Price Regulation and the Cost of Capital [#]

Fernando T. Camacho and Flavio M. Menezes
The University of Queensland, School of Economics

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Abstract: This paper investigates how price regulation under moral hazard can affect a regulated firm's cost of capital. We consider stylised versions of the two most typical regulatory frameworks that have been applied over the last decades by regulators: Price Cap and Cost of Service. We show that there is a trade-off between lower operational costs and a higher cost of capital under Price Cap regulation and higher operational costs and lower cost of capital under Cost of Service regulation. As a result, when the extent of moral hazard is not significant, Price Cap regulation generates lower welfare than the Cost of Service regulation.

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1. Introduction

Cost of Service (COS) regulation has been the basic framework for price setting of infrastructure services in the U.S. and it has evolved substantially over time as a result of both judicial review and conceptual developments in regulatory economics. In its purest form, COS regulation is an *ex post* mechanism whereby the costs incurred by the firm in providing the service (including the opportunity cost of capital) are computed and the price is set by the regulator to cover these costs. While this type of regulation could in principle restrict the firm's ability to extract economic rents – as prices are set so that the firm can only recover costs, including a competitive rate of return on capital – and avoid adverse selection, as prices are based on actual costs, it can lead to moral hazard in that firms face a diminished incentive to minimise costs.[1]

Price Cap (PC) regulation, which has been used to set regulated prices in the UK since it was proposed by Stephen Littlechild (1983), offers a solution to the moral hazard problem. PC regulation is an *ex ante* mechanism that sets a fixed price. When faced with a fixed price, the firm's incentive is to produce at the lowest possible cost. In practice, however, regulators cannot fix prices for the entire life of regulated assets, which are typically long-lived. Thus, PC regulation involves setting maximum prices for a regulatory period (typically 5 years). As a result, although PC regulation successfully addresses moral hazard, its implementation reintroduces adverse selection. This follows as the prospect of a price revision at the end of the regulatory period might distort the firm's behaviour.

The incentive effects of these two different types of price regulation regarding informational asymmetries are well documented.[2] However, their impact on the incentives to invest and more specifically on the firm's cost of capital is not as well understood. This is an important gap in the literature that this paper addresses.

The importance of understanding the relationship between PC regulation and the firm's cost of capital is reflected, for instance, in the project RPI@20 undertaken by OFGEM, the regulator for the UK gas and electricity network services. The objective of RPI@20 is to propose a new regulatory regime that is friendlier to investment than PC regulation (http://www.ofgem.gov.uk/Networks/rpix20/Pages/RPIX20.aspx). PC regulation will also have to provide the correct incentives for the deployment of fibre-optic infrastructure and so-called Next

[1] In addition, COS regulation has been associated with gold plating of assets; when the allowed rate of return, which is the regulator's estimate of the cost of capital, is greater than the actual cost of capital, then the firm has an incentive to increase profits by increasing its asset base. This is the so called Averch-Johnson (1962) effect.

[2] See, for example, Laffont and Tirole (1993), Crew and Kleindorfer (2002), Armstrong and Sappington (2007), and Joskow (2006, 2007)).

Generation Networks (NGNs), a subject that is currently under debate by OFCOM, the regulator for the UK communications industries. As stated in Ofcom (2006), the challenge for *ex ante* regulation is to balance the dual aims of both promoting competition and ensuring efficient investment incentives are not distorted.

The theoretical regulatory economics literature (e.g., Laffont and Tirole (1993)) typically (implicitly) assumes that the firm's cost of capital is exogenous and independent of the nature of price regulation. Therefore, when PC and COS are compared, the costs and benefits associated with each type of regulation are not adequately considered.

An earlier literature has investigated the relationship between price regulation and cost of capital. Peltzman (1976) develops a one-period model where demand and cost are stochastic and uncertainty is resolved at the end of the period. The regulator waits until uncertainty is resolved and then sets the regulated price to maximise welfare. As Peltzman's model considers a COS-type regulation (that is, prices are set in line with realised demand and cost), it predicts that any economic shock will be buffered by the regulator. Peltzman concludes that the variability of profits (and stock prices) should be lower under regulation than under an unregulated market.

Alexander, Mayer and Weeds (1996), Robinson and Taylor (1998), Alexander, Estache and Oliveri (2000), Binder and Norton (1999), Nwaeze (2000) and Paleari and Redondi (2005) use econometric methods and/or financial models such as the Capital Asset Pricing Model (CAPM) to investigate the firm's cost of capital under regulation. More specifically, this literature examines the difference in the beta parameter or stock price volatility across regulatory systems and pre and post regulatory events. The basic conclusion is that the firm's cost of capital under PC regulation is higher than under COS regulation. Consistent with this literature, we develop a theoretical model that confirms this result. We show that there is a trade-off between lower cost efficiency under COS regulation and a higher cost of capital under PC regulation.

