MANDATE AND PATERNALISM: A THEORY OF LARGE ELECTIONS*

MARCO FARAVELLI†, PRISCILLA MAN‡, AND RANDALL WALSH§

ABSTRACT. We propose a game theoretic model of large elections that incorporates the assumption that mandate matters. This innovation is motivated by empirical evidence that US Representatives with larger victory margins on average vote in a more partisan manner. Without relying on preference for voting, this new model predicts strictly positive limiting turnout rates in a costly voting environment as the number of paternalistic voters grows arbitrarily large. The model also preserves stylized comparative statics results of costly voting models, including the underdog effect and the competition effect.

Keywords: Costly Voting, Large Elections, Turnout, Mandate, Paternalism.
JEL codes: D72, C72.

†School of Economics, University of Queensland. Email: m.faravelli@uq.edu.au.
‡School of Economics, University of Queensland. Email: t.man@uq.edu.au.
§Dept. of Economics, University of Pittsburgh & National Bureau of Economic Research. Email: walshr@pitt.edu.

*This paper was originally circulated under the title "Smooth Politicians and Paternalistic Voters: A Theory of Large Elections".

1. Introduction

In the original formulation of his rational choice theory, Anthony Downs (1957) draws attention to the "paradox of voting": if voting is costly, turnout in large elections should be negligible. Downs proposes a decision theoretic framework in which the probability of being pivotal is exogenous. In later work, Ledyard (1981, 1984) and Palfrey and Rosenthal (1983, 1985) demonstrate that the paradox of voting remains even when the problem is recast in game theoretic terms. As game theoretic costly voting models have continued to proliferate,[1,2] the prediction that turnout converges to zero as the size of the electorate grows has remained typical. When the literature has attempted to overcome this grim and unrealistic result it has done so by assuming either a preference for the act of voting itself (Riker and Ordeshook, 1968), or the existence of some coordination mechanism (such as a leader who can mobilize supporters) that effectively reduces the voting game to one with a small number of players (Harsanyi, 1977, 1992; Morton, 1991; Shachar and Nalebuff, 1999).

We adopt a new approach. We maintain the standard game theoretic costly voting model in which voting is purely instrumental and no coordination device is available. Novel to our setting is the assumption that mandate — the margin of victory — matters. We argue, supported by empirical evidence from US Congressional voting records, that the policies adopted by elected politicians are closer to the "center" when their margin of victory is smaller and more "extreme" when their seats are won with a landslide victory. This new, yet plausible, assumption, together with the assumption of paternalistic voters — that a voter derives spillover benefits from the impact of policies on other individuals — delivers a strictly positive limiting turnout as the size of the electorate grows without bounds.[3] In other words, the paradox of voting disappears.

At first blush, this result might seem obvious: if voters care about the margin of victory and such benefit is multiplied by the size of the population, one might think that the benefit from voting would go to infinity as the size of the electorate grows — leading any agent with a finite voting cost to choose to vote. This "intuition" is fallacious as it ignores the fact that the effect of a single vote on the mandate also goes to zero when the number of votes cast goes to infinity. Rather, the positive limiting turnout result is driven by a delicate balance between the significance of one vote on the mandate and the magnitude of the paternalism spillover. The rates at which these two effects converge to zero and infinity, respectively, balance out in equilibrium, prompting a positive proportion of the electorate to vote. We demonstrate that this positive proportion need not be one. Given

[1]See, for instance, Campbell (1999); Börgers (2004); Goeree and Großer (2007); Krasa and Polborn (2009); Taylor and Yildirim (2010); Krishna and Morgan (2012).

[2]There are also information aggregation voting models that analyze voters' participation. See McMurray (2012).

[3]To be precise, by 'strictly positive limiting turnout' we mean that the proportion of potential voters who turn out is bounded away from zero — as opposed to the case where the absolute number of voters is bounded away from zero, but the proportion goes to zero.

any policy rule, the limiting turnout is strictly less than one whenever voters' paternalism is not too high.

More importantly, as we can characterize the limiting turnout, we are also able to analyze the relationship between the expected sizes of political parties and turnout. Under mild symmetry assumptions (so that supporters of neither party are ex-ante more motivated to vote), we obtain the "underdog effect" — that members of the minority party turn out to vote at higher rates than those of the majority; as well as the "competition effect" — that closer elections generate higher turnout.[4] These effects have been well-documented, both in laboratory experiments and empirically in large elections (Levine and Palfrey, 2007; Shachar and Nalebuff, 1999; Blais, 2000). However, theoretical predictions for such effects have been notoriously difficult to derive in game theoretical models of large elections (Taylor and Yildirim, 2010). This is largely due to the fact that most of these models predict zero turnout in large elections, thereby stripping off the possibility for any meaningful analysis of factors affecting turnout. Our ability to predict strictly positive turnout in large elections enables us to be one of the few game theoretical papers to generate predictions matching stylized empirical findings without reliance on preference for voting or coordination devices.

We conclude our introductory comments by noting that our work is not the first attempt to overcome the "paradox of voting" in large elections. Building on early attempts to generate positive limiting turnouts, Feddersen and Sandroni (2006) propose a model that yields a strong set of comparative statics predictions and provides an important stepping stone for understanding large elections.[5] In their model, a proportion of the voters are "ethical" in that they receive a benefit for "doing their part". Specifically, a voter is "doing her part" if she follows the voting strategy that would be adopted by a social planner whose party preference is aligned with hers. These voters, like those in our model, are paternalistic in that an individual of a given party projects her own preferred election outcome onto the entire measure of voters. However, under Feddersen and Sandroni's conceptualization, agents are rule utilitarians. Thus, voters within the same party do not interact strategically among themselves. Our model departs from this setting by assuming that all voters interact with each other strategically, thereby allowing voters within the same party to free-ride on each other.

Closest in spirit to our paper is the concurrent and independent work by Evren (2012), who develops a game theoretic framework that delivers Feddersen and Sandroni's results. Evren's key insight is to introduce uncertainty over the proportion of other-regarding voters in a winner-take-all election. This uncertainty "smooths" the expected utility of each voter in the sense of slowing the rate at which the benefit from voting approaches zero, just as mandate does in our model. In fact, removing the mandate effect from our

[4]The terms underdog effect and competition effect are introduced by Levine and Palfrey (2007).

[5]Coate and Conlin (2004) use a version of this model to explain turnout in the Texas liquor referenda.

model yields results identical to those obtained in Evren's model under the assumption of no uncertainty (see Section 6).

Finally, Castanheira (2003) also incorporates the notion that mandate matters in an otherwise standard costly voting model. He shows that under population uncertainty turnout converges to zero as the population grows without bound, albeit at a slower rate than in a winner-take-all election. Voters in his model are selfish while those in ours are paternalistic. Nonetheless, we do demonstrate in Section 6 that when voters of at least one party are selfish, limiting turnout would be zero. This finding can be viewed as an extension of the famous zero turnout result of Palfrey and Rosenthal (1985) to a general class of policy rules beyond winner-take-all elections.

The remainder of the paper is organized as follows. Section 2 leverages empirical evidence to motivate the assumption that mandate matters. Section 3 presents the model. Section 4 proves that limiting turnout is strictly positive and provides its characterization, which is then analyzed in Section 5. Section 6 discusses the necessity of our key assumptions. Section 7 concludes. All proofs are relegated to the Appendices.

2. Mandate Matters: Empirical Evidence

The key innovation of our approach is accounting for the possibility that margin of victory matters. This notion has received attention in the political science literature and various explanations for the presence of a mandate effect have been put forward.[6] The two leading arguments are that candidates may find an extreme partisan position risky for the next election unless they are supported by a large mandate and/or the margin of victory may signal the position of the center of the electorate, toward which candidates lean. Formally, Razin (2003) proposes a model of "responsive candidates" who infer information on an imperfectly observed common preference shock from the election result, and position themselves optimally given this inference.

We do not intend to offer a theoretical foundation of why elected officials respond to their mandate. However, we do wish to provide an interpretation of this crucial assumption and provide empirical support for the presence of a mandate effect. For simplicity, suppose that the policy position of a given elected official can be mapped into the interval $[0, 1]$ and that members of party A strictly prefer outcomes closer to 1, while members of party B strictly prefer outcomes closer to 0. Assume that if elected candidates respond to the margin of victory in terms of their policy positions. Thus, the proportion of votes received by party A can be mapped into a policy outcome through an increasing function G — which we assume to be smooth. This G function can be viewed as a "policy rule". We plot several possible policy rules against the mandate of party A in Figure 1.[7] The dotted line presents a "proportional rule" under which the winning

[6]See Smirnov and Fowler (2007) for a survey.

[7]All three policy rules in Figure 1 are symmetric around 1/2. This is not necessary for obtaining our positive limiting turnout result. See Section 3 for the precise restrictions on the G function.

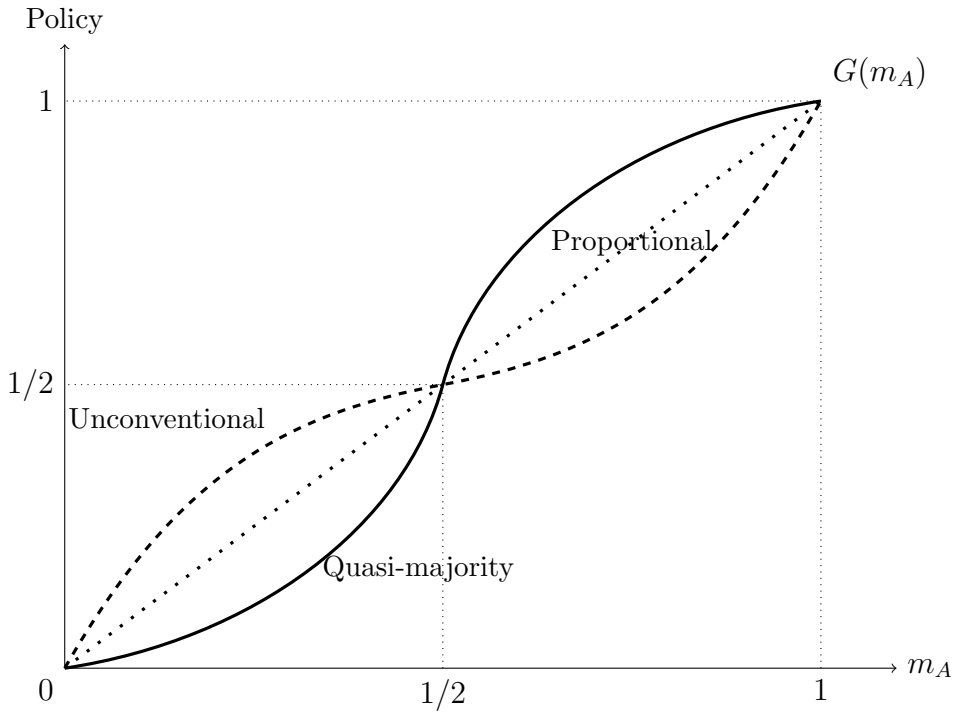


FIGURE 1. Examples of Smooth Policy Rule

politician responds to the mandate at a constant rate. It approximates a proportional representation democracy. The solid line represents a "quasi-majority rule" under which the winning politician responds to the mandate at a decreasing rate. It approximates policy outcomes under direct election regimes such as the US Congress. Notice that winner-take-all majority rule can be viewed as the extreme case of a quasi-majority rule when G becomes a step function. The dashed line is an "unconventional rule" under which the winning politician responds to the mandate at an increasing rate. It is rarely observed empirically but is still admitted by the model.

Do politicians adopt a smooth policy rule? Fowler (2005) shows that candidates in US Senate races respond to increases in previous election's Republican vote by adopting a more conservative position in the current race. Using a sample of 23 Democracies between 1945 and 1988 from the Comparative Manifesto Project, Somer-Topcu (2009) finds that political parties respond to declining vote shares by changing the policies they support. Peterson, Grossback, Stimson, and Gangl (2003) proxy "mandate" to the winning party by analyzing newspaper coverage in US Presidential and off-year election and provide empirical evidence that Members of Congress deviate from their historical voting pattern in the direction of the mandate following a "mandate" election.

