the seller, and sometimes it is just a guide for further negotiations. The most relevant one to the present study is the housing market, and specifically the Australian housing market. In several countries, such as the United States and Australia, one of the most popular selling methods in the housing market is when the seller of a property posts an asking price to attract offers from potential buyers over time. After negotiation between the seller and buyers, the sale price for the object is sometimes lower than the asking price, sometimes the same, and sometimes higher. What is the role of the asking price in this specific market? What we observe more in Australia comparing to the U.S are the cases in which the transaction prices are higher than the asking prices. Some researchers (Horowitz (1992) and Chen and Rosenthal (1996a), among others) argue that the asking price acts as a ceiling price or a commitment device from the monopolist. Yet these theoretical models do not result in a convincing argument for those cases in which the sale price is higher than the asking price. Indeed, a theoretical model that explains the role of the asking price and at the same time predicts all possible outcomes with respect to that asking price has not been studied before. The purpose of this analysis is to examine a selling mechanism that has a role for the asking price and that at the same time can result in sale prices higher than the asking price. In this case the asking price no longer functions as a ceiling price or a commitment device from the seller.

Several researchers have attempted to study the behaviour of sellers and buyers in the housing market from a theoretical point of view, and some have also tested their models empirically. From a theoretical point of view, the behaviour of parties in the housing market has been mainly studied using search theory. Some papers have studied one-sided search models, in which only one party, mainly the seller, is searching for potential traders, whereas others have attempted to study two-sided search models. Two-sided search models are more complicated in terms of the equilibrium analysis because there are two active parties in the game. This makes it even more difficult to empirically study two-sided search models.

Yinger (1981) is one theoretical study of the real estate market. This paper studies the search behaviour of a real estate broker when there is uncertainty about the number of buyers and available listings. The role of the real estate broker in this market is to find matches between buyers and sellers. Yinger (1981) also studies the behaviour of real estate brokers in the Multiple Listing Service in the United States.

They claim the Multiple Listing Service increases the efficiency of the market and reduces commissions.

Some studies have mainly focused on the strategic role of the asking price in the housing market. Horowitz (1992) attempts to model and estimate the behaviour of a seller in the housing market. He considers an infinite-horizon stationary search framework in which a seller posts an asking price and waits for offers from potential buyers who arrive over time. The asking price in this model acts as a ceiling price or a commitment device from the seller. Thus, at any time during the game, if a buyer asks to buy the object at the asking price, the seller accepts the offer. Horowitz (1992) finds the optimal asking price and the reservation price when the price offers are drawn from a known distribution. Consequently, it is not optimal for a seller to vary the asking price over time, which can explain why a seller who has not sold her house for a long time may not change the asking price. Finally, he estimates the parameters of the model using data on list price, transaction price, and time on the market. There are some limitations to his model. First, sellers in his model are identical, so he characterises the behaviour of only one seller. Second, it is not possible to have more than one buyer at any given time, and it is also not possible to have a sale price higher than the asking price. Another limitation concerns the exogenous rate of offers arriving to a seller, which can not characterise the search behaviour of the buyers in the housing market. Yavas and Yang (1995) study the strategic role of the asking price in a single-period model. In their theoretical study they examine how the choice of list price affects the broker's incentive to search and the length of time the property is on the market. Their study also attempts to show empirically the effect of higher asking prices on time on the market.

Chen and Rosenthal (1996a) and Chen and Rosenthal (1996b) are two theoretical attempts to show the optimal behaviour of a monopolist using an asking price mechanism to attract buyers. They assume that, in an infinite-horizon setup, the asking price is the seller's commitment device to attract potential buyers to incur the search cost. Chen and Rosenthal (1996a) show the optimal reservation price and asking price of a seller in an environment in which the buyers pay a cost to inspect the object and after the inspection bargain with the seller over a share of the surplus (if any). They also study duopolistic competition. Yet there are some limitations to their model. First, they only study a monopolistic case with the possibility of shared bargaining powers between parties. In duopolistic cases the seller has complete bargaining power, and the objects are identical. Second, at any period of time only one buyer can make an offer; thus, it is not possible to have a sale price higher than the asking price. Chen and Rosenthal (1996b) argue that under some specific assumptions this asking price is the optimal mechanism within the class of incentive

compatible mechanisms. The critical assumption is that the seller can extract the entire surplus in the bargaining game. In other words, the seller has all the bargaining power.