A more recent literature includes De Fraja and Stones (2004) and Stones (2007) who investigate how different types of *ex ante* price regulation that are contingent on *ex post* costs can alter the firm's cost of capital. In their model regulated prices are set before the firm's investment and financing decisions, but prices are contingent on future realised costs. Also, prices are set so that the firm faces no risk of bankruptcy and the cost of debt is equal to the risk-free rate.

De Fraja and Stones (2004) assume that the cost of equity always exceeds the cost of debt and the cost of equity increases with the level of debt. Consequently, optimal regulated prices are such that the firm issues a positive level of debt, as this yields a lower expected price (due to the lower cost of capital) and a higher consumer surplus. In Stones (2007), the cost of equity is determined by the covariance of the return to shareholders and the market return, and its value

depends on the nature of regulation. For instance, when prices are set *ex ante* to cover the firm's *ex post* costs (a form of COS regulation), the firm's cost of capital is equal to the risk-free rate. This follows as the return on equity is constant and does not depend on the state of nature. Thus, the covariance of the return to shareholders and the market return is zero and the cost of equity equals the cost of debt and the risk-free return. In contrast to De Fraja and Stones, the cost of debt is endogenously determined in our model and, therefore, may be affected by the nature of regulation. [3]

In contrast to the existing literature, our paper investigates the implication of PC regulation for the regulated firm's cost of capital. Consistent with regulatory practice, we assume that under PC the regulator sets an *ex ante* price cap before the firm's investment and financing decisions and uncertainty resolution. In order to evaluate the welfare generated by PC, we consider as a benchmark the COS regulation whereby price is set *ex post* to firm's investment and financing decisions and uncertainty resolution so that the regulated revenue covers exactly the firm's operational and capital costs. This modelling choice allows us to explore the contrast between the fixed price and cost-plus nature of regulatory contracts.

While we show that the entrepreneur can be more efficient under PC regulation than under COS regulation, we find that the former type of regulation may yield either a higher cost of capital or a higher rent to the firm – and, therefore, lower welfare – than the latter type of regulation. This result is consistent with the existing empirical evidence described. Which regulation is superior will depend on a comparison of the extent of moral hazard and the effect on the cost of capital that arises from setting prices *ex ante* and creating, therefore, the risk of bankruptcy.

This paper is organised as follows. In section 2 we present the modelling framework and the optimal choices made by an unregulated monopolist. In section 3 we characterise the entrepreneur's optimal choice and welfare under COS regulation and then we solve the problem of a regulator that uses PC regulation to set the price of the regulated firm. Section 4 compares the welfare generated by the two different types of price regulation. Section 5 concludes.

[3] Taggart (1981), Spiegel (1994, 1996) and Spiegel and Spulber (1994, 1997) also develop models that take into account different types of funding but they focus on the firm's capital structure instead of cost of capital.

2. The Benchmark Model

An infrastructure project requires a fixed investment I and it is completed in one period. The project is undertaken by a risk-neutral entrepreneur who holds cash on hand $H < I$. To fund the project, the entrepreneur must borrow an amount $D \geq I - H$ from risk-neutral lenders.

Lenders behave competitively and are subject to a zero profit constraint; the rate of return expected by lenders is the risk free rate k_f. If the entrepreneur borrows $D > I - H$, then the entrepreneur invests the amount $H - (I - D)$ in a Treasury bond that provides the risk-free return k_f, that is, the entrepreneur's opportunity cost is equal to k_f. Also, the entrepreneur's liability is limited and thus the income he derives from the project is non-negative.

The cost to operate the infrastructure project is $C \in \{\alpha c, c\}$, where $0 < \alpha < 1$ and $c > 0$. The cost depends on the level of effort $E \in \{0, \varepsilon\}$ undertaken by the entrepreneur as follows:

$$C = \begin{cases} \alpha c, \text{with probability } p(E) \\ c, \text{with probability } 1\text{- } p(E) \end{cases}, \text{ where } p(\varepsilon) > p(0) \ .$$

The effort level chosen by the entrepreneur is not observable. Consumers are risk-neutral. The infrastructure service provider is a monopolist and faces an inverse demand function characterised by a choke price P. At any price less than or equal to P demand is equal to Q. Without loss of generality we set *Q = 1*. At any price greater than P demand is equal to zero.

