On testing for non-linear and time irreversible probabilistic structure in high frequency ASX financial time series data

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SUMMARY

In this paper, we present three nonparametric trispectrum tests that can establish whether the spectral decomposition of kurtosis of high frequency financial asset price time series is consistent with the assumptions of Gaussianity, linearity and time reversiblility. The detection of nonlinear and time irreversible probabilistic structure has important implications for the choice and implementation of a range of models of the evolution of asset prices, including Black-Sholes-Merton (BSM) option pricing model, ARCH/GARCH and stochastic volatility models. We apply the tests to a selection of high frequency Australian (ASX) stocks.

Keywords: Bonferroni Test, Gaussianity, Linearity, Time Reversibility, Trispectrum.

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1. INTRODUCTION

There has been a lot of debate centered on the nature and extent to which the data generating process explaining the dynamics of financial and currency exchange time series data is a linear or nonlinear phenomenon. More recently, there has been a rise in interest in whether the data generating mechanism is a time reversible or irreversible process. To-date, most attention has centered upon modeling nonlinear structure associated with multiplicative nonlinearity or nonlinearity-in-variance. In this case, the main transmission mechanism is viewed as one where nonlinearity affects the process through its variance. The most common type of time series models used to address this type of process are ARCH/GARCH and stochastic volatility models (Engle, 1982; Bollerslev, 1986; Shephard, 1996).

In this paper, we focus our investigation upon asset price returns which are defined as

$$y(t_n) = \ln\left(\frac{z(t_n)}{z(t_n - 1)}\right) * 100 \tag{1}$$

where $y(t_n)$ is the log return for time period t_n, $z(t_n)$ is the observed stock price time series data and t_n is a discrete time index where $t_n = n\tau$ for sampling interval τ. It should be noted that the transformation in (1) will not affect the time reversibility properties of the process generating the source stock price series $\{z(t_n)\}$. This is an important consideration because applying many linear time series models including AR or ARMA models can induce time irreversibility even if the probabilistic structure of the source data is time reversible (Weiss, 1975; Hallin *et al*, 1988; Lawrance, 1991; Ramsey and Rothman, 1996; Chen, 1999; Tong and Zhang, 2005). This will particularly arise because linear time series models can be viewed as causal (or physically realizable) filters that are capable of changing the phase of the process, thereby generating time irreversible probabilistic structure (Priestley, 1981, Sect 4.12).

We can view the observed stock price series in (1) as a discrete-time sampled process that was sampled from an underlying continuous time process $\{z(t)\}$ and subsequently 'discretized' by some sampling procedure. In principle, the ideal sampling procedure is derived from application of the sampling theorem. This involves filtering the continuous time process $z(t)$ using a low-pass filter to remove all frequency components above the Nyquist frequency f_o and then sampling the filtered output at a rate equal to $2f_o$ in order to avoid aliasing the sampled process (Priestley, 1981, Sect. 7.1.1; Hinich *et al*, 2010). This procedure ensures that the discrete-time values are completely determined from the continuous time process without any loss of information (Bracewell, 1978, Chapter 10). Furthermore, the sampled process is band-limited and thus has finite support. This property ensures that all moments of the sampled process are bounded (finite) (Hinich *et al.* (2010). This, in principle, removes concerns about the potential applicability of bispectrum and trispectrum tests in the context of possible infinite moments associated with high frequency finance data (Jansen and de Vries, 1991; Loretan and Phillips, 1994; de Lima, 1997; Chen et al, 2000).

However, it should still be recognised that finite but very large moments can still pose problems for computer based numeric arithmetic.

The trispectrum tests we present here are generalizations of the Hinich (1982) bispectrum tests of Gaussianity and linearity and the Hinich and Rothman (1998) bispectrum test of time reversibility. In the context of high frequency financial data, the bispectrum linearity test was originally employed by Hinich and Patterson (1985) to confirm that daily stock rates of return are nonlinear. Hinich and Patterson (1989) went on to falsify the hypothesis that intraday stock returns are linear. Hinich (2008) used the bispectrum time reversibility test to falsify ARCH/GARCH models. Barnett *et al.* (1997) also provides an overview and critique of the performance of different tests for nonlinearity in detecting different types of artificially generated nonlinear structure.

The trispectrum tests are generalizations of the above-mentioned bispectrum tests. The trispectrum constitutes a spectral decomposition of kurtosis whereas the bispectrum represents a spectral decomposition of skewness. The generalization follows because all distributions except the Gaussian have non-zero kurtosis whereas all symmetric distributions have zero skewness. Therefore, as long as the sample size of the observed time series is sufficiently large, the trispectrum tests offers much more powerful set of diagnostic tests than the corresponding bispectrum tests.

Another key consideration favouring the use of the trispectrum test in the context of high frequency financial and currency exchange time series data, in particular, is that this data is universally characterised as being predominantly leptokurtotic. Because the trispectrum involves the spectral decomposition of kurtosis, it can be used to investigate, directly, how leptokurtosis will affect the underlying linearity and time reversibility properties of the data generating mechanism.

In the finance literature, leptokurtosis is associated with volatility clustering effects. However, there has been much less interest in examining how this observed leptokurtosis might influence key properties governing the time evolution of asset price rates of return, such as linearity and time reversibility. Here we use the trispectrum to test whether leptokurtosis is associated with nonlinear or time irreversible probabilistic structure in the returns data. Thus, we identify 'stylised facts' that have to be captured in models constructed to explain the time evolution of an asset price.

For example, in the context of the Black-Scholes-Merton option pricing framework, asset price rates of return are modeled either as 1) a Wiener process in the Black-Scholes option pricing model, 2) a Wiener process with stationary Poisson jumps in the Black-Scholes-Merton (BSM) pricing model (Black and Scholes, 1973; Merton, 1973,1976; Campbell *et al*, 1997; Gourieroux and Jasiak, 2001). The first difference of a discrete-time Wiener process is a random walk which is Gaussian, linear and time reversible. The first difference of a Wiener process with stationary Poisson jumps is also linear and time reversible. So discovery of nonlinearity and time irreversibility in asset price returns invalidates these models and more recent variants of them as parsimonious representations of the data.

Further, a key aspect of both ARCH/GARCH and Stochastic Volatility (SV) modeling frameworks is that the time series is assumed to be a conditional and unconditional zero mean process. In the usual case where the source time series has a mean or drift component, this then requires that the mean component be removed, typically by recourse to estimation or data transformations based on linear time series estimation or filter techniques. An example of the latter would be the use of a first difference filter to remove a stochastic trend. The residuals then constitute a 'zero mean' process and the modeling of the conditional variance by both ARCH/GARCH/SV models can then be used to test hypotheses drawn from theories of financial volatility. However, if the mean of the source time series is nonlinear, erroneous conclusions of 'nonlinearity in variance' can emerge when the prime source of serial dependence in the residuals is nonlinear structure that was not successfully extracted when using conventional linear time series models or filtering techniques to remove the mean of the source time series. In particular, Ashley *et al.* (1986) formally prove that if the data generating mechanism is nonlinear and a linear filter (i.e. linear time series model) is fitted to the nonlinear data, then the nonlinear structure will end up in the residuals of the fitted linear model.

Furthermore, any dependence between different model components, such as diffusion and jump components in the case of SV models, are usually defined and modeled strictly in terms of second order moments – namely correlations. However, if the evolution of stock price returns is non-Gaussian, it is possible that higher order serial dependencies exist, being linked to statistically significant bi-correlation and tri-correlation structures associated with the decomposition of skewness and kurtosis. If this is the case, important structural information can be overlooked by focusing solely on second order serial dependencies.

From a diagnostic testing perspective, 'nonlinear in mean' structure can be detected using higher order analogues of conventional Portmanteau tests for serial correlation. The trispectrum test of linearity can be viewed as a frequency domain analogue of a Portmanteau test for serial tri-correlation (i.e. 4^{th} order serial dependence). It is essential to discover as much information of this kind as possible about the nature of any nonlinearity present in time series data prior to applying techniques such as GARCH. Here, we apply these tests to a number of minute-to-minute Australian stocks chosen from the top 50 companies in terms of market capitalization.

In Section 2, we briefly explain how the trispectrum is defined. In Section 3 we explain how the trispectrum can be estimated. In Section 4 we discuss how the Gaussianity, linearity and time reversibility tests are implemented. In Section 5, we present a Bonferonni test framework that allows us to measure the pervasiveness of possible rejection patterns across valid tri-frequency combinations. In Section 6, we apply the tests in the context of high frequency stock rate of return data to see if the probabilistic structure of the empirical distribution functions of these data is consistent with the properties of Gaussianity, linearity and time reversibility. Section 7 contains some conclusions.

2. THE TRISPECTRUM

The trispectrum captures the values of the fourth order cumulant spectrum for four frequencies that sum to zero. Details of the definitions of the trispectrum and the fourth order cumulant spectrum and their asymptotic properties are presented by Dalle Molle and Hinich (1995). For our purposes here we selected a simpler, yet mathematically correct, definition of the trispectrum $T_{yyyy}\left(f_{k_1}, f_{k_2}, f_{k_3}\right)$ of observed values of $y(t_n)$ represented by (1) for a segment $\{y(0), y(t_1), y(t_2), \cdots, y(t_{L-1})\}$ of L successive observations and for the discrete (Fourier) frequencies $f_k = k/T$ where $T = L\tau$ where from Section 1, τ is the sampling interval. To economize on notation we use the index k to denote the k-th Fourier frequency rather than f_k.

The trispectrum value at the triple frequency (k_1, k_2, k_3) is

$$T_{yyyy}(k_1, k_2, k_3) = \lim_{L\to\infty} E\left[\frac{Y(k_1)\,Y(k_2)\,Y(k_3)\,Y(k_4)}{L}\right] \qquad (2)$$

where $Y(k) = \sum_{n=0}^{L-1} y(t_n)\exp(-i2\pi f_k t_n)$ is the discrete Fourier transform of the segment and $k_4 = L - k_1 - k_2 - k_3$ for the discrete frequencies in the principal domain, except for frequencies in the three linear (index) subsets $S_1 = \{k_1 + k_2 = 0 \ \&\ k_3 + k_4 = 0\}$, $S_2 = \{k_1 + k_3 = 0 \ \&\ k_2 + k_4 = 0\}$ and $S_3 = \{k_1 + k_4 = 0 \ \&\ k_2 + k_3 = 0\}$.

It should be noted that the restriction involving the exclusion of the three subsets means that we are focusing analysis off of any proper submanifold in the principle domain of the trispectrum. Estimation methods and distribution theory for this region of the principal domain is well established dating back to seminal contributions of (Brillinger, 1965; Brillinger and Rosenblatt, 1967a, 1967b) and also adopted in (Walden and Williams, 1993; Dalle Molle and Hinich, 1995).

When attention is focused on the region of the principal domain off of the proper submanifolds, the trispectral density estimates associated with taking the fourth order periodograms of fourth order moments and cumulants coincide. It is only on the proper submanifolds where these results diverge with the trispectral density associated with the fourth order moments not being well defined – i.e. being singular. This contrasts with the situation of the trispectral density of the fourth order cumulant which are well defined on the proper submanifolds (Brillinger and Rosenblatt, 1967a, Section II.C; Kim, 1989, Sections 1 and 4).

The region in the principal domain off of the proper submanifolds and where the four frequency indices are different is where the statistical structure associated with 4^{th} order nonlinear serial dependence would be located – the statistical structure we are principally interested in. The statistical structure on the proper submanifolds, on the other hand, is of lower order involving sums of products of power spectrum.

In general, two approaches have been proposed to deal with contributions associated with frequency components appearing on proper submanifolds. The first dates back to Brillinger and Rosenblatt (1967)a and involved a methodology whereby estimates on and off proper submanifolds were constructed as weighted averages of the fourth order periodogram avoiding the proper submanifolds. As such, the contribution from components on proper submanifolds was made by taking averages of values near, but <u>not on</u> the proper submanifolds. This approach was encapsulated in equation (2.55) in (Brillinger and Rosenblatt, 1967a, Section C) in which the phi variable in (2.55) effectively suppressed or ‘zeroed out’ any contribution to estimate (2.55) from proper submanifolds, as well as zeroing out any contributions to the expected value and variance/covariance of the estimate on proper submanifolds.

The second approach involved constructing estimates that explicitly included contribution of components on proper submanifolds. This class of estimator, both on and off the proper submanifolds, has been proven to be unbiased, consistent and asymptotically normal (Kim, 1989, 1991, 1994; Lii and Rosenblatt, 1990). However, the resulting expression for large sample variance is more complicated than with the case arising when attention is restricted solely to regions in the principal domain off of proper submanifolds.

For example, from the discussion of equation (31) in (Dalle Molle and Hinich, 1995, Section IV), for analysis off of the proper submanifolds, the scaling implied in equation (31) is $N^{(3c-1)}B(k_1,k_2,k_3,k_4) = N^{(3c-1)}$ because $B(k_1,k_2,k_3,k_4) = 1$ when off of the proper submanifolds and the four frequency indices k_1, k_2, k_3 and k_4 are different from each other. For each such estimate, this scaling would be the same because N and c are constants.

Things, however, become more complex when the proper submanifolds are included and $B(k_1, k_2, k_3, k_4)$ can subsequently take on many values different from unity as listed in Table II of Dalle Molle and Hinich (1995). These values depend on the nature of equality and complex conjugate relationship between the different frequency indices k_1, k_2, k_3 and k_4 and whether any of the four frequency indices are equal to the upper cut-off frequency (Dalle Molle and Hinich, 1995, Section III.B). Therefore, the scaling factor $N^{(3c-1)}B(k_1, k_2, k_3, k_4)$ could vary quite significantly in this latter case for different trispectral estimates. The results listed in (Kim, 1991, Sections 2 and 3; Kim, 1994, Sections 3 and 4) also indicate that the asymptotic variance becomes much more complicated when account is taken of the proper submanifolds.

Another related issue is whether some proper submanifolds can be legitimately excluded from trispectral analysis without any loss of information relating to the spectral information inherent to the fourth order cumulant spectral function of the stationary random process. Specifically, some loss of information relating to the fourth order cumulant spectral density function would arise with the exclusion of the three above subsets, but this might not be as significant as first imagined. Dalle Molle and Hinich (1995, Section C.II.C) demonstrate that there are 48 distinct nonredundant replicates of the principal manifold of the trispectrum – in the terminology above, each distinct replicate can be termed a principal domain. They also demonstrated that for each of the 48 principal manifolds (domains), at least two of the three proper submanifolds contained redundant information and only a subset (e.g. a section) of the third proper submanifold contained nonredundant information. This followed because each of the 48 equivalent nonredundnant replicates of the principal manifold intersected with only a subset of only one of the three proper submanifolds. Thus, within each principal domain, most of the information contained on the three proper submanifolds would constitute redundant information. Moreover, the questions of which two proper submanifolds contain completely redundant information and which proper submanifold (and section therein) contains nonredundant information varied with each of the 48 principal domains – thus complicating trispectral analysis.