Arnold (1999) analyses not only the search behaviour but also the bargaining game between the seller and potential buyers. In his model, the asking price, which is chosen by the seller, can influence the number of buyers who want to inspect the house. Yet there is another role for the asking price in this work as well: as the initial offer in the bargaining game. Arnold (1999) introduces a different bargaining game than the one in Chen and Rosenthal (1996b). In Arnold (1999), the outcome of bargaining no longer is a fixed share of the surplus but depends on the discount rates of the buyers and the seller. He claims that because this change makes the seller's surplus a non linear function of the total surplus, unlike in Chen and Rosenthal (1996b), the comparative statistics analysis will also change.

To our knowledge Carrillo (2012) is the only empirical study of a two-sided search model in the housing market. He presents an environment in which both sellers and buyers search for potential traders. He introduces an asking price mechanism like the one in Chen and Rosenthal (1996a,b) and Horowitz (1992) as a ceiling price and a commitment device from the seller. In his model there is simple negotiation between two parties, in which the potential buyers have a random chance to make a one-time take-it-or-leave-it counteroffer to the asking price. Carrillo (2012) argues that a buyer's optimal counteroffer, given that she has a chance to make one, is the seller's reservation price. He solves the buyer-seller search problem and finds the condition for the seller's optimal reservation and asking prices. To estimate the model, he uses an arbitrary function as a starting point to solve the baseline model and to show the convergence of equilibrium. Finally, he estimates the parameters of the model using the maximum likelihood method. In this study it is not possible to have a final price higher than the asking price. There is also no possibility of multiple buyers arriving at any stage of the game. The aim of this paper is to answer how the amount of information on the house and the real estate agent's commission can change the outcome.

Albrecht, Gautierz, and Vroman (2012) is a working paper that models buyers' and sellers' direct search behaviour in the housing market. It is unique in the literature because it considers the possibility of multiple offers from buyers at any stage of the game given the seller's asking price. Their model explains cases in which the house is sold below, above, or at the asking price. In their model a buyer can accept the asking price or make a counteroffer. If a seller receives more than one request at the asking price, she runs a second-price auction with the asking price as the reserve price. In the first part of the paper the authors assume that all sellers are

homogeneous, keeping the ratio of the number of sellers and buyers exogenous. Then any configuration of an asking price higher than the seller's reservation value forms an equilibrium. Therefore, in the case of homogeneous sellers there is no specific role for the asking price. They also study the efficiency of this model with free entry. In their heterogeneous seller model there are two types of sellers in the market: low and high. In this model, they show that under some conditions there exists a separating equilibrium at which sellers signal their type via the asking price; thus, the asking price plays a signalling role in the heterogeneous seller model.

Wang (2011) studies a game in which a seller posts a price and buyers may pay the price or bargain. Wang (2011) studies how a seller signals the quality of her house through the list price. His results suggest that in the separating equilibrium high-quality sellers signal with higher prices, and the higher prices induce more bargaining. In his setting buyers need to pay a cost to realise their type and the quality of the house. In the separating equilibrium buyers infer the true type by the list price.

The nature of the environment under study here often results in a negotiation between a buyer and the seller. But because this is a dynamic game and the seller may be uncertain about future demand, it is sometimes the case that more than one potential buyer arrives in some periods. If the seller is aware of the fact that she might have more than one interested buyer in some periods, she can set the asking price to optimise both events. For example, consider a seller who hires a real estate agent to sell her property and pays the agent a fixed commission. The seller will not negotiate directly with the buyers, and hence before the property goes to market she will need to specify two prices to the real estate agent: first, the advertisement price, or the asking price; and second, the minimum price she will accept in the event of any negotiation with buyers, or the reservation price.[1] The agent is not allowed to sell the house at a lower price than the reservation price. However, it might be the case that she sells the house for more than the advertisement price.