To provide more direct support for the presence of a mandate effect, we analyze the impact of margins of victory on voting behaviors of US Members of Congress for the 105^{th} (1997-1998) through the 111^{th} Congress (2009-2010). The political position of a

TABLE 1. Estimations of the effect of Margin of Victory on Degree of Partisanship, US 105th to 111th Congress[a]

Congress	**105th**	**106th**	**107th**	**108th**	**109th**	**110th**	**111th**	**105th–111th**	**105th–111th**
Victory Margin	0.1368***	0.0572**	0.1754***	0.0599**	0.1431***	0.2443***	0.2604***	0.1507***	0.0746***
	(0.0338)	(0.0263)	(0.0343)	(0.0271)	(0.0275)	(0.0257)	(0.0299)	(0.0111)	(0.0095)
Years in Office	−0.0008	−0.0010	−0.0007	−0.0013	−0.0001	0.0001	0.0011	−0.0002	0.0049***
	(0.0010)	(0.4261)	(0.3867)	(0.0009)	(0.0009)	(0.0008)	(0.0009)	(0.0004)	(0.0006)
Candidate FE	·	·	·	·	·	·	·	No	Yes
Congress FE	·	·	·	·	·	·	·	Yes	Yes
Marginal Effect[b]	0.2011	0.1060	0.2452	0.1082	0.2479	0.4338	0.3981	0.2473	0.1224
N	423	431	428	430	431	424	432	2999	2999
No. Legislators	423	431	428	430	431	424	432	753	753
Partisanship									
Mean	0.4253	0.4426	0.4496	0.4661	0.4793	0.4881	0.5017	0.4647	0.4647
S.D.	0.1498	0.1551	0.1778	0.1439	0.1445	0.1465	0.1705	0.1578	0.1578
Victory Margin									
Mean	0.3033	0.4264	0.3953	0.4253	0.4070	0.3603	0.3740	0.3847	0.3847
S.D.	0.2201	0.2876	0.2487	0.2597	0.2503	0.2602	0.2608	0.2590	0.2590

[a] The dependent variable in each regression is the degree of partisanship which is constructed using Poole and Rosenthal's DW-Nominate Scores (see text). Numbers in parentheses are standard errors. *, ** and *** indicate statistical significance at 10%, 5% and 1% levels, respectively.

[b] Measures the change in partisanship (expressed in standard deviations) associated with a one standard deviation increase in the margin of victory.

given Member of Congress in a given year is identified by the first dimension of her DW-Nominate Score (Poole and Rosenthal, 2001).[8] This component of the DW-Nominate Score measures liberal-conservative leaning based on all roll-call votes during each Congressional Session. We construct the degree of partisanship for each Congressperson in each year as the distance between her DW-Nominate Score and the mid-point between the scores of the most liberal Republican and the most conservative Democrat (expressed in terms of absolute value).

We regress the degree of partisanship on the margin of victory in the election in which the Congressperson won her seat, as well as on years in office. The first seven columns of Table 1 report the results for each of the seven included sessions. The eighth column pools the data for the seven congressional sessions into a single regression with the incorporation of congress-level fixed effect. In all cases the coefficient on margin of victory is highly statistically significant. To provide a more intuitive interpretation of the results, we also report the predicted marginal effect associated with a one standard deviation increase in the margin of victory — expressed in terms of standard deviations in partisanship. For example, in the 105$^{\text{th}}$ Congress, a one standard deviation increase in the margin of victory will be associated with a 0.2 standard deviation increase in degree of partisanship. The estimated marginal effects across all seven congresses range between 0.11 and 0.44, with a pooled estimate of 0.25.

Of course, these results do not necessarily imply a direct causal link. One alternative explanation is that more ideologically extreme candidates select into districts with larger partisan majorities. However, even in the presence of such a selection process, as long as election outcomes can be considered as the most direct measure of a given district's voter ideology, voters would still benefit by increasing their supported candidate's margin of victory as it would eventually lead to preferred policy outcomes through the recruitment of candidates with preferable policy positions.

Nonetheless, we also use the panel structure of the data to investigate the direct, within representative, connection between margin of victory and partisan voting behavior. To this end, the final column of Table 1 reports the results of regressing the degree of partisanship on our explanatory variables with both Congress fixed effects and Congressperson fixed effects, using the pooled data from the seven Congresses. This allows us to evaluate the way that the voting behavior of a given candidate evolves in response to changes in her margin of victory. Again the coefficient on the margin of victory is significant at the 1% level, with an estimated marginal effect of 0.12. Thus, even when we focus solely on the within candidate variation in ideological voting patterns, there is clear evidence supporting the assumption that mandate matters.

[8]See Poole and Rosenthal (2001) for details on the construction of the DW-Nominate Score. A complete set of scores for the 1$^{\text{st}}$ through 111$^{\text{th}}$ Congress are available online at `http://pooleandrosenthal.com/dwnominate.asp`. We restrict our analysis to the 105$^{\text{th}}$ to 111$^{\text{th}}$ Congress because of the limited availability of digitized data in the earlier periods.

3. Model

We model an election with $N+1$ citizens and two parties, A and B. Denote a generic citizen by i and a generic party by P. Each citizen has, independently, the same ex-ante probability $\alpha \in (0,1)$ of being a supporter of party A and $1-\alpha$ of supporting party B. Citizens choose simultaneously to vote for the candidate of party A, the candidate of party B or to abstain (note: for ease of exposition, we use the terms "party P" and "candidate of party P" interchangeably). If citizen i votes (instead of abstaining), she bears a voting cost of c_i. The voting costs of citizens are identically and independently (regardless of their party alliance) drawn from a non-degenerate interval $C \subseteq \mathbb{R}_+$ according to the cumulative distribution function F with a density function f, where $f(c) > 0$ for all $c \in C$. Each citizen's party alliance and voting cost are her private information, but their distributions are commonly known.

Let n_A and n_B be the numbers of votes cast for parties A and B respectively. Party P's mandate is equal to the proportion of the votes received by its candidate, except in the case when no one votes in which case each party receives a mandate of $1/2$. Thus,

$$m_P(n_A, n_B) = \begin{cases} \frac{n_P}{n_A+n_B} & \text{if } n_A + n_B > 0 \\ \frac{1}{2} & \text{if } n_A + n_B = 0 \end{cases}. \tag{1}$$

To operationalize the mandate effect we assume that each member of party P receives a direct benefit as a result of her party's mandate. This benefit is denoted by $G(m_P)$ where $G : [0,1] \to [0,1]$ is a continuous, strictly increasing, twice differentiable function whose first derivative is bounded away from 0 and infinity.[9]

Citizens are also paternalistic. If a supporter of party P receives a direct benefit of b from the mandate, she will also enjoys an indirect benefit equal to $\gamma_P b$ for each other citizen in the society. Additionally, she may place greater weight on members of her own party and thus receives an extra benefit of $\hat{\gamma}_P b$ for each other member of her own party. For each party P, both γ_P and $\hat{\gamma}_P$ are weakly positive, with at least one of them being strictly positive. All payoff information is common knowledge.

Let i be a member of party P with voting cost c_i. Let v_P and v_Q be the number of votes cast for parties P and Q, both exclusive of i. Likewise, let N_P be the number of party P supporters exclusive of i. Then i's utility is

$$\begin{cases} [1 + \gamma_P N + \hat{\gamma}_P N_P] G(m_P(v_P + 1, v_Q)) - c_i & \text{if } i \text{ votes for party } P; \\ [1 + \gamma_P N + \hat{\gamma}_P N_P] G(m_P(v_P, v_Q + 1)) - c_i & \text{if } i \text{ votes for party } Q; \text{ and} \\ [1 + \gamma_P N + \hat{\gamma}_P N_P] G(m_P(v_P, v_Q)) & \text{if } i \text{ abstains.} \end{cases}$$

[9] One interpretation of the model's "preference for mandate" is that it captures the behavioral notion that voters simply enjoy winning with a larger mandate. However, we prefer to consider the model within the context of our previous discussion and interpret $G(.)$ as representing the process through which larger voting majorities are translated into increasingly partisan (and thus more preferred) policy outcomes.

As specified, the model defines a Bayesian game. A pure strategy for an individual is a pair of (measurable) voting rules, (s_i^A, s_i^B) where $s_i^P : C \to \{A, B, \text{Abstain}\}$ assigns a choice for each voting cost when i belongs to party P. Notice that for each party affiliation and voting cost realization, voting for the opposite party is always *strictly* worse than abstaining regardless of the number of votes cast for the own and opposite party. Thus at no voting cost would any citizen vote for the opposite party if she is best-responding to the strategy profiles of other citizens. In addition, conditional on party alliance and other citizens' voting rules, the difference between the expected payoffs from voting for own party and abstaining is decreasing in individual i's cost of voting. Thus any best response of citizen i to any profile of other citizens' strategy must be monotone — i.e., for each party P, there exists a cut-off cost c_P such that when i is a P-supporter, she votes for P for all $c < c_P$ and abstains for all $c > c_P$.

In our analysis,we consider symmetric Bayesian-Nash equilibria in which all citizens use the same pair of voting rules, i.e., $(s_i^A, s_i^B) = (s^A, s^B)$ for all i. By our argument above, this equilibrium pair of voting rules involves two cut-off costs (c_A, c_B).[10] Define the turnout of party P as the probability that a randomly chosen supporter of P votes. In equilibrium, the turnout of party P is $F(c_P)$. Since there is a one-to-one relationship between c_P and party P's turnout, we will refer to an equilibrium solely by $(a, b) = (F(c_A), F(c_B))$.

4. Turnout in Large Elections

When N is finite, an equilibrium exists by a simple application of the Brouwer Fixed-Point Theorem:[11]

Proposition 1. *For finite N, a pure strategy symmetric monotone Bayesian-Nash equilibrium exists.*

We are, however, more interested in large elections. Let (a_N, b_N) be an equilibrium turnout profile in an election with $N + 1$ citizens.[12] Notice that $\{(a_N, b_N)\}_N \subset [0, 1]^2$ possesses a convergent subsequence.

Theorem 1. *Suppose a convergent subsequence of $\{(a_N, b_N)\}_N$ converges to (a^*, b^*). Then $a^* > 0$ and $b^* > 0$.*

In other words, any limiting turnout (regardless of the subsequence picked) is strictly positive. One may be tempted to think that this follows directly from the preference for mandate — that an individual always makes a difference — magnified by paternalism. This view is incorrect since the difference that a single voter makes on the mandate vanishes as the number of votes cast gets large. The intuition for the proof of Theorem 1

[10] Strictly speaking, an equilibrium should also specify the actions chosen at the two cut-off points. However, $c_i = c_A$ and $c_i = c_B$ are zero probability events. The actions at these two points would not affect ex-ante calculations. Hence we will only describe an equilibrium by the cut-offs.

[11] The proofs of all theorems and propositions are presented in the Appendices.

[12] If there are multiple equilibria at N, just pick any one of them.

is as follows. Suppose by contradiction that a_N and b_N approach zero as N goes to infinity. Then the marginal benefit from mandate will vanish at a rate slow enough such that the gross benefit from voting will explode, leading a significant proportion of the electorate to vote. This contradicts the assumption that a_N and b_N converge to zero. Conversely, a^* and b^* need not be 1, which would be the case if the expected gross benefit from voting went to infinity with N. Indeed, the sequence of turnout profiles is interior in many settings.