The limit in (2) exists from Theorem 4.3.1 in (Brillinger, 1981, Chapter 4) if we assume that (1) is strictly stationary and satisfies the Brillinger mixing condition 2.6.1 (Brillinger, 1981, Chapter 2). The stationarity and mixing conditions are necessary to obtain the asymptotic properties of the trispectrum estimates presented in the next section.

The principal domain Ω, defined within the cube given by the indices $\{-k_o < k_1 < k_o,\ -k_o < k_2 < k_o,\ -k_o < k_3 < k_o\}$ where $k_o = f_o T$, is given by $\Omega = \Omega_+ \cup \Omega_- - (S_1 \cup S_2 \cup S_3)$, where

$$\Omega_+ = \{1 \le k_1 \le k_0,\ 1 \le k_2 \le k_0,\ 1 \le k_3 \le \min(k_2, k_0 - k_1 - k_2)\ \&\ k_1 + k_2 < k_0\} \tag{3}$$

$$\Omega_- = \{1 \le k_1 \le k_0,\ 1 \le k_2 \le k_0,\ k_0 - k_1 - k_2 \le k_3 \le -k_2\}. \tag{4}$$

The normalized trispectrum is defined to be

$$K_{yyyy}(k_1,k_2,k_3)=\frac{T_{yyyy}(k_1,k_2,k_3)}{\sqrt{S_y(k_1)\,S_y(k_2)\,S_y(k_3)\,S_y(k_4)}} \tag{5}$$

where $S_y(k)$ is the spectrum of $\{y(t_n)\}$ at frequency k and $k_4 = L - k_1 - k_2 - k_3$, (Dalle Molle and Hinich, 1995, Section III.B-C). Note that we have excluded the scale factor $\left[\frac{L^2}{N_F}\right]B(k_1,k_2,k_3,k_4)$ contained in the large sample variance equation (31) of Dalle Molle and Hinich (1995) from the definition of the normalized trispectrum in (3) but will include the $\left[\frac{L^2}{N_F}\right]$ term appropriately in the development of the theory and tests in the next two sections. It should also be noted that because we are restricting attention to trispectral estimates that are off the proper submanifolds and whose four frequency indices are different, then $B(k_1,k_2,k_3,k_4)=1$ and the scale factor reduces to $\left[\frac{L^2}{N_F}\right]$, (Dalle Molle and Hinich, 1995, Section IV).

If the process defined in (1) is linear, then the trispectrum is $T_{yyyy}(k_1,k_2,k_3)=\sigma_y^4\kappa_{y4}H(k_1)H(k_2)H(k_3)H(k_4)$ and $S_y(k)=\sigma_y^2H^2(k)$ where κ_{y4} is the kurtosis, σ_y^2 is the variance of $y(t_n)$ and $H(k)=\sum_{t=-\infty}^{\infty}h(t)\exp(-i2\pi kt)$ is the linear filter's complex transfer function. Thus, for a linear process, the normalized kurtosis values are $\mathrm{K}_{yyyy}(k_1,k_2,k_3)=\kappa_{y4}$, a real number for each triple index.

3. ESTIMATING THE TRISPECTRUM

Suppose that we observe the process $\{y(t_n)\}$ for a time period of length $N\tau$ and divide this period into M non-overlapping frames each of length L, assuming for simplicity that N is divisible by L. The estimate of the trispectrum for each frequency triple in the principal domain, but not in the three linear subsets identified above, is the frame averaged estimate given in Dalle Molle and Hinich (1995)

$$\hat{T}_{yyyy}(k_1,k_2,k_3) = \frac{1}{M}\sum_{m=1}^{M}\frac{Y_m(k_1)\,Y_m(k_2)\,Y_m(k_3)\,Y_m(L-k_1-k_2-k_3)}{L} \tag{6}$$

where $Y_m(k)$ is the k-th term of the discrete Fourier transform of the m-th frame. Using the frame averaged estimate of the spectrum

$$\hat{S}_y(k) = \frac{1}{M}\sum_{m=1}^{M}\frac{|Y_m(k)|^2}{L} \tag{7}$$

then the sample normalized trispectrum is given by

$$\hat{K}_{yyyy}(k_1,k_2,k_3) = \left[\hat{S}_y(k_1)\,\hat{S}_y(k_2)\,\hat{S}_y(k_3)\,\hat{S}_y(L-k_1-k_2-k_3)\right]^{-1/2}\,\hat{T}_{yyyy}(k_1,k_2,k_3)\,. \tag{8}$$

It follows from (Dalle Molle and Hinich, 1995, Section III.C) that $L = N^c$ and $M = N_F = N^{(1-c)}$, where the bandwidth frame coefficient c is assumed to be in the interval $0 < c < 1/3$ and scale factor $\left(L^2/N_F\right) = N^{(3c-1)}$.

Given the definition of $\left(L^2/N_F\right)$, the asymptotic results in Brillinger and Rosenblatt (1967a) and Brillinger (1981, Sections 4.4 and 5.3) were implicitly used in Dalle Molle and Hinich (1995, Section IV) to get the following large sample results:

For the set of indices (k_1, k_2, k_3) in the principal domain, except for the three linear subsets, $\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right)$ is approximately jointly complex Normal with zero mean and variance $N^{(3c-1)} S_y(k_1) S_y(k_2) S_y(k_3) S_y(k_4)$, for $k_4 = L - k_1 - k_2 - k_3$ and have zero covariances across the frequency triples and between the real and imaginary parts of the trispectrum values. We can make use of the result that if $\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right) \approx N^C(0,\Sigma)$ where 'N^C' denotes complex normal distribution, then the real and imaginary parts are distributed as

$$\mathrm{Re}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right) = \mathrm{Im}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right) \approx N\left(0, \frac{\Sigma}{2}\right),$$

(Brillinger, 1981, Section 4.2). It then follows that the real part of the trispectrum $\mathrm{Re}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right)$ and imaginary part of the trispectrum $\mathrm{Im}\left(\hat{T}_{yyyy}(k_1,k_2,k_3) - T_{yyyy}(k_1,k_2,k_3)\right)$ are approximately jointly Normal with zero means and variances $\left(N^{(3c-1)}/2\right) S_y(k_1) S_y(k_2) S_y(k_3) S_y(k_4)$.

Assuming that the $S_y(k_1), S_y(k_2), S_y(k_3), S_y(k_4)$ in (3) and (6) differ from zero so that the normalized trispectrum values are well defined numerically, we can use arguments similar to those in Hinich and Wolinsky (2005) and Hinich (2008), to establish that each complex variate $\sqrt{2N^{(3c-1)}}\left[\hat{K}_{yyyy}(k_1,k_2,k_3) - K_{yyyy}(k_1,k_2,k_3)\right]$ is asymptotically independent complex

Normal with zero mean and unit variance as $N \to \infty$ and $0 < c < \frac{1}{3}$. It then follows that the real part of the normalized trispectrum $\sqrt{2N^{(3c-1)}}\,\mathrm{Re}\left[\hat{K}_{yyyy}(k_1,k_2,k_3) - K_{yyyy}(k_1,k_2,k_3)\right]$ and the imaginary part of the normalized trispectrum $\sqrt{2N^{(3c-1)}}\,\mathrm{Im}\left[\hat{K}_{yyyy}(k_1,k_2,k_3) - K_{yyyy}(k_1,k_2,k_3)\right]$ are asymptotically independent Normal with zero mean and variance equal to $1/2$, using the results in (Brillinger, 1981, Section 4.2).

4. TESTING FOR GAUSSIANITY, LINEARITY AND TIME REVERSIBILITY

Under the null hypothesis that the observed values $y(t_n)$ are Gaussian, then the kurtosis is identically zero across all triple frequencies in the principal domain. Under the null hypothesis of Gaussianity, the large sample distribution of $2N^{(3c-1)}\left|\hat{K}_{yyyy}(k_1,k_2,k_3)\right|^2$ is $\chi_2^2(0)$, a central chi square with two degree-of-freedom. Note that the non-centrality parameter is zero under the null hypothesis since the kurtosis is zero. Since the trispectrum values are approximately independently distributed over the triple frequencies in the principal domain (Ω), the statistic

$$G = \sum_{(k_1,k_2,k_3)\in\Omega} 2N^{(3c-1)}\left|\hat{K}_{yyyy}(k_1,k_2,k_3)\right|^2 \tag{9}$$

has a central chi square $\chi_{2D}^2(0)$ distribution, where D is the number of triple frequencies in Ω (Dalle Molle and Hinich (1995, Section IV).

Let $F_1(r \mid 2D,0)$ denote the cumulative distribution function of a $\chi^2_{2D}(0)$ random variable. Furthermore, let $U_G = F_1(G \mid 2D,0)$ denote transforming the statistic G into a uniform $(0,1)$ variate under the null hypothesis of Gaussianity. The trispectrum program computes the p-values given by $\left[1-U_G(k_1,k_2,k_3)\right]$ for all the triple frequencies in the principal domain, as well as the CUSUM test p-value associated with $[1-U_G]$.

Under the null hypothesis that the observed values $y(t_n)$ are generated by a linear model, the large sample distribution of $W(k_1,k_2,k_3) = 2N^{(3c-1)}\left|\hat{K}_{yyyy}(k_1,k_2,k_3)\right|^2$ is $\chi^2_2(\lambda)$, a non-central chi square with two degree-of-freedoms and a non-central parameter $\lambda = \kappa^2_{y4}$ (Dalle Molle and Hinich (1995, Section IV). Let $F_2(r \mid 2,\lambda)$ denote the cumulative distribution function of a $\chi^2_2(\lambda)$ random variable. Let $U_L(k_1,k_2,k_3) = F_2(W(k_1,k_2,k_3) \mid 2,\lambda)$ denote transforming the statistic for the scaled square magnitude of the normalized trispectrum value to a uniform (0,1) distribution under the null hypothesis of linearity. The estimate of the non-centrality parameter is the mean value of the $W(k_1,k_2,k_3)$'s in Ω, which are approximately independently distributed. The trispectrum program computes the cumulative distribution function for $U_L(k_1,k_2,k_3)$ for all combinations of (k_1,k_2,k_3) in Ω and then calculates the value $U_L^q(k_1,k_2,k_3)$ associated with a user specified quantile value q such that $(0 \le q \le 1)$ and with associated p-value given by $\left[1-U_L^q(k_1,k_2,k_3)\right]$ associated with a $(1-q)$ level of significance.

If the observed values $y(t_n)$ are generated by a time reversible process, we can use the trispectrum version of the Hinich and Rothman (2008) test. Under the null hypothesis of time reversibility, the large sample distribution of $2N^{(3c-1)}\left|\operatorname{Im}\hat{K}_{yyyy}(k_1,k_2,k_3)\right|^2$ is $\chi_1^2(0)$, a central chi square with one degree-of-freedom. Note that the non-centrality parameter is zero under the null hypothesis of time reversibility because the imaginary parts of the normalized trispectral values are identically zero. Since the trispectrum values are approximately independently distributed over the triple frequencies in Ω, the statistic

$$R = \sum_{(k_1,k_2,k_3)\in\Omega} 2N^{(3c-1)}\left|\operatorname{Im}\hat{K}_{yyyy}(k_1,k_2,k_3)\right|^2 \tag{10}$$

has a central chi square $\chi_D^2(0)$ distribution where D is the number of triple frequencies in Ω.

Let $F_3(r \mid D,0)$ denote the cumulative distribution function of a $\chi_D^2(0)$ random variable and let $U_R = F_3(R \mid D,0)$ denote transforming the statistic R into a uniform $(0,1)$ variate under the null hypothesis of time reversibility. The trispectrum program computes the p-values given by $\left[1-U_R(k_1,k_2,k_3)\right]$ for all the triple frequencies in Ω as well as the CUSUM test p-value associated with $[1-U_R]$.

5. A BONFERRONI METHODOLOGY TO GAUGE PERVASIVENESS OF REJECTION PATTERNS ACROSS VALID TRI-FREQUENCY COMBINATIONS

In order to gauge how pervasive the rejection patterns of the above test statistics are across valid tri-frequencies in the principal domain, we also employ a testing procedure based on the Classical Bonferroni procedure. This procedure can, in principle, be used to construct alternative joint tests of the Gaussianity, linearity and time reversibility hypotheses. The details of this test methodology are outlined in Appendix A.

To implement this test procedure in accordance with the classical Bonferroni inequality, we adopt a critical value for test purposes that is given by (α / D) where α is a user specified level of significance and D is the total number of tri-frequency Gaussianity, linearity or time reversibility tests performed in implementing the methodology outlined in Section 4. In general, the testing procedure leads to the rejection of the joint null hypothesis H_0 if $P_1 \leq (\alpha / D)$, where P_1 is the smallest p-value associated with one of the individual tri-frequency tests. Here, we set $\alpha = 0.1,\ 0.05$ and 0.01 signifying ten percent, five percent and one percent levels of significance, respectively.

We implement the Bonferroni test procedure and calculate the number of individual tri-frequency tests that would have led to the rejection of the joint null of Gaussianity, linearity or time reversibility. We express these rejections as a percentage of the total number of p-values (i.e. tests) that were admissible and subsequently performed. The larger this percentage value is the more pervasive will be the rejection of the null hypotheses across admissible tri-frequency combinations.

6. APPLICATION TO STOCK RATES OF RETURN

The trispectrum tests were applied to the rates of return of an assortment of companies that are in the top 50 companies, in terms of market capitalization, traded on the ASX (Australian Stock Exchange). We report the test results for sixteen stocks that form a good representative sample of large cap Australian stocks (e.g. large companies in terms of market capitalization). Company details relating to all sixteen ASX stocks are listed in Table 1. These tests can be viewed as a generalization of the statistical analysis of Lo and MacKinlay (1988).

[INSERT TABLE 1 ABOUT HERE]

It should be noted that adjustments were made to the original spot price data for the following stocks. First, to BHP to correct for a stock split that arose on 29/6/2001 (from $21.418 to $10.495 at 10:00:00 on 29/6/2001) associated with the BHP-Billiton merger. Second, to Brambles (BIL) to correct for a stock split associated with a merger with UK services group GFN on 8/8/2001 (from $45.679 to $10.703 at 10:00:00 on 8/8/2001). Third, to Lend Lease (LLC) for a stock split associated with a one-to-one bonus share offer in December 1998 (from $38.675 to $19.983 at 10:00:00 on 1/12/1998).

The data were provided by the industry and academic research center 'Securities Industry Research Centre of Asia-Pacific' Limited (SIRCA) – information about this organization can be found at: http://www.sirca.org.au/. The data is minute-by-minute time series of spot price data over the time interval 10:00:00 to 16:06:00 for the period January 2, 1996 to December 30, 2005. Rates of return were calculated from these spot price data as time weighted mid-point prices, calculated as averages of asks and bids over the time segments that together constituted a time interval of a minute. Thus, the spot prices dynamics were not unduly influenced by micro-structure phenomenon such as 'Bid-Ask Bounce'.

