OPTIMAL CONTRACTS FOR LOSS AVERSE CONSUMERS

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Abstract. We enrich the standard model of optimal contract design between a monopolist and a continuum of potential buyers under asymmetric information by assuming that consumers have reference-dependent preferences and loss aversion. In our model, consumers are endowed with quasi-linear utilities over the quality parameter of a good sold by a monopolist. The total valuation for quality is composed of the standard consumption valuation, which is affected by privately known types, and an additional reference valuation that depends on deviations of purchased quality from the reference point. Potential buyers are loss-averse, so that deviations from the reference point are evaluated differently depending on whether they are gains or losses. We maintain Kőszegi and Rabin's (2006) basic framework and let gains and losses relative to the reference point be evaluated in terms of changes in the consumption valuation. However, we consider different ways in which reference quality levels are formed. In particular, we consider reference qualities as a monotone function of types. The presence of reference points creates downward kinks in the total valuation for the good, as a function of the privately known types, which renders the standard techniques based on revenue equivalence moot. Nonetheless, we apply a generalization of the standard envelope techniques to derive, for monotone reference functions, the optimal selling contract between the monopolist and loss averse consumers. We find that, depending on how reference points are formed, there is substantial difference between optimal contracts for loss averse consumers and optimal contracts for loss neutral buyers, both in terms of expected revenue generated to the monopolist and quality–price offered to each type of consumer.

1. Introduction

This paper studies how a revenue maximizing firm solves the problem of optimal contract design in the presence of consumers with heterogeneous tastes, asymmetric information, and reference-dependent preferences. Building on the classic monopoly pricing models under asymmetric information developed by Mussa and Rosen (1978) and Maskin and Riley (1984), we introduce consumers whose preferences exhibit loss aversion in the product attribute dimension. In our model, consumers are endowed with quasi-linear utilities over the quality parameter of a good sold by a monopolist. A consumer's total valuation for quality is composed of the standard consumption valuation, which is affected by a privately known type via the marginal willingness to pay, and an additional loss valuation that depends on deviations of purchased quality from the consumer's reference point. Our key assumption is that buyers exhibit loss aversion biases; i.e., deviations from reference quality levels are evaluated differently depending on whether they are gains or losses, with losses counting more heavily than comparable gains.

Loss aversion in riskless environments was introduced by Thaler (1980) and Tversky and Kahneman (1991), and has been since validated, with few exceptions, in various experimental setting.[1] In this paper we adopt instead the line of inquiry pioneered by della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), and Heidhues and Kőszegi (2008), among others, and study how revenue maximizing firms deal with optimal contract

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[1]For a recent account, see for example Novemsky and Kahneman (2005).

design in a market context where consumers have systematic deviations from traditional preferences.

In our model, consumers with quasi-linear utilities and heterogeneous tastes derive utility from the consumption of a unit of a good of different qualities and from comparison of actual consumption with a reference point. We let consumers with different tastes differ in terms of their reference quality levels as well. As a first step in our theory, we assume that reference points are formed via a weakly increasing *reference function*. The way we model the firm's behavior is standard: a monopolist with a convex cost function maximizes expected revenues, given that consumers' tastes are private information. The monopolist is, on the other hand, aware of the loss aversion biases and of the consumers' reference points, and takes these facts into consideration. From a technical point of view, the presence of reference points creates downward kinks in the total consumer's valuation for the good, as a function of the privately known types, which renders the standard contract theory techniques based on revenue equivalence moot. Nonetheless, we apply a generalization of the standard envelope techniques to derive, for monotone reference functions, the optimal menu of posted contracts between the monopolist and loss averse consumers. We find that, depending on how reference points are formed, there is substantial difference between optimal contracts for loss averse consumers and optimal contracts for loss neutral buyers, both in terms of expected revenue generated to the monopolist and quality–price offered to each type of consumer.

Our paper is not the first to combine loss aversion and asymmetric information in a market context with a revenue-maximizing firm. However, we depart from previous work in behavioral contract theory in several important ways. First, while we follow Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010) in modelling the consumer's gain-loss function in terms of the difference between valuation at consumption and valuation at the reference point, we differ in terms of how comparisons with the reference point occur. Specifically, we assume that reference points are type-dependent (in a manner to be explained below) and that consumption for each consumer type is compared with the reference point for that consumer alone. If the H-consumer has high marginal valuation for a good and the L-consumer has low marginal valuation, then actual consumption of the H-consumer is to be compared with his (possibly high) reference point r_H, and *not* with the (possibly low) reference point of the L-consumer.

Second, while in our model, again as in the models of Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010), a reference point is interpreted as the recent expectation of consumption outcome, we do not impose a priori rational expectations in consumers' behavior. We endow consumers with a reference formation process that depends on consumers' types, which are private information. Aside from standard technical continuity and differentiability conditions, the reference point formation process is only assumed to be weakly increasing so that a higher consumer's type (i.e., a higher marginal consumption valuation) corresponds to a weakly higher reference point. Thus, our model allows us to explore different ways in which reference points are generated and their non-trivial implications in terms of optimal contract design. Our model allows for reference point formation processes that could be influenced partly by advertising campaigns, fashion, the nature of the product under consideration, etc. Thus our model bears some resemblance to the aspirational considerations of Gilboa and Schmeidler (2001). On the other hand, we do explore implications of consistent reference point formation processes in terms of correct expectations of immediate future consumption.

Third, we model reference points in terms of expected or aspirational consumption levels that affect the valuation for the object offered by the revenue-maximizing monopolist. We depart from recent work in the area, for instance Herweg and Mierendorff (2011) and Spiegler (2011), that specifies reference points in terms of prices. Thus, our model combines private consumer heterogeneity in the valuation function for the object, captured by the one-dimensional type space, with a reference formation process that delivers type-dependent reference consumption levels. This specification goes in line with empirical evidence from the marketing literature that suggests that loss aversion on quantity/quality attributes is at least as important as, if not more important than, loss aversion in monetary transactions; see Hardie, Johnson, and Fader (1993). This point is also suggested by some experimental data as reported recently by Novemsky and Kahneman (2005).

Fourth, our model, while parsimonious in terms of deviations from standard preferences, allows us to derive optimal contracts that differ substantially from the optimal contract without reference-dependent preferences (even under complete information). As we shall demonstrate, the optimal contract design problem of the revenue-maximizing firm is heavily influenced by the reference formation process. In general, for a range of intermediate types, the shape of the optimal quality schedule will be determined by the shape of the reference quality function. Since we consider weakly monotone increasing, continuous reference functions, the optimal quality schedule may exhibit flat parts among increasing parts for intermediate types. The additional flexibility in contract design could partially explain complex contract schemes (e.g., three part tariff schemes) that have become increasingly popular among wireless phone providers, Internet providers, and other subscription services; see Lambrecht, Seim, and Skiera (2007) among others. The optimal quality/quantity provided by the monopolist is increasing for low types, constant for intermediate types, and then reverts to being increasing for higher types. Moreover, depending on the reference function, the optimal quality/quantity schedule may exhibit two or more flat parts, which seems to be consistent with two-tier tariff systems used by electricity providers, as reported by Reiss and White (2005). Our model also allow us to precisely describe the consequences of rational expectations in terms of contract design. We find that, if the reference point formation process is required to be consistent, then the preferred quality schedule for the firm provides excessive quality levels for all buyers with positive consumption, compared to the optimal quality schedule without loss aversion, whereas the preferred quality schedule for the consumers under delivers quality for the intermediate types.

The rest of the paper is organized as follows. Section 2 contains the formal details of our model and a brief analysis of the complete information case. In Section 3, we present the analysis of optimal contract design for loss averse consumers. As mentioned, we find that optimal contracts depend on the reference function: for types with high or low reference quality levels (compared to the quality level that maximizes virtual surplus when loss aversion is taken into account), the optimal quality schedule for the monopolist maximizes virtual surplus. For types with reference quality in the intermediate region, the optimal quality schedule coincides with the reference function. Note that in this setting we have let optimal contracts differ from reference qualities, as long as they are individually rational and incentive compatible. That is, a consumer's actual consumption need not coincide with her expectations, given by the reference point, as long as the menu of contracts is incentive feasible: optimal contracts may not be self-confirming. We impose the consistency of beliefs assumption in Section 4 and construct the most preferred contracts from the point of view of the firm and from the point of view of consumers. Section 5 offers some concluding remarks. Omitted proofs from the text are gathered in Section 6.

2. Monopolist's Problem under Reference-Dependent Preferences

In this section we lay out the notation and main assumptions of our framework, which builds on monopoly pricing models originally studied by Mussa and Rosen (1978) and Maskin and Riley (1984), among others. The main difference is that in our framework consumers derive utility from consumption and, in addition, from comparisons between actual consumption and a reference point. Following the insights of Tversky and Kahneman (1991), we consider consumers who evaluate negative deviations from their reference points more heavily than similar positive deviations; i.e., we consider loss-averse consumers.

2.1. **The firm.** A revenue-maximizing monopolist produces a commodity of different characteristics, which are captured by the product attribute parameter $q \geq 0$. This parameter can be interpreted as either a one-dimensional measure of quality (speed of connection offered by an Internet provider, exclusive features of a luxury good) or a measure of quantity (minutes included in a mobile phone plan, electricity units offered by a local power supplier). We pay tribute to Mussa and Rosen (1978) and maintain the first interpretation, thus referring to q as the *quality* attribute. The cost of producing one unit of the good with quality $q \geq 0$ is represented by $c(q)$, where the *cost function* $q \mapsto c(q)$ is non-decreasing, twice continuously differentiable and strongly convex[2], with $c(0) = 0$. The firm's problem is to design a revenue maximizing menu of posted quality–price pairs to offer to potential buyers with differentiated demands.