In the first period ($t = 0$), the entrepreneur chooses the level of effort E and the level of debt $D(E)$ to maximise profit given price P and taking into account the cost of debt $k_D^E(D(E), \bullet)$ determined in the debt market. It takes one period to build the network. In the second period ($t = 1$), the infrastructure project is completed, the demand and the operational cost to run the infrastructure are realised and the service is provided to consumers. The sales revenue is then used to cover expenditures in the same order of priority as defined in basic financial statements, namely (1) the operational cost C; (2) bondholders; and (3) the entrepreneur.

The expected total welfare at $t = 1$ is equal to $CS + \lambda\pi(E)$, where CS is the expected consumer surplus, $\pi(E)$ is the entrepreneur's expected profit and $0 < \lambda < 1$ is the weight assigned to the entrepreneur's profit. The entrepreneur's expected profit must be non-negative, that is, it must satisfy the participation constraint (otherwise, no investment would take place). To simplify the

analysis we assume that the minimum price that satisfies the firm's participation constraint for any level of effort E and level of debt $D(E)$ is $P \geq c$. This assumption allows us to focus on the effects of price regulation on the cost of capital and it is consistent with the notion that capital costs represent the bulk of total costs of infrastructure businesses.

We now turn to the case of an unregulated monopolist service provider. We solve the monopolist's problem backwards. We start at period $t = 1$ and calculate the entrepreneur's net payoff for the two states of nature:

$$F(E) = Max\{(P - C) - (1 + k_D^E(D(E),\bullet))D(E) - (1 + k_f)(I - D(E)) - E, -(1 + k_f)(I - D(E)) - E\}, \quad for \quad C \in \{\alpha c, c\} \tag{1}$$

The expression for $F(E)$ follows from the limited liability constraint; if the firm's revenue is insufficient to pay debt plus interest, the entrepreneur's maximum loss is equal to the equity and effort used in the project. We can determine the lenders' payoff in a similar vein:

$$R(E) = Max\{(1 + k_D^E(D(E),\bullet))D(E), (P - C)\}, \quad for \quad C \in \{\alpha c, c\} \tag{2}$$

The expression for $R(E)$ again reflects the limited liability constraint; if the firm's revenue is insufficient to pay debt plus interest, lenders receive the total revenue as payment.

We now determine the cost of debt. If the firm's revenue is sufficient to pay debt plus interest in all states of nature (i.e., when $P \geq (1 + k_D^E(D(E),\bullet))D(E) + c$), there is no default risk and, therefore:

$$k_D^E(D(E),\bullet) = k_f \ . \tag{3}$$

In contrast, when the firm's revenue is insufficient to pay debt plus interest when the realised operational cost is c (i.e., when $P \leq (1 + k_D^E(D(E),\bullet))D(E) + c$), the cost of default is determined by using expression (2) as follows:[4]

$$D(E) = \frac{p(E)(1 + k_D^E(D(E),\bullet))D(E) + (1 - p(E))(P - c)}{(1 + k_f)}$$

Rearranging this expression yields:

[4] We dismiss the case where the firm's revenue is insufficient to pay debt in all states of nature as in this case bondholders would not provide capital and the entrepreneur would not invest.

$$k_D^E(D(E),\bullet) = k_f + \frac{(1-p(E))}{p(E)}\left[(1+k_f) - \frac{(P-c)}{D(E)}\right]. \quad (4)$$

Note that the cost of equity is equal to k_f. Having established the cost of debt and equity, we now determine the monopolist's choice of capital structure, as per the following Lemma:

Lemma 1: *The monopolist firm always (weakly) chooses the minimum amount of debt* $D(E) = I - H$ *, independently of being unregulated or regulated.*

The proof is straightforward. Recall that the cost of equity is always less than or equal to the cost of debt. Thus the monopolist firm is either indifferent between debt and equity (when the cost of debt is equal to the cost of equity) or strictly prefers equity to debt (when the cost of debt is larger than the cost of equity). For simplicity, we assume henceforth that the entrepreneur chooses $D(E) = D = I - H$. We anticipate that the level of debt does not depend on whether the market is regulated or not since the entrepreneur does not have the power (under PC) or the incentives (under COS) to use the level of debt strategically to increase regulated prices.[5]

Having determined that the amount of debt does not depend on the level of effort, we can now proceed to analyse the monopolist's optimal choice of effort. We also anticipate that the level of effort does depend on whether the market is regulated and also on the type of regulation. In this section we focus on the unregulated monopolist's decision.