The summary statistics for the sixteen ASX Australian stock returns are reported in Table 2. It is noticeable that these high frequency rates of return are very fat tailed, with kurtosis values ranging from a low of 688.0 (for AGL) to a high of 7380.0 (for BHP) – see the data in the $\kappa = 0$ rows in Table 2. Furthermore, 75% of the stock returns listed in Table 2 are left (negatively) skewed. The kurtosis values in this range point to the presence of fat tails in the empirical distribution function of these stock rates of return and, together with the skewness results, point to substantial deviations from the Gaussian distribution. However, the large sample convergence properties of the trispectral estimates are problematical in the case of such fat tailed time series. To eliminate this problem, the rates of returns series were trimmed in order to improve the validity of the use of the asymptotic properties of the trispectral tests.

[INSERT TABLE 2 ABOUT HERE]

Trimming data to make sample means less sensitive to outliers has been used in applied statistics for many years. Some of the earliest discussion of trimming appears in (Galton, 1879; McAlister, 1879). Subsequently, Edgeworth (1898) and Johnson (1949) contributed to the understanding of this technique for examining data. Trimming is a simple data transformation that makes statistics based on the trimmed sample more normally distributed. To trim the upper and lower $\omega/2$ % values of the sample $\{y(t_1),\ldots,y(t_N)\}$ we ordered the data and computed the $\omega/200$ quantile $y_{\omega/200}$ and the $1-\omega/200$ quantile $y_{1-\omega/200}$ of the order statistics. We then set all sample values less than the $\omega/200$ quantile to $y_{\omega/200}$ and all sample values greater than the $1-\omega/200$ quantile to $y_{1-\omega/200}$. The remaining $(100-\omega)\%$ data values were left unchanged.

In performing different rates of trimming, it was possible to see how robust our findings were to more stringent reductions of outlier effects. This allowed us to determine if any findings of nonlinearity and time irreversibility that might be attributed to outliers survived in the 'central' inter-quartile range of the empirical distribution function of the returns data. This permitted direct testing for 'nonlinear in mean' structure in the returns data.

In the tables cited in this paper, the degree of trimming is denoted by parameter 'κ'. As indicated above, $\kappa=0$ indicates that no trimming was performed on the source returns data. A value of $\kappa=1$ indicates that the bottom and top 1% of the empirical distribution function was trimmed while $\kappa=5$ indicates that the bottom and top 5% of the empirical distribution function of the returns was trimmed. This means that any returns that are smaller than the 1% and 5% quantiles of the order statistics are set to the respective 1% and 5% quantile values. Similarly, any returns that exceed the 95% and 99% quantiles of the order statistics are set to the respective 95% and 99% quantile values.

The statistics for the trimmed Australian returns are also presented in Table 2. As expected, the skewness and kurtosis values of the trimmed returns are reduced significantly together with the range of the returns data as the rate of trimming increases. There is also evidence that trimming rates of 10% are excessive for ANZ, BHP, CBA, NAB, and WBC while a trimming rate of 15% appears excessive for AGL, BIL, LLC, QBE, SGB, TAH, WOW and WPL – for example, excessive trimming leading to the onset of negative kurtosis values listed in Table 2 in bold font. This would suggest that trimming of 5% for ANZ, BHP, CBA, NAB, and WBC and trimming of 10% for AGL, BIL, LLC, QBE, SGB, TAH, WOW and WPL is acceptable. The remaining stocks (e.g. AMC, NCM and STO) have trimming of 15%.

We used a frame length of 60 one minute returns which yields a bandwidth frame coefficient of $c = 0.3$. The overall sample size is 927,776 observations and, with a frame length of 60 one minute returns, this produces frame averaging over 15,462 frames. There are also 2,920 trispectral estimates in the principal domain for this frame length. The pass band implied in trispectral analysis is (60.00, 2.00) minutes, encompassing periodicities whose cycle length is inclusively between one hour and two minutes in duration.

The CUSUM Gaussianity, linearity and time reversibility tests presented in Section 4 were applied to the trimmed returns data and the results are reported in Table 3. It is apperent from inspection of Table 3 that the hypotheses of Gaussianity, linearity and time reversibility are strongly rejected by these tests for all sixteen Australian stocks. Note also that, because of the larger sample sizes involved, the linearity test was computed at the 99% quantile for the Australian returns.

[INSERT TABLE 3 ABOUT HERE]

The results from the application of the Bonferroni test procedure for the Gaussianity, linearity and time reversibility tests are documented in Tables 4, 5 and 6, respectively. Note from inspection of these tables that a significant number of individual p-values are less than the associated Bonferroni critical values of 0.000034, 0.000017 and 0.000003 for the 10%, 5% and 1% levels of significance. Note that the critical values are quite small in magnitude. This follows because of the large number of trispectral estimates in the principal domain and total number of individual tests performed. Essentially, because of the large number of individual tests involved and the higher probability that random chance leads to a false rejection of the null hypothesis when it is true, smaller critical values are needed to ensure that the probability of false rejection is not greater than 10%, 5% and 1%, respectively.

The test outcomes can be interpreted in the following manner. In Table 4, for the BHP stock returns and a trimming rate of 5%, the application of the Bonferroni based Gaussianity test resulted in 55.07%, 53.15% and

48.90% of all the tri-frequencies in the principal domain having p-values less than or equal to the 10%, 5%, and 1% Bonferroni critical values cited above. Furthermore, in Table 5, for the CBA returns and 2% rate of trimming, the Bonferroni linearity test resulted in 16.99%, 16.71% and 16.51% of all the tri-frequencies in the principal domain having p-values less than or equal to the 10%, 5%, and 1% Bonferroni critical values. Similarly, in Table 6, for the WPL returns and trimming rate of 10%, the application of the Bonferroni time reversibility test resulted in 3.15%, 2.64% and 1.71% of all the tri-frequencies in the principal domain having p-values less than or equal to the above-mentioned 10%, 5%, and 1% Bonferroni critical values.

Recall that according to the Bonferroni test procedure, only one p-value being less than or equal to the Bonferroni critical values is required to secure rejection of the joint null hypotheses at the relevant levels of significance. So the results reported in Tables 4, 5 and 6 clearly point to rejections of the null hypotheses of Gaussianity, linearity and time reversibility for all sixteen stocks and across all trimming rates specified in the tables.

[INSERT TABLES 4, 5 AND 6 ABOUT HERE]

Further evidence of excessive trimming at the 15% trimming rate is apparent from inspection of Table 4 in relation to ANZ, BHP, CBA, NAB, NCM, QBE (at 10% level of significance), WBC, WOW and WPL stocks with the increase in the value of percentage rejections over and above the values observed for the trimming rate of 10% - for example, the increase from 41.30% to 44.01% at the 10% level of significance for WOW. These results are highlighted in Table 4 in bold font for all relevant stocks except NCM which is highlighted in bold red font (for a trimming rate of 15%). It should also be recognized that the strong rejections reported in Tables 4-6 are consistent with the results obtained from the CUSUM based test statistics outlined in Table 3 for all sixteen stocks and various trimming rates and test considered.

If we adopt trimming rates of 5% for ANZ, BHP, CBA, NAB, and WBC, trimming rates of 10% for AGL, BIL, LLC, QBE, SGB, TAH, WOW and WPL and trimming rates of 15% for AMC, NCM and STO, then we secure strong rejections of the various null hypotheses for the sixteen stocks in accordance with the Bonferroni test procedure. Note in this context, that these results are highlighted in red font in Tables 4-6, while the NCM results associated with a 10% trimming rate has also been highlighted in blue font. Specifically, in the case of the linearity test, it is apparent from inspection of Table 5 that we secure rejections of the null of linearity with rejection percentages ranging from a low of (5.62%, 5.45%, 5.31%) for WPL and a high of (8.56%, 8.29%, 7.98%) for CBA. For the time reversibility test, the findings reported in Table 6 indicate that we secure rejections of the null of time reversibility with percentage rejections ranging from lows of (2.57%, 2.12%, 1.47%) for AMC and (2.57%, 2.40%, 1.82%) for SGB and a high of (10.07%, 8.97%, 6.75%) for BHP. Furthermore, from inspection of Table 4, it is apparent we secure strong rejection of the null hypothesis of Gaussianity with rejection percentages ranging from a low of (32.98%, 31.54%, 27.81%) for SGB and a high of (55.07%, 53.15%, 48.90%) for BHP.

The results in Table 4 indicate that, in the case of the Gaussianity test, the total number of rejections is of an order of magnitude larger than the comparable numbers associated with the linearity and time reversibility test results cited in Tables 5 and 6. This result also holds against the backdrop of the more stringent trimming of the upper and lower tails of the empirical distribution function of the returns data, implying that the Gaussianity test rejections are not being driven principally by extreme outliers in the returns data.

In order to assess how sensitive our results are to the time frequency of observation of the underlying data, we also applied the tests to a more aggregated time series obtained by averaging across ten one minute returns to obtain a time series of average ten minute returns. We used a frame length of 18 ten minute returns which yields a bandwidth frame coefficient of c = 0.25. The overall sample size is 92,777 observations and, with a frame length of 18 ten minute average returns, produces frame averaging over 5,154 frames. There are also 73 trispectral estimates in the principal domain for this particular frame length. The pass band implied in trispectral analysis is (18.00, 2.00) ten minute periods, thereby encompassing periodicities whose cycle length is inclusively between three hours and twenty minutes in duration.

The summary statistics for the aggregated sixteen ASX Australian stock returns are reported in Table 7. It is evident from inspection of this table that the ten minute average returns data are still fat tailed, although at a lower order of magnitude than was the case with results reported in Table 2 for the one minute returns data. Specifically, kurtosis values in Table 7 range from a high of 813.0 for STO to a low of 70.50 for BHP. Note that the latter result contrasts with the results listed in Table 2 where BHP had the largest kurtosis value of all sixteen stocks of 7380.0. Moreover, 87% of the stock returns listed in Table 7 are left (negatively) skewed, with particular changes in the nature of skewness occurring for BHP, NCM and WPL when compared with skewness results for these particular stocks listed in Table 2.

[INSERT TABLE 7 ABOUT HERE]

The statistics of the trimmed aggregated ten minute average returns are also presented in Table 7. The skewness and kurtosis values of the trimmed returns are reduced significantly as the rate of trimming increases. It is also apparent from examination of Table 7 that a trimming rate of 10% appears excessive for all sixteen stocks – trimming at 10% leads to the onset of negative kurtosis values as outlined in Table 7 in bold font for all sixteen stocks.

The CUSUM Gaussianity, linearity and time reversibility test results are presented in Table 8 for the trimmed average ten minute returns data. It is evident from inspection of Table 8 that, for all sixteen stocks, the hypotheses of Gaussianity and time reversibility are strongly rejected while the hypothesis of linearity is rejected at the 1% level of significance. Note that because of the smaller sample size produced by the aggregation process used to calculate the ten minute average returns and the resulting smaller number of tri-frequencies in the principal domain (and individual tests employed in the Bonferroni tests), the linearity test was computed at the 90% quantile.

[INSERT TABLE 8 ABOUT HERE]

The other noticeable feature of Table 8 is the similarity in the size of the p-values obtained for the CUSUM linearity test for all sixteen stocks and all trimming rates considered. The reason for this is that the output quantiles obtained in all cases for the input quantile of 90% are the same. This is shown in Figure 1 which contains the cumulative distribution function of the linearity test for stock CBA and for the various trimming rates considered. It is evident from examination of Figure 1 that the cumulative distribution functions of the linearity test varies with trimming rate, with the transition becoming more gradual over the percentile range 0.40 to 0.80 as the rate of trimming increases. The results in Figure 1 are representative in qualitative terms of the results obtained for ten minute average returns of all sixteen stocks and for all trimming rates considered. It should be noted that in Figure 1 the horizontal axis contains the percentiles of the cumulative distribution function of the individual linearity tests $U_L(k_1,k_2,k_3)$ that make up the CUSUM linearity test documented in Section 4. The cumulative distribution function value corresponding to an input quantile of 0.9 (as read from along the horizontal axis in Figure 1) produces the same output quantile for all trimming rates considered. Because the linearity test statistic depends on the input and output quantiles and number of individual linearity tests performed which do not vary with trimming rates considered, the p-values obtained as a function of trimming rate remain the same across all sixteen stocks.

This is an interesting outcome because the non-centrality parameter associated with the non-central chi-square test underpinning the linearity test changes with trimming rate as shown in Figure 2. Furthermore, the ascending order of the individual tests that make up the cumulative distribution functions listed in Figure 1 also vary as a function of trimming rate – see Table 9. In Table 9, columns 3 to 6 contain the listing in ascending order of the array indexing of the individual tests making up the CUSUM linearity test. Each index value listed in Table 9 denotes a trispectral estimate with different tri-frequency combination. The sorting was done according to the confidence levels obtained from each individual application of the non-central chi-squared test with two degrees of freedom and non-centrality parameter that varied with trimming rate as shown in Figure 2. The difference in index values listed in columns 3 to 6 of Table 9 for the different trimming rates indicates that the contribution of the trispectral density as a function of tri-frequency changes with the trimming rate. Thus, notwithstanding the variation in non-centrality parameter value and tri-spectral structure (e.g. density) as a function of trimming rate, we still obtain very similar p-values for the CUSUM linearity tests. Recall that this arises because the output quantiles associated with the input quantile of 0.9 are the same for all trimming rates considered, as indicated in Figure 1. These observations extend to all sixteen ASX stocks and trimming rates considered.

[INSERT TABLE 9 ABOUT HERE]

It is also instructive to compare the cumulative distribution function of the linearity test associated with the ten minute average returns data listed in Figure 1 with that associated with the one minute returns data which is depicted in Figure 3. Recall that the main difference between the two returns time series is the much larger sample size associated with the one minute returns data (e.g. 927,776 observations) when compared with the ten minute averaged returns data (e.g. 92,777 observations). This produces, in turn, more individual tests in the case of the one minute returns data when compared with the ten minute returns data – e.g. 2,920 versus 73 trispectral estimates or individual tests, respectively. It is clear from inspection of both figures that the cumulative distribution function associated with the linearity test statistic applied to the one minute returns data (i.e. Figure 3) is much smoother when compared with the cumulative distribution function associated with the ten minute average returns data displayed in Figure 1. This finding also extends to all sixteen stocks.