Proposition 2. *If C, the support of voting cost, is unbounded from above, then $a_N, b_N < 1$ for all N. If C is bounded from above, then given α and G, there exist γ and $\hat{\gamma}$ sufficiently small that $a_N, b_N < 1$ for all N sufficiently large.*

When the convergent sequence of equilibrium voting probabilities is interior, the limiting probabilities can be characterized:

Proposition 3. *If $\{(a_N, b_N)\}_N \subset [0,1)^2$ converges to (a^*, b^*), then (a^*, b^*) satisfies*

$$\begin{aligned}(\gamma_A + \hat{\gamma}_A \alpha)\frac{1 - m_A^*}{T^*} G'(m_A^*) &= F^{-1}(a^*) \\ (\gamma_B + \hat{\gamma}_B (1-\alpha))\frac{m_A^*}{T^*} G'(1 - m_A^*) &= F^{-1}(b^*)\end{aligned} \tag{2}$$

where $T^* = \alpha a^* + (1-\alpha) b^*$ is the limiting total turnout (the probability that a citizen votes) and $m_A^* = \alpha a^*/T^*$ is the limiting mandate of party A.

Theorem 1 and Proposition 3 are statements about the limiting turnout only. They do not imply higher turnout for larger society. Nothing prohibits $\{a_N\}$ and $\{b_N\}$ from being decreasing sequences.

5. Equilibrium Analysis

The characterization of the limiting equilibrium in Proposition 3 is rather general. For a meaningful equilibrium analysis, however, we would like to impose some symmetry across the two parties. Otherwise, the comparison across the two parties could arise simply out of the asymmetries and may have little to do with the election game per se.

To this end, we require that all citizens, conditional on their party alliance, are ex-ante identical in terms of their utilities. First, we assume that the two parties are identical in terms of their "paternalism loading factors". All citizens care about all citizens in the same way (i.e., $\gamma_A = \gamma_B = \gamma$); and all citizens care about their own party's members in the same way (i.e., $\hat{\gamma}_A = \hat{\gamma}_B = \hat{\gamma}$). Second, we assume that the benefit from mandate is symmetric around 1/2. That is, for any mandate $m \in [0,1]$, $G(1-m) = 1 - G(m)$.[13] Using the interpretation in Section 2, this is saying that, in case of victory, either candidate responds to mandate in the same magnitude. Under the above symmetry assumptions,

[13]This immediately implies $G'(1-m) = G'(m)$ and $G''(1-m) = -G''(m)$ for all $m \in [0,1]$.

Equation (2) can be rewritten as

$$(\gamma + \hat{\gamma}\alpha)\,\frac{1 - m_A^*}{T^*} G'(m_A^*) = F^{-1}(a^*) \tag{3}$$

$$(\gamma + \hat{\gamma}(1-\alpha))\,\frac{m_A^*}{T^*} G'(1 - m_A^*) = F^{-1}(b^*). \tag{4}$$

The only asymmetry we allow is the (expected) size of the parties. Without loss of generality we assume that party A is the larger party. That is, $\alpha \geq 1/2$.

We begin by proving the existence of an *underdog effect*: that supporters of the smaller party vote more frequently, but the larger party still secures a greater mandate.

Proposition 4 (Underdog Effect)**.** *Suppose $\alpha \geq 1/2$. Then $a^* \leq b^*$ and $m_A^* \geq 1/2$. Moreover, if $\gamma > 0$, both inequalities are strict whenever $\alpha > 1/2$.*

The term underdog effect was coined by Levine and Palfrey (2007). It is documented in large elections by Shachar and Nalebuff (1999); Blais (2000). Taylor and Yildirim (2010) formalize the notion in a general game theoretic framework for small elections. However, in their model the underdog effect disappears when $N \to \infty$, since turnout converges to zero. In contrast, our Proposition 4 offers a theoretical foundation for the underdog effect in a large election.

The underdog effect compares voting behaviors across parties given fixed party sizes. It is caused by the relatively higher incentive to free-ride experienced by the majority supporters, but it depends neither on the curvature of G (the smooth policy rule), nor on the distribution of voting costs. All it requires is that supporters across parties have the same preference parameters (so that none of them are more zealous about voting ex-ante) and that the preference for mandate be symmetric around $1/2$.

On the contrary, the effect of a change in α on turnout is affected by both the curvature of G and the distribution of voting costs. The former governs the rate of returns to voting arising from changes in the expected number of votes for each party (and the associated change in mandate benefits). The latter affects responsiveness to changes in expected votes through changes in the gross benefits of voting. To make our analysis tractable, we impose two mild assumptions. First, we assume that the marginal benefit from mandate when a party is winning is decreasing. This is the usual "decreasing returns" assumption in economics. In elections, though, it has the additional interpretation that the benefit from voting is greater when a vote closes the gap with the other party than when it widens the gap. We think of this as a realistic feature of most elections. Mathematically this translates to G being concave on $[1/2, 1]$. That is, G is the quasi-majority rule described in Section 2. Adopt also the notation

$$\sigma_G(m) = \frac{-G''(m)m}{G'(m)} \quad \text{for } m \in [0,1]$$

as the elasticity of G' at m. We say that G' is *elastic at* m if $\sigma_G(m) \geq 1$.[14] Second, we assume that the distribution F satisfies the following condition:

$$\frac{f(c)c}{F(c)} \quad \text{is weakly decreasing in } c.$$

In other words, the *elasticity* of the cumulative distribution function is decreasing in c. This assumption is satisfied by many common distributions with support in $\mathbb{R}_+$, including log-normal distributions, exponential distributions, Pareto distributions on $[\underline{c}, \infty)$ for $\underline{c} > 0$, uniform distributions on $[\underline{c}, \overline{c}]$ where $\underline{c} \geq 0$, Weibull distributions and single parameter Fréchet distributions.

Given the above assumptions, we obtain the *competition effect*, which states that total turnout decreases as the election becomes more lopsided.

Proposition 5 (Competition Effect)**.** *When $\alpha \in [1/2, 1)$ increases,*

(1) *Party A's turnout, a^*, decreases;*
(2) *Expected proportion of votes for party B (out of all citizens), $(1-\alpha)b^*$, decreases;*
(3) *Mandate for party A, m_A^*, increases; and*
(4) *If either $\sigma_G(m_A^*) \geq \frac{\alpha\hat{\gamma}}{\gamma+\alpha\hat{\gamma}}$ or $\gamma = 0$, then total turnout, T^*, decreases.*

While the first three comparative statics in this proposition deal with each party's turnout and mandate, the last result is commonly known in the voting literature as the competition effect. Like the underdog effect, the term competition effect is coined by Levine and Palfrey (2007) and documented in large elections by Shachar and Nalebuff (1999); Blais (2000). The competition effect has been difficult to obtain in models of large elections (Krasa and Polborn, 2009). Indeed, Taylor and Yildirim (2010, p. 464) concede, "the widely held intuition that elections with a more evenly split electorate should generate a greater expected turnout appears to be a property of small elections". It should be pointed out, though, that the standard costly voting model fails to predict the competition effect in large election because it predicts zero limiting turnout. Thus, by adopting the assumptions that mandate matters and voters are paternalistic, we are able to overcome this obstacle and provide a theoretical prediction for the competition effect in large elections.

Notice that $\alpha\hat{\gamma}/(\gamma + \alpha\hat{\gamma}) \leq 1$. Thus if G' is elastic at m_A^*, total turnout must decrease as α increases. The intuition behind this result is fairly straightforward. Under our decreasing returns assumption, members of the majoritarian party have less incentives to vote when they expect to win by a landslide than in a close election. If G is sufficiently "curved", their benefits from voting diminish quickly, causing a decrease in their turnout that offsets the increase in the size of the majority party. Meanwhile, the proportion of votes for the smaller party decreases. The total turnout is therefore reduced. Notice that

[14] If G were a von Neumann-Morgenstern utility function $\sigma_G(m)$ would be the coefficient of relative risk aversion.

the condition we give is sufficient but not necessary. For instance, if $\gamma = 0$ or if $\hat{\gamma} = 0$, then weak concavity of G on $[1/2, 1]$ is sufficient for the competition effect to arise.

6. Importance of Mandate and Paternalism

The above results demonstrate that two assumptions — that mandate matters and voters are paternalistic — generate strictly positive turnout in large elections. Moreover, the model preserves the underdog effect and the competition effect, which are intuitive equilibrium predictions of costly voting models but have only been proved theoretically for small elections.

One may question if both of our assumptions are needed. To dispel such doubts, we conclude the analysis by showing that the positive turnout result disappears when any one of the two main assumptions are lifted.

Proposition 6 (No Paternalism)**.** *If voters in at least one party are not paternalistic (i.e., $\gamma_P = \hat{\gamma}_P = 0$ for at least one $P \in \{A, B\}$), then $a^* = b^* = 0$.*

Intuitively, if voters of one party are not paternalistic, the supporters of this party are motivated to vote only for their direct mandate benefit. Yet if their own party's limiting turnout were strictly positive, in the limit one vote would have made no difference on the mandate. Thus the limiting turnout for the non-paternalistic party is zero. But then voters from the other party would have no incentives to vote either, resulting in a zero limiting total turnout.

Conversely, if mandate does not matter, limiting turnout will also converge to zero except in a knife-edge case. Thus:

Proposition 7 (No Preference for Mandate)**.** *Consider a winner-take-all election in which*

$$G(m_P) = \begin{cases} 0 & \text{if } m_P < 1/2 \\ 1/2 & \text{if } m_P = 1/2 \\ 1 & \text{if } m_P > 1/2. \end{cases}$$

If $\alpha \neq 1/2$, then $a^* = b^* = 0$.

The intuition is the same as for the zero turnout results present in the extant literature on large elections: as the population increases, the probability that a voter becomes pivotal goes to zero *exponentially*. Even if this benefit is multiplied by the size of the population due to paternalism, the resulting gross benefit from voting would still approach zero. The knife-edge case of $\alpha = 1/2$ is an exception. In this case, if the same proportion of voters from each party votes, the probability that each voter is pivotal vanishes at the rate of $1/\sqrt{N}$. With paternalism, the expected gross benefit goes to infinity.

Interestingly, Evren (2012) obtains the same result as Proposition 7 as the limiting case when the uncertainty on the proportion of altruistic voters disappears. This is because,

while our conceptualizations of the problem are different, uncertainty in his framework serves the same purpose as mandate does in our model — smoothing out the expected utility of each citizen. As a result, the two models coincide when the election is winner-take-all and all preference parameters are known. It would be an interesting research question to investigate a more general framework encompassing both Evren's and our model.

7. Conclusion

This paper is one of two recent theoretical advances that provide new approaches to overcoming the "paradox of voting" (Downs, 1957). It relies neither on the explicit assumption that individuals have a preference for the act of voting itself, nor on coordination mechanisms which effectively collapse the problem of voting into a game between a small number of players. Our solution is built on the assumption — for which we provide empirical support — that when elections are decided, the mandate (magnitude of the victory) has consequences. The complement to our work is that of Evren (2012), who obtains positive turnout ratios in large elections with a model that incorporates uncertainty about the proportion of voters who exhibit paternalistic preferences.[15]

In concluding, we draw attention to several positive aspects of our approach for overcoming the paradox. First, the novel feature of our model (mandate matters) is both intuitive and empirically relevant. Second, our proposed framework maintains the characterization of elections as a game theoretic setting with an unbounded number of players. Third, we demonstrate that interior equilibria exist over a wide range of preference and cost assumptions. A straightforward characterization of these equilibria is given in proposition 3. Finally, the model predicts the presence of both an "underdog effect" (supporters of the smaller party vote more frequently than do supporters of the larger party, but fail to achieve a majority) and a "competition effect" (turnout is higher in more competitive races). The presence of each of these effects has been empirically documented in large elections and their prediction has been a touchstone for the extant literature.

Our model's performance on these key dimensions suggests that incorporating a mandate effect is an attractive alternative for extending existing game theoretic results from small elections into the large election context.

Appendix A. Proofs of Results in Section 4

This appendix contains the proofs of all theorems and propositions in Section 4. We first prove Proposition 1, Theorem 1 and Proposition 3. Proposition 2 will be proved last using intermediate results from the proof of Proposition 3.

[15] In his work, Evren uses the term "altruism" when referring to the concept that we term "paternalism".