Before defining the selling mechanism and explaining the model it is important to note some facts about the arrival of buyers in a dynamic game. If one assumes that there is at most one potential buyer at each period who negotiates with the seller, it is conceivable to support the role of the asking price as a ceiling price, such as in Chen and Rosenthal (1996a,b) and Horowitz (1992). Although this is the case for most transactions in the housing market, sometimes transaction prices are higher than the asking prices. A single-buyer-arrival assumption does not support these situations. In the next section we relax some of the assumptions of the standard model common in the literature on the housing market to include cases in which the

[1] The reservation price is different from the reserve price of an auction.

transaction price is above the asking price. This is not possible without multiple buyers arriving in at least some periods. We then examine a selling method with an asking price and analyse whether there is still a role for the asking price in this situation. In Section 3 a more general model is introduced to examine the possible extension of the previous results. Section 4 concludes the results in this paper and discuss the possibility of further extensions.

2 A Model With a Maximum of Two Bidders

The seller of an indivisible object posts an asking price p_a to sell the object in an infinite sequence of time until it is sold. The seller discounts the future at the rate δ. The seller *ex ante* believes that at each period with some probability ρ_1 there will be only one buyer; with probability ρ_2 there will be two buyers; and with probability $1 - \rho_1 - \rho_2$ there will be no potential buyers, so she has to wait for the next period. Arriving buyers leave at the end of each period, and the seller faces a new set of buyers in the next period. Suppose each buyer i's value for the object is a random variable V_i, independently and identically distributed according to $F(.)$ on the interval $[0, \bar{v}]$, and $F(.)$ is continuous and differentiable with density f. After arriving, each buyer realises the match-specified value v of V. We assume there is no cost for the realisation of v. Furthermore, we assume that the *hazard rate* function of $F(.)$ is increasing.

The selling mechanism is as follows. The seller posts an asking price at period zero. Then at each period of the game, if there is only one buyer, the buyer has the option to buy the object at the asking price or make a counteroffer, which would trigger a bargaining game between the seller and the buyer. If there are two buyers, the seller runs a sealed-bid second-price auction with a reserve price, from which one of the three possible outcomes results:

- If both buyers have values lower than the reserve price, the seller waits for the next period.
- If only one buyer has a value higher than the reserve price, then that buyer wins the object and pays the reserve price.
- If both buyers have values higher than the reserve price, then the highest bidder wins and pays the second highest bid.

Suppose the seller's outside value for the object is zero. Define the seller's reservation price p_r, which is the minimum price that she will accept to sell the object at

any stage of the game. Suppose that at this stage the reserve price in the auction game is equal to the reservation price p_r. In fact, this assumption does not necessarily maximise the seller's expected revenue, but if the seller is restricted to choosing only two prices, this is what will eventually happen. The seller's problem is to choose an optimal asking price and reservation price to maximise her expected payoff for the game.

We next define the bargaining game as follows. If the buyer's value is v and the seller's reservation price is p_r, then the transaction price resulting from bargaining is between v and p_r. To simplify the game like the one in Chen and Rosenthal (1996b), suppose that a fixed fraction θ of the surplus $v - p_r$ goes to the seller and the remainder goes to the buyer. This happens as long as the expected transaction price is lower than the asking price. Therefore, if a buyer has a value high enough that the outcome of bargaining would result in a transaction price higher than the asking price, then she buys the object at the asking price. Hence, we can define the transaction price as follows:

$$p = \begin{cases} \theta v + (1-\theta)p_r & \text{if} \quad p_r < v < p_l \\ p_a & \text{if} \quad p_l < v, \end{cases} \tag{1}$$

where $p_l = (p_a - (1-\theta)p_r)/\theta$. Here we suppose that at the time of bargaining v is revealed to the seller. Although this is a common assumption in the literature [2], even if the values were unknown we could argue that there exists a θ such that the outcomes are the same as the full information case. In fact, the assumption of *ex post* complete information simplifies the bargaining game.