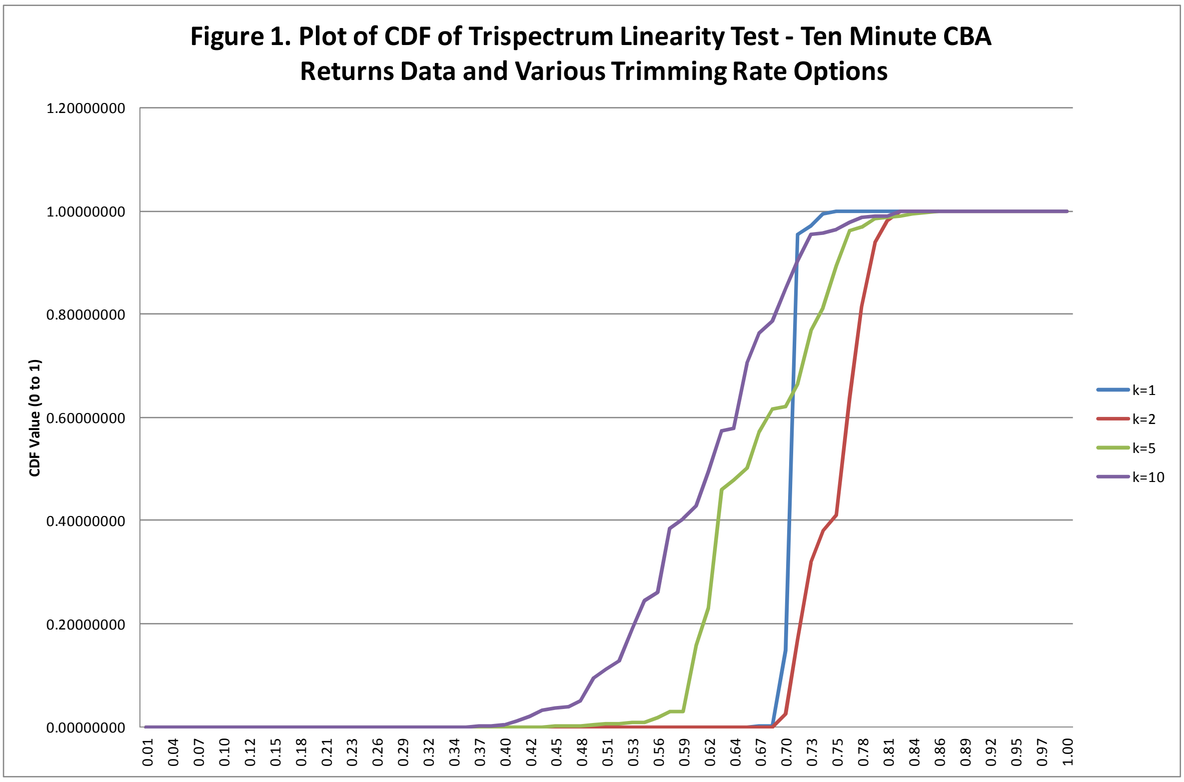


Figure 1. Plot of CDF of Trispectrum Linearity Test - Ten Minute CBA Returns Data and Various Trimming Rate Options

Figure 2. Plot of Trispectrum Linearity Test Chi-Square Noncentrality Parameter as a Function of Trimming Rate - CBA Ten Minute Returns Data

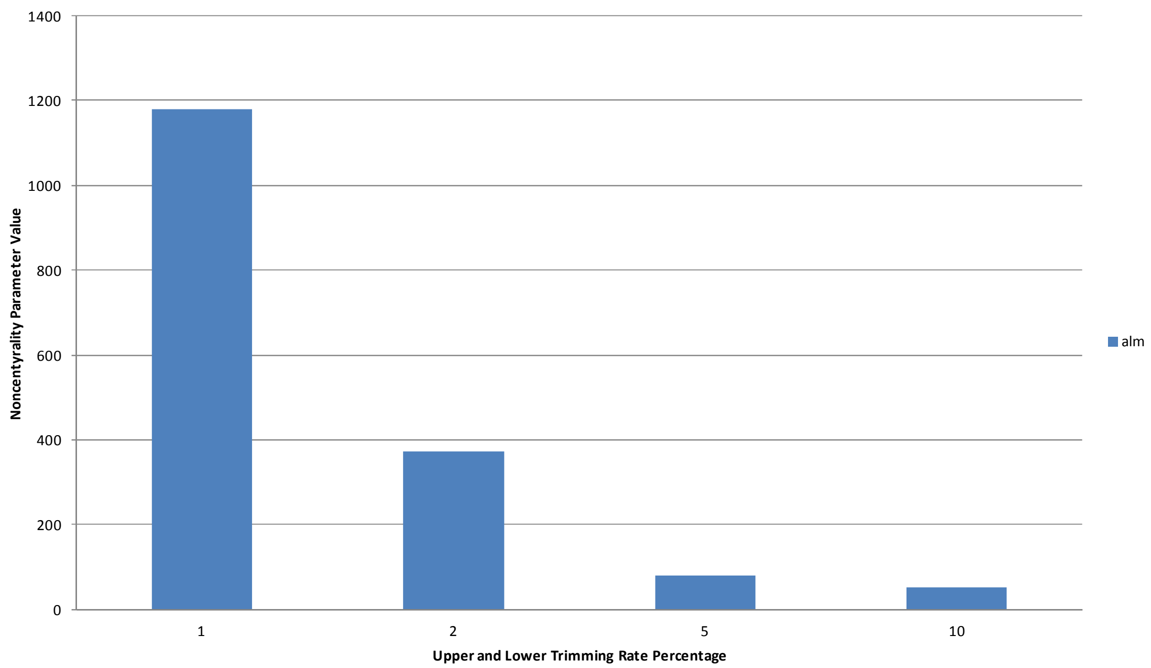
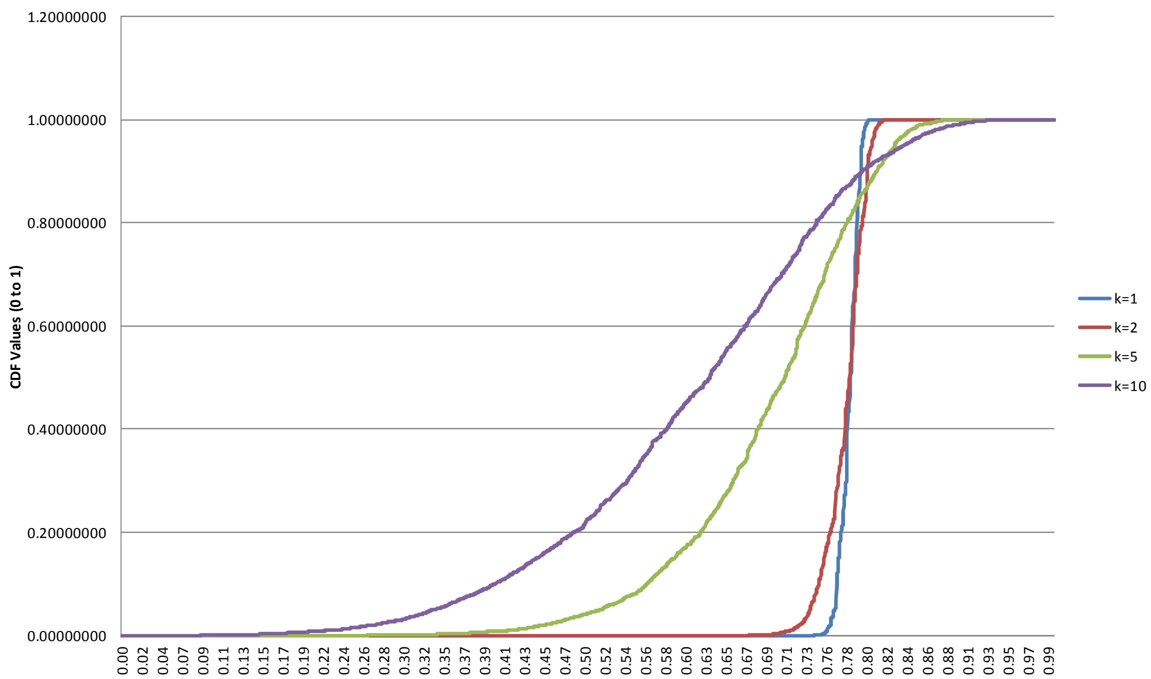


Figure 3. Plot of CDF of Trispectrum Linearity Test - One Minute CBA Returns Data and Various Trimming Rate Options



The results from the application of the Bonferroni test procedure for the Gaussianity, linearity and time reversibility tests to the trimmed ten minute average returns data are documented in Tables 10, 11 and 12, respectively. It is evident from inspection of these tables that a significant number of individual p-values are less than the associated Bonferroni critical values of 0.001370, 0.000685 and 0.000137 for the 10%, 5% and 1% levels of significance. Note that these Bonferroni critical values are larger in magnitude (i.e. less significant) than the critical values associated with the one minute returns data which were 0.000034, 0.000017 and 0.000003, respectively. The larger critical values in the case of the ten minute average returns reflect the smaller number of trispectral estimates in the principal domain and total number of individual tests performed when compared with the one minute returns data.

The results reported in Tables 10, 11 and 12 clearly point to rejections of the null hypotheses of Gaussianity, linearity and time reversibility for all sixteen stocks and across all trimming rates specified in the tables, except in certain cases relating to the time reversibility test and trimming rate of 10% which will be discussed below.

[INSERT TABLES 10, 11 AND 12 ABOUT HERE]

Recall from inspection of the kurtosis values in Table 7 that trimming rates of 5% seemed to be indicated for all sixteen stocks. If we adopt a trimming rate of 5%, then we secure strong rejections of the various null hypotheses for the sixteen stocks in accordance with the Bonferroni test procedure. Note that these particular cases are highlighted in red font in Tables 10, 11 and 12. Specifically, in the case of the linearity test, it is apparent from inspection of Table 11 that we secure rejections of the null hypothesis of linearity with rejection percentages ranging from a low of (12.33%, 10.96%, 10.96%) for AMC to highs of (27.40%, 27.40%, 23.29%) for BHP and (27.40%, 26.03%, 24.66%) for BIL. For the time reversibility test, the findings reported in Table 12 indicate that we secure rejections of the null hypothesis of time reversibility with percentage rejections ranging from lows of (6.85%, 6.85%, 6.85%) for NCM and (8.22%, 5.48%, 4.11%) for SGB to highs of (32.88%, 31.51%, 27.40%) for BHP and (30.14%, 30.14%, 28.77%) for BIL. Furthermore, from inspection of Table 10, we secure strong rejection of the null hypothesis of Gaussianity with rejection percentages ranging from a low of (75.34%, 71.23%, 69.86%) for ANZ to highs of (89.04%, 89.04%, 87.67%) for LLC and (89.04%, 87.67%, 83.56%) for NCM.

Once again, the results in Table 10 indicate that, in the case of the Gaussianity test, the total number of rejections is of an order of magnitude larger than the comparable numbers associated with the linearity and time reversibility test results cited in Tables 11 and 12. Moreover, comparison of Tables 4 and 10 indicate that the process of aggregation to obtain the ten minute average returns did not appear to extinguish the presence of non-Gaussianity in the aggregated returns data. Specifically, the larger magnitude of the percentage rejections outlined in Table 10 when compared with those listed in Table 4 suggest that non-Gaussianity became more pronounced in qualitative terms particularly at trimming rates of 5% and 10% in the case of the ten minute average returns. This conclusion can also be extended to the results from the linearity test when comparative assessment is made of the percentage rejection rates in Tables 5 and 11 especially at the 5% and 10% rates of trimming.

In the case of the time reversibility test, the percentage rejection rates for trimming rates in the range of 1% to 5% for the ten minute average returns remained broadly comparable to those in the range 1% to 10% in the case of the one minute returns. However, in the case of trimming at 10% for the ten minute average returns, the possibility of failing to reject time reversibility in the case of a number of stocks is raised – notably, CBA (at 5% and 1% levels of significance), NAB, SGB, and STO and WPL (at 1% levels of significance). This indicates that excessive trimming could potentially destroy time irreversible probabilistic structure.

This can be contrasted with the case of the Gaussianity test where excessive trimming appears to lead to increased rejections of the null hypothesis of Gaussianity as the excessive trimming produces increasingly thin tailed empirical distributions, at least in the case of the one minute returns. Evidence supporting this can be found in bold font in Table 4 for a trimming rate of 15%. There is also some evidence of a similar situation arising with the Bonferroni linearity test in the case of the ten minute average returns – for example, see the increased percentage rejections associated with 10% rate of trimming over the results associated with 5% trimming rates for AGL, AMC, ANZ, CBA, LLC, NAB, NCM, QBE, SGB, TAH and WPL in Table 11. These results are also highlighted in bold font in Table 11.

Note, however, that the results associated with the Bonferroni linearity test only emerge in the aggregated ten minute average returns and is not evident at all in the one minute return results listed in Table 5. Thus, this could be a consequence of the aggregation process and also might reflect the much less smooth cumulative distribution functions associated with the aggregated ten minute average returns indicated in Figure 1 when compared with the much smoother results cited in Figure 3 for the higher frequency one minute returns data. Indeed, this performance can be more generally seen also in the patterns of percentage rejections associated with the Bonferroni tests listed in Table 10-12 when compared with those listed in Tables 4-6. Specifically, in the former tables, there is much less variability in percentage rejection patterns ***across*** significance levels associated with particular trimming rates than is evident in the same patterns in Tables 4-6. For example, it not uncommon to experience the same percentage rejection rates for different levels of significance for especially trimming rates of 1% and 2% in the case of the ten minute returns. This can be observed, for example, in the results for BIL in Table 10, AMC in Table 11 and CBA in Table 12 for trimming rates of 1% and 2%. Moreover, many other similar examples also occur in these particular tables.

In the case of the linearity test, it is also apparent from inspections of Tables 5 and 11 that there is also much less variability or even perverse movements in percentage rejection patterns ***across*** trimming rates associated with particular levels of significance. Examples of the former include the results for ANZ and STO for trimming rates of 2% and 5% and

5% and 10%, respectively, in Table 11 and an example of the latter is the results for BHP in Table 11 for trimming rates of 2%, 5% and 10%. In this latter case, the perverse movement encompasses percentage rejection rates that increase for 5% trimming over the results associated with 2% trimming and then fall for a trimming rate of 10%. It should also be noted that this particular phenomenon is not observed in the results associated with the Gaussianity and time reversibility tests applied to the ten minute average returns data.

More generally, it should be recognized that the smoothness in the cumulative distribution function of the linearity test as well as the superior statistical performance of the tests at especially the 5% and 1% levels of significance and also at trimming rates of 1%, 2% and 5% for the one minute returns data is linked to the much larger sample size associated with the one minute returns data. This larger sample size infers three particular benefits. First, the larger number of tri-frequencies (and individual tests) in the principal domain of the trispectrum is advantageous for calculating cumulative distribution functions. Second, the larger sample size also provides a better approximation to asymptotic theory with a greater degree of frame averaging involved when applying the analysis to the higher frequency one minute returns data (e.g. 15,462 frames) when compared with the ten minute average returns data (e.g. 5,154 frames). The greater degree of frame averaging works to reduce the variance of each trispectral estimate. Third, the frame length associated with the one minute returns of 60 is greater than that associated with the ten minute averaged returns of 18. The larger frame size will increase the resolution bandwidth in the case of the one minute returns when compared with the ten minute average returns, enhancing discrimination between adjacent tri-frequencies in the case of the one minute returns. Further discussion of the choice of frame length and trade-off between variance reduction and resolution bandwidth is contained in Dalle Molle and Hinich (1995, Section C). Overall, these factors combine to provide better discrimination of both upper tail performance and performance at lower trimming rates for the Bonferroni tests, in particular, in the case of the one minute returns.