A.1. **Proof of Proposition 1.** Fix a finite N. From the point of view of a party A supporter, the number of supporters of party A other than himself, as well as the number of votes others cast for party A and B, are random variables. Given N, denote these random variables as $\tilde{N}_A^N$, $\tilde{v}_A^N$ and $\tilde{v}_B^N$, respectively. The dependence on N will be suppressed when no confusion may arise. For generic realizations v_A and v_B of $\tilde{v}_A$ and $\tilde{v}_B$, denote the "marginal mandate benefit from voting" for a party A supporter as

$$u_A(v_A, v_B) = G(m_A(v_A + 1, v_B)) - G(m_A(v_A, v_B)),$$

where m_A is defined in Equation (1).

Given a turnout profile (a, b), the expected gross benefit to a party A supporter from voting (for A) over abstaining is

$$U_A(a,b) = \sum_{N_A}^{N} \Bigg(\Pr\left[\tilde{N}_A = N_A\right](1 + \gamma_A N + \hat{\gamma}_A N_A) \times \sum_{v_A=0}^{N_A} \sum_{v_B=0}^{N-N_A} \Pr\left[\tilde{v}_A = v_A \mid N_A\right] \Pr\left[\tilde{v}_B = v_B \mid N_A\right] u_A(v_A, v_B) \Bigg), \quad (5)$$

where

$$\begin{aligned} \Pr\left[\tilde{N}_A = N_A\right] &= \binom{N}{N_A} \alpha^{N_A} (1-\alpha)^{N-N_A} \\ \Pr\left[\tilde{v}_A = v_A \mid N_A\right] &= \binom{N_A}{v_A} a^{v_A} (1-a)^{N_A - v_A} \\ \Pr\left[\tilde{v}_B = v_B \mid N_A\right] &= \binom{N-N_A}{v_B} b^{v_B} (1-b)^{N-N_A-v_B}. \end{aligned}$$

The expected gross benefit to a party B supporter from voting over abstaining, $U_B(a, b)$, can be similarly defined.

As argued in Section 3, any best response of citizen i is monotone in the voting cost. In equilibrium, the turnout of party P is the probability that its supporter has a voting cost below $U_P(a, b)$. Hence a turnout profile (a, b) can be supported as a symmetric Bayesian Nash equilibrium if and only if

$$\begin{aligned} a &= F\left(U_A(a,b)\right) \\ b &= F\left(U_B(a,b)\right). \end{aligned} \quad (6)$$

Define a function $\varphi : [0,1]^2 \to [0,1]^2$ by

$$\varphi(a,b) = \left(F(U_A(a,b)), F(U_B(a,b))\right).$$

Since U_A, U_B are jointly continuous in (a, b), and F is a continuous function, φ is a continuous function mapping from a compact Euclidean space into itself. By the Brouwer Fixed-Point Theorem, φ has a fixed-point. By construction, this fixed-point satisfies Equations (6) and is therefore a symmetric Bayesian Nash equilibrium.

A.2. **Proof of Theorem 1.** Let $\{(a_N, b_N)\}_N$ be a sequence of turnout profiles converging to (a^*, b^*). We establish the proposition with four lemmas: First we show that it is not possible that $a_N = b_N = 0$ for N sufficiently large. Next we establish that the expected sum of votes cast is infinite in the limit. We then prove that the limiting turnout of at least one party has to be strictly positive. Finally we demonstrate that if the turnout of one party is strictly positive, so must be the other.

Throughout this proof, we will always be considering the expected gross benefit to a party A supporter from voting (for A) over abstaining (Equation (5)). For notational simplicity, therefore, we will drop the party subscript on γ_A, $\hat{\gamma}_A$ and u_A. We will denote the expression in Equation (5) as U^N, with the superscript indicating the number of citizens.

Lemma A.1. *There is no $M > 0$ such that $a_N = b_N = 0$ for all $N > M$.*

Proof. Suppose not. Then at some $N > M$,

$$\begin{aligned} U^N(a_N, b_N) &= \sum_{N_A=0}^{N} \Pr\left[\tilde{N}_A = N_A\right] (1 + \gamma N + \hat{\gamma} N_A)\, u(0,0) \\ &= \sum_{N_A=0}^{N} \Pr\left[\tilde{N}_A = N_A\right] (1 + \gamma N + \hat{\gamma} N_A) \left[G(1) - G(\frac{1}{2})\right], \end{aligned}$$

which goes to infinity as $N \to \infty$. Voting is therefore a profitable deviation for a party A supporter with a finite c at sufficiently large N'. □

For the other results, we introduce some notations. For each N, define the random variable

$$\tilde{\alpha}^N = \frac{\tilde{N}_A^N}{N}.$$

And given $\tilde{\alpha}^N \in (0,1)$, define

$$\begin{aligned} \tilde{a}^N &= \frac{\tilde{v}_A^N}{\tilde{\alpha}^N N} \\ \tilde{b}^N &= \frac{\tilde{v}_B^N}{(1-\tilde{\alpha}^N) N}. \end{aligned}$$

The following fact will be used in many proofs of this paper. It is an application of the Hoeffding's Inequality (Hoeffding, 1963).

Fact 1. For any $\varepsilon > 0$,

$$(1) \qquad \Pr\left[\left|\tilde{\alpha}^N - \alpha\right| \geq \varepsilon\right] \leq 2e^{-2\varepsilon^2 N} \quad \text{for all } N;$$

and for all $\hat{\alpha} \in (0,1)$ and all N sufficiently large,

$$(2) \qquad \Pr\left[\left|\tilde{a}^N - a^*\right| \geq \varepsilon \mid \tilde{\alpha}^N = \hat{\alpha}\right] \leq 2e^{-2\hat{\alpha}^2(\varepsilon - |a_N - a^*|)^2 N}$$

$$(3) \qquad \Pr\left[\left|\tilde{b}^N - b^*\right| \geq \varepsilon \mid \tilde{\alpha}^N = \hat{\alpha}\right] \leq 2e^{-2(1-\hat{\alpha})^2(\varepsilon - |b_N - b^*|)^2 N}.$$

Proof. (1): We can write

$$\Pr\left[\left|\tilde{\alpha}^N - \alpha\right| \geq \varepsilon\right] = \Pr\left[\left|\tilde{N}_A^N - \alpha N\right| \geq \varepsilon N\right].$$

Since $\tilde{N}_A^N$ is the sum of N independent Bernoulli random variables with mean α, Hoeffding's Inequality requires

$$\begin{aligned}\Pr\left[\left|\tilde{N}_A^N - \alpha N\right| \geq \varepsilon N\right] &\leq 2e^{-\frac{2\varepsilon^2 N^2}{N}} \\ &= 2e^{-2\varepsilon^2 N}.\end{aligned}$$

(2): Notice that

$$\begin{aligned}\Pr\left[\left|\tilde{a}^N - a^*\right| \geq \varepsilon \mid \tilde{\alpha}^N = \hat{\alpha}\right] &\leq \Pr\left[\left|\tilde{a}^N - a_N\right| + |a_N - a^*| \geq \varepsilon \mid \tilde{\alpha}^N = \hat{\alpha}\right] \\ &= \Pr\left[\left|\tilde{v}_A^N - a_N\hat{\alpha}N\right| \geq (\varepsilon - |a_N - a^*|)\hat{\alpha}N \mid \tilde{\alpha}^N = \hat{\alpha}\right].\end{aligned}$$

For N sufficiently large, $|a_N - a^*| < \varepsilon$, so $\varepsilon - |a_N - a^*|$ would be a positive constant.

Conditional on $\tilde{\alpha}^N = \hat{\alpha}$, $\tilde{v}_A^N$ is the sum of N independent Bernoulli random variables with mean $a_N\hat{\alpha}N$. By Hoeffding's Inequality,

$$\begin{aligned}\Pr\left[\left|\tilde{v}_A^N - a_N\hat{\alpha}N\right| \geq (\varepsilon - |a_N - a^*|)\hat{\alpha}N \mid \tilde{\alpha}^N = \hat{\alpha}\right] &\leq 2e^{-\frac{2\hat{\alpha}^2(\varepsilon - |a_N - a^*|)^2 N^2}{N}} \\ &= 2e^{-2(\hat{\alpha}^2(\varepsilon - |a_N - a^*|)^2 N}.\end{aligned}$$

The proof of (3) is similar and is therefore omitted. □

An immediate consequence of Fact 1 is that the limiting expected gross benefit to voting is governed by the values of $\tilde{\alpha}$, $\tilde{a}$ and $\tilde{b}$ around their means.

Fact 2. Given an $\varepsilon \in (0, \alpha)$, write $I_\alpha^\varepsilon = [\alpha - \varepsilon, \alpha + \varepsilon]$. Denote $\overline{\gamma}^N(\tilde{\alpha}^N) = (1 + (\gamma + \hat{\gamma}\tilde{\alpha}^N)N)$.[16] Let $\varepsilon', \varepsilon'' > 0$.

$$\begin{aligned}&\lim_N U^N(a_N, b_N) \\ &\quad = \lim_N \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \qquad (7) \\ &\quad = \lim_N \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{\alpha}^N, |\tilde{a}^N - a^*| < \varepsilon', |\tilde{b}^N - b^*| < \varepsilon''\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right]. \qquad (8)\end{aligned}$$

Proof. We can write

$$\begin{aligned}U^N(a_N, b_N) = \Pr\left[\tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\ + \Pr\left[\tilde{\alpha}^N \notin I_\alpha^\varepsilon\right] \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{\alpha}^N \notin I_\alpha^\varepsilon\right].\end{aligned}$$

[16] Strictly speaking, $\overline{\gamma}$ should be subscripted by the party. Again we drop the subscript since we will only be concerned about that of party A.

Using (1) of Fact 1,

$$\Pr\left[\tilde{\alpha}^N \notin I_\alpha^\varepsilon\right] \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N \notin I_\alpha^\varepsilon\right] \leq 2e^{-2\varepsilon^2 N}\left(1+(\gamma+\hat{\gamma})N\right)\mathrm{E}\left[u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N \notin I_\alpha^\varepsilon\right].$$

Since u is bounded, the right hand side of this inequality goes to 0 as $N \to \infty$. Fact 1 also implies $\Pr\left[\tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \to 1$. Taking limits on $U^N(a_N, b_N)$ gives Equation (7).

Let E_1 be the event that $\left|\tilde{a}^N - a^*\right| < \varepsilon'$ and $\left|\tilde{b}^N - b^*\right| < \varepsilon''$, and E_2 be its complement (i.e., either $\tilde{a}^N$ is more than ε' away from a^* or $\tilde{b}^N$ is more than ε'' away from b^*). Given $\tilde{\alpha}^N \in (0,1)$,

$$\begin{aligned}\mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N\right] &= \Pr\left[E_1 \mid \tilde{\alpha}^N\right]\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N, E_1\right] \\ &\quad + \Pr\left[E_2 \mid \tilde{\alpha}^N\right]\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N, E_2\right]. \quad (9)\end{aligned}$$

Using (2) and (3) of Fact 1,

$$\lim_N \Pr\left[E_2 \mid \tilde{\alpha}^N\right]\overline{\gamma}^N(\tilde{\alpha}^N) = 0.$$

Moreover, $\Pr\left[E_1 \mid \tilde{\alpha}^N\right] \to 1$. Substituting the limit of the expressions in Equation (9) into Equation (7) we obtain Equation (8). □

We can now proceed to prove the other three lemmas for the proof of Theorem 1. Lemma A.1 then ensures that there is at least one party whose turnout is not converging to zero at a rate faster than that of the other party. The proofs of the next two lemmas illustrate that had the the turnout of this party been going to zero (which means the other party's limiting turnout must also be zero), the expected gross benefit of voting to the other party would have grown without bounds, prompting voters to vote.

Lemma A.2. *If* $\lim_N(b_N/a_N) \neq 0$,[17] $\lim_N b_N N \to \infty$.