The entrepreneur chooses to undertake $E = \varepsilon$ only if the following two constraints are satisfied:

$$\pi(\varepsilon) = p(\varepsilon)(P - \alpha c) + (1 - p(\varepsilon))(P - c) - (1 + k_D^{\varepsilon}(I - H, \bullet))(I - H) - (1 + k_f)H - \varepsilon \geq$$
$$\pi(0) = p(0)(P - \alpha c) + (1 - p(0))(P - c) - (1 + k_D^{0}(I - H, \bullet))(I - H) - (1 + k_f)H \quad \text{(IC)}$$

and

$$\pi(\varepsilon) \geq 0. \quad \text{(PR)}$$

If the incentive compatibility (IC) constraint is not satisfied, then the entrepreneur undertakes $E = 0$ as long as $\pi(0) \geq 0$. Note that we can rewrite the IC constraint as follows:

[5] Taggart (1981), Spiegel (1994, 1996) and Spiegel and Spulber (1994, 1997) consider a different type of regulation where the firm can use the level of debt to increase regulated prices.

$$\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c + \left[k_D^0(I-H,\bullet) - k_D^{\varepsilon}(I-H,\bullet)\right](I-H). \quad (5)$$

The term ε is the direct cost whereas $(1-\alpha)(p(\varepsilon)-p(0))c$ is the direct benefit of undertaking a positive effort in the form of a lower expected marginal cost. The term $\left[k_D^0(I-H,\bullet) - k_D^{\varepsilon}(I-H,\bullet)\right](I-H)$ is the difference between the total cost of debt (debt plus interest) when $E=0$ and when $E=\varepsilon$. Expression (5) then states that it will be incentive compatible for the unregulated monopolist to undertake level of effort ε as long as the resource cost of undertaking such effort is less than or equal to the sum of the expected benefit in terms of lower operational costs plus the change in the total cost of debt associated with a positive effort.

Whether or not the IC constraint is satisfied depends on the choke price. For a sufficiently large P (i.e., when $P \geq (1+k_f)(I-H)+c$), the cost of debt is equal to the risk-free rate under both levels of effort and therefore (5) is reduced to:

$$\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c. \quad (6)$$

In contrast, if the risk of default is positive regardless of the level of effort (i.e., $P < (1+k_f)(I-H)+c$), the cost of debt is higher than the risk-free rate and then (5) can be rewritten as:

$$\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(0)p(\varepsilon)}\tau, \quad (7)$$

Where $\tau \in (0,(1+k_f)(I-H)]$.

Note that $\frac{(p(\varepsilon)-p(0))}{p(0)p(\varepsilon)}\tau > 0$, that is, a positive effort decreases the total cost of debt. The reason is that a positive effort increases the probability of a low cost scenario ($p(\varepsilon) > p(0)$), that is, it decreases the probability of default and also the cost of debt (see equation (4)). Note also that the lower the P (the higher τ) the higher ε can be for the entrepreneur to undertake $E=\varepsilon$. That is, the difference between the total cost of debt when $E=0$ and when $E=\varepsilon$ increases as P decreases. We have therefore established the following result:

Lemma 2: *Table 1 summarises the threshold levels that ε must satisfy in order for the entrepreneur to undertake $E=\varepsilon$.*

Table 1: Threshold Level for $E = \varepsilon$

P	$\varepsilon \leq$
$P = (1+k_f)(I-H)+c-\tau$, where $\tau \in (0,(1+k_f)(I-H)]$	$(1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(0)p(\varepsilon)}\tau$
$P \geq (1+k_f)(I-H)+c$	$(1-\alpha)(p(\varepsilon)-p(0))c$

The characterisation of the unregulated monopolist's choices of D and E allows us to determine total welfare as follows:

Lemma 3: *If the entrepreneur undertakes $E = \varepsilon$, then overall welfare in an unregulated industry W_M is equal to $\lambda\pi(\varepsilon)$. Otherwise, overall welfare is equal to $\lambda\pi(0)$ if $\pi(0) \geq 0$ or 0 if the entrepreneur does not invest.*

In the next Section we will compare the outcome of both COS and PC regulation with the outcome of the unregulated monopolist. In particular, under PC regulation the regulator will make full use of the information on Lemma 2 (with P replaced by the regulated price P_R) to explore the trade-off between rent extraction and satisfying the IC and PR constraints in order to maximise total welfare.

3. Infrastructure Regulation

In this Section, we assume that the price is set by a risk-neutral regulator who has perfect information about I, P and Q. At $t = 0$, the regulator reveals whether it will apply a COS or PC regulatory framework. In the case of the former, prices will be set *ex post* and will be conditional on the realisation of costs, while in the case of the latter, a single price P_R is announced at $t = 0$. At $t = 1$, the regulator observes C but does not observe E. The regulator's objective function is to maximise expected overall welfare at $t = 1$.