2.2. **Consumers.** There is a population of consumers with quasi-linear preferences who have unit demands for the good offered by the firm but heterogeneous tastes in terms of product attribute. This heterogeneity is captured by the type parameter $\theta \in \Theta = [\theta_L, \theta_H]$, with $0 \leq \theta_L < \theta_H < +\infty$. Types are private information, so that the firm knows the distribution F, with full support on Θ and density $f > 0$, but not the actual realization of types. The *inverse hazard rate* $\theta \mapsto h(\theta) \equiv (1 - F(\theta))/f(\theta)$ is assumed to be non-increasing and twice continuously differentiable.

A θ-consumer has a *consumption valuation* for quality $q \geq 0$ given by

$$\theta m(q),$$

where the function $q \mapsto m(q)$ is strictly increasing, twice continuously differentiable and concave, with $m(0) = 0$. In addition, we assume that

$$\lim_{q \to 0} (m'(q) - c'(q)) > 0 \quad \text{and} \quad \lim_{q \to +\infty} (m'(q) - c'(q)) < 0.$$

The specification of consumption surplus $\theta m(q) - c(q)$ includes, among others, Mussa and Rosen's (1978) monopoly model with linear consumption valuation and quadratic costs. These assumptions ensure that the quality schedule that maximizes (virtual) consumption surplus is continuously differentiable on Θ. An alternative to accommodate Maskin and Riley's (1984) model is to impose strong concavity of m and convexity of c.

We depart from standard contract theoretic work in assuming that, aside from classic consumption preferences, potential buyers exhibit reference-dependent preferences for the product attribute (but not for monetary transactions). Specifically, we assume that in addition to his consumption valuation, a θ-consumer derives utility from evaluating quality q relative to a reference quality level $r(\theta) \geq 0$. Reference qualities encompass different concepts: a reference point reflects subjective expectations of future consumption that may be influenced by past experiences as well as by current aspirational considerations, which in turn can be affected by fashion, advertising campaigns, social influences, etc. At this stage, we represent the reference formation process via a *reference function* $\theta \mapsto r(\theta) \geq 0$, which is assumed to be non-decreasing, Lipschitz and piecewise continuously differentiable on Θ. Following Kőszegi and Rabin (2006), we let deviations from the reference points be evaluated in terms of changes in the consumption valuation. On the

[2]A function $f\colon \mathbb{R} \to \mathbb{R}$ is strongly convex if there exists a number $\rho > 0$ such that $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha)f(y) - (\rho/2)\alpha(1-\alpha)|x-y|^2$, for all $x, y \in \mathbb{R}$, for all $\alpha \in (0,1)$. For twice differentiable functions, strong convexity is obtained whenever $f''(x) \geq \rho > 0$, for all x. See Montrucchio (1987).

other hand, while in some economic contexts it may be reasonable to assume, as in Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010), that all potential buyers share the same reference point distribution, one can consider environments in which buyers with different marginal consumption valuation have different reference quality levels and assess their consumption relative to their individual reference points. We take this last approach: a θ-consumer has a *reference valuation* for quality q given by

$$\mu\big(\theta m(q) \;-\; \theta m(r(\theta))\big)$$

where μ equals 0 if $q > r(\theta)$ and μ equals the loss aversion coefficient $\lambda > 0$ if $q \leq r(\theta)$. Thus, the *total valuation* for the θ-consumer derived from purchasing one unit of the good with quality $q \geq 0$ written as:

$$v(q,\theta) \;\equiv\; \theta m(q) \;+\; \mu\big(\theta m(q) \;-\; \theta m(r(\theta))\big). \tag{1}$$

Since consumers' preferences are quasi-linear, the utility that the θ-consumer obtains from buying one unit with product quality q at price p is $v(q,\theta) - p$. The value of the outside option for each consumer is taken to be the payoff derived from consuming zero quality at null price. That is, the outside option of the θ-consumer is $\underline{u}(\theta) = -\lambda\theta m(r(\theta))$.

2.3. **Comment.** The interpretation of our framework is the following. Consumers and monopolist interact for a period of time. At the beginning of the period, each consumer receives a type that affects his marginal consumption valuation for the good. The firm knows the distribution of types but not each buyer's realization. After types are privately observed, but before the monopolist can post any menu of contracts, each θ-consumer forms subjective beliefs about immediate future quality consumption that act as his reference point. We capture the reference formation process via the reference function $\theta \mapsto r(\theta)$. Once reference qualities are established, the firm offers a menu of posted contracts to maximize expected revenue. At this stage, each θ-consumer selects the quality-price combination that maximizes his utility given $r(\theta)$, as long as the total utility derived from the chosen contract is higher than the value of the outside option.

Throughout the paper, it is assumed that the functional forms of the consumption valuation, reference valuations, and reference function are common knowledge. One can consider many ways in which reference qualities are formed (consistent with the properties imposed on the reference function), each of which may generate a different optimal contract menu for the firm. We derive, in the next section, the optimal menu of contracts for a general monotone reference function. In Section 4 we focus on quality expectations that are consistent with the optimal contracts generated. Before proceeding, we briefly consider the case of an informed monopolist. It will be shown that, even in this case, the actual form optimal contracts adopt is influenced by the reference function.

2.4. **First best contracts.** When types are known, the firm makes specific take-it-or-leave-it offers to each θ-consumer to extract all surplus by setting a price $p(\theta) = v(q(\theta),\theta) - \underline{u}(\theta)$ and choosing a quality level $q(\theta)$ that maximizes:

$$\Pi^{fb} \;=\; (1+\mu)\theta m(q) \;-\; c(q) \;+\; (\lambda - \mu)\theta m(r(\theta)). \tag{2}$$

Clearly, the firm's objective function depends on whether the quality selected is greater or smaller than the reference quality level. To find the solution to the firm's problem for the θ-consumer, define the quality schedules $\theta \mapsto q^e(\theta)$ and $\theta \mapsto q^e_\lambda(\theta)$ as the solution to the following optimization problems:

$$q^e(\theta) \;\equiv\; \arg\max_{q\geq 0}\,\{\theta m(q) - c(q)\} \qquad \text{and} \qquad q^e_\lambda(\theta) \;\equiv\; \arg\max_{q\geq 0}\,\{(1+\lambda)\theta m(q) - c(q)\}.$$

The consumption surplus $(1+\mu)\theta m(q)-c(q)$ is twice differentiable in q and θ, and strongly concave in q for each θ. It follows from standard arguments[3] that both quality schedules q^e, q^e_λ are differentiable and non-decreasing. Moreover, since m is strictly increasing, $q^e(\theta) < q^e_\lambda(\theta)$ for all types $\theta > 0$.

If the reference quality $r(\theta)$ is sufficiently low (respectively, sufficiently high) compared to the quality level $q^e(\theta)$ (respectively, $q^e_\lambda(\theta)$), the monopolist ignores the effects of the reference point and sets quality at its efficient level, charging a price that leaves the θ-consumer with zero surplus. If the reference point $r(\theta)$ lies in intermediate ranges, any deviation from the reference quality will hurt the firm's profits. Providing a quality level higher than the reference level will position the monopolist in the downward sloping portion of its profit function, so the monopolist has incentives to reduce quality level. A similar effect takes place when quality level q is below the reference point $r(\theta)$, only now the profit function considers the loss aversion coefficient λ. This leads us to the following observation.

Proposition 1. *Given the reference formation process $\theta \mapsto r(\theta)$, the complete information optimal menu of contracts offered by the monopolist consists of quality–price pairs $\{(q^{fb}(\theta), p^{fb}(\theta)) \mid \theta \in \Theta\}$ such that:*

$$\begin{aligned} q^{fb}(\theta) &= \begin{cases} q^e(\theta) & : \ r(\theta) < q^e(\theta), \\ r(\theta) & : \ q^e(\theta) \leq r(\theta) \leq q^e_\lambda(\theta), \\ q^e_\lambda(\theta) & : \ q^e_\lambda(\theta) < r(\theta); \end{cases} \\ p^{fb}(\theta) &= (1+\mu)\theta m(q^{fb}(\theta)) + (\lambda-\mu)\theta m(r(\theta)), \end{aligned}$$

where in the expression for prices one has $\mu = 0$ if $q^{fb}(\theta) > r(\theta)$ and $\mu = \lambda$ otherwise.

Proof. See Section 6. □

While straightforward, Proposition 1 stresses the relevance of the reference function for optimal contract design under loss aversion, even in the complete information case, as it entirely determines the shape of first-best quality schedule for consumers for which $q^e(\theta) \leq r(\theta) \leq q^e_\lambda(\theta)$. Since the reference function may in principle be very general, as long as it is monotone increasing a piecewise continuously differentiable, first-best contracts may take various forms. On the other hand, for low and high reference points relative to the efficient qualities, the first best quality level $q^{fb}(\theta)$ differs from expectations or aspirational levels as captured by $r(\theta)$. Since the derivation of the optimal contract menu includes individual rationality constraints, in principle there is no reason to exclude such possibility without imposing a consistency requirement in the way consumers form beliefs about immediate future consumption.

A reference function $\theta \mapsto r(\theta)$ is called *consistent* if the optimal quality schedule offered by the firm agrees with consumers' expectations: $q(\theta) = r(\theta)$ for each θ-consumer. A consistent reference function generates *consistent contract menus*. Our notion of consistent contracts is stronger than the personal equilibrium notion of Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010), as the latter involves agreement in expectations between a buyer's consumption plan and various individual reference points held at every state of the world. In our model, each consumer has a unique reference point and evaluates his quality consumption with respect to this reference quality level alone.