Now we can redefine the game as follows. After the seller posts the asking price, she observes the number of buyers arriving at each period. If there is only one buyer, then they negotiate according to the aforementioned bargaining process. If there are two buyers, the seller runs a second-price auction with a reserve price equal to p_r. Clearly, p_r might not be the best reserve price for the auction, but because we assume that the seller can only optimise the situation with two prices-namely, the asking price and the reservation price-then p_r itself becomes the reserve price for the auction. We will also study the case in which the seller can separately identify an optimal reserve price for any possible auction, but in reality this might not be an option for the seller, although it could result in a higher expected payoff. We are also going to examine a case in which the seller combines the asking price and the reservation price into a single price that is a take-it-or-leave-it offer if only one buyer

[2]See Arnold and Lippman (1995), Albrecht, Anderson, Smith, and Vroman (2007) or Chen and Rosenthal (1996b).

arrives and the reserve price for the auction if multiple buyers arrive. In this case the seller does not engage in negotiation with the buyer.

In the model with only two prices (an asking price and a reservation price), although buyers only observe the asking price, according to the seller's optimal decision in equilibrium, the reservation price would also be revealed to them. We continue with the seller's problem to find an optimal asking price and the reservation price. At each period t the seller's expected payoff is as follows:

$$\begin{aligned} U_t^e =& \rho_1 \Big[\int_{p_r}^{p_l} (\theta v + (1-\theta) p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \Big] \\ &+ \rho_2 \Big[\int_{p_r}^{\bar{v}} v f_2(v) dv + (F_2(p_r) - F_1(p_r)) p_r \Big] \\ &+ \big[\rho_1 F(p_r) + \rho_2 F_1(p_r) + (1 - \rho_1 - \rho_2) \big] \delta U_{t+1}^e. \end{aligned} \tag{2}$$

According to the model, ρ_1 and ρ_2 are exogenous, and δU_{t+1}^e is the discounted expected payoff from going to the next period. Because the model is infinite horizon, the seller's expected profit is independent of time. This stationary model implies that the reservation price and the asking price are also independent of time [3]. Thus, there exists a steady state in which the expected payoffs converge to a payoff independent of time:

$$U_t^e = U_{t+1}^e = U^e. \tag{3}$$

In this case the seller's optimal decision is to set the reservation price equal to the discounted reservation value, i.e.,

$$p_r^* = \delta U^e. \tag{4}$$

Substituting this condition in (2), then we have

$$\begin{aligned} U^e =& \rho_1 \Big[\int_{p_r}^{p_l} (\theta v + (1-\theta) p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \Big] \\ &+ \rho_2 \Big[\int_{p_r}^{\bar{v}} v f_2(v) dv + (F_2(p_r) - F_1(p_r)) p_r \Big] \\ &+ \big[\rho_1 F(p_r) + \rho_2 F_1(p_r) + (1 - \rho_1 - \rho_2) \big] p_r. \end{aligned} \tag{5}$$

[3] This is a standard approach in infinite-horizon search models (see Lippman and McCall (1976) for a survey).

The seller chooses a reservation price p_r^* and an asking price p_a^* to maximise the total value of the search. We obtain the following expression for the optimal asking price by differentiating (5) with respect to p_a:

$$\rho_1(1 - F(p_l)) = 0. \tag{6}$$

Proposition 2.1. *The optimal asking price and the reservation price are a pair* (p_a^*, p_r^*) *that solve* (6) *and* (5) *simultaneously.*

Proof. See Appendix

The results for proposition(2.1) suggest that the seller sets the asking price in such a way that the buyer with the highest value in the interval is indifferent about entering the negotiation or buying at the asking price as long as she is the only buyer.