3.1 Cost of Service Regulation

We proceed to describe the COS regulation that will be used as a benchmark to evaluate the welfare generated by PC regulation. Under COS regulation, at $t=1$, after the entrepreneur's choices of D and E, and subsequently to the resolution of cost uncertainty, the regulator sets a price $\underline{P}$ when $C=\alpha c$ or a price $\overline{P}$ when $C=c$. Thus, the regulated price always covers the operational and capital costs. This is known in advance.

As operational and capital costs are always covered, there is no risk of default and the firm's cost of capital is always equal to k_f. Thus, the entrepreneur is indifferent between any level of debt $D\in[I-H,I]$. As said in the previous section, we assume that the entrepreneur chooses $D=I-H$. The following proposition characterises the choice of effort by the entrepreneur and the optimal (*ex post*) prices under COS regulation.

Proposition 1: *Under COS regulation the entrepreneur always chooses* $E=0$. *The optimal* ex post *prices are given by:*

$$\underline{P}=(1+k_f)I+\alpha c \text{ when } C=\alpha c \text{ and } \overline{P}=(1+k_f)I+c \text{ when } C=c.$$

The proof of this proposition is in the appendix. This proposition simply states that society bears the full extent of moral hazard under COS regulation but the cost of capital is minimised and equal to the risk-free rate.

Proposition 1 allows us to compute the expected overall welfare from COS regulation evaluated at $t=1$ as follows:

$$W_{COS}=p(0)(P-\underline{P})+(1-p(0))(P-\overline{P})=P-(1+k_f)I-(1-(1-\alpha)p(0))c. \quad (8)$$

3.2 Price Cap Regulation

Under PC regulation, a single price P_R is announced at $t=0$, before the entrepreneur's choices of D and E, and prior to the resolution of cost uncertainty. The regulator chooses the regulated price P_R to maximise total welfare given by (9) below:

$$\underset{P_R}{Max} \quad W_{PC} = (P - P_R) + \lambda \pi_R(E) \quad (9)$$

Subject to:

$\pi_R(E) \geq 0$. (PR)

$\pi_R(E) \geq \pi_R(E^*)$, where $E, E^* \in \{0, \varepsilon\}$ and $E \neq E^*$. (IC)

where

$$\pi_R(E) = \begin{cases} Max\{p(E)(P_R - \alpha c) + (1 - p(E))(P_R - c) - (1 + k_f)I - E, (1 - p(E))(-(1 + k_f)H - E)\} & if \quad P_R \geq (1 + k_f)(I - H) + c \\ Max\left\{ p(E)(P_R - \alpha c) + (1 - p(E))(P_R - c) - \left(\frac{(1 + k_f)}{p(E)} - \frac{(1 - p(E))}{p(E)} \frac{(P_R - c)}{(I - H)} \right)(I - H) - (1 + k_f)H - E, -(1 + k_f)H - E \right\} & if \quad c \leq P_R \leq (1 + k_f)(I - H) + c \end{cases}$$

To sum up, P_R is fixed *ex ante* to the firm's investment and financing decision to maximise expected welfare, the regulator offers a regulatory contract that anticipates a capital structure where $D = I - H$, and the cost of debt is higher than or equal to the cost of equity.

Table 2 below shows the optimal price caps for all possible parameter values. The proof is in the appendix.

Proposition 2: *Table 2 below summarises the optimal price cap given the IC and PR conditions:*