7. CONCLUSIONS

In this paper, we have introduced three nonparametric trispectrum based tests that are capable of testing the hypothesis that an observed time series is Gaussian, linear and time reversible. Such tests, we believe, are essential to conduct in the context of statistical analysis of high frequency financial and currency exchange time series data. The trispectrum Gaussianity and linearity tests we propose are novel extensions of the Hinich (1982) bispectrum tests for Gaussianity and linearity. The trispectrum time reversibility test builds upon the Hinich and Rothman (1998) bispectrum test for time reversibility.

We applied these tests to investigate whether asset price returns data are consistent with the assumptions of Gaussianity, linearity and time reversibility. Because the trispectrum involves the spectral decomposition of kurtosis, these tests enable us to investigate, directly, how leptokurtosis in the source asset price returns time series affects the underlying linearity and time reversibility properties of the data generating mechanism.

Three trispectrum tests were applied to a number of minute-to-minute Australian stocks selected from the top 50 companies in terms of market capitalization. The tests showed clearly that the returns were non-Gaussian, nonlinear and were not time reversible. Results such as these falsify, for example, the Black-Sholes-Merton model of option pricing as a valid and parsimonious representation of asset price returns data. The rejections of linearity and time reversibility also falsify the 'jump-diffusion' model (e.g. Weiner process with additive Poisson jump process) used in the Black-Sholes-Merton option pricing model. More recent variants of such models are also subject to this problem.

All of the returns data displayed significant evidence of kurtosis which poses problems for the large sample convergence properties of the trispectral estimates. Trimming was therefore employed in order to improve the validity of the asymptotic properties of the trispectral estimates and tests. The possibility and potential characteristics of excessive trimming was also identified. While, as might be expected, the trimming operations moved the skewness and kurtosis values significantly closer to their Gaussian ideals, the tests applied to this data confirmed strongly that the data generating mechanism of the returns was non-Gaussian, nonlinear and time-irreversible.

A Bonferroni test procedure was also used to investigate the pervasiveness of the rejection patterns. The classical Bonferroni inequality was used to enumerate the test. While rejection of the joint null hypotheses of Gaussianity, linearity or time reversibility according to this test procedure only required one p-value from a marginal (i.e. individual tri-frequency) test to be less than or equal to the relevant Bonferroni critical value, the empirical results indicated that many individual tests fulfilled this criterion especially after consideration was taken into account of the possibility of excessive rates of trimming.

We also examined how sensitive our results were to the time frequency of observation of the underlying data set by applying the tests to a more aggregated time series obtained by averaging across ten one minute returns in order to obtain a time series of average ten minute returns. In general, conclusions made in relation to trispectral analysis of higher frequency one minute returns data continued to hold although some evidence emerged relating to the potential of excessive trimming to destroy time irreversible probabilistic structure. We also found that in the case of the aggregated ten minute average returns, the kurtosis values were of a lower order of magnitude than was the case with the one minute returns data, thereby also requiring lower rates of trimming. Finally, the larger sample size associated with the high frequency one minute returns data enabled better discrimination of both upper tail performance and performance at lower trimming rates for the Bonferroni tests.

Overall, these results indicate that the rejections were not secured by a small number of individual test outcomes driven by outliers (i.e. jumps) in the returns data. The scope of the rejections, instead, point to significant statistical structure operating over the inter-quartile range associated with mean or 'diffusion' properties of the process. This is supported further by the use of reasonably stringent rates of trimming which has the effect of controlling for the presence of outliers in the data. As such, the rejection of the hypotheses of Gaussianity, linearity and time reversibility pointed to statistically significant 'nonlinear-in-mean' structure operating over 'central' portions of the empirical distribution function of the returns data.

The finding of 'nonlinear in mean' structure poses serious implications for how 'volatility' modeling using ARCH/GARCH or SV models is implemented. In particular, application of volatility models in which the mean or 'drift' properties are modeled by linear processes can constitute model misspecification. At the very least, models of the mean behavior of the process have to be able to accommodate the statistically significant nonlinear structure found to be present in the returns data. Failure to do so will mean that this nonlinear structure ends up in the residuals, making it very difficult (if not impossible) to disentangle it from genuine multiplicative non-linear structure, conventionally associated with conditional heteroscedasticity.

The finding of nonlinear dependencies in the returns data that is analogous to statistically significant tri-correlation structure also poses problems when focusing purely on the second order volatility measure encompassed in conventional standard deviation based measures. Not only does the higher order structure significantly contribute to the time evolution of the returns data itself, but, in amplitude terms, can dominate the contribution of the conventional second order standard deviation measure emphasized in the broader finance literature. For example, the use of correlations to capture dependencies between the evolution of asset price returns and volatility, as typically used in SV models, is no longer sufficient if there is evidence of higher order dependencies in the data pointing to the presence of statistically significant tri-correlation structure.

APPENDIX A: OUTLINE OF THE BONFERRONI TEST METHODOLOGY.

In Section 5 we utilize a test methodology based upon the Classical Bonferroni procedure to gauge how pervasive the rejection patterns of the test statistics outlined in Section 4 are across valid tri-frequencies in the principal domain. The statistical methodology underpinning the test procedure is based on the Union-Intersection (UI) method proposed by Roy (1953) that can be used to construct alternative joint tests of the Gaussianity, linearity and time reversibility hypotheses – also see (Miller, 1966, Chapter 2; Savin, 1980, 1984; Hochberg and Tamhane, 1987, Chapters 1 and 2). In this Appendix, we provide a formal outline of the test methodology that was employed in Section 5.

The (UI) procedure involves the construction of a joint hypothesis test from a series of individual hypothesis tests. Suppose we have a number of individual hypotheses of the form

$$\begin{aligned} H_{0_k} &= TS_k \\ H_{1_k} &\neq TS_k \end{aligned} \quad for\ k = \{1,2,\dots,D\}. \tag{A.1}$$

The individual tests refer to those that are involved in the CUSUM tests defined in (9) and (10) or each $W(k_1,k_2,k_3)$ associated with the linearity test defined in Section 4. Note that the number of individual tests D corresponds to the number of triple frequencies in the principal domain Ω.

The (UI) procedure involves combining the individual hypotheses to form an overall joint hypothesis H_0, defined as an intersection of a family of individual hypotheses. In the current context, this is given by

$$H_0 = \bigcap_{k=1}^{D} H_{0_k} \tag{A.2}$$

where D is the total number of tri-frequency tests to be performed. Note that H_0 is interpreted as a 'global' null in which all H_{0_k} are true against the alternative H_1 which is interpreted as 'not H_0', (Hochberg and Tamhane, 1987, p. 28; Rom, 1990; Samuel-Cahn, 1996).

The rejection region for H_0 corresponds to the union of rejection regions for the individual hypotheses H_{0_k}, giving $H_1 = \bigcup_{k=1}^{D} H_{1_k}$. This means that H_0 is rejected if and only if at least one of the H_{0_k}'s is rejected (Hochberg and Tamhane, 1987, p. 29). Given the individual tri-frequency tests alluded to above that underpin the individual tests of H_{0_k} versus H_{1_k}, we can express the resulting rejection region for each test as

$$TS_k > \xi_k \ for\ 1 \le k \le D \tag{A.3}$$

with the critical constants ξ_k constrained so that the rejection region of the (UI) test of H_0 is the union of rejection regions of (A.3). For the rejection region of the joint test to have size α, the ξ_k's must satisfy

$$\Pr_{H_0}\{TS_k > \xi_k\} = \alpha \tag{A.4}$$

for some $k = 1,2,\ldots,D$ (Hochberg and Tamhane, 1987, p. 29).

We can simplify (A.3)-(A.4) further by assuming that the 'D' testing problems are to be treated symmetrically with regard to the relative importance attached to Type I and Type II errors, thereby implying that the marginal levels $\Pr_{H_{0k}}\{TS_k > \xi_k\}$ should be the same for $k = 1,2,\ldots,D$. Moreover, the TS_k's have the same marginal distribution under H_{0_k}, corresponding to either a $\chi_2^2(0)$, $\chi_2^2(\lambda)$ or $\chi_1^2(0)$ distribution, for the Gaussianity, linearity or time reversibility tests, respectively, and which are subsequently transformed into uniform (0,1) variates under these null hypotheses. We can therefore set the $\xi_k's$ to the same value (Hochberg and Tamhane, 1987, pp. 29-30). Thus after setting $\xi_k = \xi,$ for all k in (A.3), the (UI) test will reject if

$$\max_{1\le k\le D} TS_k > \xi \tag{A.5}$$

with ξ being chosen so that

$$\Pr_{H_0}\left\{\max_{1\le k\le D} TS_k > \xi\right\} = \alpha \tag{A.6}$$

(Hochberg and Tamhane, 1987, p. 30).

In this paper we use the classical Bonferroni inequality to enumerate (A.6). This involves using the inequality

$$\Pr\left\{\bigcup_{i=1}^{D}\left(P_i \le \alpha/D\right)\right\} \le \alpha \tag{A.7}$$

which guarantees that the probability of rejecting at least one hypothesis when all are true is not greater than α (Hochberg and Tamhane, 1987, p. 3, 7, 28; Simes, 1986). This procedure leads to a critical value for ξ given by (α / D) where α is a user specified level of significance and D is the total number of tri-frequency tests.

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Table 1 Details of Sixteen Australian ASX Companies

ASX Code	Name	Core Activities	Stock/Dividend Split Details
AGL	Australian Gas and Lighting Ltd	Retail Energy and Fuel	N.A.
AMC	Amcor Ltd	Packaging	N.A.
ANZ	Australian and New Zealand Banking Group Ltd	Banking and Financial Services	N.A.
BHP	BHP-Billiton Ltd	Mineral and Hydrocarbon Exploration, Production and Processing	BHP-Billiton merger on 29/6/2001 (from $21.418 to $10.495 at 10:00:00 on 29/6/2001).
BIL	Brambles Ltd	Commercial and Business Support (e.g. Packaging, Containers, Storage)	merger with UK services group GFN on 8/8/2001 (from $45.679 to $10.703 at 10:00:00 on 8.8.2001)
CBA	Commonwealth Bank of Australia Ltd	Banking and Financial Services	N.A.
LLC	Lend Lease Ltd	Retail Property and Asset Management and Development and Construction	1 to 1 bonus share offer in December 1998 (from $38.675 to $19.983 at 10:00:00 on 1/12/1998)
NAB	National Australia Bank Ltd	Banking and Financial Services	N.A.
NCM	Newcrest Mining Ltd	Exploration, Development, Mining and Sale of Gold	N.A.
QBE	QBE Insurance Group Ltd	Insurance and Financial Services	N.A.
SGB	St George Bank	Banking and	N.A.

	Ltd	Financial Services	
STO	Santos Group Ltd	Exploration, Development, Mining, Transportation and Marketing of Petroleum and Gas Products	N.A.
TAH	Tabcorp Holdings Ltd	Provision of Gambling and Other Entertainment Services	N.A.
WBC	Westpac Banking Corporation Ltd	Banking and Financial Services	N.A.
WOW	Woolworths Ltd	Food, General Merchandise and Specialty Retailing	N.A.
WPL	Woodside Petroleum Ltd	Exploration, Development, Mining,	N.A.

Table 2. Descriptive Statistics of the Sixteen Australian Returns.

ASX Code	Mean	Sigma	Skew	Kurtosis	Max	Min
AGL						
$\kappa = 0$	0.132E-03	0.084	-1.940	688.00	6.530	-7.430
$\kappa = 1$	-0.105E-03	0.050	0.030	7.80	0.214	-0.210
$\kappa = 2$	-0.174E-03	0.043	-0.040	4.63	0.142	-0.143
$\kappa = 5$	-0.170E-03	0.031	-0.060	1.93	0.075	-0.077
$\kappa = 10$	-0.104E-03	0.018	-0.050	0.55	0.036	-0.037
$\kappa = 15$	0.251E-04	0.009	0.012	**-0.06**	0.015	-0.015
AMC						
$\kappa = 0$	-0.265E-04	0.101	-4.370	4270.00	17.400	-15.200
$\kappa = 1$	0.407E-04	0.054	0.047	6.37	0.218	-0.213
$\kappa = 2$	-0.119E-04	0.047	0.005	4.01	0.152	-0.152
$\kappa = 5$	-0.448E-04	0.036	-0.013	2.01	0.090	-0.090
$\kappa = 10$	-0.946E-05	0.022	-0.009	0.74	0.044	-0.044
$\kappa = 15$	-0.130E-04	0.008	-0.008	0.07	0.014	-0.014
ANZ						
$\kappa = 0$	0.143E-03	0.093	-4.060	1660.00	8.250	-11.500
$\kappa = 1$	0.176E-03	0.046	0.006	4.10	0.167	-0.166
$\kappa = 2$	0.160E-03	0.042	-0.006	2.43	0.123	-0.123
$\kappa = 5$	0.163E-03	0.034	-0.004	0.79	0.077	-0.077
$\kappa = 10$	0.180E-03	0.025	0.004	**-0.20**	0.046	-0.046
$\kappa = 15$	0.195E-03	0.018	0.013	**-0.68**	0.029	-0.029
BHP						
$\kappa = 0$	0.617E-04	0.090	3.300	7380.00	21.200	-20.100
$\kappa = 1$	0.494E-04	0.038	0.028	5.20	0.147	-0.146
$\kappa = 2$	0.474E-04	0.034	0.027	2.84	0.103	-0.102
$\kappa = 5$	0.189E-04	0.026	0.008	0.70	0.059	-0.059
$\kappa = 10$	-0.217E-05	0.020	0.001	**-0.29**	0.035	-0.035
$\kappa = 15$	-0.538E-05	0.015	-0.002	**-0.73**	0.023	-0.023
BIL						
$\kappa = 0$	0.118E-03	0.077	-4.160	3460.00	12.900	-13.000
$\kappa = 1$	0.190E-03	0.037	0.079	11.50	0.172	-0.170
$\kappa = 2$	0.182E-03	0.029	0.080	7.06	0.109	-0.107