2.1 Example

Suppose buyers' values are distributed uniformly from [0,1]. ρ_1 and ρ_2 are equal to 0.4. Also assume that $\theta = 0.5$, and the discount factor is 0.9. Then the optimal asking price and the reservation price are

$$p_a^* = 0.72 \quad \text{and} \quad p_r^* = 0.45.$$

Figure 1 and 2 show how the asking price and the reservation price change when θ changes. When the seller has all of the bargaining power, she sets the asking price at the highest level and extracts the entire surplus in the bargaining game. In fact, this situation has the highest expected payoff for the seller when the distributional assumptions and the discount factor are kept the same.

2.2 Optimally Chosen Reserve Price

As we mentioned before, if the seller has the option to choose a reserve price for the auction, she might have a higher expected payoff. In this case the seller optimises the expected payoff with respect to three prices-an asking price p_a, a reservation price p_r, and a reserve price r-for the auction event only. The seller chooses the optimal reserve price, and the equation in (2) becomes

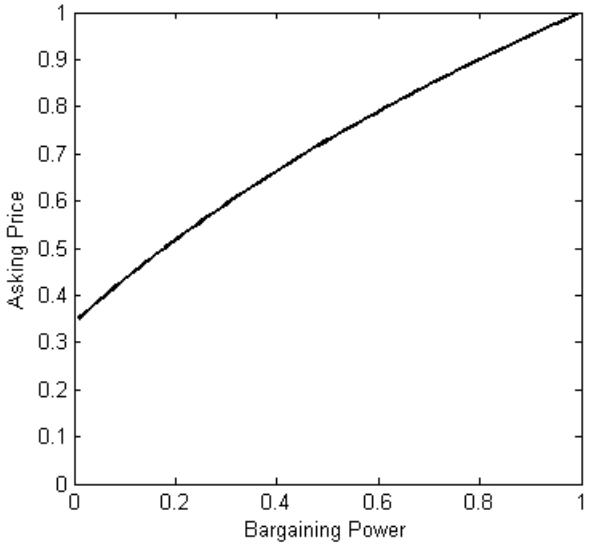

Figure 1: Asking Prices for Different Bargaining Powers

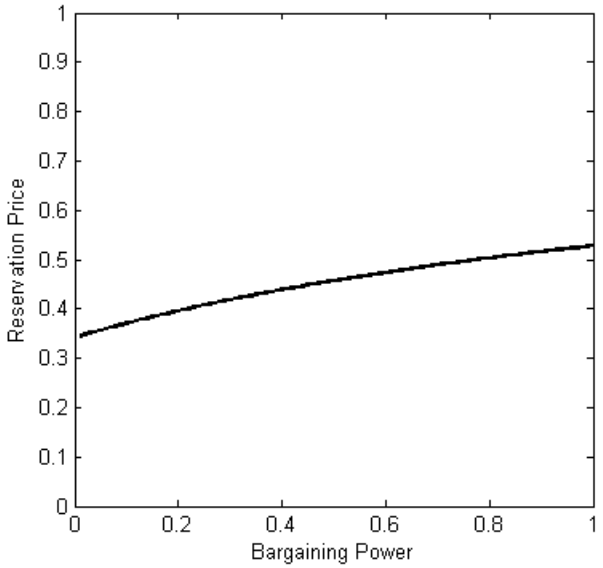

Figure 2: Reservation Prices for Different Bargaining Powers

$$
\begin{aligned}
U_t^e =& \rho_1 \Big[\int_{p_r}^{p_l} (\theta v + (1-\theta) p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \Big] \\
&+ \rho_2 \Big[\int_{r}^{\bar{v}} v f_2(v) dv + (F_2(r) - F_1(r)) r \Big] \\
&+ \big[\rho_1 F(p_r) + \rho_2 F_1(r) + (1 - \rho_1 - \rho_2) \big] \delta U_{t+1}^e.
\end{aligned} \tag{7}
$$