From Proposition 1, any reference function r satisfying $q^e(\theta) \leq r(\theta) \leq q^e_\lambda(\theta)$ generates consistent optimal contracts. Under complete information, any consistent menu of contract leaves each θ-consumer with zero surplus, and thus buyers are indifferent between

[3]See Montrucchio (1987) and Araujo (1991).

any two such contracts. On the other hand, the monopolist has a unique preferred consistent menu of contracts, as a higher reference quality level never decreases the monopolist profits.

Corollary 1. *The monopolist's net revenue from every θ-consumer under complete information and loss aversion is increasing in the reference quality level $r(\theta)$, for all $r(\theta) \leq q^e_\lambda(\theta)$, and constant afterwards.*

Proof. See Section 6. □

Thus, the preferred consistent optimal menu of contracts for the firm has $q^{fb}(\theta) = q^e_\lambda(\theta) = r(\theta)$, for each θ-consumer. Suppose the monopolist publicly announces a quality schedule at a fixed cost prior to the market interaction. If each θ-type consumer forms consistent beliefs on the reference quality accordingly, then the monopolist cannot do better than announcing $q^e_\lambda(\theta)$ for each $\theta \in \Theta$. These announcements are credible, in the sense that consumers form quality expectations $r(\theta) = q^e_\lambda(\theta)$ that coincide with their actual quality purchases, and maximize revenue among all possible announcements.

3. Optimal Contract Design under Reference-Dependent Preferences

In this section we consider the asymmetric information case, where consumers' types are private information. Since the monopolist only knows the type distribution F, its problem is to design menu of contracts that maximizes expected revenue subject to the incentive compatibility and individually rationality constraints.

3.1. Optimal contracts for loss averse consumers.

The problem of the monopolist can be stated as follows: choose a menu of posted contracts $\{(q(\theta), p(\theta)) \mid \theta \in \Theta\}$ that maximizes expected profits

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \Big(p(\theta) \,-\, c(q(\theta))\Big) f(\theta)\, d\theta$$

subject to

$$v(q(\theta),\theta) \,-\, p(\theta) \;\geq\; v(q(\hat\theta),\theta) \,-\, p(\hat\theta), \qquad \text{for all } \theta, \hat\theta \in \Theta; \tag{3}$$

$$v(q(\theta),\theta) \,-\, p(\theta) \;\geq\; -\lambda\theta m(r(\theta)), \qquad \text{for all } \theta \in \Theta. \tag{4}$$

The incentive and participation constraints capture in (3) and (4) contain the total valuation as formulated in Equation 1. Thus, we consider loss-averse consumers whose outside option does not ignore the effects of reference qualities in our formulation of feasibility constraints for the optimization problem of the firm.

Notice that for a fixed quality level $\hat q$, the reference valuation $\mu\big(\theta m(q) - \theta m(r(\theta))\big)$ is neither convex nor everywhere differentiable with respect to types. In particular, differentiability may fail for a subset of consumer types of positive measure. When such possibility arises, we cannot apply standard techniques[4] based on the envelope theorem to express expected profits in terms of the virtual surplus. This is because the standard integral representation of the indirect utility function (and thus revenue equivalence) may fail. To see this, fix a quality level $\hat q$ and write the total valuation as a function of types:

$$\theta \mapsto v(\hat q, \theta) \;=\; \theta m(\hat q) \,+\, \mu\,\theta\big(m(\hat q) \,-\, m(r(\theta))\big), \tag{5}$$

where $\mu = 0$ if $\hat q > r(\theta)$ and $\mu = \lambda$ if $\hat q \leq r(\theta)$. Since r is piecewise continuously differentiable, we focus on points of non-differentiability originated in sudden changes in the the valuation function around $\hat q = r(\theta)$, and omit discussion of kinks in the valuation function due to kinks in the reference function.

[4]As proposed first by Myerson (1981), and more recently by Williams (1999), and Milgrom and Segal (2002), among others.

For a fixed quality level $\hat{q}$, define the right and left subderivatives of the function $\theta \mapsto v(\hat{q},\theta)$ evaluated at $\hat{\theta} \in \Theta$, respectively, as:

$$\overline{\mathrm{d}}v(\hat{q},\hat{\theta}) \;\equiv\; \liminf_{\delta\downarrow 0} \frac{v(\hat{q},\hat{\theta}+\delta) \,-\, v(\hat{q},\hat{\theta})}{\delta}, \quad \text{and}$$

$$\underline{\mathrm{d}}v(\hat{q},\hat{\theta}) \;\equiv\; \limsup_{\delta\uparrow 0} \frac{v(\hat{q},\hat{\theta}+\delta) \,-\, v(\hat{q},\hat{\theta})}{\delta}.$$

The function $v(\hat{q},\cdot)$ defined in Equation 5 is Lipschitz, hence these subderivatives exist for all consumer types $\theta \in \Theta$ and quality levels $\hat{q} \geq 0$ (in fact, except for finitely many types at which differentiability of r may fails, the right and left subderivatives coincide with the right and left derivatives, but we keep the first notation for expositional convenience). Define now the subderivative correspondence $\theta \rightrightarrows S(\hat{q},\theta)$ associated with quality level $\hat{q}$ as:

$$S(\hat{q},\theta) \;\equiv\; \left\{ s \in \mathbb{R} \mid \overline{\mathrm{d}}v(\hat{q},\theta) \,\leq\, s \,\leq\, \underline{\mathrm{d}}v(\hat{q},\theta) \right\}.$$

The subderivative correspondence is by definition closed-valued. It is empty-valued at consumer types where the right subderivative of $v(\hat{q},\cdot)$ is greater than the left subderivative, and single-valued when these two expressions are equal. One can easily find the following expressions for each θ-consumer:

$$S(\hat{q},\theta) \;=\; \begin{cases} m(\hat{q}) & : \; \hat{q} > r(\theta), \\ (1+\lambda)m(\hat{q}) \,-\, \big(\lambda\,\theta m(r(\theta))\big)' & : \; \hat{q} < r(\theta), \\ \big[(1+\lambda)m(\hat{q}) \,-\, \big(\lambda\,\theta m(r(\theta))\big)'\,,\, m(\hat{q})\big] & : \; \hat{q} = r(\theta). \end{cases} \tag{6}$$

Notice that if there are types $\theta, \hat{\theta}$, $\theta \neq \hat{\theta}$, satisfying $r(\theta) = r(\hat{\theta}) = \hat{q}$, then since r is piecewise differentiable and weakly increasing, it follows that $r'(\theta) = r'(\hat{\theta}) = 0$. Thus, the subderivative correspondence $S(\hat{q},\cdot)$ is single-valued on Θ except when there exists a unique type θ for which $r(\theta) = \hat{q}$.

We stress the fact that the monopolist choice of quality schedule affects the subderivative correspondence. To be more precise, the monopolist may find optimal to select a quality schedule $q(\cdot)$ such that $q(\theta) = r(\theta)$ for a subset of Θ of positive measure. If for these consumers one has $r'(\theta) > 0$, then the subderivative correspondence will be multi-valued for a non-negligible subset of the consumer type space. This implies that the valuation may fail to be differentiable in types on a non-negligible subset of Θ. To overcome these difficulties, we employ the results reported in Carbajal and Ely (2011) to characterize incentive compatible contracts based on an integral monotonicity condition and a generalization of the Mirrlees representation of the indirect utility. Given a (measurable) quality schedule $\theta \mapsto q(\theta)$ and associated subderivative correspondence $\theta \rightrightarrows S(q(\theta),\theta)$, we use $\theta \mapsto s(q(\theta),\theta) \in S(q(\theta),\theta)$ to indicate an integrable selection of the subderivative correspondence (whenever one such selection exists). Say that $\theta \rightrightarrows S(q(\theta),\theta)$ is a *regular correspondence* if it is non empty-valued almost everywhere on Θ, closed-valued, measurable and integrably bounded. Given the incentive feasible menu of contracts $\{(q(\theta), p(\theta) \mid \theta \in \Theta\}$ satisfying the informational constraints (3) and (4), its indirect utility is

$$U(\theta) \;=\; v(q(\theta),\theta) \,-\, p(\theta), \qquad \text{for all } \theta \in \Theta.$$

Proposition 2. *Under incomplete information, the (measurable) quality schedule $\theta \mapsto q(\theta) \geq 0$ designed by a monopolist facing loss-averse consumers is incentive compatible and individually rational, with associated indirect utility U, if and only if $\theta \rightrightarrows S(q(\theta),\theta)$ is regular and admits an integrable selection $\theta \mapsto s(q(\theta),\theta)$ for which the following conditions are satisfied.*

(a) *Integral monotonicity: for all* $\theta', \theta'' \in \Theta$,

$$v(q(\theta''), \theta'') \;-\; v(q(\theta''), \theta') \;\geq\; \int_{\theta'}^{\theta''} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta} \;\geq\; v(q(\theta'), \theta'') \;-\; v(q(\theta'), \theta').$$

(b) *Generalized Mirrlees representation: for all* $\theta \in \Theta$,

$$U(\theta) \;=\; U(\theta_L) \;+\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}.$$

(c) *Participation of the* θ_L*-consumer:*

$$U(\theta_L) \;+\; \lambda \theta_L m(r(\theta_L)) \;\geq\; 0.$$

Proof. The equivalence between incentive compatibility of the quality schedule q and parts (a) and (b) follow from Theorem 1 in Carbajal and Ely (2011), since we can restrict the set of quality outcomes so that each valuation is Lipschitz continuous with uniformly bounded Lipschitz constants. Condition (c) clearly holds when the quality schedule q with associated indirect utility U is individually rational. Suppose now that (c) is in place. We must show that for each θ-consumer, the following inequality holds:

$$U(\theta_L) \;+\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta} \;+\; \lambda \theta\, m(r(\theta)) \;\geq\; 0.$$