κ = 5	0.154E-03	0.017	0.080	2.63	0.045	-0.043
κ = 10	0.983E-04	0.008	0.056	0.52	0.016	-0.016
κ = 15	0.598E-04	0.004	0.049	**-0.15**	0.007	-0.007
CBA						
κ = 0	0.148E-03	0.116	-1.790	1260.00	9.200	-9.470
κ = 1	0.300E-03	0.040	0.052	4.76	0.152	-0.149
κ = 2	0.278E-03	0.036	0.032	2.62	0.108	-0.106
κ = 5	0.250E-03	0.029	0.018	0.76	0.065	-0.064
κ = 10	0.244E-03	0.021	0.019	**-0.29**	0.038	-0.037
κ = 15	0.236E-03	0.015	0.021	**-0.78**	0.025	-0.024
LLC						
κ = 0	0.571E-04	0.066	-4.470	711.00	3.800	-6.810
κ = 1	0.113E-03	0.033	0.074	9.13	0.149	-0.146
κ = 2	0.879E-04	0.027	0.036	5.06	0.094	-0.093
κ = 5	0.560E-04	0.019	0.006	1.77	0.046	-0.046
κ = 10	0.578E-04	0.011	0.009	0.24	0.022	-0.022
κ = 15	0.114E-03	0.007	0.065	**-0.38**	0.011	-0.011
NAB						
κ = 0	0.106E-03	0.129	-2.460	2200.00	13.900	-15.400
κ = 1	0.139E-03	0.040	0.009	4.43	0.152	-0.152
κ = 2	0.140E-03	0.036	0.011	2.35	0.108	-0.107
κ = 5	0.149E-03	0.030	0.016	0.59	0.066	-0.065
κ = 10	0.127E-03	0.022	0.009	**-0.39**	0.040	-0.039
κ = 15	0.111E-03	0.017	0.006	**-0.85**	0.026	-0.026
NCM						
κ = 0	0.158E-03	0.170	1.320	737.00	15.700	-14.500
κ = 1	0.112E-03	0.089	0.042	10.60	0.409	-0.405
κ = 2	0.638E-04	0.072	0.014	6.09	0.255	-0.254
κ = 5	0.181E-04	0.046	-0.005	2.43	0.117	-0.117
κ = 10	0.457E-04	0.023	0.007	0.75	0.047	-0.047
κ = 15	0.507E-04	0.009	0.027	0.09	0.016	-0.015
QBE						
κ = 0	0.123E-03	0.133	-1.340	1950.00	18.000	-20.100
κ = 1	0.755E-04	0.066	0.039	9.61	0.298	-0.294
κ = 2	0.394E-04	0.054	0.007	5.44	0.188	-0.187
κ = 5	-0.163E-04	0.036	-0.030	2.10	0.090	-0.091
κ = 10	0.697E-04	0.021	0.006	0.63	0.041	-0.041
κ = 15	0.104E-03	0.009	0.054	**-0.03**	0.015	-0.149

SGB						
$\kappa = 0$	0.149E-03	0.099	1.590	2710.00	14.000	-15.500
$\kappa = 1$	0.236E-03	0.047	0.071	7.77	0.201	-0.197
$\kappa = 2$	0.228E-03	0.041	0.065	4.76	0.138	-0.135
$\kappa = 5$	0.158E-03	0.029	0.027	1.90	0.071	-0.070
$\kappa = 10$	0.114E-03	0.017	0.012	0.41	0.034	-0.034
$\kappa = 15$	0.193E-03	0.009	0.082	**-0.23**	0.016	-0.015
STO						
$\kappa = 0$	0.122E-03	0.125	-0.589	1250.00	11.000	-11.800
$\kappa = 1$	0.262E-04	0.064	0.011	6.36	0.254	-0.252
$\kappa = 2$	-0.140E-04	0.056	-0.017	4.19	0.182	-0.183
$\kappa = 5$	-0.123E-04	0.043	-0.019	2.05	0.105	-0.106
$\kappa = 10$	0.327E-04	0.024	-0.005	0.71	0.049	-0.049
$\kappa = 15$	0.560E-04	0.009	0.024	0.09	0.017	-0.016
TAH						
$\kappa = 0$	0.155E-03	0.108	-1.870	741.00	8.50	-8.720
$\kappa = 1$	0.134E-03	0.056	0.082	8.98	0.251	-0.245
$\kappa = 2$	0.849E-04	0.047	0.036	5.24	0.162	-0.160
$\kappa = 5$	0.850E-05	0.032	-0.014	2.04	0.080	-0.080
$\kappa = 10$	0.400E-04	0.018	-0.003	0.56	0.037	-0.037
$\kappa = 15$	0.215E-03	0.009	0.131	**-0.03**	0.016	-0.015
WBC						
$\kappa = 0$	0.144E-03	0.118	-0.307	1210.00	9.870	-9.530
$\kappa = 1$	0.198E-03	0.047	0.032	4.94	0.179	-0.177
$\kappa = 2$	0.187E-03	0.042	0.024	2.87	0.128	-0.127
$\kappa = 5$	0.165E-03	0.033	0.014	0.97	0.077	-0.076
$\kappa = 10$	0.162E-03	0.024	0.016	**-0.07**	0.045	-0.044
$\kappa = 15$	0.172E-03	0.017	0.024	**-0.55**	0.028	-0.027
WOW						
$\kappa = 0$	0.178E-03	0.140	-2.730	3930.00	21.100	-20.900
$\kappa = 1$	0.261E-03	0.056	0.024	7.08	0.230	-0.229
$\kappa = 2$	0.236E-03	0.049	0.010	4.44	0.161	-0.160
$\kappa = 5$	0.299E-03	0.035	0.053	1.81	0.087	-0.085
$\kappa = 10$	0.245E-03	0.021	0.052	0.45	0.042	-0.041
$\kappa = 15$	0.150E-03	0.011	0.049	**-0.15**	0.019	-0.018
WPL						
$\kappa = 0$	0.187E-03	0.120	0.710	1380.00	12.100	-10.500

$\kappa = 1$	-0.602E-04	0.053	0.071	7.10	0.225	-0.218
$\kappa = 2$	-0.145E-03	0.046	-0.008	4.13	0.151	-0.150
$\kappa = 5$	-0.179E-03	0.034	-0.037	1.51	0.081	-0.082
$\kappa = 10$	-0.180E-03	0.022	-0.052	0.19	0.041	-0.042
$\kappa = 15$	-0.673E-04	0.013	-0.024	**-0.38**	0.022	-0.023

Table 3. Gaussianity, '99 Percent Quantile' Linearity and Time Reversibility 'CUSUM' Test p – Values for the Australian BHP Stock.

Stock Code	Trispectrum Quantile Gaussianity Test p-values	Trispectrum Quantile Nonlinearity Test p-values	Trispectrum Time Reversibility Test p-values
AGL			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
AMC			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
ANZ			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
BHP			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
BIL			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000

CBA			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
LLC			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
NAB			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
NCM			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
QBE			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
SGB			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
STO			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000
$\kappa=5$	0.00000	0.00000	0.00000
$\kappa=10$	0.00000	0.00000	0.00000
$\kappa=15$	0.00000	0.00000	0.00000
TAH			
$\kappa=1$	0.00000	0.00000	0.00000
$\kappa=2$	0.00000	0.00000	0.00000

$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
WBC			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
WOW			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000
WPL			
$\kappa = 1$	0.00000	0.00000	0.00000
$\kappa = 2$	0.00000	0.00000	0.00000
$\kappa = 5$	0.00000	0.00000	0.00000
$\kappa = 10$	0.00000	0.00000	0.00000
$\kappa = 15$	0.00000	0.00000	0.00000

Table 4. Classical Bonferroni Gaussianity Test Statistics for the Sixteen Australian Returns.

ASX Code	10% BF - % of p-values in PD < 0.000034	5% BF - % of p-values in PD < 0.000017	1% BF - % of p-values in PD < 0.000003
AGL			
$\kappa = 1$	94.86%	94.59%	93.77%
$\kappa = 2$	83.56%	83.01%	81.13%
$\kappa = 5$	55.38%	53.87%	50.79%
$\kappa = 10$	35.82%	34.18%	30.45%
$\kappa = 15$	34.97%	32.74%	28.63%
AMC			
$\kappa = 1$	89.04%	88.22%	86.68%
$\kappa = 2$	75.68%	74.55%	72.12%
$\kappa = 5$	55.07%	53.66%	50.00%
$\kappa = 10$	43.94%	42.26%	38.97%
$\kappa = 15$	43.25%	41.75%	37.60%
ANZ			

κ = 1	77.64%	76.23%	73.70%
κ = 2	62.77%	61.13%	57.40%
κ = 5	42.64%	40.82%	36.61%
κ = 10	34.73%	32.26%	27.19%
κ = 15	**39.73%**	**37.67%**	**32.53%**
BHP			
κ = 1	85.89%	85.17%	83.63%
κ = 2	75.27%	74.11%	71.06%
κ = 5	55.07%	53.15%	48.90%
κ = 10	41.44%	39.49%	35.38%
κ = 15	**43.29%**	**40.96%**	**35.68%**
BIL			
κ = 1	96.30%	95.89%	95.27%
κ = 2	89.86%	89.32%	87.95%
κ = 5	72.50%	71.13%	67.71%
κ = 10	50.68%	48.60%	43.94%
κ = 15	46.37%	43.84%	39.11%
CBA			
κ = 1	82.26%	81.51%	78.90%
κ = 2	66.51%	64.73%	61.34%
κ = 5	46.03%	43.87%	39.79%
κ = 10	35.68%	33.42%	28.56%
κ = 15	**40.14%**	**36.92%**	**31.82%**
LLC			
κ = 1	95.17%	94.73%	94.04%
κ = 2	84.69%	83.94%	81.88%
κ = 5	56.88%	54.66%	50.21%
κ = 10	36.61%	34.25%	30.24%
κ = 15	36.20%	34.21%	29.59%
NAB			
κ = 1	86.30%	85.62%	83.60%
κ = 2	70.38%	68.84%	65.07%
κ = 5	46.54%	43.87%	39.45%
κ = 10	36.85%	34.45%	30.51%
κ = 15	**40.48%**	**38.25%**	**32.67%**
NCM			
κ = 1	97.16%	97.09%	96.61%

κ = 2	91.06%	90.55%	89.18%
κ = 5	67.60%	66.13%	62.57%
κ = 10	47.77%	45.99%	42.23%
κ = 15	**49.90%**	**47.98%**	**43.73%**
QBE			
κ = 1	96.64%	96.40%	95.86%
κ = 2	90.45%	89.97%	88.56%
κ = 5	62.95%	61.37%	58.36%
κ = 10	41.78%	40.31%	36.06%
κ = 15	**42.33%**	40.14%	35.48%
SGB			
κ = 1	95.79%	95.41%	94.62%
κ = 2	86.68%	85.96%	84.04%
κ = 5	58.77%	57.12%	53.97%
κ = 10	32.98%	31.54%	27.81%
κ = 15	31.40%	28.84%	24.79%
STO			
κ = 1	91.92%	91.47%	90.48%
κ = 2	81.82%	81.10%	79.38%
κ = 5	60.21%	58.39%	54.69%
κ = 10	42.57%	40.89%	36.51%
κ = 15	41.71%	40.24%	35.82%
TAH			
κ = 1	96.82%	96.61%	96.06%
κ = 2	87.77%	86.99%	85.51%
κ = 5	60.65%	59.21%	56.20%
κ = 10	39.21%	36.71%	32.77%
κ = 15	38.12%	36.23%	32.36%
WBC			
κ = 1	84.76%	83.46%	81.51%
κ = 2	69.55%	68.39%	65.82%
κ = 5	44.83%	43.01%	38.42%
κ = 10	32.57%	30.82%	26.54%
κ = 15	**36.85%**	**34.35%**	**28.63%**
WOW			
κ = 1	91.37%	90.99%	89.38%
κ = 2	79.93%	78.80%	76.51%

κ = 5	56.10%	54.38%	50.89%
κ = 10	41.30%	38.97%	35.03%
κ = 15	**44.01%**	**41.99%**	**37.57%**
WPL			
κ = 1	92.12%	91.68%	90.72%
κ = 2	80.21%	79.49%	76.88%
κ = 5	55.48%	53.32%	50.10%
κ = 10	35.68%	33.01%	28.94%
κ = 15	**36.47%**	**34.18%**	**29.62%**

Table 5. Classical Bonferroni Linearity Test Statistics for the Sixteen Australian Returns.

ASX Code	10% BF - % of p-values in PD < 0.000034	5% BF - % of p-values in PD < 0.000017	1% BF - % of p-values in PD < 0.000003
AGL			
κ = 1	24.11%	23.77%	23.49%
κ = 2	20.82%	20.58%	20.31%
κ = 5	13.84%	13.49%	13.05%
κ = 10	7.09%	6.78%	6.64%
κ = 15	5.82%	5.51%	5.34%
AMC			
κ = 1	21.75%	21.47%	21.20%
κ = 2	19.45%	19.35%	19.11%
κ = 5	14.90%	14.55%	14.21%
κ = 10	9.79%	9.35%	9.21%
κ = 15	7.67%	7.40%	7.12%
ANZ			
κ = 1	18.46%	18.32%	18.22%
κ = 2	16.30%	15.99%	15.75%
κ = 5	7.53%	7.12%	7.02%
κ = 10	3.66%	3.39%	2.95%
κ = 15	2.16%	1.88%	1.71%
BHP			
κ = 1	19.21%	19.18%	18.87%
κ = 2	16.23%	15.96%	15.68%

$\kappa = 5$	8.39%	7.81%	7.43%
$\kappa = 10$	4.79%	4.14%	3.84%
$\kappa = 15$	3.18%	2.91%	2.81%
BIL			
$\kappa = 1$	23.84%	23.53%	23.01%
$\kappa = 2$	21.27%	20.99%	20.72%
$\kappa = 5$	13.90%	13.70%	13.42%
$\kappa = 10$	7.19%	6.88%	6.54%
$\kappa = 15$	5.31%	5.10%	5.03%
CBA			
$\kappa = 1$	19.18%	19.04%	18.90%
$\kappa = 2$	16.99%	16.71%	16.51%
$\kappa = 5$	8.56%	8.29%	7.98%
$\kappa = 10$	3.73%	3.12%	2.91%
$\kappa = 15$	2.26%	2.05%	1.61%
LLC			
$\kappa = 1$	21.75%	21.54%	21.27%
$\kappa = 2$	19.76%	19.55%	19.38%
$\kappa = 5$	12.84%	12.47%	12.16%
$\kappa = 10$	6.51%	6.23%	5.99%
$\kappa = 15$	3.80%	3.70%	3.56%
NAB			
$\kappa = 1$	16.75%	16.68%	16.58%
$\kappa = 2$	14.73%	14.38%	14.25%
$\kappa = 5$	6.30%	5.82%	5.72%
$\kappa = 10$	3.77%	3.25%	3.05%
$\kappa = 15$	2.88%	2.50%	2.12%
NCM			
$\kappa = 1$	23.32%	23.05%	22.77%
$\kappa = 2$	21.06%	20.96%	20.62%
$\kappa = 5$	15.48%	15.27%	15.03%
$\kappa = 10$	8.87%	8.56%	8.18%
$\kappa = 15$	6.82%	6.51%	6.37%
QBE			
$\kappa = 1$	22.88%	22.53%	22.26%
$\kappa = 2$	21.13%	20.65%	20.38%
$\kappa = 5$	14.83%	14.21%	13.87%

κ = 10	7.64%	7.40%	7.23%
κ = 15	6.10%	5.86%	5.82%
SGB			
κ = 1	23.84%	23.46%	23.18%
κ = 2	20.58%	20.27%	19.97%
κ = 5	12.64%	12.16%	12.05%
κ = 10	7.05%	6.92%	6.68%
κ = 15	5.21%	5.03%	4.83%
STO			
κ = 1	22.26%	21.75%	21.30%
κ = 2	19.59%	19.21%	18.87%
κ = 5	13.42%	13.12%	12.84%
κ = 10	9.01%	8.56%	8.42%
κ = 15	7.26%	6.95%	6.75%
TAH			
κ = 1	25.14%	24.69%	24.42%
κ = 2	22.40%	22.09%	21.82%
κ = 5	14.38%	14.01%	13.49%
κ = 10	7.53%	7.23%	6.99%
κ = 15	6.13%	5.99%	5.82%
WBC			
κ = 1	19.55%	19.28%	19.01%
κ = 2	16.64%	16.23%	15.96%
κ = 5	7.40%	7.12%	6.88%
κ = 10	3.73%	3.39%	3.22%
κ = 15	2.98%	2.71%	2.50%
WOW			
κ = 1	23.18%	22.71%	22.33%
κ = 2	20.45%	20.24%	19.90%
κ = 5	12.98%	12.50%	12.26%
κ = 10	6.64%	6.44%	6.34%
κ = 15	5.21%	4.90%	4.73%
WPL			
κ = 1	23.08%	22.81%	22.29%
κ = 2	20.34%	20.03%	19.69%
κ = 5	11.82%	11.37%	10.89%
κ = 10	5.62%	5.45%	5.31%

κ = 15	4.01%	3.84%	3.60%

Table 6. Classical Bonferroni (BF) Time Reversibility Test Statistics for the Sixteen Australian Returns.