Proof. Suppose by contradiction that $b_N N \to r_B < \infty$. Since $\lim(b_N/a_N) \neq 0$, $a_N N \to r_A < \infty$ as well. Hence, conditional on $\tilde{\alpha}^N = \hat{\alpha}$, the distributions of $\tilde{v}_A^N$ and $\tilde{v}_B^N$ converge to Poisson distributions with means $\hat{\alpha}r_A$ and $(1-\hat{\alpha})r_B$ respectively. Using the probability mass function of the Poisson distribution,

$$\begin{aligned}\lim_N \Pr\left[\tilde{v}_A = 0 \mid \tilde{\alpha}^N = \hat{\alpha}\right] &= e^{-\hat{\alpha}r_A} \\ \lim_N \Pr\left[\tilde{v}_B = 0 \mid \tilde{\alpha}^N = \hat{\alpha}\right] &= e^{-(1-\hat{\alpha})r_B}.\end{aligned}$$

Since u is positive,

$$\mathrm{E}\left[u(\tilde{v}_A,\tilde{v}_B) \mid \tilde{\alpha}^N = \hat{\alpha}\right] \geq \Pr\left[\tilde{v}_A = 0 \mid \tilde{\alpha}^N = \hat{\alpha}\right]\Pr\left[\tilde{v}_B = 0 \mid \tilde{\alpha}^N = \hat{\alpha}\right]u(0,0) \neq 0.$$

[17] As a convention, if $a_N = 0$ for all N sufficiently large, define $\lim_N(b_N/a_N) = \infty \neq 0$. The same remark applies to Lemma A.3.

The right hand side of this inequality converges to $e^{-(\hat{\alpha}r_A+(1-\hat{\alpha})r_B)}u(0,0)$. Using Equation (7),

$$\begin{aligned}\lim_N U^N &\geq \lim_N \mathrm{E}\left[(1+(\gamma+\hat{\gamma}\tilde{\alpha})N)\, e^{-(\tilde{\alpha}^N r_A+(1-\tilde{\alpha}^N)r_B)}u(0,0) \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right]\\ &= \infty.\end{aligned}$$

Thus voting is a profitable deviation for a party A supporter with a finite cost c at sufficiently large N. □

Lemma A.3. *If* $\lim_N(b_N/a_N) \neq 0$*, then* $b^* = \lim_N b_N > 0$.

Proof. Suppose by contradiction that $b_N \to 0$. Since $\lim(b_N/a_N) \neq 0$, $a_N \to 0$ as well. Fix $\varepsilon > 0$. Since u is always positive,

$$\begin{aligned}\mathrm{E}\left[u(\tilde{v}_A^N, \tilde{v}_B^N) \mid \tilde{\alpha}^N\right] \geq \Pr\left[\tilde{a}^N \leq (1+\varepsilon)a_N \mid \tilde{\alpha}^N\right] \Pr\left[|\tilde{b}^N - b_N| \leq \varepsilon b_N \mid \tilde{\alpha}^N\right]\\ \times \mathrm{E}\left[u(\tilde{v}_A^N, \tilde{v}_B^N) \mid \tilde{a}^N \leq (1+\varepsilon)a_N, |\tilde{b}^N - b_N| \leq \varepsilon b_N, \tilde{\alpha}\right] \quad (10)\end{aligned}$$

We prove that the conditional probability on the right hand side is strictly positive at the limit. First note that if $a_N = 0$, then $\tilde{a}^N = 0 \leq (1+\varepsilon)a_N$ with probability 1. Otherwise, by Markov's Inequality,

$$\Pr\left[\tilde{a}^N > (1+\varepsilon)a_N \mid \tilde{\alpha}^N\right] \leq \frac{a_N}{(1+\varepsilon)a_N} = \frac{1}{1+\varepsilon}.$$

Hence $\Pr\left[\tilde{a}^N \leq (1+\varepsilon)a_N \mid \tilde{\alpha}^N\right] \geq \varepsilon/1-\varepsilon > 0$ (we will not take ε to zero in this proof).

Meanwhile, by Chebychev's Inequality,

$$\begin{aligned}\Pr\left[|\tilde{b}^N - b_N| > \varepsilon b_N \mid \tilde{\alpha}^N\right] &\leq \frac{\mathrm{E}\left[(\tilde{b}^N - b_N)^2\right]}{(\varepsilon b_N)^2}\\ &= \frac{b_N(1-b_N)}{(1-\tilde{\alpha}^N)N}\frac{1}{(\varepsilon b_N)^2}\\ &= \frac{1-b_N}{\varepsilon^2(1-\tilde{\alpha})b_N N}.\end{aligned}$$

By Lemma A.2, $b_N N \to \infty$. Hence $\Pr\left[|\tilde{b}^N - b_N| \leq \varepsilon b_N \mid \tilde{\alpha}^N\right] \to 1$. For notational simplicity, we will refer to the event that $\tilde{a}^N \leq (1+\varepsilon)a_N$ and $|\tilde{b}^N - b_N| \leq \varepsilon b_N$ as E_1.

Next we consider the magnitude of the conditional expectation in Equation (10). For any (v_A, v_B) such that $v_A + v_B > 0$,

$$\begin{aligned}u(v_A, v_B) &= G\left(\frac{v_A+1}{v_A+v_B+1}\right) - G\left(\frac{v_A}{v_A+v_B}\right)\\ &= G'\left(\frac{v_A}{v_A+v_B}\right)\Delta(v_A, v_B) + o\left(\Delta(v_A, v_B)\right), \quad (11)\end{aligned}$$

where

$$\Delta(v_A, v_B) = \frac{v_A+1}{v_A+v_B+1} - \frac{v_A}{v_A+v_B} = \frac{v_B}{(v_A+v_B)^2+(v_A+v_B)}. \quad (12)$$

If $\tilde{b}^N \geq (1-\varepsilon)b_N$, then $\tilde{v}_B^N \geq (1-\varepsilon)(1-\tilde{\alpha})b_N N$. By Lemma A.2, $\tilde{v}_B^N > 0$ for large N. The above Taylor approximation is applicable. Given $\tilde{\alpha}$,

$$\begin{aligned}\Delta(\tilde{v}_A^N, \tilde{v}_B^N) &= \frac{\tilde{b}^N(1-\tilde{\alpha})N}{(\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))^2 N^2 + (\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))N} \\ &= \frac{(1-\tilde{\alpha})}{(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha}))^2 \tilde{b}^N N + (\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha}))}.\end{aligned}$$

Conditional on E_1,

$$\frac{\tilde{a}^N}{\tilde{b}^N} \leq \frac{(1+\varepsilon)a_N}{(1-\varepsilon)b_N}.$$

Since $\lim(b_N/a_N) \neq 0$, the limiting $\tilde{a}^N/\tilde{b}^N$ is finite. Also, for each N, define the random variable $\tilde{r}_N = \tilde{b}^N/b_N$. (This is well-defined since, by Lemma A.1, $b_N \neq 0$.) For the range of $\tilde{b}^N$ concerned, $\tilde{r}$ is distributed on the bounded interval $[1-\varepsilon, 1+\varepsilon]$. We can now write

$$\Delta(\tilde{v}_A^N, \tilde{v}_B^N) = \frac{(1-\tilde{\alpha})}{\left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)^2 \tilde{r}_N b_N N + \left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)}.$$

Since $b_N N \to \infty$ (Lemma A.2), $\Delta(\tilde{v}_A^N, \tilde{v}_B^N)$ goes to zero conditional on E_1. This allows us to ignore the smaller order term in Equation (11).

More importantly, given $\tilde{\alpha}$,

$$\begin{aligned}\overline{\gamma}^N(\tilde{\alpha})\Delta(\tilde{v}_A^N, \tilde{v}_B^N) &= \frac{(1-\tilde{\alpha})(1+(\gamma+\hat{\gamma}\tilde{\alpha})N)}{\left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)^2 \tilde{r}_N b_N N + \left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)} \\ &= \frac{(1-\tilde{\alpha})\left(\frac{1}{N} + (\gamma+\hat{\gamma}\tilde{\alpha})\right)}{\left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)^2 \tilde{r}_N b_N + \left(\frac{\tilde{a}^N}{\tilde{b}^N}\tilde{\alpha} + (1-\tilde{\alpha})\right)\frac{1}{N}},\end{aligned}$$

which goes to infinity since $b_N \to 0$.

Using Equations (7) and (10), then

$$\begin{aligned}\lim_N U^N &\geq \lim_N \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\Pr\left[E_1 \mid \tilde{\alpha}\right]\mathrm{E}\left[G'\left(\frac{\tilde{v}_A^N}{\tilde{v}_A^N + \tilde{v}_B^N}\right)\Delta(\tilde{v}_A^N, \tilde{v}_B^N) \mid E_1, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\ &= \infty\end{aligned}$$

as G' is bounded away from zero. Thus voting is a profitable deviation for a party A supporter with a finite cost c at sufficiently large N. □

By now we have shown that the turnout of at least one party has to be strictly positive. We complete the proof of Theorem 1 by arguing that if the turnout of one party is strictly positive, so must the other party's.

Lemma A.4. *If $a_N \to a^* > 0$, then $b_N \to b^* > 0$.*

Proof. Suppose by contradiction that $a^* > 0$ and $b^* = 0$. Pick an $\varepsilon < \alpha, a^*$ and an $\varepsilon' > 0$. Denote E_1 as the event that (1) $\tilde{\alpha}^N \in I_\alpha^\varepsilon$; (2) $\left|\tilde{a}^N - a^*\right| < \varepsilon$; and (3) $\tilde{b}^N < \varepsilon'$.

Given E_1, $\tilde{a}^N \geq a^* - \varepsilon > 0$. Hence $\tilde{v}_A^N > 0$ and the Taylor approximation in Equation (11) is applicable. Given $\tilde{\alpha}$ and using Equation (12)

$$\Delta(\tilde{v}_A^N, \tilde{v}_B^N) = \frac{\tilde{b}^N(1-\tilde{\alpha})}{(\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))^2 N + (\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))}.$$

Given E_1, $\tilde{a}^N > a^* - \varepsilon > 0$ and $\tilde{\alpha} > \alpha - \varepsilon > 0$. Thus $\Delta(\tilde{v}_A^N, \tilde{v}_B^N)$ goes to 0 as $N \to \infty$. Again we can ignore the smaller order term in the Taylor approximation.

Also, given E_1,

$$\begin{aligned}
\overline{\gamma}^N(\tilde{\alpha})\Delta(\tilde{v}_A^N, \tilde{v}_B^N) &= \frac{\tilde{b}^N(1-\tilde{\alpha})(1+(\gamma+\hat{\gamma}\tilde{\alpha})N)}{(\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))^2 N + (\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))} \\
&\leq \frac{(b^*+\varepsilon')(1-\alpha+\varepsilon)(1+(\gamma+\hat{\gamma}(\alpha+\varepsilon))N)}{(a^*-\varepsilon)^2(\alpha-\varepsilon)^2 N + (a^*-\varepsilon)(\alpha-\varepsilon)} \\
&\to \frac{\varepsilon'(1-\alpha+\varepsilon)(\gamma+\hat{\gamma}(\alpha+\varepsilon))}{(a^*-\varepsilon)^2(\alpha-\varepsilon)^2}.
\end{aligned}$$

Using Equation (8),

$$\begin{aligned}
\lim_N U^N &= \lim_N \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{\alpha}^N, |\tilde{a}^N - a^*| < \varepsilon, |\tilde{b}^N - b^*| < \varepsilon'\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\
&= \lim_N \mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N)\mathrm{E}\left[G'\left(\frac{\tilde{v}_A^N}{\tilde{v}_A^N + \tilde{v}_B^N}\right)\Delta(\tilde{v}_A^N, \tilde{v}_B^N) \mid E_1, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\
&\leq \lim_N \frac{\varepsilon'(1-\alpha+\varepsilon)(\gamma+\hat{\gamma}(\alpha+\varepsilon))}{(a^*-\varepsilon)^2(\alpha-\varepsilon)^2}\mathrm{E}\left[G'\left(\frac{\tilde{v}_A^N}{\tilde{v}_A^N + \tilde{v}_B^N}\right) \mid E_1\right].
\end{aligned}$$

Since G' is bounded away from infinity, its conditional expectation is finite. Call the last expression $c(\varepsilon')$. It is continuous and strictly increasing in ε' and is zero when $\varepsilon' = 0$. Thus for ε' sufficiently small, $F(c(\varepsilon')) < a^*$. But this means abstaining is a profitable deviation for a party A supporter whose voting cost is between $c(\varepsilon')$ and $F^{-1}(a^*)$ for sufficiently large N. □

A.3. **Proof of Proposition 3.** Let $\{(a_N, b_N)\}_N \subset [0,1)^2$ be a sequence of turnout profiles converging to (a^*, b^*). By Theorem 1 we can choose this sequence such that $a_N, b_N > 0$ for all N. This implies

$$\begin{aligned}
U_A^N(a_N, b_N) &= F_A^{-1}(a^N) \\
U_B^N(a_N, b_N) &= F_A^{-1}(b^N)
\end{aligned} \quad \text{for all } N.$$

Our goal is to determine the limiting expressions on the left hand side as $N \to \infty$. We will do so only for U_A^N in this proof (U_B^N is analogous). Thus, as in the proof of Theorem 1, we will drop all party subscripts on U_A, γ_A, $\hat{\gamma}_A$ and u_A.