Using (6), we can write

$$\begin{aligned}
&U(\theta_L) \;+\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta} \;+\; \lambda \theta\, m(r(\theta)) \\
&\geq\; U(\theta_L) \;+\; \int_{\theta_L}^{\theta} \big[(1+\lambda) m(q(\tilde{\theta})) - \big(\lambda \tilde{\theta} m(r(\tilde{\theta}))\big)'\,\big] d\tilde{\theta} \;+\; \lambda \theta m(r(\theta) \\
&=\; U(\theta_L) \;+\; \int_{\theta_L}^{\theta} (1+\lambda) m(q(\tilde{\theta})) d\tilde{\theta} \;+\; \lambda_L \theta_L m(r(\theta_L)) \;\geq\; 0,
\end{aligned}$$

where the last inequality follows from (c). □

To find the incentive feasible, revenue maximizing menu of contracts, we use the generalized Mirrlees representation to express payments in terms of the integral of the subderivative:

$$p(\theta) \;=\; v(q(\theta), \theta) \;-\; U(\theta_L) \;-\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \qquad \text{for all } \theta \in \Theta, \tag{7}$$

where the selection $s(q(\cdot), \cdot)$ is associated to the indirect utility U. We shall use (7) and the fact that the firm puts $U(\theta_L) = -\lambda \theta_L m(r(\theta_L))$, which follows from Proposition 2, to write expected profits. Integration by parts leads to:

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \Big((1+\mu)\theta m(q(\theta)) - \mu \theta m(r(\theta)) + \lambda \theta_L m(r(\theta_L)) - h(\theta) s(q(\theta), \theta) - c(q(\theta))\Big) f(\theta)\, d\theta.$$

The first three terms of the integrand in the above equation denote the direct impact on surplus due to the presence of loss aversion and endogenous participation. The next term, involving the inverse hazard rate and the selection $s(q(\theta), \theta)$, captures the additional flexibility gained by the monopolist due to potential downward kinks in the consumers' valuation function, as the subderivative correspondence $S(q(\cdot), \cdot)$ depends on the choice of $q(\cdot)$.

From Equation 6 and the fact that $h \geq 0$, we express any revenue maximizing selection as

$$s(q(\theta), \theta) \;=\; m(q(\theta))$$

for all θ such that $r(\theta) < q(\theta)$, and

$$s(q(\theta), \theta) \;=\; (1+\lambda)m(q(\theta)) \;-\; \lambda\big(\theta m(r(\theta))\big)'$$

for all θ such that $r(\theta) \geq q(\theta)$. Let $\theta \mapsto \phi(\theta) \equiv \theta - h(\theta)$ be the marginal virtual consumption valuation, which is twice continuously differentiable and non-decreasing. We obtain:

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \Big((1+\mu)\phi(\theta) m(q(\theta)) \;-\; c(q(\theta)) \;+\; \lambda \theta_L m(r(\theta_L)) \\ \;-\; \mu\, \theta m(r(\theta)) \;+\; \mu\, h(\theta)\big(\theta m(r(\theta))\big)' \Big) f(\theta)\, d(\theta), \tag{8}$$

with $\mu = 0$ whenever $q(\theta) > r(\theta)$ and $\mu = \lambda$ otherwise. Using Proposition 2, the problem of the monopolist can now be stated as follows. Choose a quality schedule $\theta \mapsto q^{sb}(\theta) \geq 0$ to maximize Π^{sb} in (8), subject to the integral monotonicity condition. Here we present the solution to this problem ignoring the constraints. In Section 6, we argue that this solution is incentive compatible.

Denote by $\underline{\Theta}$ the subset of consumer types for which $\phi(\theta)$ is non-positive. By monotonicity, $\underline{\Theta}$ is either empty or a subinterval $[\theta_L, \underline{\theta}] \subseteq \Theta$.

Lemma 1. *For all $\theta \in \underline{\Theta}$, the optimal quality schedule is $q^{sb}(\theta) = 0$.*

Proof. Fix any $\theta \in \underline{\Theta}$ and assume $r(\theta) > 0$ (the case of $r(\theta) = 0$ is similar). Choosing a quality level $q^{sb}(\theta) = 0$ performs better than choosing an alternative quality level $0 < q \leq r(\theta)$, as the value of $\mu = \lambda$ does not change and the first term of the integrand in (8) becomes negative. Now consider the alternative quality level $\hat{q} > r(\theta) > 0$, so that $\mu = 0$. The difference between profits at $q^{sb}(\theta) = 0$ and profits at $\hat{q}$ is given by:

$$-\,\lambda\phi(\theta)m(r(\theta)) \;+\; \lambda h(\theta)\theta m'(r(\theta))r'(\theta) \;-\; \phi(\theta)m(\hat{q}) \;+\; c(\hat{q}).$$

This expression is nonnegative, as $\phi(\theta) \leq 0$, $m' > 0$ and $r' \geq 0$. □

Thus, just as in the standard case, a revenue maximizing monopolist assigns zero quality to types with negative marginal virtual consumption valuation, which leaves every θ-consumer in $\underline{\Theta}$ experiencing a total utility of $-\lambda\theta m(r(\theta))$.

Next we argue that for high reference quality levels, the optimal quality schedule maximizes virtual surplus under reference-dependent preferences, whereas for intermediate reference quality levels, optimal quality coincides with the reference point. For all $\theta \in \Theta \setminus \underline{\Theta}$, define the quality schedules $\theta \mapsto q^*(\theta)$ and $\theta \mapsto q^*_\lambda(\theta)$ as the solution to the following problems:

$$q^*(\theta) \;\equiv\; \arg\max_{q \geq 0} \{\phi(\theta)m(q) - c(q)\} \qquad \text{and} \qquad q^*_\lambda(\theta) \;\equiv\; \arg\max_{q \geq 0} \{(1+\lambda)\phi(\theta)m(q) - c(q)\}.$$

As before, both q^* and q^*_λ are differentiable and non-decreasing, with $q^*(\theta) < q^*_\lambda(\theta)$ for all $\theta \in \Theta \setminus \underline{\Theta}$. To simplify notation, let Θ_m, Θ_r and Θ_l denote subsets of $\Theta \setminus \underline{\Theta}$ for which the following holds:

$$\begin{aligned} r(\theta) \;<\; q^*(\theta) \quad & : \text{ for each } \theta\text{-consumer in } \Theta_m, \\ q^*(\theta) \;\leq\; r(\theta) \;\leq\; q^*_\lambda(\theta) \quad & : \text{ for each } \theta\text{-consumer in } \Theta_r, \\ q^*_\lambda(\theta) \;<\; r(\theta) \quad & : \text{ for all } \theta \in \Theta_l. \end{aligned}$$

By continuity, these subsets are either empty or subintervals. For the remainder of the paper, we make the following simplifying assumption to ensure that Θ is partitioned in at most four subintervals: $\theta \to \operatorname{sgn}(r(\theta) - q^*(\theta))$ and $\theta \to \operatorname{sgn}(r(\theta) - q^*_\lambda(\theta))$ are weakly monotonic on $\Theta \setminus \underline{\Theta}$. This rules out the reference function r and q^* (respectively, q^*_λ) cross more than once on the relevant parts of the type space, although the values of r and q^* (respectively, q^*_λ) could coincide for a continuum of consumer types. This weak single crossing condition can be relaxed to a finite crossing condition, but doing so complicates the notation and the arguments without providing additional insights to the results.

Lemma 2. *For each θ-consumer in Θ_l, the optimal quality schedule is $q^{sb}(\theta) = q^*_\lambda(\theta)$; for each θ-consumer in Θ_r, the optimal quality schedule is $q^{sb}(\theta) = r(\theta)$.*

Proof. Fix a type $\theta \in \Theta \setminus \underline{\Theta}$ and assume $r(\theta) > q^*_\lambda(\theta)$. The unique maximizer of the integrand in the profit function of (8) with $\mu = \lambda$ is precisely $q^*_\lambda(\theta)$. Let then $q^{sb}(\theta) = q^*_\lambda(\theta)$. Any deviation to an alternative quality level $q \leq r(\theta)$ will only hurt profits as it decreases the first term of the objective function in (8) without changing the rest of the expression. Now consider a deviation to $\hat{q} \geq r(\theta)$, which changes μ in the objective function from λ to 0. Since $\hat{q} \geq r(\theta) > q^*(\theta)$, it follows that the optimal deviation in this case is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ and $\mu = \lambda$, and profits at $r(\theta)$ and $\mu = 0$ is given by:

$$\lambda h(\theta)\theta m'(r(\theta)r'(\theta) \;\geq\; 0.$$

It follows that profits at $q^*_\lambda(\theta)$ and $\mu = \lambda$ are strictly greater than profits at $r(\theta)$ with $\mu = \lambda$, which are greater than profits at $r(\theta)$ with $\mu = 0$.

Assume now $q^*(\theta) \leq r(\theta) \leq q^*_\lambda(\theta)$ and let $q^{sb}(\theta) = r(\theta)$. Consider the quality level $\hat{q}$, such that $\hat{q} > r(\theta) \geq q^*(\theta)$. The integrand of (8) has $\mu = 0$. Clearly, profits are strictly decreasing in quality as long as $\hat{q} > r(\theta)$, so that there is no upward profitable deviation. One can use a similar argument to show that there is no downward profitable deviation from $q^{sb}(\theta) = r(\theta)$, and therefore this is the optimal quality schedule. □

We now find the optimal quality schedule for consumers with reference points below $q^*(\theta)$.