ASX Code	10% BF - % of p-values in PD < 0.000034	5% BF - % of p-values in PD < 0.000017	1% BF - % of p-values in PD < 0.000003
AGL			
κ = 1	35.86%	33.80%	30.14%
κ = 2	20.03%	18.49%	15.14%
κ = 5	7.47%	6.44%	4.69%
κ = 10	3.39%	2.64%	1.71%
κ = 15	2.36%	2.12%	1.54%
AMC			
κ = 1	36.10%	34.62%	31.23%
κ = 2	20.86%	18.77%	15.72%
κ = 5	9.38%	8.15%	5.99%
κ = 10	4.21%	3.60%	2.47%
κ = 15	2.57%	2.12%	1.47%
ANZ			
κ = 1	31.85%	29.86%	25.92%
κ = 2	17.88%	16.37%	13.36%
κ = 5	6.27%	5.82%	4.62%
κ = 10	2.05%	1.68%	1.20%
κ = 15	0.72%	0.55%	0.27%
BHP			
κ = 1	45.24%	43.70%	40.31%
κ = 2	30.72%	29.04%	25.72%
κ = 5	10.07%	8.97%	6.75%
κ = 10	2.26%	1.99%	1.40%
κ = 15	1.03%	0.92%	0.58%
BIL			
κ = 1	61.16%	59.55%	57.02%
κ = 2	48.73%	47.50%	43.84%
κ = 5	25.58%	24.04%	21.61%
κ = 10	6.75%	5.96%	4.42%

κ = 15	2.84%	2.43%	1.68%
CBA			
κ = 1	43.53%	41.85%	37.98%
κ = 2	24.59%	22.91%	19.76%
κ = 5	7.29%	6.13%	4.79%
κ = 10	1.95%	1.58%	1.10%
κ = 15	0.89%	0.72%	0.48%
LLC			
κ = 1	55.96%	54.38%	51.20%
κ = 2	36.03%	34.08%	30.58%
κ = 5	11.58%	10.03%	7.40%
κ = 10	3.29%	2.53%	1.44%
κ = 15	1.47%	1.06%	0.79%
NAB			
κ = 1	46.06%	44.04%	40.99%
κ = 2	27.36%	25.41%	21.95%
κ = 5	9.08%	8.25%	6.68%
κ = 10	2.60%	2.09%	1.61%
κ = 15	0.99%	0.79%	0.65%
NCM			
κ = 1	53.97%	52.19%	49.28%
κ = 2	35.38%	33.36%	29.55%
κ = 5	14.42%	12.67%	9.86%
κ = 10	6.88%	5.96%	4.18%
κ = 15	5.89%	5.41%	4.18%
QBE			
κ = 1	57.33%	56.20%	52.98%
κ = 2	36.78%	35.03%	31.23%
κ = 5	12.74%	11.47%	8.94%
κ = 10	4.35%	3.53%	2.50%
κ = 15	3.29%	2.67%	1.51%
SGB			
κ = 1	38.97%	37.26%	33.66%
κ = 2	23.84%	22.23%	19.04%
κ = 5	8.77%	7.50%	5.48%
κ = 10	2.57%	2.40%	1.82%
κ = 15	1.37%	1.10%	0.62%

STO			
κ = 1	32.60%	30.79%	27.33%
κ = 2	19.49%	17.84%	14.62%
κ = 5	7.84%	6.92%	5.17%
κ = 10	4.14%	3.36%	2.12%
κ = 15	2.81%	2.40%	1.75%
TAH			
κ = 1	45.38%	44.04%	40.24%
κ = 2	27.53%	25.86%	21.78%
κ = 5	8.87%	7.88%	6.03%
κ = 10	4.14%	3.36%	2.47%
κ = 15	3.08%	2.47%	1.78%
WBC			
κ = 1	36.78%	34.97%	31.34%
κ = 2	20.65%	19.18%	16.61%
κ = 5	7.74%	6.82%	5.07%
κ = 10	2.98%	2.29%	1.37%
κ = 15	1.40%	1.23%	0.45%
WOW			
κ = 1	38.77%	37.12%	33.12%
κ = 2	24.93%	23.08%	19.11%
κ = 5	10.14%	8.77%	6.64%
κ = 10	4.69%	3.42%	2.36%
κ = 15	2.91%	2.40%	1.51%
WPL			
κ = 1	42.77%	40.72%	36.95%
κ = 2	26.34%	25.07%	21.23%
κ = 5	10.62%	9.38%	7.36%
κ = 10	3.15%	2.64%	1.71%
κ = 15	1.92%	1.27%	0.75%

Table 7. Descriptive Statistics of the Sixteen Australian Returns: Ten Minute Returns.

ASX	Mean	Sigma	Skew	Kurtosis	Max	Min

Code						
AGL						
κ = 0	0.132E-02	0.199	-1.220	96.90	5.100	-8.450
κ = 1	0.121E-02	0158	0.236	3.67	0.599	-0.536
κ = 2	0.663E-03	0.145	0.111	1.95	0.428	-0.404
κ = 5	0.666E-04	0.119	0.029	0.30	0.260	-0.255
κ = 10	-0.180E-03	0.092	0.006	**-0.63**	0.160	-0.159
AMC						
κ = 0	-0.267E-03	0.213	-4.760	272.00	4.610	-12.300
κ = 1	0.416E-03	0.160	0.119	3.38	0.585	-0.554
κ = 2	0.988E-04	0.147	0.052	1.72	0.421	-0.411
κ = 5	-0.269E-06	0.122	0.034	0.17	0.263	-0.257
κ = 10	-0.398E-03	0.096	0.001	**-0.68**	0.164	-0.165
ANZ						
κ = 0	0.144E-02	0.202	-4.610	342.00	5.540	-14.500
κ = 1	0.198E-02	0.151	0.053	3.41	0.549	-0.534
κ = 2	0.196E-02	0.136	0.042	1.54	0.390	-0.377
κ = 5	0.174E-02	0.114	0.009	0.01	0.241	-0.236
κ = 10	0.148E-02	0.091	-0.009	**-0.78**	0.156	-0.154
BHP						
κ = 0	0.616E-03	0.183	-0.823	70.50	3.100	-7.500
κ = 1	0.716E-03	0.138	0.074	4.90	0.537	-0.530
κ = 2	0.810E-03	0.122	0.099	2.45	0.376	-0.360
κ = 5	0.420E-03	0.097	0.032	0.37	0.215	-0.211
κ = 10	0.217E-03	0.074	0.011	**-0.66**	0.128	-0.127
BIL						
κ = 0	0.119E-02	0.161	-4.29	245.00	3.860	-9.060
κ = 1	0.150E-02	0.120	0.074	5.64	0.471	-0.463
κ = 2	0.149E-02	0.107	0.079	3.51	0.344	-0.335
κ = 5	0.154E-02	0.082	0.105	1.20	0.198	-0.187
κ = 10	0.107E-02	0.055	0.048	**-0.13**	0.105	-0.101
CBA						
κ = 0	0.148E-02	0.180	-2.330	119.00	3.430	-8.070
κ = 1	0.204E-02	0.135	0.045	3.83	0.504	-0.491
κ = 2	0.196E-02	0.122	0.023	1.75	0.353	-0.345
κ = 5	0.186E-02	0.100	0.009	0.055	0.213	-0.209
κ = 10	0.187E-02	0.080	0.013	**-0.76**	0.138	-0.133

LLC						
$\kappa = 0$	0.571E-03	0.152	-2.400	149.00	4.220	-7.340
$\kappa = 1$	0.916E-03	0.113	0.055	5.25	0.444	-0.435
$\kappa = 2$	0.929E-03	0.100	0.054	2.87	0.314	-0.305
$\kappa = 5$	0.693E-03	0.078	0.003	0.58	0.175	-0.173
$\kappa = 10$	0.712E-03	0.058	0.008	**-0.53**	0.102	-0.100
NAB						
$\kappa = 0$	0.106E-02	0.252	-2.240	290.00	10.300	-13.800
$\kappa = 1$	0.151E-02	0.156	0.034	5.16	0.618	-0.616
$\kappa = 2$	0.168E-02	0.135	0.074	2.09	0.407	-0.391
$\kappa = 5$	0.139E-02	0.109	0.026	0.09	0.232	-0.226
$\kappa = 10$	0.119E-02	0.086	0.008	**-0.76**	0.147	-0.144
NCM						
$\kappa = 0$	0.158E-02	0.403	-0.104	72.40	11.500	-11.800
$\kappa = 1$	0.196E-02	0.308	0.175	5.18	1.230	-1.160
$\kappa = 2$	0.151E-02	0.273	0.117	2.82	0.857	-0.817
$\kappa = 5$	0.563E-03	0.214	0.042	0.65	0.486	-0.475
$\kappa = 10$	-0.146E-04	0.158	0.014	**-0.46**	0.280	-0.278
QBE						
$\kappa = 0$	0.123E-02	0.323	-0.888	446.00	20.800	-17.300
$\kappa = 1$	0.200E-02	0.218	0.101	4.54	0.835	-0.807
$\kappa = 2$	0.192E-02	0.196	0.087	2.47	0.598	-0.575
$\kappa = 5$	0.148E-02	0.157	0.043	0.50	0.351	-0.341
$\kappa = 10$	0.876E-03	0.118	0.004	**-0.53**	0.207	-0.205
SGB						
$\kappa = 0$	0.149E-02	0.221	1.630	229.00	11.700	-7.330
$\kappa = 1$	0.171E-02	0.154	0.066	3.85	0.569	-0.554
$\kappa = 2$	0.172E-02	0.139	0.067	1.95	0.410	-0.396
$\kappa = 5$	0.159E-02	0.114	0.050	0.25	0.250	-0.241
$\kappa = 10$	0.141E-02	0.089	0.036	**-0.64**	0.157	-0.151
STO						
$\kappa = 0$	0.122E-02	0.299	-0.214	813.00	22.500	-21.400
$\kappa = 1$	0.979E-03	0.201	0.103	3.73	0.742	-0.717
$\kappa = 2$	0.925E-03	0.182	0.089	1.88	0.536	-0.513
$\kappa = 5$	0.295E-03	0.150	0.022	0.18	0.322	-0.317
$\kappa = 10$	0.871E-04	0.118	0.005	**-0.69**	0.202	-0.201
TAH						

κ = 0	0.155E-02	0.253	-1.430	146.00	7.700	-10.800
κ = 1	0.191E-02	0.189	0.117	4.30	0.714	-0.687
κ = 2	0.184E-02	0.171	0.103	2.40	0.520	-0.497
κ = 5	0.146E-02	0.137	0.060	0.48	0.308	-0.297
κ = 10	0.102E-02	0.104	0.030	**-0.54**	0.184	-0.179
WBC						
κ = 0	0.145E-02	0.262	-0.905	200.00	10.000	-11.200
κ = 1	0.170E-02	0.167	-0.013	4.90	0.650	-0.653
κ = 2	0.187E-02	0.146	0.026	2.12	0.435	-0.429
κ = 5	0.179E-02	0.117	0.018	0.13	0.252	-0.246
κ = 10	0.164E-02	0.093	0.006	**-0.74**	0.159	-0.155
WOW						
κ = 0	0.178E-02	0.308	-1.660	577.00	18.900	-18.700
κ = 1	0.241E-02	0.187	0.084	4.94	0.740	-0.714
κ = 2	0.219E-02	0.165	0.039	2.29	0.495	-0.485
κ = 5	0.214E-02	0.132	0.036	0.33	0.291	-0.282
κ = 10	0.191E-02	0.102	0.023	**-0.60**	0.180	-0.174
WPL						
κ = 0	0.187E-02	0.272	0.179	131.00	8.840	-8.120
κ = 1	0.190E-02	0.188	0.271	4.59	0.745	-0.683
κ = 2	0.161E-02	0.167	0.207	2.24	0.519	-0.473
κ = 5	0.564E-03	0.135	0.082	0.29	0.298	-0.285
κ = 10	-0.159E-03	0.105	0.031	**-0.63**	0.183	-0.180

Table 8. Gaussianity, '90 Percent Quantile' Linearity and Time Reversibility 'CUSUM' Test p – Values for the 16 Australian Stocks: Ten Minute Returns.

Stock Code	Trispectrum Quantile Gaussianity Test p-values	Trispectrum Quantile Nonlinearity Test p-values	Trispectrum Time Reversibility Test p-values
AGL			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
AMC			

$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0023	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
ANZ			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
BHP			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
BIL			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
CBA			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
LLC			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
NAB			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
NCM			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
QBE			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
SGB			

$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0002
STO			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
TAH			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
WBC			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
WOW			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000
WPL			
$\kappa = 1$	0.0000	0.0022	0.0000
$\kappa = 2$	0.0000	0.0022	0.0000
$\kappa = 5$	0.0000	0.0022	0.0000
$\kappa = 10$	0.0000	0.0022	0.0000

Table 9. Change in Ascending Order Structure of Individual Tests of the CUSUM Linearity Test as a Function of Trimming Rate: CBA Ten Minute Returns.