Choose $\varepsilon < \alpha, a^*, b^*$ Let $I_\alpha^\varepsilon = [\alpha - \varepsilon, \alpha + \varepsilon]$, $I_a^\varepsilon = [a^* - \varepsilon, a^* + \varepsilon]$ and $I_b^\varepsilon = [b^* - \varepsilon, b^* - \varepsilon]$. Given $\tilde{a} \in I_a^\varepsilon$, $\tilde{b} \in I_b^\varepsilon$ and $\tilde{\alpha} \in I_\alpha^\varepsilon$, define

$$h_N(\tilde{a}, \tilde{b}, \tilde{\alpha}) = \frac{\tilde{b}(1-\tilde{\alpha})}{\left(\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha}) + \frac{1}{N}\right)\left(\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha})\right)N}.$$

We can also write

$$\begin{aligned}
&\overline{\gamma}(\tilde{\alpha}N, N)u(\tilde{v}_A, \tilde{v}_B) \\
&= \left(\frac{1}{N} + \gamma + \hat{\gamma}\tilde{\alpha}\right) N \left[G\left(\frac{\tilde{a}\tilde{\alpha}N}{\tilde{a}\tilde{\alpha}N + \tilde{b}(1-\tilde{\alpha})N} + h_N(\tilde{a}, \tilde{b}, \tilde{\alpha})\right) - G\left(\frac{\tilde{a}\tilde{\alpha}N}{\tilde{a}\tilde{\alpha}N + \tilde{b}(1-\tilde{\alpha})N}\right)\right] \\
&= \left(\frac{1}{N} + \gamma + \hat{\gamma}\tilde{\alpha}\right) h_N(\tilde{a}, \tilde{b}, \tilde{\alpha}) N \left[\frac{G\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha}+\tilde{b}(1-\tilde{\alpha})} + h_N(\tilde{a}, \tilde{b}, \tilde{\alpha})\right) - G\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha}+\tilde{b}(1-\tilde{\alpha})}\right)}{h_N(\tilde{a}, \tilde{b}, \tilde{\alpha})}\right].
\end{aligned}$$

Call the last expression $\pi_N(\tilde{a}, \tilde{b}, \tilde{\alpha})$.

As N goes to infinity, $h_N \to 0$. Hence

$$\lim_N \frac{G\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha}+\tilde{b}(1-\tilde{\alpha})} + h_N(\tilde{a}, \tilde{b}, \tilde{\alpha})\right) - G\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha}+\tilde{b}(1-\tilde{\alpha})}\right)}{h_N(\tilde{a}, \tilde{b}, \tilde{\alpha})} = G'\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha})}\right).$$

Meanwhile,

$$\lim_N h_N(\tilde{a}, \tilde{b}, \tilde{\alpha}) N = \frac{\tilde{b}(1-\tilde{\alpha})}{(\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha}))^2},$$

which is bounded for $\tilde{\alpha} \in I_\alpha^\varepsilon$, $\tilde{a} \in I_a^\varepsilon$ and $\tilde{b} \in I_b^\varepsilon$. Therefore $\{\pi_N\}$ is a sequence of bounded functions converging pointwise to

$$\pi(\tilde{a}, \tilde{b}, \tilde{\alpha}) = (\gamma + \hat{\gamma}\tilde{\alpha}) \frac{\tilde{b}(1-\tilde{\alpha})}{(\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha}))^2} G'\left(\frac{\tilde{a}\tilde{\alpha}}{\tilde{a}\tilde{\alpha} + \tilde{b}(1-\tilde{\alpha})}\right).$$

This π function is jointly continuous in all its arguments.

Using Equation (8),

$$\begin{aligned}
\lim_N U^N(a_N, b_N) &= \lim_N \mathrm{E}\left[\mathrm{E}\left[\overline{\gamma}^N(\tilde{\alpha}^N) u(\tilde{v}_A^N, \tilde{v}_B^N) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\
&= \lim_N \mathrm{E}\left[\mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \tilde{\alpha}) \mid \tilde{a}^N \in I_a^\varepsilon \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right]. \qquad (13)
\end{aligned}$$

The following version of the continuous mapping theorem is useful for determining this limit:

Lemma A.5. *Let $\{X_n\}$ be random variables defined on a metric space S. Let $g : S \to S'$ (where S' is another metric space) be a continuous function. If $\{g_n\}$ is a sequence of functions converging pointwise to g, then $\lim_n \operatorname{plim} g_n(X_n) = g(\operatorname{plim} X)$.*

Proof. Fix an $\varepsilon > 0$ and take a $\delta > 0$. For each n, construct the set

$$B_\delta^n = \{x \in S : \exists y \in S,\ |x - y| < \delta \text{ s.t. } |g_n(y) - g(x)| > \varepsilon\}.$$

Pointwise convergence of g implies

$$\lim_n B_\delta^n = B_\delta = \{x \in S : \exists y \in S,\ |x - y| < \delta \text{ s.t. } |g(y) - g(x)| \geq \varepsilon\}.$$

By the continuity of g, B_δ becomes empty as $\delta \to 0$.

Let $X = \operatorname{plim} X_n$. If $|g_n(X_n) - g(X)| \geq \varepsilon$, either $|X_n - X| \geq \delta$ or $X_n \in B_\delta^n$. Hence,

$$\Pr\left[|g_n(X_n) - g(X)| \geq \varepsilon\right] \leq \Pr\left[|X_n - X| \geq \delta\right] + \Pr\left[X_n \in B_\delta^n\right].$$

By the definition of plim, $\lim_n \Pr\left[|X_n - X| \geq \delta\right] = 0$ for all $\delta > 0$. Therefore

$$\lim_{\delta \to 0} \lim_n \Pr\left[|g_n(X_n) - g(X)| \geq \varepsilon\right] \leq \lim_{\delta \to 0} \Pr\left[X_n \in B_\delta\right]$$
$$\lim_n \Pr\left[|g_n(X_n) - g(X)| \geq \varepsilon\right] \leq 0. \qquad \square$$

By Fact 1, $\operatorname{plim} \tilde{\alpha}^N = \alpha$; and conditional on $\tilde{\alpha}$, $\operatorname{plim} \tilde{a}^N = a^*$ and $\operatorname{plim} \tilde{b}^N = b^*$. Now $\mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \tilde{\alpha}) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}\right]$ is a continuous, bounded function for all $\tilde{\alpha} \in I_\alpha^\varepsilon$. Using Lemma A.5,

$$\operatorname{plim} \mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \tilde{\alpha}) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}\right] = \lim_N \mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \alpha) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon\right].$$

Moreover, as each π_N is bounded, the right hand side of the above equation is a finite constant. Using Equation (13),

$$\begin{aligned}\lim_N U^N(a_N, b_N) &= \lim_N \mathrm{E}\left[\mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \tilde{\alpha}) \mid \tilde{a}^N \in I_b^\varepsilon, \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right] \\ &= \lim_N \mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \alpha) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon\right]. \qquad (14)\end{aligned}$$

Using Lemma A.5 once more,

$$\operatorname{plim}_N \pi_N(\tilde{a}^N, \tilde{b}^N, \alpha) = \pi(a^*, b^*, \alpha).$$

The right hand side of this equation is a finite constant. Thus,

$$\lim_N U^N(a_N, b_N) = \lim_N \mathrm{E}\left[\pi_N(\tilde{a}^N, \tilde{b}^N, \alpha) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon\right] = \pi(a^*, b^*, \alpha). \qquad (15)$$

Substituting this into Equation (14) and adopting the shorthands

$$T^* = \alpha a^* + (1 - \alpha) b^*$$
$$m_A^* = \frac{\alpha a^*}{T^*},$$

we can characterize the limiting (a^*, b^*) by

$$(\gamma_A + \hat{\gamma}_A \alpha) \frac{1 - m_A^*}{T^*} G'(m_A^*) = F^{-1}(a^*)$$
$$(\gamma_B + \hat{\gamma}_B (1 - \alpha)) \frac{m_A^*}{T^*} G'(1 - m_A^*) = F^{-1}(b^*)$$

as in Proposition 3.

A.4. **Proof of Proposition 2.** First suppose C is unbounded from above. Given N, $U_A^N(a_N, b_N)$ and $U_B^N(a_N, b_N)$ are finite for any $(a_N, b_N) \in [0, 1]^2$. Thus $a_N = F(U_A^N(a_N, b_N))$ and $b_N = F(U_B^N(a_N, b_N))$ are both strictly less than 1.

Now suppose C is bounded above by $\bar{c}$. We will consider party A. Suppose by contradiction that $a_N = 1$ for all N large and let $\{b_N\}$ converges to b^*. Notice that the

derivation of the limiting expression for $U_A^N(a_N, b_N)$ in Equation (15) is valid for all $(a_N, b_N) \in (0,1]^2$. Hence,

$$\lim_N U_A^N(1, b_N) = (\gamma_A + \hat{\gamma}_A \alpha) \frac{b^*(1-\alpha)}{(\alpha + b^*(1-\alpha))^2} G'\left(\frac{\alpha}{\alpha + b^*(1-\alpha)}\right).$$

For γ_A and $\hat{\gamma}_A$ sufficiently small, the above limit is strictly smaller than $\bar{c}$. Thus for N sufficiently large, party A's supporters with voting cost in an open interval below $\bar{c}$ would prefer abstaining, contradicting $a_N = 1$. The case for party B is analogous.

Appendix B. Proofs of Equilibrium Analysis Results

B.1. **Underdog Effect (Proposition 4).** Suppose by contradiction that $\alpha \geq 1/2$ and $a^* > b^*$. Dividing Equation (3) by (4) and using the symmetry of G we get

$$\frac{[\gamma + \hat{\gamma}\alpha](1-\alpha)b^*}{[\gamma + \hat{\gamma}(1-\alpha)]\alpha a^*} = \frac{F^{-1}(a^*)}{F^{-1}(b^*)}$$
$$\frac{\gamma(1-\alpha)b^* + \hat{\gamma}\alpha(1-\alpha)b^*}{\gamma\alpha a^* + \hat{\gamma}\alpha(1-\alpha)a^*} = \frac{F^{-1}(a^*)}{F^{-1}(b^*)}.$$

Since $\alpha \geq 1/2$ and $a^* > b^*$, the left hand side is strictly less than 1. Meanwhile, the right hand side is strictly greater than 1 since F^{-1} is a strictly increasing function. Contradiction. Moreover, if $\gamma > 0$, $\alpha > 1/2$ and $a^* = b^*$, then the left hand side is still strictly less than 1 while the right hand side is exactly 1, which is also a contradiction.