Lemma 3. *For each θ-consumer in Θ_m, the optimal quality schedule $q^{sb}(\theta)$ is $q^*(\theta)$ or $r(\theta)$.*

Proof. Consider a θ-consumer in $\Theta \setminus \underline{\Theta}$ with $0 \leq r(\theta) < q^*(\theta)$. From first order conditions, the unique maximizer of the integrand in (8) with $\mu = 0$ is $q^*(\theta)$. Any deviation to a quality level $\hat{q} > r(\theta)$ will not change the value of μ from 0 to λ and thus will only decrease profits. Among deviations from $q^*(\theta)$ to quality levels $\hat{q} \leq r(\theta)$ that change the parameter μ to λ in (8), the one with highest profits is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ with associated $\mu = \lambda$ and profits at $q^*(\theta)$ with associated $\mu = 0$ is given by the expression:

$$\triangle(r(\theta)) \;=\; \lambda h(\theta)\theta m'(r(\theta)r'(\theta) - \big(\phi(\theta)m(q^*(\theta)) - c(q^*(\theta)) - \phi(\theta)m(r(\theta)) + c(r(\theta))\big). \quad (9)$$

The sign of the above expression depends on the difference between gains associated with a change in the value of μ from 0 to λ in the profit function, and efficiency gains in virtual surplus derived from shifting quality from $r(\theta)$ to $q^*(\theta)$. □

If the reference formation function r is flat on Θ_m, then clearly one has $q^{sb}(\theta) = q^*(\theta)$ as Equation 9 becomes negative. Suppose r is strictly increasing. By continuity, for consumers with q^* sufficiently close to $r(\theta)$, the term in brackets in the right-hand side of (9) is approximately zero. Thus, in principle, one may have a revenue-maximizing quality schedule that assigns $r(\theta)$ for types at both the low and high ends of Θ_m, and $q^*(\theta)$ for consumers in the middle range. This pattern violates the incentive compatibility constraints.

Lemma 4. *Consider a quality schedule $\theta \mapsto q(\theta)$ such that for types $\theta' < \theta''$ in Θ_m sufficiently closed to each other, it assigns $q(\theta') = q^*(\theta') > r(\theta')$ and $q(\theta'') = r(\theta'') < q^*(\theta'')$. Then q is not incentive compatible.*

Proof. From Proposition 2, it suffices to show a violation of monotonicity for the θ', θ''-consumers: $v(q(\theta''), \theta'') - v(q(\theta''), \theta') < v(q(\theta'), \theta'') - v(q(\theta'), \theta')$. The left-hand side of the inequality is equal to $(\theta'' - \theta')m(r(\theta'))$, whereas its right-hand side is equal to $(\theta'' - \theta')m(q^*(\theta'))$. The result follows by strict monotonicity of m. $\square$

One concludes from Lemma 3 and Lemma 4 that if Θ_m is non-empty, the optimal quality schedule corresponds to one of the following three cases: either it assigns $q^*(\theta)$ for all θ-consumers in Θ_m; or $r(\theta)$ for all θ-consumers in Θ_m; or there exists a cutoff type $\theta_c \in \Theta_m$ such that $q^{sb}(\theta) = r(\theta)$ for all $\theta \leq \theta_c$, and $q^{sb}(\theta) = q^*(\theta)$ for all $\theta > \theta_c$. The existence and position of the cutoff type θ_c depends on the details of the model (i.e., marginal virtual consumption valuation, distribution of types, reference function r, and the value of the gain-loss and loss-aversion coefficients). We are now ready to derive the optimal quality schedule.

Proposition 3. *Consider the contract design problem of a monopolist facing loss averse consumers. The optimal menu of incentive compatible and individually rational contracts consists of pairs* $\{(q^{sb}(\theta), p^{sb}(\theta)) \mid \theta \in \Theta\}$ *such that*

$$q^{sb}(\theta) \;=\; \begin{cases} 0 & : \ \theta \in \underline{\Theta}, \\ q^*_\lambda(\theta) & : \ \theta \in \Theta_l, \\ r(\theta) & : \ \theta \in \Theta_r, \ \theta \in \Theta_m, \ \theta \leq \theta_c \ (\theta_c \in \operatorname{cl}\Theta_m), \\ q^*(\theta) & : \ \theta \in \Theta_m, \ \theta > \theta_c \ (\theta_c \in \operatorname{cl}\Theta_m), \end{cases} \tag{10}$$

$$p^{sb}(\theta) \;=\; v(q^{sb}(\theta), \theta) + \lambda\theta_L m(r(\theta_L)) - \int_{\theta_L}^{\theta} s(q^{sb}(\tilde\theta), \tilde\theta)\, d\tilde\theta, \tag{11}$$

where the selection $s(q^{sb}(\theta), \theta) = (1+\mu)m(q^{sb}(\theta)) - \mu\big(\theta m(r(\theta))\big)'$ *has* $\mu = \lambda$ *for* $q^{sb}(\theta) \leq r(\theta)$ *and* $\mu = 0$ *otherwise.*

Proof. The construction of the optimal quality schedule $\theta \mapsto q^{sb}(\theta)$ described above follows from Lemmas 1 to 4. To show that the informational constraints are satisfied, it suffices to show that conditions (a) to (c) of Proposition 2 are in place. Using (11), we write the indirect utility of the lowest type consumer as $U(\theta_L) = -\lambda\theta_L m(r(\theta_L))$, which guarantees that condition (c) is in place. From the equation for prices one also has that the Mirrlees representation of the indirect utility associated with q^{sb} is in place. We prove part (a) of Proposition 2 in Section 6. $\square$

The verification of the integral monotonicity condition is complicated by the fact that the selection $s(q^{sb}(\theta), \theta)$ changes value depending on whether the optimal quality $q^{sb}(\theta)$ is greater or less than the reference level $r(\theta)$. The economic intuition however is preserved: the optimal quality schedule is non-decreasing on the entire type interval Θ, and corresponds to the quality that maximizes virtual profits (for some consumers, this quality coincides with the reference quality level). We gather the monotonicity and other properties of q^{sb} in the following corollary.

Corollary 2. *The optimal quality schedule* $\theta \mapsto q^{sb}(\theta)$ *is non-decreasing, piecewise differentiable and continuous on* Θ*, except possibly at* $\theta_c \in \Theta_m$*.*

From Proposition 3 one concludes that lower types with zero quality allocation receive zero rent, as in the standard monopoly pricing model, while higher types receive a positive rent that is weakly increasing in types. Note also that while the optimal quality schedule q^{sb} under loss aversion exhibits common elements with the optimal schedule under standard consumption valuation, the actual shape of q^{sb} depends on the reference formation process. On the one hand, the informational constraints have an effect on contract design similar to the case without loss aversion, as the optimal menu of contracts distorts qualities for lower types and allocates qualities for higher types according to the marginal virtual surplus, which is modified to consider reference dependent preferences. On the other hand, it is possible that for a subset of types of positive measure, the optimal quality schedule be determined entirely by the reference function. Since r is assumed to be weakly increasing, this implies that optimal quality schedules may exhibit flat parts among increasing parts for intermediate and higher types. The pricing structure takes informational incentives but also considers the effects of consumers' loss aversion to enhance profits.

Thus, Proposition 3, which isolates the effect of loss aversion biases onto optimal contract design under asymmetric information, stresses both the new opportunities for the firm created by reference-dependent preferences and the limits imposed by the informational constraints. It also highlights the relevance of the reference point formation process. We explore these possibilities next.

3.2. **Application: optimal contracts under linear valuation and quadratic cost.** Here we impose additional assumptions to explicitly derive optimal contracts under loss aversion: consumers are uniformly distributed on $\Theta = [0, 1]$, m is linear and c is quadratic, so that $m(q) = q$ and $c(q) = q^2/2$ for all $q \geq 0$. Under these assumptions, the marginal virtual consumption valuation is $\phi(\theta) = 2\theta - 1$, with $\underline{\Theta} = [0, 1/2]$. Readily, one obtains $q^*(\theta) = 2\theta - 1$ and $q^*_\lambda(\theta) = (1 + \lambda)(2\theta - 1)$, for all $\theta \in \Theta \setminus \underline{\Theta}$.

After learning his type, but before observing the menu of contracts offered by the firm, a θ-type consumer forms aspirational quality levels according to a reference point formation process. We shall consider different ways in which consumers form beliefs about reference quality levels. Implicitly, we are assuming in all cases explored here that potential buyers are unaware of their loss aversion, prior to the market interaction with the firm. That is, the effects of the reference valuation in consumer welfare become tangible only after the menu of contracts is observed, immediately before a purchasing decision is made. The question of consistency of beliefs is addressed in Section 4; here we want to focus on the effects of different beliefs in terms of optimal contract design.

3.2.1. *Optimal contract under r_1.* Consider first a constant reference function $\theta \mapsto r_1(\theta)$, with

$$r_1(\theta) \;=\; 1/2, \qquad \text{for each } \theta \in \Theta.$$

That is, the reference point shared by all consumers is the quality level that maximizes expected consumer surplus. We readily observe that the consumer type space is partitioned in four subintervals: $\underline{\Theta}$ as before, $\Theta_{l1} = (1/2, (3 + 2\lambda)/4(1 + \lambda))$, $\Theta_{r2} = [(3+2\lambda)/4(1+\lambda), 3/4]$, and $\Theta_{m2} = (3/4, 1]$. Note that $r'(\theta) = 0$ everywhere. Hence, from Proposition 3, the optimal quality schedule q_1^{sb} associated with r_1 is given by

$$q_1^{sb}(\theta) \;=\; \begin{cases} 0 & : \; \theta \in \underline{\Theta}, \\ (1+\lambda)(2\theta - 1) & : \; \theta \in \Theta_{l1}, \\ 1/2 & : \; \theta \in \Theta_{r1}, \\ 2\theta - 1 & : \; \theta \in \Theta_{m1}. \end{cases}$$

In this case, the quality levels purchased by consumers differ from their reference points except for buyers in the midrange interval Θ_{r1}. As mentioned before, a divergence between actual consumption and expectations is allowed, as long as the optimal menu of contracts satisfies individual rationality.