Index Order	**CDF Percentile Value**	**Trimming Rate** $\kappa = 1$	**Trimming Rate** $\kappa = 2$	**Trimming Rate** $\kappa = 5$	**Trimming Rate** $\kappa = 10$
1	0.0137	47	3	61	70
2	0.0274	37	4	51	65
3	0.0411	4	35	26	52
4	0.0548	23	59	32	49
5	0.0685	8	21	54	58
6	0.0822	6	57	63	68
7	0.0959	7	31	41	62
8	0.1096	50	26	25	56
9	0.1233	17	53	48	27
10	0.1370	21	50	67	24
11	0.1507	12	41	44	69
12	0.1644	31	45	46	34
13	0.1781	57	5	28	42
14	0.1918	11	8	30	44
15	0.2055	10	6	68	36
16	0.2192	40	40	42	33
17	0.2329	14	7	60	46
18	0.2466	3	10	1	1
19	0.2603	18	38	39	54
20	0.2740	9	23	59	67
21	0.2877	22	48	4	55
22	0.3014	5	1	58	39
23	0.3151	13	29	34	60
24	0.3288	41	22	45	43
25	0.3425	1	12	56	61
26	0.3562	26	17	36	30
27	0.3699	45	47	38	4
28	0.3836	35	37	29	32
29	0.3973	53	11	40	63
30	0.4110	15	18	27	71
31	0.4247	29	14	53	26
32	0.4384	2	61	3	41
33	0.4521	48	32	31	25
34	0.4658	16	2	21	28
35	0.4795	59	28	49	51
36	0.4932	20	51	52	53

37	0.5068	19	16	10	40
38	0.5205	38	15	24	48
39	0.5342	44	44	8	10
40	0.5479	32	46	70	31
41	0.5616	56	42	65	59
42	0.5753	67	9	35	45
43	0.5890	68	19	50	21
44	0.6027	28	13	69	8
45	0.6164	51	68	62	50
46	0.6301	42	54	57	3
47	0.6438	61	63	22	64
48	0.6575	46	20	23	66
49	0.6712	54	67	33	29
50	0.6849	58	56	12	9
51	0.6986	34	58	47	72
52	0.7123	24	60	5	47
53	0.7260	27	30	9	38
54	0.7397	36	25	15	15
55	0.7534	30	34	6	23
56	0.7671	25	27	7	57
57	0.7808	63	36	18	16
58	0.7945	62	24	37	22
59	0.8082	60	69	17	35
60	0.8219	55	39	14	12
61	0.8356	52	49	16	18
62	0.8493	49	70	11	5
63	0.8630	43	62	43	73
64	0.8767	39	55	19	37
65	0.8904	73	52	55	14
66	0.9041	72	43	13	7
67	0.9178	71	73	20	13
68	0.9315	70	72	73	6
69	0.9452	69	71	72	11
70	0.9589	66	66	71	20
71	0.9726	33	33	66	19
72	0.9863	65	65	64	17
73	1.0000	64	64	2	2

Table 10. Classical Bonferroni Gaussianity Test Statistics for the Sixteen Australian Returns: Ten Minute Returns.

ASX Code	10% BF - % of p-values in PD < 0.001370	5% BF - % of p-values in PD < 0.000685	1% BF - % of p-values in PD < 0.000137

AGL			
κ = 1	94.52%	93.15%	87.67%
κ = 2	93.15%	90.41%	87.67%
κ = 5	80.82%	79.45%	71.23%
κ = 10	71.23%	68.49%	64.38%
AMC			
κ = 1	97.26%	95.89%	94.52%
κ = 2	86.30%	86.30%	84.93%
κ = 5	75.34%	73.97%	69.86%
κ = 10	71.23%	68.49%	64.38%
ANZ			
κ = 1	97.26%	95.89%	94.52%
κ = 2	93.15%	93.15%	93.15%
κ = 5	75.34%	71.23%	69.86%
κ = 10	71.23%	65.75%	63.01%
BHP			
κ = 1	98.63%	97.26%	95.89%
κ = 2	91.78%	91.78%	91.78%
κ = 5	86.30%	84.93%	79.45%
κ = 10	76.71%	75.34%	69.86%
BIL			
κ = 1	98.63%	98.63%	98.63%
κ = 2	98.63%	98.63%	98.63%
κ = 5	86.30%	86.30%	83.56%
κ = 10	83.56%	80.82%	80.82%
CBA			
κ = 1	91.78%	91.78%	89.04%
κ = 2	90.41%	90.41%	89.04%
κ = 5	80.82%	76.71%	71.23%
κ = 10	69.86%	68.49%	63.01%
LLC			
κ = 1	94.52%	94.52%	93.15%
κ = 2	**95.89%**	**95.89%**	**95.89%**
κ = 5	89.04%	89.04%	87.67%
κ = 10	75.34%	73.97%	73.97%
NAB			
κ = 1	93.15%	93.15%	93.15%

$\kappa = 2$	89.04%	89.04%	87.67%
$\kappa = 5$	78.08%	78.08%	76.71%
$\kappa = 10$	69.86%	67.12%	64.38%
NCM			
$\kappa = 1$	98.63%	98.63%	97.26%
$\kappa = 2$	86.30%	84.93%	84.93%
$\kappa = 5$	**89.04%**	**87.67%**	83.56%
$\kappa = 10$	69.86%	67.12%	67.12%
QBE			
$\kappa = 1$	94.52%	93.15%	93.15%
$\kappa = 2$	93.15%	90.41%	89.04%
$\kappa = 5$	89.04%	87.67%	79.45%
$\kappa = 10$	69.86%	69.86%	63.01%
SGB			
$\kappa = 1$	97.26%	97.26%	95.89%
$\kappa = 2$	90.41%	90.41%	90.41%
$\kappa = 5$	78.08%	76.71%	72.60%
$\kappa = 10$	64.38%	64.38%	60.27%
STO			
$\kappa = 1$	97.26%	97.26%	95.89%
$\kappa = 2$	94.52%	91.78%	90.41%
$\kappa = 5$	80.82%	79.45%	75.34%
$\kappa = 10$	65.75%	63.01%	58.90%
TAH			
$\kappa = 1$	95.89%	95.89%	95.89%
$\kappa = 2$	95.89%	94.52%	91.78%
$\kappa = 5$	84.93%	84.93%	84.93%
$\kappa = 10$	68.49%	67.12%	63.01%
WBC			
$\kappa = 1$	97.26%	97.26%	97.26%
$\kappa = 2$	95.89%	94.52%	94.52%
$\kappa = 5$	76.71%	76.71%	73.97%
$\kappa = 10$	73.97%	72.60%	64.38%
WOW			
$\kappa = 1$	90.41%	90.41%	90.41%
$\kappa = 2$	89.04%	89.04%	87.67%
$\kappa = 5$	82.19%	80.82%	76.71%

κ = 10	67.12%	61.64%	57.53%
WPL			
κ = 1	98.63%	98.63%	97.26%
κ = 2	90.41%	89.04%	86.30%
κ = 5	84.93%	80.82%	78.08%
κ = 10	71.23%	68.49%	61.64%

Table 11. Classical Bonferroni Linearity Test Statistics for the Sixteen Australian Returns: Ten Minute Returns.

ASX Code	10% BF - % of p-values in PD < 0.001370	5% BF - % of p-values in PD < 0.000685	1% BF - % of p-values in PD < 0.000137
AGL			
κ = 1	28.77%	28.77%	28.77%
κ = 2	24.66%	23.29%	21.92%
κ = 5	16.44%	13.70%	12.33%
κ = 10	**19.18%**	**17.18%**	**15.07%**
AMC			
κ = 1	27.40%	27.40%	27.40%
κ = 2	24.66%	24.66%	21.92%
κ = 5	12.33%	10.96%	10.96%
κ = 10	**16.44%**	**16.44%**	**13.70%**
ANZ			
κ = 1	24.66%	24.66%	24.66%
κ = 2	19.18%	17.81%	16.44%
κ = 5	19.18%	17.81%	16.44%
κ = 10	**20.55%**	**20.55%**	**16.44%**
BHP			
κ = 1	30.14%	30.14%	30.14%
κ = 2	23.29%	21.92%	20.55%
κ = 5	**27.40%**	**27.40%**	**23.29%**
κ = 10	23.29%	20.55%	16.44%
BIL			
κ = 1	31.51%	31.51%	30.14%
κ = 2	23.29%	23.29%	20.55%

κ = 5 κ = 10	**27.40%** 20.55%	**26.03%** 20.55%	**24.66%** 19.18%
CBA κ = 1 κ = 2 κ = 5 κ = 10	 26.03% 19.18% 15.07% **19.18%**	 26.03% 19.18% 15.07% **19.18%**	 23.29% 17.81% 13.70% **15.07%**
LLC κ = 1 κ = 2 κ = 5 κ = 10	 28.77% 24.66% 20.55% 19.18%	 28.77% 24.66% 17.81% **19.18%**	 28.77% 24.66% 17.81% **19.18%**
NAB κ = 1 κ = 2 κ = 5 κ = 10	 26.03% 20.55% 13.70% **23.29%**	 26.03% 19.18% 13.70% **16.44%**	 26.03% 19.18% 10.96% **15.07%**
NCM κ = 1 κ = 2 κ = 5 κ = 10	 32.88% 27.40% 19.18% **20.55%**	 32.88% 26.03% 19.18% **20.55%**	 32.88% 26.03% 16.44% 15.07%
QBE κ = 1 κ = 2 κ = 5 κ = 10	 28.77% 23.29% 13.70% **19.18%**	 28.77% 21.92% 10.96% **16.44%**	 27.40% 21.92% 10.96% **16.44%**
SGB κ = 1 κ = 2 κ = 5 κ = 10	 30.14% 26.03% 15.07% **19.18%**	 30.14% 26.03% 15.07% **17.81%**	 30.14% 24.66% 12.33% **16.44%**
STO κ = 1 κ = 2 κ = 5 κ = 10	 31.51% 24.66% 16.44% 16.44%	 31.51% 24.66% 15.07% 15.07%	 30.14% 21.92% 15.07% 10.96%

TAH			
κ = 1	32.88%	31.51%	30.14%
κ = 2	24.66%	23.29%	21.92%
κ = 5	16.44%	16.44%	15.07%
κ = 10	**19.18%**	**19.18%**	**16.44%**
WBC			
κ = 1	27.40%	27.40%	27.40%
κ = 2	23.29%	23.29%	23.29%
κ = 5	21.92%	19.18%	17.81%
κ = 10	19.18%	17.81%	16.44%
WOW			
κ = 1	30.14%	30.14%	30.14%
κ = 2	23.29%	21.92%	21.92%
κ = 5	19.18%	19.18%	17.81%
κ = 10	19.18%	17.81%	16.44%
WPL			
κ = 1	27.40%	27.40%	27.40%
κ = 2	23.29%	23.29%	21.92%
κ = 5	19.18%	17.81%	13.70%
κ = 10	**24.66%**	**23.29%**	**20.55%**

Table 12. Classical Bonferroni (BF) Time Reversibility Test Statistics for the Sixteen Australian Returns: Ten Minute Returns.

ASX Code	**10% BF - % of p-values in PD < 0.001370**	**5% BF - % of p-values in PD < 0.000685**	**1% BF - % of p-values in PD < 0.000137**
AGL			
κ = 1	50.68%	46.58%	39.73%
κ = 2	34.25%	32.88%	23.29%
κ = 5	12.33%	12.33%	9.59%
κ = 10	6.85%	4.11%	2.74%
AMC			
κ = 1	41.10%	39.73%	31.51%
κ = 2	23.29%	21.92%	20.55%
κ = 5	8.22%	6.85%	5.48%

$\kappa = 10$	5.48%	5.48%	1.37%
ANZ			
$\kappa = 1$	52.05%	50.68%	38.36%
$\kappa = 2$	28.77%	26.03%	19.18%
$\kappa = 5$	10.96%	10.96%	9.59%
$\kappa = 10$	1.37%	1.37%	1.37%
BHP			
$\kappa = 1$	65.75%	64.38%	54.79%
$\kappa = 2$	49.32%	49.32%	46.58%
$\kappa = 5$	32.88%	31.51%	27.40%
$\kappa = 10$	13.70%	5.48%	1.37%
BIL			
$\kappa = 1$	67.12%	67.12%	58.90%
$\kappa = 2$	54.79%	54.79%	49.32%
$\kappa = 5$	30.14%	30.14%	28.77%
$\kappa = 10$	16.44%	12.33%	10.96%
CBA			
$\kappa = 1$	32.88%	32.88%	28.77%
$\kappa = 2$	27.40%	23.29%	17.81%
$\kappa = 5$	9.59%	8.22%	4.11%
$\kappa = 10$	2.74%	**0.00%**	**0.00%**
LLC			
$\kappa = 1$	60.27%	57.53%	52.05%
$\kappa = 2$	52.05%	47.95%	41.10%
$\kappa = 5$	30.14%	27.40%	16.44%
$\kappa = 10$	8.22%	6.85%	2.74%
NAB			
$\kappa = 1$	38.36%	34.25%	31.51%
$\kappa = 2$	24.66%	19.18%	12.33%
$\kappa = 5$	10.96%	9.59%	4.11%
$\kappa = 10$	**0.00%**	**0.00%**	**0.00%**
NCM			
$\kappa = 1$	38.36%	32.88%	27.40%
$\kappa = 2$	20.55%	15.07%	10.96%
$\kappa = 5$	6.85%	6.85%	6.85%
$\kappa = 10$	2.74%	2.74%	1.37%

QBE			
$\kappa = 1$	56.16%	54.79%	47.95%
$\kappa = 2$	36.99%	34.25%	28.77%
$\kappa = 5$	16.44%	13.70%	9.59%
$\kappa = 10$	8.22%	5.48%	1.37%
SGB			
$\kappa = 1$	41.10%	39.73%	34.25%
$\kappa = 2$	27.40%	24.66%	19.18%
$\kappa = 5$	8.22%	5.48%	4.11%
$\kappa = 10$	**0.00%**	**0.00%**	**0.00%**
STO			
$\kappa = 1$	30.14%	28.77%	26.03%
$\kappa = 2$	23.29%	19.18%	16.44%
$\kappa = 5$	10.96%	8.22%	6.85%
$\kappa = 10$	2.74%	1.37%	**0.00%**
TAH			
$\kappa = 1$	53.42%	52.05%	46.58%
$\kappa = 2$	35.62%	34.25%	31.51%
$\kappa = 5$	9.59%	9.59%	8.22%
$\kappa = 10$	1.37%	1.37%	1.37%
WBC			
$\kappa = 1$	47.95%	43.84%	35.62%
$\kappa = 2$	32.88%	28.77%	21.92%
$\kappa = 5$	15.07%	9.59%	5.48%
$\kappa = 10$	5.48%	4.11%	2.74%
WOW			
$\kappa = 1$	32.88%	32.88%	26.03%
$\kappa = 2$	23.29%	19.18%	15.07%
$\kappa = 5$	10.96%	9.59%	6.85%
$\kappa = 10$	5.48%	5.48%	2.74%
WPL			
$\kappa = 1$	32.88%	30.14%	26.03%
$\kappa = 2$	28.77%	26.03%	19.18%
$\kappa = 5$	8.22%	8.22%	4.11%
$\kappa = 10$	2.74%	2.74%	**0.00%**