Now if $a^* \leq b^*$, we need

$$[\gamma + \hat{\gamma}\alpha](1 - m_A^*) \leq [\gamma + \hat{\gamma}(1-\alpha)]m_A^*$$

with a strictly inequality if $a^* < b^*$. Since $(\gamma + \hat{\gamma}\alpha) \geq (\gamma + \hat{\gamma}(1-\alpha))$, we must have $1 - m_A^* \leq m_A^*$, with a strictly inequality if $a^* < b^*$. Therefore $m_A^* \geq 1/2$, with a strict inequality when $\gamma > 0$ and $\alpha > 1/2$.

B.2. **Competition Effect (Proposition 5).** This proof is mainly about elasticities. Thus for any variable x to be determined, write

$$\varepsilon_x = \frac{dx}{d\alpha}\frac{\alpha}{x}$$

as the elasticity of x with respect to α (evaluated at the original value of α). For example, ε_a would be the elasticity of a^* with respect to α. Since we will only consider party A's mandate, m_A^*, we will write ε_m as the elasticity of m_A^* with respect to α.

For each party P, let $c_P^* = F^{-1}(p^*)$ be the limiting cut-off voting cost below which a P-supporter would vote. Write also:

$$\sigma_G = \frac{-G''(m_A^*)m_A^*}{G'(m_A^*)} = \sigma_G(m_A^*),$$
$$\sigma_F^A = \frac{F(c_A^*)}{f(c_A^*)c_A^*} \qquad \text{and} \qquad \sigma_F^B = \frac{F(c_B^*)}{f(c_B^*)c_B^*}.$$

By Proposition 4, $b^* \geq a^*$. Hence $c_B^* \geq c_A^*$. Given the assumption that $f(c)c/F(c)$ is decreasing in c, $\sigma_F^B \geq \sigma_F^A$.

Differentiate Equations (3) and (4) with respect to α. Completing elasticities we obtain:

$$\frac{\alpha\hat{\gamma}}{\gamma+\hat{\gamma}\alpha} - \varepsilon_T - \left(\frac{m_A^*}{1-m_A^*} + \sigma_G\right)\varepsilon_m = \sigma_F^A \varepsilon_a \tag{16}$$

$$\frac{-\alpha\hat{\gamma}}{\gamma+\hat{\gamma}\alpha} - \varepsilon_T + (1-\sigma_G)\varepsilon_m = \sigma_F^B \varepsilon_b. \tag{17}$$

Using the expressions for T^* and m_A^*,

$$\begin{aligned}
\varepsilon_T &= m_A^*(1+\varepsilon_a) + (1-m_A^*)\left(-\frac{\alpha}{1-\alpha} + \varepsilon_b\right) \\
\varepsilon_m &= 1 + \varepsilon_a - \varepsilon_T \\
&= (1-m_A^*)\left[(1+\varepsilon_a) - \left(-\frac{\alpha}{1-\alpha} + \varepsilon_b\right)\right].
\end{aligned}$$

Adopt the following shorthands:

$$x = \frac{d\alpha a^*}{d\alpha}\frac{\alpha}{\alpha a^*} = 1+\varepsilon_a; \qquad y = \frac{d(1-\alpha)b^*}{d\alpha}\frac{\alpha}{(1-\alpha)b^*} = -\frac{\alpha}{1-\alpha} + \varepsilon_b;$$

$$k_A = \frac{\alpha\hat{\gamma}}{\gamma+\alpha\hat{\gamma}}; \qquad k_B = \frac{(1-\alpha)\hat{\gamma}}{\gamma+(1-\alpha)\hat{\gamma}};$$

$$z = 2m_A^* - 1 + \sigma_G(1-m_A^*).$$

Notice that since $1/2 \leq m_A^* < 1$ and $\sigma_G \geq 0$, $z \geq 0$.

Using the expressions above, we can rewrite Equations (16) and (17) as the following system of linear equations:

$$\begin{pmatrix} -z-(1+\sigma_F^A) & z \\ -z & z-(1+\sigma_F^B) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -(\sigma_F^A + k_A) \\ \frac{\alpha}{1-\alpha}(\sigma_F^B + k_B) \end{pmatrix}.$$

The determinant of this system is

$$\begin{aligned}
D &= \left[-z-(1+\sigma_F^A)\right]\left[z-(1+\sigma_F^B)\right] + z^2 \\
&= z(\sigma_F^B - \sigma_F^A) + (1+\sigma_F^A)(1+\sigma_F^B) \\
&> 0.
\end{aligned}$$

Using Cramer's rule:

$$\begin{aligned}
x &= \frac{1}{D}\left[-(\sigma_F^A + k_A)(z-(1+\sigma_F^B)) - \frac{\alpha}{1-\alpha}(\sigma_F^B + k_B)z\right] \\
&= \frac{1}{D}\left[-z\left(\sigma_F^A + k_A + \frac{\alpha}{1-\alpha}(\sigma_F^B + k_B)\right) + (k_A + \sigma_F^A)(1+\sigma_F^B)\right] \\
&\leq \frac{(1+\sigma_F^A)(1+\sigma_F^B)}{z(\sigma_F^B - \sigma_F^A) + (1+\sigma_F^A)(1+\sigma_F^B)} \\
&\leq 1.
\end{aligned}$$

Therefore $x = 1+\varepsilon_a \leq 1$, meaning that $\varepsilon_a \leq 0$. In other words, a^* falls when α increases. However, it is not possible to determine the sign of x without further assumption.

Similarly,

$$\begin{aligned} y &= \frac{1}{D}\left[-(z+(1+\sigma_F^A))\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B) - z(\sigma_F^A+k_A)\right] \\ &= \frac{1}{D}\left[-z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) - \frac{\alpha}{1-\alpha}(1+\sigma_F^A)(k_B+\sigma_F^B)\right] \\ &< 0. \end{aligned}$$

Hence $(1-\alpha)b^*$ falls as α increases. However, it is not possible to tell whether y is greater than or smaller than $-\alpha/(1-\alpha)$. The sign of ε_b is unknown.

Recall that (the relation $\overset{\text{sgn}}{=}$ means the two expressions are of the same sign):

$$\begin{aligned} \varepsilon_m &\overset{\text{sgn}}{=} x - y \\ &\overset{\text{sgn}}{=} -z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) + (k_A+\sigma_F^A)(1+\sigma_F^B) \\ &\quad + z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) + \frac{\alpha}{1-\alpha}(1+\sigma_F^A)(k_B+\sigma_F^B) \\ &> 0. \end{aligned}$$

Therefore m_A^* increases as α increases.

Finally we would like to consider the turnout.

$$\begin{aligned} \varepsilon_T &\overset{\text{sgn}}{=} m_A^*\left[-z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) + (k_A+\sigma_F^A)(1+\sigma_F^B)\right] \\ &\quad +(1-m_A^*)\left[z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) + \frac{\alpha}{1-\alpha}(1+\sigma_F^A)(k_B+\sigma_F^B)\right] \\ &\overset{\text{sgn}}{=} -z\left(\sigma_F^A+k_A+\frac{\alpha}{1-\alpha}(\sigma_F^B+k_B)\right) \\ &\quad + m_A^*(k_A+\sigma_F^A)(1+\sigma_F^B) - (1-m_A^*)\frac{\alpha}{1-\alpha}(1+\sigma_F^A)(k_B+\sigma_F^B). \end{aligned} \tag{18}$$

If $\gamma = 0$, $k_A = k_B = 1$. The last two terms of expression (18) becomes

$$(1+\sigma_F^A)(1+\sigma_F^B)\left[\frac{\alpha a^*}{T^*} - \frac{\alpha}{1-\alpha}\frac{(1-\alpha)b^*}{T^*}\right] \leq 0.$$

The first term of expression (18) is weakly negative. Thus if $\gamma = 0$, total turnout falls as α increases.

If $\sigma_G \geq k_A$, expression (18) can be rearranged as

$$\begin{aligned} &\left[(1-m_A^*)(1-\sigma_G)(\sigma_F^A+k_A) - m_A^*\frac{\alpha}{1-\alpha}\left(\sigma_F^B+k_B\right)\right] \\ &\qquad + \left[m_A^*\sigma_F^B(k_A+\sigma_F^A) - (1-m_A^*)\frac{\alpha}{1-\alpha}(k_B+\sigma_F^B)(\sigma_G+\sigma_F^A)\right]. \end{aligned} \tag{19}$$

Recall that $1 - m_A^* \leq m_A^*$, $1 - \sigma_G \leq \alpha/(1-\alpha)$ and $\sigma_F^A \leq \sigma_F^B$. Also

$$k_A = \frac{\alpha\hat{\gamma}}{\gamma + \alpha\hat{\gamma}} \leq \frac{\alpha\hat{\gamma}}{\gamma + (1-\alpha)\hat{\gamma}} = \frac{\alpha}{1-\alpha} k_B.$$

Therefore the first term in Equation (19) is negative. Meanwhile,

$$m_A^* = \frac{\alpha a^*}{T^*} \leq \frac{\alpha b^*}{T^*} = \frac{\alpha}{1-\alpha}(1 - m_A^*).$$

With the assumption $\sigma_G \geq k_A$, then, the second term is also negative. Total turnout falls as α increases.

Appendix C. Proofs of Results in Section 6

C.1. **Proof of Proposition 6.** As in Section A.2, we would only be concerned about the payoff to a party A member and would drop the party subscript on the preference parameters when no confusion may arise.

We will first show that if voters in one party are not paternalistic, the turnout for that party is zero. Formally,

Lemma C.1. *If $\gamma_A = \hat{\gamma}_A = 0$, then $a^* = 0$.*

Proof. Suppose by contradiction that $a^* > 0$. Pick an $\varepsilon < \alpha, a^*$. Using Equation (8), the limiting expected gross payoff of voting for party A (to a party A supporter) is

$$\lim_N U^N = \mathrm{E}\left[\mathrm{E}\left[u(\tilde{v}_A, \tilde{v}_B) \mid \tilde{a}^N \in I_a^\varepsilon, \tilde{b}^N \in I_b^\varepsilon, \tilde{\alpha}^N\right] \mid \tilde{\alpha}^N \in I_\alpha^\varepsilon\right],$$

Given this range of $\tilde{\alpha}^N$ and $\tilde{a}^N$, $\tilde{v}_A > 0$. Hence we can perform the Taylor approximation for u as in Equation (11). The order of U^N (in terms of N) is then governed by the order of

$$\Delta(\tilde{v}_A^N, \tilde{v}_B^N) = \frac{\tilde{b}^N(1-\tilde{\alpha})}{(\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))^2 N + (\tilde{a}^N\tilde{\alpha} + \tilde{b}^N(1-\tilde{\alpha}))}.$$

This expression goes to 0 as $N \to \infty$ for each $(\tilde{\alpha}, \tilde{a}^N) \in I_\alpha^\varepsilon \times I_a^\varepsilon$. Therefore abstaining is a profitable deviation for each party A supporter with a strictly positive voting cost below $F^{-1}(a^*)$ at sufficiently large N. □

The next lemma completes the proof.

Lemma C.2. *If $\gamma_B = \hat{\gamma}_B = 0$, then $a^* = 0$.*

Proof. If $\gamma_A = \hat{\gamma}_A = 0$ as well, the desired result follows from Lemma C.1.

If at least one of γ_A and $\hat{\gamma}_A$ is strictly positive, the payoff to a party A supporter is the same as Section 3. Lemma A.4 therefore applies (its proof relies only on the payoff to a party A supporter). By Lemma C.1, $b^* = 0$ when $\gamma_B = \hat{\gamma}_B = 0$. The contra-positive of Lemma A.4 then implies $a^* = 0$. □

C.2. **Proof of Proposition 7.** This proof consists of two parts. First we show that when the election is winner-take-all, at least one party has a zero limiting turnout. We then show that if the limiting turnout of one party is zero, so must the other party's.