3.2.2. *Optimal contract under r_2.* Consider a reference function $\theta \mapsto r_2(\theta)$ defined by

$$r_2(\theta) \;=\; q^e(\theta) \;\equiv\; \theta, \qquad \text{for all } \theta \in \Theta.$$

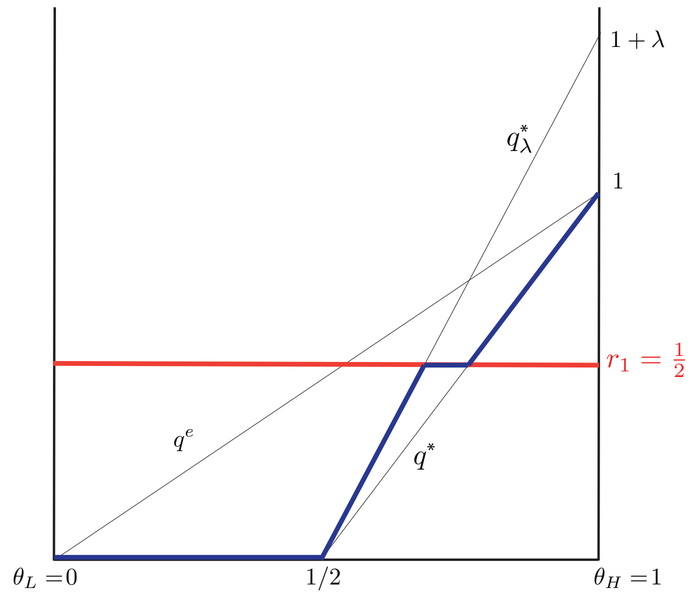

FIGURE 1. Optimal quality schedules q_1^{sb} for reference function r_1.

In this scenario, each θ-consumer has as his reference point the quality level that maximizes consumer surplus at θ. Then, $\Theta_{l2} = (1/2, (1+\lambda)/(1+2\lambda))$ and $\Theta_{r2} = [(1+\lambda)/(1+2\lambda), 1]$. From Proposition 3, the optimal quality schedule given r_2 is

$$q_2^{sb}(\theta) \;=\; \begin{cases} 0 & : \;\theta \in \underline{\Theta}, \\ (1+\lambda)(2\theta-1) & : \;\theta \in \Theta_{l2}, \\ \theta & : \;\theta \in \Theta_{r2}. \end{cases}$$

Note that the optimal quality schedule consists of three differentiated parts. Intermediate consumers see their optimal quality levels distorted downward, but not as much as the quality schedule without loss aversion. In other words, the firm is exploiting intermediate consumers' loss aversion to restrict the distortion from efficient quality levels. This also induces the firm to offer efficient quality levels to consumers with high marginal willingness to pay.

3.2.3. *Optimal contract under r_3.* Consider a reference function $\theta \mapsto r_3(\theta)$ given by

$$r_3(\theta) \;=\; \theta/2 \,+\, 1/4, \qquad \text{for all } \theta.$$

An interpretation for this reference function is that each θ-consumer puts equal weight into his reference point being the efficient quality level, given his type, and the expected efficient quality level. As in the previous case, one can immediately see that the type space is partitioned into four subintervals: $\underline{\Theta}$, $\Theta_{l3} = (1/2, (5+4\lambda)/(6+8\lambda))$, $\Theta_{r3} = [(5+4\lambda)/(6+8\lambda), 5/6]$, and $\Theta_{m3} = (5/6, 1]$. Notice that $r'(\theta) > 0$ on Θ_{m3}. Thus, we shall use Equation 9 to evaluate the shape of the optimal quality schedules in that subinterval. For each θ-consumer in Θ_{m3}, the difference between profits at $r_3(\theta)$ and $\mu = \lambda$ and profits at $q^*(\theta)$ and $\mu = 0$ is:

$$\triangle(r_3(\theta), q^*(\theta)) \;=\; \lambda(1-\theta)\theta/2 \;-\; (3\theta/2 - 5/4)^2/2.$$

One has that $\triangle(r_3(\theta), q^*(\theta)) > 0$ for types sufficiently closed to $\theta = 5/6$, and $\triangle(r_3(1), q^*(1)) < 0$. Since the profit difference, as a function of consumer types, is continuous and strictly decreasing on Θ_{3l}, there exists a unique cutoff type θ_c such that $\triangle(r_3(\theta_c), q^*(\theta_c)) = 0$. For types to the left of θ_c, the optimal quality schedule coincides with reference levels, even

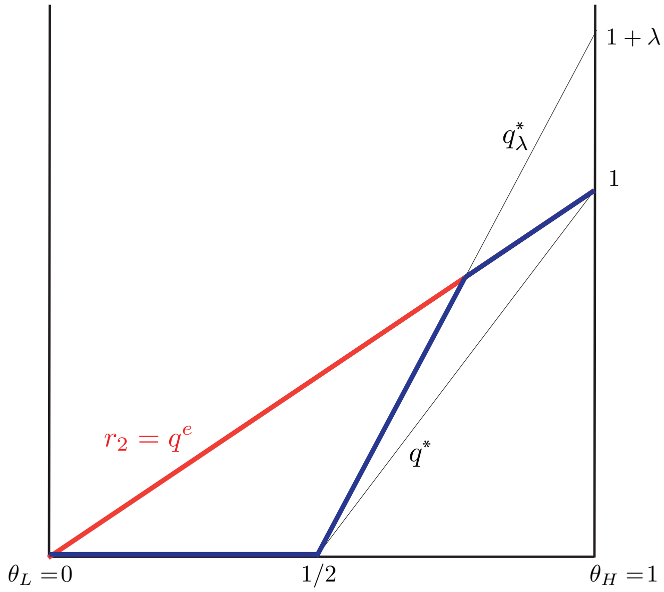

FIGURE 2. Optimal quality schedules q_2^{sb} for reference function r_2.

though these reference points lie below the marginal virtual valuation. This is because the monopolist gains more flexibility in terms of contract design, as it now can put $\mu = \lambda$ in the selection $s(q_3^{sb}(\cdot), \cdot)$ that is used to provide incentives. Using the above observations, we construct the optimal quality schedule q_3^{sb} under the reference function r_3:

$$q_3^{sb}(\theta) \;=\; \begin{cases} 0 & : \;\theta \in \underline{\Theta}, \\ (1+\lambda)(2\theta-1) & : \;\theta \in \Theta_{l3}, \\ \theta/2 + 1/4 & : \;\theta \in \Theta_{r3},\;\; \theta \in \Theta_{m3}, \theta \leq \theta_c, \\ 2\theta - 1 & : \;\theta \in \Theta_{m3}, \theta > \theta_c. \end{cases}$$

As with r_2, the optimal menu of contracts associated with r_3 involves quality levels that differ from the reference quality levels for consumers with low and high types.

4. Self-Confirming Contracts

Our results so far illustrate two related points. First, optimal contract design in the presence of loss aversion is heavily influenced by the reference function. Indeed, while the broad insights of contract theory prevail — i.e., the allocation of qualities is monotone and downward distorted for lower types — for intermediate and high types optimal contracts can exhibit various new features, including discontinuities, flat parts, and no distortion from efficient levels. Second, the quality schedule in an optimal menu of contracts can be generated by a multiplicity of reference functions, and this may affect the way prices are structured. In particular, it may be possible to have two different pricing schemes implementing the same quality schedule (and assigning the same payoff to the lowest type).

As a simple illustration of the latter point, consider the optimal quality schedule derived for the linear valuation and quadratic cost function of Section 3.2 under different reference functions: $\underline{r}(\theta) = 0$ for all θ-consumers, and $r(\theta) = 0$ for $\theta \in \underline{\Theta}$, $r(\theta) = q^*(\theta)$ for $\theta \in \Theta \setminus \underline{\Theta}$. In both cases, the optimal quality schedule is $q^{sb}(\theta) = 0$ for $\theta \in \underline{\Theta}$, and $q^{sb}(\theta) = q^*(\theta) = 2\theta - 1$ for $\theta \in \Theta \setminus \underline{\Theta}$. Also, in both cases, the total valuation for each θ-consumer equals $\theta q^{sb}(\theta)$. Prices however differ for consumers with positive quality consumption, with

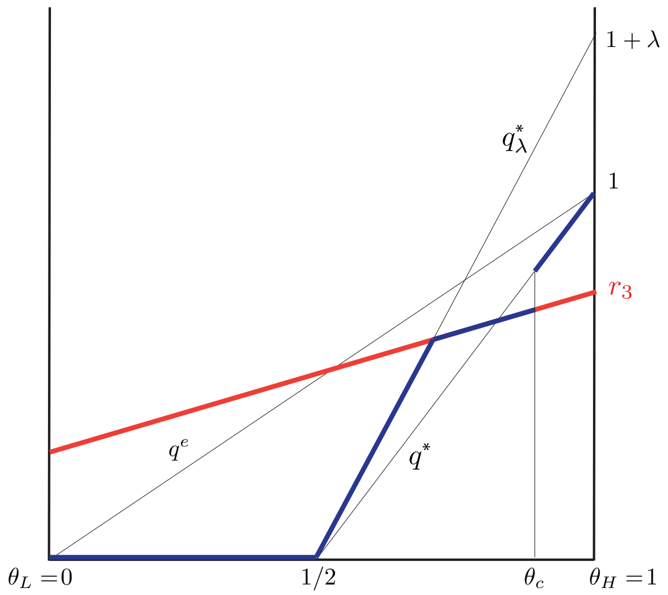

FIGURE 3. Optimal quality schedules q_3^{sb} for reference function r_3.