Table 2: Optimal Price Cap

Constraints	$p(E)$	Optimal Price Cap
IC $\varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))c$ **PR** $(1 + k_f)H + \varepsilon \geq (1 - \alpha)p(\varepsilon)c$	-	$(1 + k_f)I + \varepsilon + (1 - (1 - \alpha)p(\varepsilon))c$
IC $\varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))c$ **PR** $(1 + k_f)H + \varepsilon < (1 - \alpha)p(\varepsilon)c$	$p(\varepsilon) < \lambda$	$(1 + k_f)(I - H) + c$
	$p(\varepsilon) \geq \lambda$	$(1 + k_f)(I - (1 - p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1 - (1 - \alpha)p^2(\varepsilon))c$
IC $\varepsilon > (1 - \alpha)(p(\varepsilon) - p(0))c$ **PR** $(1 + k_f)H \geq (1 - \alpha)p(0)c$	-	$(1 + k_f)I + (1 - (1 - \alpha)p(0))c$
IC $\varepsilon > (1 - \alpha)(p(\varepsilon) - p(0))c + \frac{(p(\varepsilon) - p(0))}{p(\varepsilon)}[(1 - \alpha)p(0)c - (1 + k_f)H]$ **PR** $(1 + k_f)H < (1 - \alpha)p(0)c$	$p(0) < \lambda$	$(1 + k_f)(I - H) + c$
	$p(0) \geq \lambda$	$(1 + k_f)(I - (1 - p(0))H) + (1 - (1 - \alpha)p^2(0))c$
IC $(1 - \alpha)(p(\varepsilon) - p(0))c < \varepsilon \leq (1 - \alpha)(p(\varepsilon) - p(0))c + \frac{(p(\varepsilon) - p(0))}{p(0)}[(1 - \alpha)p(\varepsilon)c - (1 + k_f)H - \varepsilon]$ **PR** $(1 + k_f)H < (1 - \alpha)p(0)c$	$p(\varepsilon) < \lambda$	$(1 + k_f)(I - H) + c$
	$p(0) \geq \lambda$	$(1 + k_f)(I - (1 - p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1 - (1 - \alpha)p^2(\varepsilon))c$
	$p(0) < \lambda \leq p(\varepsilon)$	If $p(\varepsilon)\varepsilon < (1 - \alpha)(p^2(\varepsilon) - \lambda p(0))c + (\lambda - p(\varepsilon))(1 + k_f)H$, then $(1 + k_f)(I - (1 - p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1 - (1 - \alpha)p^2(\varepsilon))c$. Otherwise, $(1 + k_f)(I - H) + c$.

Proposition 2 has important implications for the trade-off between rent extraction, optimal effort induction and the regulated firm's cost of capital. To understand the nature of this trade-off, note that as $\lambda < 1$ the regulator will extract the entire entrepreneur's rent when the rate by which the firm's cost of capital increases is sufficiently low. Conversely, when the negative impact on the entrepreneur's rent is higher than the positive impact on consumer surplus, the regulator then sets the minimum price such that the firm's cost of capital is equal to the risk-free rate. This price leaves a positive rent for the entrepreneur.

Of course, the actual nature of the trade-off will depend on parameter values. Consider P_R such that $(1+k_f)(I-H)+c \le P_R \le P$. It is easy to see that $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -(1-\lambda) < 0$; a decrease in P_R increases welfare as the positive impact on consumer surplus (1) is higher than the negative impact on the entrepreneur's profit $(-\lambda)$; a decrease in P_R does not impact the cost of debt, only the expected revenue, which is always sufficient to pay total cost of debt in all states of nature.

When parameter values are such that debt is paid under all states of nature, the regulated firm's cost of capital is equal to the risk-free rate, and it does not depend on the level of effort. In this case, a price decrease does not cause any change in the firm's cost of capital, and thus the regulator will set the lowest price possible, which is $P_R = (1+k_f)I + E + (1-(1-\alpha)p(E))c$, for $E \in \{0, \varepsilon\}$, to extract all expected rent (so that the entrepreneur's participation constraint is always binding). There is no trade-off between rent extraction, optimal effort choice and the cost of capital. The total welfare is then given by:

$$W_{PC} = P - (1+k_f)I - c + (1-\alpha)p(E)c - E \ . \qquad (10)$$

The optimal choice of effort in equation (10) will depend on parameter values as described in rows one ($E = \varepsilon$) and three ($E = 0$) of Table 2.

Alternatively, for other parameter values, by reducing the regulated price to extract rents, the regulator affects the firm's cost of capital and may affect the optimal choice of effort. In fact, consider P_R such that $c \le P_R < (1+k_f)(I-H)+c$. In this case, $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -\left(1 - \frac{\lambda}{p(E)}\right)$; whether a decrease in P_R increases welfare depends on $p(E)$. The rationale is as follows: a decrease in P_R has a positive impact on consumer surplus (1) and a negative impact on the entrepreneur's

profit ($-\frac{\lambda}{p(E)}$). Furthermore, the entrepreneur's profit is reduced because the expected revenue decreases ($-\lambda$) and the cost of debt increases ($-\lambda\frac{(1-p(E))}{p(E)}$); the sum of these two effects is equal to $-\frac{\lambda}{p(E)}$. Thus, if $p(E)<\lambda$, then a decrease in P_R reduces welfare. In this case, the rate by which the cost of debt increases ($-\frac{(1-p(E))}{p(E)}$) is sufficiently high and compensates the fact that the consumers surplus' weight is higher than the entrepreneur profit's weight in the welfare function ($\lambda<1$). The regulator then sets the minimum price such that the firm's cost of capital is equal to k_f, that is, $P_R=(1+k_f)(I-H)+c$, for $E\in\{0,\varepsilon\}$. The welfare is given by:

$$W_{PC}=P-(1+k_f)(I-H)-c+\lambda\left[(1-\alpha)p(E)c-(1+k_f)H-E\right] \qquad (11)$$

where $(1+k_f)H+E<(1-\alpha)p(E)c$. Note that in this case the entrepreneur's expected profit is positive.