Suppose for each party P,

$$G(m_P) = \begin{cases} 0 & \text{if } m_P < 1/2 \\ 1/2 & \text{if } m_P = 1/2 \\ 1 & \text{if } m_P > 1/2. \end{cases}$$

To a party P members, voting yields a marginal benefit (over abstaining) of $1/2$ if and only if his vote is pivotal. That is, when either $v_P = v_Q$ or $v_P + 1 = v_Q$, where Q is the other party. Write

$$\Pr\left[\text{piv}_A^N\right] = \Pr\left[\tilde{v}_A^N = \tilde{v}_B^N \text{ or } \tilde{v}_A^N + 1 = \tilde{v}_B^N\right]$$
$$\Pr\left[\text{piv}_B^N\right] = \Pr\left[\tilde{v}_B^N = \tilde{v}_A^N \text{ or } \tilde{v}_B^N + 1 = \tilde{v}_A^N\right]$$

as the probability that a vote for party A and B, respectively, is pivotal when there are N citizens. The expected gross benefits of voting over abstaining to members of the two parties are therefore

$$U_A^N(a_N, b_N) = \frac{1}{2}\mathrm{E}\left[\left(1 + \gamma_A N + \hat{\gamma}_A \tilde{N}_A\right)\Pr\left[\text{piv}_A^N\right]\right] \tag{20}$$
$$U_B^N(a_N, b_N) = \frac{1}{2}\mathrm{E}\left[\left(1 + \gamma_B N + \hat{\gamma}_B \tilde{N}_B\right)\Pr\left[\text{piv}_B^N\right]\right]. \tag{21}$$

Suppose by contradiction that there is a sequence of equilibria $\{(a_N, b_N)\}_N \to (a^*, b^*)$ where $a^*, b^* > 0$. Given N, the ex-ante probability that any citizen would vote is $T_N = \alpha a_N + (1-\alpha) b_N$. When N is large, the number of citizens who vote has a probability distribution that is approximately[18] Poisson with mean T_N. Conditional on voting, each citizen has a probability $m_A^N = \alpha a_N / T_N$ of voting for party A. Hence we can think of $\tilde{v}_A^N$ and $\tilde{v}_B^N$ being generated from a Poisson game with mean population $T_N N$ in which each player votes for A with probability m_A^N. Using the approximation for large Poisson games (see Myerson, 2000, Eq. (5.5)),

$$\Pr\left[\text{piv}_A^N\right] \approx \frac{e^{-2T_N N\left(\sqrt{m_A^N} - \sqrt{1-m_A^N}\right)^2}}{4\sqrt{\pi T_N N \sqrt{m_A^N(1-m_A^N)}}}\left(\frac{\sqrt{m_A^N} + \sqrt{1-m_A^N}}{\sqrt{m_A^N}}\right)$$
$$\Pr\left[\text{piv}_B^N\right] \approx \frac{e^{-2T_N N\left(\sqrt{m_A^N} - \sqrt{1-m_A^N}\right)^2}}{4\sqrt{\pi T_N N \sqrt{m_A^N(1-m_A^N)}}}\left(\frac{\sqrt{m_A^N} + \sqrt{1-m_A^N}}{\sqrt{1-m_A^N}}\right).$$

[18] In this proof, two functions of N are approximately equal if their ratio converges to 1 as N goes to infinity. This equivalent relation is denoted by the symbol $\approx$.

We can plug these equations into Equations (20) and (21):

$$U_A^N \approx \frac{1}{2}\mathrm{E}\left[(1+\gamma_A N+\hat{\gamma}_A\tilde{N}_A)\frac{e^{-2T_N N\left(\sqrt{m_A^N}-\sqrt{1-m_A^N}\right)^2}}{4\sqrt{\pi T_N N\sqrt{m_A^N(1-m_A^N)}}}\left(\frac{\sqrt{m_A^N}+\sqrt{1-m_A^N}}{\sqrt{m_A^N}}\right)\right]$$

$$U_B^N \approx \frac{1}{2}\mathrm{E}\left[(1+\gamma_B N+\hat{\gamma}_B\tilde{N}_B)\frac{e^{-2T_N N\left(\sqrt{m_A^N}-\sqrt{1-m_A^N}\right)^2}}{4\sqrt{\pi T_N N\sqrt{m_A^N(1-m_A^N)}}}\left(\frac{\sqrt{m_A^N}+\sqrt{1-m_A^N}}{\sqrt{1-m_A^N}}\right)\right].$$

Lemma C.3. *If $\alpha \neq 1/2$, then $\lim_N N(\sqrt{m_A^N}-\sqrt{1-m_A^N})^2 > 0$.*

Proof. Suppose $\lim_N N(\sqrt{m_A^N}-\sqrt{1-m_A^N})^2 = 0$. Then

$$\lim_N U_A^N = \lim_N \mathrm{E}\left[\frac{1+\gamma_A N+\hat{\gamma}_A\tilde{N}_A}{4\sqrt{\pi T_N N(1/2)}}\right] = \infty$$

$$\lim_N U_B^N = \lim_N \mathrm{E}\left[\frac{1+\gamma_B N+\hat{\gamma}_B\tilde{N}_B}{4\sqrt{\pi T_N N(1/2)}}\right] = \infty.$$

Thus $(a^*, b^*) = (1,1)$. But this means

$$\frac{1}{2} = m_A^* = \lim_N \frac{\alpha a_N}{\alpha a_N + (1-\alpha)b_N} = \frac{\alpha}{\alpha+(1-\alpha)} = \alpha. \qquad \square$$

If $\alpha \neq 1/2$, U_A^N is of the same order as

$$e^{-2T_N N(\sqrt{m_A^N}-\sqrt{1-m_A^N})^2}\sqrt{N} \to 0$$

as $N \to \infty$. But then a party A supporter with a strictly positive voting cost would find it profitable to deviate to abstaining for sufficiently large N.

By now we have shown that the turnout of at least one party is 0. Without loss assume $b^* = 0$. We would like to show that $a^* = 0$.

Suppose not. Choose an $\varepsilon < \alpha, a^*$ and pick

$$\varepsilon' < \frac{(\alpha-\varepsilon)(a^*-\varepsilon)}{1-(\alpha-\varepsilon)}.$$

Using Fact 2,

$$\lim_N U_A^N = \lim_N \mathrm{E}\left[\mathrm{E}\left[\frac{1}{2}\overline{\gamma}^N(\tilde{\alpha})\Pr\left[\mathrm{piv}_A^N\right] \mid \tilde{a} \in I_a^\varepsilon, \tilde{b} < \varepsilon', \tilde{\alpha}\right] \mid \tilde{\alpha} \in I_\alpha^\varepsilon\right].$$

However, given $\tilde{\alpha}$, when $\tilde{a} > a^* - \varepsilon$ and $\tilde{b} < \varepsilon'$,

$$\tilde{v}_A > (a^*-\varepsilon)\tilde{\alpha}N$$

$$\tilde{v}_B < \varepsilon'(1-\tilde{\alpha})N.$$

Given our choice of ε', this means $\tilde{v}_A > \tilde{v}_B$. In other words,

$$\Pr\left[\mathrm{piv}_A^N\right] = \Pr\left[\tilde{v}_A^N = \tilde{v}_B^N \text{ or } \tilde{v}_A^N + 1 = \tilde{v}_B^N\right] = 0$$

for all N given the conditional events. Therefore $\lim_N U_A^N = 0$. Any party A supporter with a strictly positive voting cost would find it profitable to deviate to abstaining for sufficiently large N.

References

Blais, A. (2000): *To Vote or Not to Vote: The Merits and Limits of Rational Choice Theory*. University of Pittsburgh Press, Pittsburgh.

Börgers, T. (2004): "Costly Voting," *American Economic Review*, 94(1), 57–66.

Campbell, C. (1999): "Large Electorates and Decisive Minorities," *Journal of Political Economy*, 107(6), 1199–1217.

Castanheira, M. (2003): "Victory Margins and the Paradox of Voting," *European Journal of Political Economy*, 19(4), 817–841.

Coate, S., and M. Conlin (2004): "A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence," *American Economic Review*, 94(5), 1476–1504.

Downs, A. (1957): *An Economic Theory of Democracy*. Harper and Row, New York.

Evren, Ö. (2012): "Altruism and Voting: A Large-turnout Result that does not Rely on Civic Duty or Cooperative Behavior," *Journal of Economic Theory*, 147(6), 2124–2157.

Feddersen, T. J., and A. Sandroni (2006): "A Theory of Participation in Elections," *American Economic Review*, 96(4), 1271–1282.

Fowler, J. H. (2005): "Turnout in a Small World," in *Social Logic of Politics*, ed. by A. Zuckerman, pp. 269–287. Temple University Press, Philadelphia.

Goeree, J., and J. Grosser (2007): "Welfare Reducing Polls," *Economic Theory*, 31(1), 51–68.

Harsanyi, J. C. (1977): "Morality and the Theory of Rational Behavior," *Social Research*, 44(4), 623–656.

——— (1992): "Game and Decision Theoretic Models in Ethics," in *Handbook of Game Theory with Economic Applications*, ed. by R. Aumann, and S. Hart, vol. I, pp. 669–707. Elsevier, Amsterdam.

Hoeffding, W. (1963): "Probability Inequalities for Sums of Bounded Random Variables," *Journal of the American Statistical Association*, 58(301), 13–30.

Krasa, S., and M. K. Polborn (2009): "Is Mandatory Voting Better than Voluntary Voting?," *Games and Economic Behavior*, 66(1), 275–291.

Krishna, V., and J. Morgan (2012): "Voluntary Voting: Costs and Benefits," *Journal of Economic Theory*, 147(6), 2083–2123.

Ledyard, J. (1981): "The Paradox of Voting and Candidate Competition: A General Equilibrium Analysis," in *Essays in Contemporary Fields of Economics*, ed. by G. Horwich, and J. P. Quirk. Purdue University Press, West Lafayette.

——— (1984): "The Pure Theory of Two Candidate Elections," *Public Choice*, 44(1), 7–41.

Levine, D. K., and T. R. Palfrey (2007): "The Paradox of Voter Participation? A Laboratory Study," *American Political Science Review*, 101(1), 143–158.

McMurray, J. C. (2012): "Aggregating Information by Voting: The Wisdom of the Experts versus the Wisdom of the Masses," *Review of Economic Studies*, forthcoming.

Morton, R. B. (1991): "Groups in Rational Turnout Models," *American Journal of Political Science*, 35(3), 758–776.

Myerson, R. B. (2000): "Large Poisson Games," *Journal of Economic Theory*, 94(1), 7–45.

Palfrey, T. R., and H. Rosenthal (1983): "A Strategic Calculus of Voting," *Public Choice*, 41(1), 7–53.

——— (1985): "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 79(1), 62–78.

Peterson, D. A. M., L. J. Grossback, J. A. Stimson, and A. Gangl (2003): "Congressional Response to Mandate Elections," *American Journal of Political Science*, 47(3), 411–426.

Poole, K. T., and H. Rosenthal (2001): "D-NOMINATE after 10 Years: A Comparative Update to *Congress: A Political Economic History of Roll-Call Voting*," *Legislative Studies Quarterly*, 26(1), 5–29.

Razin, R. (2003): "Signaling and Election Motivations in a Voting Model with Common Values and Responsive Candidates," *Econometrica*, 71(4), 1083–1119.

Riker, W. H., and P. C. Ordeshook (1968): "A Theory of the Calculus of Voting," *American Political Science Review*, 62(1), 25–42.

Shachar, R., and B. Nalebuff (1999): "Follow the Leader: Theory and Evidence on Political Participation," *American Economic Review*, 89(3), 525–547.

Smirnov, O., and J. H. Fowler (2007): *Mandates, Parties, and Voters: How Elections Shape the Future*. Temple University Press, Philadelphia.

Somer-Topcu, Z. (2009): "Timely Decisions: The Effects of Past National Elections on Party Policy Change," *Journal of Politics*, 71(1), 238–248.

Taylor, C., and H. Yildirim (2010): "A Unified Analysis of Rational Voting with Private Values and Group-specific Costs," *Games and Economic Behavior*, 70(2), 457–471.