$\underline{p}^{sb}(\theta) = \theta^2 - 1/4$ in the first case and $p^{sb}(\theta) = \theta^2 - 1/4 + \lambda\theta(2\theta - 1) = \underline{p}^{sb}(\theta) + \lambda\theta q^{sb}(\theta)$ in the second one. The extra term in p^{sb} has to do with the fact that for all but the low type consumers, the quality assigned under the optimal contract under r coincides with the reference quality. Thus the firm has additional flexibility (due to the kink in the total valuation) to exploit consumers' loss aversion without violating the incentive constraints. Since the outside option under r is precisely $-\lambda\theta q^{sb}(\theta)$ for intermediate and high type-consumers, the optimal pricing scheme is also individually rational.

Until now, we have allowed for differences between the quality schedule offered by the firm and the reference quality level expected by consumers. Regardless of how subjective beliefs about quality levels are formed, a θ-type consumer will purchase the quality-price pair designed for him, even if the quality level differs from his expectations. This leads to situations similar to the previous illustration, where forward looking buyers aware of their loss aversion (and thus aware of how a revenue maximizing monopolist reacts to reference-dependent preferences) may decrease their reference points to avoid surplus extraction. To rule out these inconsistencies in belief formation, we impose the following restriction on the reference formation process. Say a reference function $\theta \mapsto r(\theta)$ is *self-confirming* if the optimal quality schedule q^{sb} generated by r is such that $q^{sb}(\theta) = r(\theta)$, for all consumer types $\theta \in \Theta$. An optimal menu of contracts $\{(q^{sb}(\theta), p^{sb}(\theta)) \mid \theta \in \Theta\}$ is self-confirming if it is generated by a self-confirming reference function. The set of self-confirming reference functions is clearly non-empty. It is difficult to provide a full characterization of this set. However, a sufficient condition for a reference function to be self-confirming is readily obtained.

Let $\theta \mapsto r^*(\theta)$ be non-decreasing and piecewise continuously differentiable. Say that r^* is a *regular* reference function if for all θ in $\underline{\Theta}$, $r^*(\theta) = 0$, and for all θ in $\Theta \setminus \underline{\Theta}$, one of the following conditions is satisfied: (i) $q^*(\theta) \leq r^*(\theta) \leq q^*_\lambda(\theta)$, or (ii) $q^*(\theta) > r^*(\theta)$ and $\triangle(r(\theta)) \geq 0$ (where $\triangle(r(\theta))$ expresses the difference between profits obtained at $r(\theta)$ and $\mu = \lambda$, and profits obtained at $q^*(\theta)$ and $\mu = 0$, see Equation 9; as this is a differential equation, the statement should be interpreted as saying that it is satisfied for all but a finite number of types).

Proposition 4. *If the reference function $\theta \mapsto r^*(\theta)$ is regular, then it is self-confirming.*

Proof. It follows immediately from Proposition 3. □

The above result implies that there is a multiplicity of solutions to the monopolist problem, as the set of self-confirming contracts is non-empty and infinite, as any regular reference function for which $q^*(\theta) \leq r^*(\theta) \leq q^*_\lambda(\theta)$ generates a self-confirming optimal menu of contracts. On the other hand, the existence of a reference function that (partially) lies below q^* depends on the existence of a solution to the differential equation in (9). Note however that any such reference function will need to satisfy $r^*(\theta_H) \geq q^*(\theta_H)$. These observations lead to the following question. Among all regular, self-confirming optimal contracts, is there any that is preferred by the firm (or by the consumers)? One conjecture is that the firm would prefer a self-confirming contract for which $q^{sb}(\theta) = r^*(\theta) = q^*_\lambda(\theta)$ whenever $q^e(\theta) > q^*_\lambda(\theta)$ and $q^{sb}(\theta) = r^*(\theta) = q^e(\theta)$ otherwise (see Figure 2 for an illustration of this case). Such reference function minimizes downward distortions from the efficient quality levels, both in terms of providing efficient quality levels to as many consumers as possible and also reducing the size of the quality distortion for intermediate consumers. This conjecture, it turns out, is incorrect. The preferred self-confirming reference function for the firm is given in the next proposition.

Proposition 5. *A necessary and sufficient condition for $\theta \mapsto r^*_F(\theta)$ to be the firm's preferred reference function among all self-confirming reference functions is that*

$$r^*_F(\theta) \;=\; \begin{cases} 0 & : \;\theta \in \underline{\Theta}, \\ q^*_\lambda(\theta) & : \;\theta \in \Theta \setminus \underline{\Theta}. \end{cases}$$

Proof. See Section 6. □

The intuition for this result is the following. From Equation 8, one can write expected revenues given a self-confirming reference function r^* as:

$$\Pi^{sb}\left[r^*\right] \;=\; \int_{\underline{\theta}}^{\theta_H} \Big(\phi(\theta)m(r^*(\theta)) \;-\; c(r^*(\theta)) \;+\; \lambda h(\theta)\theta m'(r^*(\theta)){r^*}'(\theta)\Big) f(\theta)d(\theta).$$

To decide which reference function is preferred, the firm trades off a reduction in consumption efficiency, captured by the first two terms of the integrand, and an increase in profits due to the exploitation of consumers' loss aversion, captured by the last term in the above integrand. The surprising result is that this last effect dominates, irrespective of the size of the loss aversion coefficient and of the distribution of types, because second order effects are sufficiently small: the monopolist is not tempted to exploit the loss aversion of higher type consumers without first exploiting the loss aversion of low type consumers.

5. Concluding Remarks

In this paper, we study optimal contract design by a revenue maximizing monopolist who faces consumers with heterogeneous tastes, reference-dependent preferences and loss aversion. Our paper follows the line of work pioneered by della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), Kőszegi and Rabin (2006), and Heidhues and Kőszegi (2008), among others, in studying the optimal responses of profit maximizing firms in a market context with consumers who have systematic biases.

We found that, while the general insights of contract theory and mechanism design prevail, namely, the optimal incentive compatible quality schedule is monotone and presents downward distortions from the efficient qualities for all but the high types, for intermediate types the optimal quality schedule is determined by the reference function. Thus, depending on how potential buyers form their expectations of quality consumption, the optimal contracts may exhibit various new features. This can partially explain complex contracts often found in cable, telecommunications and electricity industries.

Our research stresses the importance of understanding how the initial reference points are formed. Consumers' aspirational consumption levels may be influenced by fashion and mode cycles, by social and peer pressure, by imitation effects, but also by marketing campaigns, the characteristics of the product or the industry under analysis, etc.

6. Omitted Proofs

Proof of Proposition 1. Fix a type $\theta \in \Theta$ and suppose that $r(\theta) < q_0^e(\theta)$. In such case, first order conditions applied to (2) yield a unique maximum at $q_0^e(\theta)$ with profits equal to

$$\Pi^{fb}(\theta|r(\theta) < q_0^e(\theta)) \;=\; \theta m(q_0^e(\theta)) \;-\; c(q_0^e(\theta)) \;+\; \lambda\theta m(r(\theta)).$$

Choosing a different quality level $q > r(\theta)$ does not change the objective function of the monopolist problem and strictly reduces profits. Choosing an alternative quality level $\hat{q} \leq r(\theta)$ shifts objective function in (2) to incorporate the term $\lambda\,[\theta m(\hat{q}) - \theta m(r(\theta))]^-$. Since $r(\theta) < q_\lambda^e(\theta)$, the monopolist would choose a deviation to $\hat{q} = r(\theta)$ with associated profits equal to $(1+\lambda)\theta m(r(\theta)) - c(r(\theta))$. Subtracting this last expression from profits at $q_0^e(\theta)$, we obtain:

$$\theta m(q_0^e(\theta)) \;-\; c(q_0^e(\theta)) \;-\; \theta m(r(\theta)) \;+\; c(r(\theta)) \;>\; 0.$$

Showing that the revenue maximizing quality level is $q_\lambda^e(\theta)$ when $r(\theta) > q_\lambda^e(\theta)$ is similar and therefore omitted. In this case, profits are equal to

$$\Pi^{fb}(\theta|r(\theta) > q_\lambda^e(\theta)) \;=\; (1+\lambda)\theta m(q_\lambda^e(\theta)) \;-\; c(q_\lambda^e(\theta)).$$

Now suppose that for the θ-type consumer, $q_0^e(\theta) \leq r(\theta) \leq q_\lambda^e(\theta)$. Choosing a quality level $\hat{q} > r(\theta) \geq q_0^e(\theta)$ leaves us with $\lambda\,[\theta m(\hat{q}) - \theta m(r(\theta))]^- = 0$ in (2), and thus we are in the strictly decreasing part of the profit function. Hence the monopolist will put $\hat{q} = r(\theta)$. Similarly, choosing $\hat{q} < r(\theta) \leq q_\lambda^e(\theta)$ yields to $\lambda\,[\theta m(\hat{q}) - \theta m(r(\theta))]^- = \lambda\theta m(\hat{q}) - \lambda\theta m(r(\theta))$ in (2), so that the monopolist is now in the strictly increasing section of the profit function. It follows that, in this case, the revenue maximizing quality level is $r(\theta)$, which generates profits equal to

$$\Pi^{fb}(\theta|q^e(\theta) \leq r(\theta) \leq q^e\lambda(\theta)) \;=\; (1+\lambda)\theta m(r(\theta)) \;-\; c(r(\theta)).$$

This completes the proof. □

Proof of Corollary 1. It is clear from the expressions $\Pi^{fb}(\theta|\cdot)$ in the above proof that profits under complete information and loss aversion are strictly increasing in the reference quality level $r(\theta)$ for all $r(\theta) \leq q_\lambda^e(\theta)$, and independent of $r(\theta)$ for all $r(\theta) > q_\lambda^e(\theta)$. □

Proof of Proposition 3. We have argued in the main text that the menu $\{(q^{sb}(\theta), p^{sb}(\theta)) \mid \theta \in \Theta\}$ is individually rational and maximizes expected revenues. We show here that it is also incentive compatible. By Proposition 2(a), it suffices to show that the integral monotonicity condition is satisfied for any two pair of types.