Considering the same argument for the stationary infinite-horizon models results in the following steady-state expected payoff:

$$
\begin{aligned}
U^e =& \rho_1 \Big[\int_{p_r}^{p_l} (\theta v + (1-\theta) p_r) dF(v) + \int_{p_l}^{\bar{v}} p_a dF(v) \Big] \\
&+ \rho_2 \Big[\int_{r}^{\bar{v}} v f_2(v) dv + (F_2(r) - F_1(r)) r \Big] \\
&+ \big[\rho_1 F(p_r) + \rho_2 F_1(r) + (1 - \rho_1 - \rho_2) \big] p_r.
\end{aligned} \tag{8}
$$

As mentioned before, because the expected payoff is independent of time, the seller's optimal decision is to set $p_r^* = \delta U^e$.

Lemma 2.1. *The optimal reserve price for the auction is* $r^* = p_r + \frac{1-F(r)}{f(r)}$.

Proof. Differentiating (8) with respect to r equal to zero gives

$$
\begin{aligned}
-r f_1(r) + F_2(r) - F_1(r) + p_r f_1(r) &= 0 \\
-r + p_r + \frac{F_2(r) - F_1(r)}{f_1(r)} &= 0 \\
r^* = p_r + \frac{1 - F(r)}{f(r)}&.
\end{aligned}
$$

□

In fact, lemma 2.1 suggests that the optimal reserve price is the continuation value plus the inverse of the *hazard rate* function, which is the same as the reserve price for the optimal auction. This is not surprising because the expected payoffs are independent of time and there is a sealed-bid second-price auction.

Proposition 2.2. *As long as (6) and lemma (2.1) hold, the optimal reservation price* p_r^* *is the one that solves (8).*

Proof. (6) and lemma (2.1) are the result of the first-order conditions of maximising (8). The second-order condition is satisfied as long as the *hazard rate* function of $F(.)$ is increasing, which is an assumption of the present model. Thus, r^* and p_a^* maximise (8). Substituting the condition in (4) into (8) results in

$$\begin{aligned}\frac{p_r}{\delta} =&\rho_1\big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)\big]\\ &+\rho_2\big[\int_{r}^{\bar{v}}vf_2(v)dv+(F_2(r)-F_1(r))r\big]\\ &+\big[\rho_1F(p_r)+\rho_2F_1(r)+(1-\rho_1-\rho_2)\big]p_r.\end{aligned}\tag{9}$$

With an argument the same as Proposition 2.1, we can show that there exists a p_r that solves this equation. □

For the example in (2.1), if we calculate an optimally chosen reserve price, then we have

$$p_a^*=0.78\quad\text{and}\quad p_r^*=0.54\quad\text{and}\quad r^*=0.57.$$

If the seller is able to identify a reserve price separately for a possible auction, then she will raise the optimal asking price compared to if she uses the same reservation price at the negotiation and for the auction. The seller's expected payoff for the game will also rise in this case because now she chooses the reserve price to maximise the revenue from the case in which she faces two buyers.

2.3 Comparison With an Optimal Auction

As mentioned previously, in the types of markets being analysed here, sellers, after posting a price, may accept a counteroffer. In these markets the posted price is not necessarily the lower bound of the transaction price. For the purpose of this analysis we introduce another selling mechanism, in which the seller may never negotiate with buyers on the asking price. Suppose a seller advertises a price p at the beginning of the game. If there are multiple buyers, the seller runs a second-price auction with the reserve price p, and if there is only one buyer, then p is a take-it-or-leave-it offer to that buyer. All other assumptions are the same as in the previous section. The seller's expected payoff at any given period becomes

$$
\begin{aligned}
U_t^o =& \rho_1 \big[p(1-F(p))\big] \\
&+ \rho_2 \big[\int_p^{\bar{v}} v f_2(v) dv + (F_2(p) - F_1(p))p \big] \\
&+ [\rho_1 F(p) + \rho_2 F_1(p) + (1 - \rho_1 - \rho_2)] \delta U_{t+1}^o .
\end{aligned} \tag{10}
$$