However, if $p(E)\geq\lambda$, then a decrease in P_R increases welfare. In this case, the rate by which the cost of debt increases is sufficiently low and is welfare enhancing to reduce prices because the consumer surplus' weight is higher than the entrepreneur profit's weight in the welfare function.[6] The regulator then sets the lowest price possible extracting all rent (so that the entrepreneur's participation constraint is binding), which is $P_R=(1+k_f)(I-(1-p(E))H)+p(E)E+(1-(1-\alpha)p^2(E))c$, for $E\in\{0,\varepsilon\}$. In this case, welfare is given by:

$$W_{PC}=P-(1+k_f)(I-(1-p(E))H)-(1-(1-\alpha)p^2(E))c-p(E)E\ . \qquad (12)$$

The optimal choice of effort in equations (11) and (12) will depend on parameter values as described in rows two ($E=\varepsilon$), four ($E=0$) and five ($E=\varepsilon$ and $E=0$) of Table 2.

[6] Note that if $p(E)=\lambda$, then welfare is constant within this price range. We assume that if $p(E)=\lambda$ the regulator sets price as low as possible.

4. The benefits and costs of PC and COS regulatory regimes

We now compare the welfare generated by the two different types of regulatory regimes. First, we note that under COS regulation the entrepreneur always undertakes $E=0$, the firm's cost of capital is always equal to k_f and the entrepreneur's profit is always equal to zero. Under PC regulation, however, the entrepreneur undertakes either $E=0$ or $E=\varepsilon$, the firm's cost of capital is higher than or equal to k_f and the entrepreneur's expected profit is higher than or equal to zero. As we will see below, whether PC regulation is welfare superior depends on parameter values. Proposition 3 below compares the welfare generated by COS and PC regulatory regimes across all parameter values. The proof is in the appendix.

Proposition 3: *Table 3 below compares the PC and COS regulatory regimes for all parameter values expressed in terms of the IC and PR constraints under PC regulation:*

Table 3: Optimal Regulation

Constraints	$p(E)$	Optimal Regulation
IC $\varepsilon \le (1-\alpha)(p(\varepsilon)-p(0))c$ **PR** $(1+k_f)H+\varepsilon \ge (1-\alpha)p(\varepsilon)c$	-	$W_{PC} \ge W_{COS}$
IC $\varepsilon \le (1-\alpha)(p(\varepsilon)-p(0))c$ **PR** $(1+k_f)H+\varepsilon < (1-\alpha)p(\varepsilon)c$	$p(\varepsilon) < \lambda$	$W_{PC} \ge W_{COS}$ if $\lambda\left[(1-\alpha)p(\varepsilon)c-(1+k_f)H-\varepsilon\right] \ge \left[(1-\alpha)p(0)c-(1+k_f)H\right]$
	$p(\varepsilon) \ge \lambda$	$W_{PC} \ge W_{COS}$ if $p(\varepsilon)\varepsilon \le (1-\alpha)\left(p^2(\varepsilon)-p(0)\right)c+(1-p(\varepsilon))(1+k_f)H$
IC $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$ **PR** $(1+k_f)H \ge (1-\alpha)p(0)c$	-	$W_{PC} = W_{COS}$
IC $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c+\frac{(p(\varepsilon)-p(0))}{p(\varepsilon)}\left[(1-\alpha)p(0)c-(1+k_f)H\right]$ **PR** $(1+k_f)H < (1-\alpha)p(0)c$	-	$W_{PC} < W_{COS}$
IC $(1-\alpha)(p(\varepsilon)-p(0))c < \varepsilon \le (1-\alpha)(p(\varepsilon)-p(0))c$ $+\frac{(p(\varepsilon)-p(0))}{p(0)}\left[(1-\alpha)p(\varepsilon)c-(1+k_f)H-\varepsilon\right]$ **PR** $(1+k_f)H < (1-\alpha)p(0)c$	$p(\varepsilon) < \lambda$	$W_{PC} < W_{COS}$
	$p(0) \ge \lambda$	$W_{PC} \ge W_{COS}$ if $p(\varepsilon)\varepsilon \le (1-\alpha)\left(p^2(\varepsilon)-p(0)\right)c+(1-p(\varepsilon))(1+k_f)H$.
	$p(0) < \lambda \le p(\varepsilon)$	If $p(\varepsilon)\varepsilon < (1-\alpha)\left(p^2(\varepsilon)-\lambda p(0)\right)c+(\lambda-p(\varepsilon))(1+k_f)H$ and if $p(\varepsilon)\varepsilon \le (1-\alpha)\left(p^2(\varepsilon)-p(0)\right)c+(1-p(\varepsilon))(1+k_f)H$, then $W_{PC} \ge W_{COS}$. Otherwise $W_{PC} < W_{COS}$.