Let $\theta', \theta'' \in \Theta$ be two consumer types such that $\theta' < \theta''$ and suppose $q^{sb}(\theta') \leq r(\theta') \leq r(\theta'') < q^{sb}(\theta'')$ holds — all remaining cases are similarly proven. By construction of the optimal quality schedule, there exists a type θ^*, with $\theta' < \theta^* < \theta''$, for which one has $r(\theta') \leq r(\theta^*) = q^{sb}(\theta^*) \leq r(\theta'')$. Moreover, we can choose θ^* so that $q^{sb}(\theta) \leq r(\theta)$ for all $\theta' \leq \theta \leq \theta^*$, and $q^{sb}(\theta) > r(\theta)$ for all $\theta^* < \theta \leq \theta''$. We first write the valuation differences in a suitable form:

$$\begin{aligned} v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta') \;&=\; \theta'' m(q^{sb}(\theta'')) - \theta' m(q^{sb}(\theta'')) \pm \theta^* m(q^{sb}(\theta'')) \\ &\geq\; v(q^{sb}(\theta''), \theta'') - v(q^{sb}(\theta''), \theta^*) + \theta^* m(q^{sb}(\theta^*)) - \theta' m(q^{sb}(\theta^*)) \end{aligned}$$

thus it follows that

$$
\begin{aligned}
& v(q^{sb}(\theta''),\theta'') - v(q^{sb}(\theta''),\theta') \\
& \qquad \geq\ v(q^{sb}(\theta''),\theta'') - v(q^{sb}(\theta''),\theta^*)\ +\ v(q^{sb}(\theta^*),\theta^*) - v(q^{sb}(\theta^*),\theta'). \qquad (12)
\end{aligned}
$$

A similar argument yields to the following inequality:

$$
\begin{aligned}
& v(q^{sb}(\theta'),\theta^*) - v(q^{sb}(\theta'),\theta')\ +\ v(q^{sb}(\theta^*),\theta'') - v(q^{sb}(\theta^*),\theta^*) \\
& \qquad \geq\ v(q^{sb}(\theta'),\theta'') - v(q^{sb}(\theta'),\theta'). \qquad (13)
\end{aligned}
$$

Notice now that

$$
\begin{aligned}
v(q^{sb}(\theta''),\theta'') - v(q^{sb}(\theta''),\theta^*) &= \int_{\theta^*}^{\theta''} m(q^{sb}(\theta''))\,d\tilde\theta \\
&\geq \int_{\theta^*}^{\theta''} m(q^{sb}(\tilde\theta))\,d\tilde\theta \ =\ \int_{\theta^*}^{\theta''} s(q^{sb}(\tilde\theta),\tilde\theta)\,d\tilde\theta, \qquad (14)
\end{aligned}
$$

where the inequality follows from the monotonicity of q^{sb} and the last equality from the fact that $\mu = 0$ for all $\theta^* < \tilde\theta \leq \theta''$. Furthermore, we can write

$$
\begin{aligned}
v(q^{sb}(\theta^*),\theta^*) - v(q^{sb}(\theta^*),\theta') &\geq \theta^* m(q^{sb}(\theta^*)) + \lambda\big(\theta^* m(q^{sb}(\theta^*)) - \theta^* m(r(\theta^*))\big) \\
&\quad -\ \theta' m(q^{sb}(\theta^*)) - \lambda\big(\theta' m(q^{sb}(\theta^*)) - \theta' m(r(\theta'))\big) \\
&\geq \int_{\theta'}^{\theta^*} \big[(1+\lambda) m(q^{sb}(\tilde\theta))\ -\ \lambda\big(\tilde\theta m(r(\tilde\theta))\big)'\big]\,d\tilde\theta \\
&= \int_{\theta'}^{\theta^*} s(q^{sb}(\tilde\theta),\tilde\theta)\,d\tilde\theta, \qquad (15)
\end{aligned}
$$

where as before second inequality follow from the monotonicity of the optimal quality schedule and the last equality from the fact that $\mu = \lambda$ for all $\theta' \leq \tilde\theta \leq \theta^*$. Combining (14) and (15) with Equation 12 we obtain

$$
v(q^{sb}(\theta''),\theta'') - v(q^{sb}(\theta''),\theta')\ \geq\ \int_{\theta'}^{\theta''} s(q^{sb}(\tilde\theta),\tilde\theta)\,d\tilde\theta,
$$

which is the first inequality of the integral monotonicity condition of Proposition 2.

To obtain the second inequality, we write

$$
\begin{aligned}
v(q^{sb}(\theta'),\theta^*) - v(q^{sb}(\theta'),\theta') &= (1+\lambda)(\theta^* - \theta') m(q^{sb}(\theta')) - \lambda\big(\theta^* m(r(\theta^*)) - \theta' m(r(\theta'))\big) \\
&\leq \int_{\theta'}^{\theta^*} \big[(1+\lambda) m(q^{sb}(\tilde\theta)) - \lambda\big(\tilde\theta m(r(\tilde\theta))\big)'\big]\,d\tilde\theta \\
&= \int_{\theta'}^{\theta^*} s(q^{sb}(\tilde\theta),\tilde\theta)\,d\tilde\theta, \qquad (16)
\end{aligned}
$$

and similarly

$$
\begin{aligned}
v(q^{sb}(\theta^*),\theta'') - v(q^{sb}(\theta^*),\theta^*) &= \theta'' m(q^{sb}(\theta^*)) + \lambda\theta''\big(m(q^{sb}(\theta^*)) - m(r(\theta''))\big) - \theta^* m(q^{sb}(\theta^*)) \\
&\leq (\theta'' - \theta^*) m(q^{sb}(\theta^*))\ \leq\ \int_{\theta^*}^{\theta''} m(q^{sb}(\tilde\theta))\,d\tilde\theta \\
&= \int_{\theta^*}^{\theta''} s(q^{sb}(\tilde\theta),\tilde\theta)\,d\tilde\theta. \qquad (17)
\end{aligned}
$$

Combining (16) and (17) with Equation 13 above, we obtain the desired inequality. Thus, integral monotonicity is satisfied. □

Proof of Corollary 2. The properties of the optimal schedule $\theta \mapsto q^{sb}(\theta)$ follow readily from Equation 10. □

Proof of Proposition 5. Given a self-confirming reference function $\theta \mapsto r^*(\theta)$, we can write the monopolist's expected profits as

$$\Pi^{sb}[r^*] \;=\; \int_{\underline{\theta}}^{\theta_H} \Big(\phi(\theta) m(r^*(\theta)) \;-\; c(r^*(\theta)) \;+\; \lambda h(\theta)\theta m'(r^*(\theta)) r^{*\prime}(\theta)\Big) f(\theta) d(\theta).$$

We need to find a self-confirming reference function r_F^* that maximizes the above expression. Note that any solution will entail $r_F^*(\theta) = 0$ for all $\theta \in [\theta_L, \underline{\theta}] = \underline{\Theta}$. Our approach is to ignore the restrictions on the reference function imposed by the self-confirming requirement and treat this as a calculus of variations problem with initial boundary condition $r^*(\underline{\theta}) = 0$ and a vertical line terminal condition at $\theta = \theta_H$. We then verify ex post that the resulting reference function is indeed self-confirming.

Let then $G(\theta, r^*, r^{*\prime}) = \big(\phi(\theta) m(r^*) - c(r^*) + \lambda h(\theta)\theta m'(r^*) r^{*\prime}\big) f(\theta)$. We use the Euler equation $G_{r^*} - d(G_{r^{*\prime}})/d\theta = 0$ to find a potential solution. After some simplifications, one finds:

$$\begin{aligned} G_{r^*} \;&=\; \big(\phi(\theta) m'(r^*) - c'(r^*) + \lambda h(\theta)\theta m''(r^*) r^{*\prime}\big) f(\theta), \qquad \text{and} \\ \frac{d}{d\theta}\big(G_{r^{*\prime}}\big) \;&=\; \big(\lambda h(\theta)\theta m''(r^*) r^{*\prime} - \lambda\phi(\theta) m'(r^*)\big) f(\theta). \end{aligned}$$

Thus, the Euler equation reduces to the following expression:

$$(1+\lambda)\phi(\theta) m'(r^*(\theta)) \;-\; c'(r^*(\theta)) \;=\; 0, \qquad \text{for all } \theta \in [\underline{\theta}, \theta_H].$$

The (unique) solution to this differential equation is then $r_F^*(\theta) = q_\lambda^*(\theta)$, for all θ-consumers. As it satisfies both the initial boundary condition and the self-confirming condition, this is a necessary condition for the preferred reference function of the firm.

To verify that this condition is also sufficient, define the auxiliary function $(\theta, r^*) \mapsto \tilde{g}(\theta, r^*) = -\lambda h(\theta)\theta m(r^*) f(\theta)$. The Euler equation will become a sufficient condition provided one can show that the function $(r^*, r^{*\prime}) \mapsto G(\theta, r^*, r^{*\prime}) + \tilde{g}_{r^*}(\theta, r^*) r^{*\prime} + \tilde{g}_\theta(\theta, r^*)$ is concave (see for example Ok (2007), section K.5). After some simplifications, once again we find that for any $\theta \in \Theta \setminus \underline{\Theta}$,

$$G(\theta, r^*, r^{*\prime}) + \tilde{g}_{r^*}(\theta, r^*) r^{*\prime} + \tilde{g}_\theta(\theta, r^*) \;=\; \big((1+\lambda)\phi(\theta) m(r^*) - c(r^*)\big) f(\theta).$$

This completes the proof of the proposition. □

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