If a price p maximises U_t^o, it will also maximise U_{t+1}^o, because the seller is facing exactly the same problem in each period. Therefore, in the steady state $U_t^o = U_{t+1}^o = U^o$, which results in an optimal price that is independent of time. Then the expected payoff becomes

$$
U^o = \frac{\rho_1 [p(1-F(p))] + \rho_2 [\int_p^{\bar{v}} v f_2(v) dv + (F_2(p) - F_1(p))p]}{1 - \delta[\rho_1 F(p) + \rho_2 F_1(p) + (1 - \rho_1 - \rho_2)]}. \tag{11}
$$

The differentiation of (11) with respect to p gives the price that maximises the expected revenue for the seller. For the example in (2.1), the optimal price is equal to 0.5 and the expected payoff is 0.48. There exists a θ in which the expected payoff to the seller for the game defined in the previous section is higher than the expected payoff of this game. In fact, the greater the seller's negotiation power, the greater the chance that she accepts any counteroffer and enter the negotiation process, as the seller knows that in the bargaining game she can extract more surplus on average.

3 A More General Model

In this section we relax the assumption of a maximum of two buyers arriving at each period to generalise the results. In particular, we assume that the probability that $n \in \{1, 2, ...\}$ buyers arrive at each period is distributed geometrically, and is independent of time, with the probability of each success equal to ρ, where $0 < \rho \leq 1$. In this situation the seller expects any number of buyers, but with lower probabilities for higher numbers of arrivals. Keeping all other assumptions as in section 2 we define the game as follows. The seller posts an asking price p_a at period zero before the game starts. Then at every period after buyers arrive, according to the probability distribution explained previously, the seller observes the number of buyers. If there is only one buyer, the buyer can either offer to buy the object at the asking price or make a counteroffer, which would result in a bargaining game like the one explained in section 2. If there is more than one buyer, then the seller runs a sealed-bid second-price auction with a reserve price equal to the reservation price.

The seller's expected payoff at each period is independent of time in this model as well:

$$\begin{aligned} U^m =&\rho\Big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)+F(p_r)\delta U^m\Big] \\ &+\sum_{n=2}^{\infty}\Bigg[\int_{p_r}^{\bar{v}}v f_2^{(n)}(v)dv+(F_2^{(n)}(p_r)-F_1^{(n)}(p_r))p_r+F_1^{(n)}(p_r)\delta U^m\Bigg](1-\rho)^{n-1}\rho. \end{aligned} \tag{12}$$

This equation can be rewritten as follows:

$$\begin{aligned} U^m =&\rho\Big[\int_{p_r}^{p_l}(\theta v+(1-\theta)p_r)dF(v)+\int_{p_l}^{\bar{v}}p_a dF(v)+F(p_r)\delta U^m\Big] \\ &+\sum_{n=2}^{\infty}\Bigg[\int_{p_r}^{\bar{v}}nF^{n-1}(v)J(v)dF(v)+F^n(p_r)\delta U^m\Bigg](1-\rho)^{n-1}\rho, \end{aligned} \tag{13}$$

where $J(v)=v-\frac{1-F(v)}{f(v)}$.

Again, the seller's optimal decision is to set the reservation price such that $p_r=\delta U^m$, which is the minimum price for which the seller agrees to sell the object at any period of time. It is possible to find the sum of the series in the second term of the right-hand side of (13), but it is not necessary for the analysis at this stage. The optimal reservation price needs to satisfy (13), and the asking price needs to satisfy the first-order condition of (13) with respect to the asking price. Because the first-order condition of maximising the bargaining outcome with respect to the asking price is independent of the number of bidders, the general model has the same equation for the optimal asking price as the two-buyer model. Of course, the optimal asking price itself will not be the same for these models because the reservation price will not be the same because of the effect of ρ.

If the seller could choose a separate reserve price for the auction, again it would be independent of the number of buyers. Indeed, in the general model the reservation price is affected by the arrival rate of the buyers and changes the optimal asking price according to the first-order condition of maximising (13). With the same argument as in Proposition 2.1, it is possible to show that there exists a p_r that satisfies (13).

4 Conclusion

In some markets, like the housing market, there is a seller of an indivisible object and uncertainty about the number of potential buyers interested in buying the object.

In these markets the problem becomes even more complicated when the seller faces more than one period over time. The objective of the seller is to sell the object at the highest possible price in the shortest amount of time. Running an auction is generally a costly activity, and if there is little probability of there being more than one buyer, the auction becomes even less interesting. A selling mechanism has been proposed in which, before the game starts, the seller has to choose two prices: the asking price and the reservation price. The asking price is the price that is publicly announced in an advertisement, and the reservation price is the lowest price the seller will accept for the object, in this case her property. Buyers arrive according to a random process. Initially the number of arriving buyers was restricted to a maximum of two at each period. That is, we assume with some exogenous probabilities that there is a chance that there will be one buyer, two buyers, or zero interested buyers at each period and that these are the only three possibilities. If there is only one buyer, the seller may engage in negotiation with her with the asking price as the ceiling price for the negotiation and the reservation price as the floor price. If there are two buyers, then the seller runs an auction with a reserve price equal to the reservation price. Under some conditions this mechanism can result in a higher expected payoff for the seller than when the seller chooses a uniform price as a take-it-or-leave-it offer if there is only one buyer and the same price as the reserve price for two buyers. Indeed, sellers with greater bargaining power may prefer this mechanism and accept counteroffers, but sellers with less bargaining power may prefer the uniform price case. We also studied a mechanism in which the seller can choose the reserve price for a possible auction separately. Of course, this mechanism can do better than the proposed mechanism with the reservation price as the reserve price of a possible auction, but in practice the seller may not be able to propose three different prices to the selling agent.

In a more general model the assumption of a maximum of two buyers was relaxed to reflect any number of bidders. The setting suggests that the number of bidders is geometrically distributed, with lower probabilities for higher numbers of arrivals. This model shows that the optimality condition for the asking price is almost the same as that for the model with a maximum of two buyers. The analysis also shows the existence of an optimal reservation price for the general model.

The current study has considered the assumptions of the independent private value models. One possible extension of this work is to create a setting in which the seller has some private information about the value of the object and buyers care about this information to determine the possibility of signalling for the asking price mechanism. Another possible extension involves adding the buyer search to the game. A buyer may need to pay a cost to realise her private signal or a part of

her value. In that case a two-sided search model of buyers and sellers in the housing market would provide a more general role for the asking price in these markets.

5 Appendix

Proof of Proposition 2.1.

Since $p_l = (p_a - (1-\theta)p_r)/\theta$, setting $\bar{v} = 1$ without loss of generality, the first order condition of maximizing the expected payoff with respect to p_a in (6) would be,

$$p_a^* = \theta + (1-\theta)p_r^* \tag{14}$$

For any $0 \leq \theta \leq 1$, this implies that $p_a^* \geq p_r^*$. Rewriting (5), we have,

$$\begin{aligned} \frac{p_r}{\delta} =& \rho_1 \Big[\int_{p_r}^{1} (\theta v + (1-\theta)p_r) dF(v) + p_r F(p_r) \Big] \\ &+ \rho_2 \Big[\int_{p_r}^{\bar{v}} v f_2(v) dv + (F_2(p_r) - F_1(p_r))p_r + p_r F_1(p_r) \Big] \\ &+ (1 - \rho_1 - \rho_2)p_r \end{aligned} \tag{15}$$

At $p_r = 0$ the left hand side is equal to zero and the right hand side is positive. At $p_r = 1$ the left hand side is greater than one (since $\delta < 1$) while the right hand side at every possible scenario the maximum is p_r. Therefore, by continuity there exist at least one p_r that satisfies (15). At this stage we do not find the uniqueness condition in general and if more than one p_r satisfies (15) then p_r^* would be the highest one. □

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