Proposition 3 shows that whether PC regulation is welfare superior depends on the trade-off between a higher cost efficiency under PC and a lower cost of capital (or a lower entrepreneur's

rent) under COS. In particular, when under PC regulation the entrepreneur undertakes $E = \varepsilon$, the regulator may be able to set a lower price under PC regulation than under COS regulation. The reason is that under PC regulation the regulator is able to extract the rent differential that stems from a positive effort in comparison to a zero level of effort and transfer it to consumer surplus through a lower regulated price.

Conversely, when under PC regulation the firm's cost of capital is higher than k_f or the entrepreneur's profit is positive, then the regulator may not be able to set a lower price under PC regulation than under COS regulation The reason is that the regulator extracts less rent under PC than under COS regulation; by reducing the regulated price to extract rents under PC regulation, the regulator may affect the firm's cost of capital. When the impact on the firm's cost of capital is sufficiently low, the regulator will lower the price because the positive impact on consumer's surplus outweighs the negative impact on firm's cost of capital (and entrepreneur's profit). However, if the impact on firm's cost of capital is sufficiently high, the regulator will set the price so that there is no bankruptcy risk and the cost of capital is equal to the risk-free risk as the negative impact on the firm's cost of capital (and entrepreneur's profit) outweighs the positive impact on consumer surplus.

As with Proposition 2, the actual nature of the trade-off depends on parameter values. We have seen that when under PC regulation P_R is such that $(1+k_f)(I-H)+c \le P_R \le P$, then the regulated firm's cost of capital is equal to the risk-free rate (as debt is paid under all states of nature). The regulator then sets the lowest price possible to extract all expected rent. In this case, note that under both types of regulation the firm's cost of capital is equal to k_f and the entrepreneur's expected profit is zero. Thus, if the entrepreneur undertakes $E = 0$ under PC regulation, then both types of regulatory regimes provide the same price (consumer surplus) and welfare. However, if the entrepreneur undertakes $E = \varepsilon$ under PC regulation, then this higher level of effort allows the regulator to set a price lower than or equal to the expected price under COS regulation. The optimal choice of effort under PC will depend on parameter values as described in rows one ($E = \varepsilon$) and three ($E = 0$) of Table 3.

Alternatively, for other parameter values, we have seen that by reducing the regulated price to extract rents under PC regulation, the regulator affects the firm's cost of capital. When P_R is such that $c \le P_R < (1+k_f)(I-H)+c$, then $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -\left(1-\frac{\lambda}{p(E)}\right)$; and whether a decrease in P_R increases welfare depends on $p(E)$. Moreover, a decrease in P_R has a positive impact on

consumer surplus (1) and a negative impact on the entrepreneur's profit $(-\frac{\lambda}{p(E)})$ due to a lower revenue $(-\lambda)$ and a higher cost of capital $(-\lambda\frac{(1-p(E))}{p(E)})$.

Thus, if $p(E)<\lambda$, then the rate by which the cost of debt increases $(-\frac{(1-p(E))}{p(E)})$ is sufficiently high and compensates for the fact that the consumers surplus' weight is higher than the entrepreneur profit's weight in the welfare function $(\lambda<1)$. The regulator then sets the minimum price such that the firm's cost of capital is equal to k_f. In this case, under both types of regulatory regimes the firm's cost of capital is equal to k_f. However, the entrepreneur's expected profit is positive under PC regulation and equal to zero under COS regulation. Thus, we have the following: If the entrepreneur undertakes $E=0$ under PC regulation, then COS regulation is welfare superior. The reason is that under COS the regulator is able to extract the entire economic rent from the entrepreneur and the cost of capital is equal to the risk-free rate whereas under PC the regulator must give a minimum positive rent for the entrepreneur in order to keep the cost of capital equal to the risk-free return. Thus, the regulator extracts less rent under PC than under COS. This rent differential allows the regulator to set a lower expected price under COS than the price under PC regulation. However, if the entrepreneur undertakes $E=\varepsilon$ under PC regulation, then there is a trade-off between a higher level of effort under PC regulation and no economic rent left for the entrepreneur under COS regulation. Whether the regulator will be able to set a lower price under PC than the expected price under COS, will depend on the parameter values.

If $p(E)\geq\lambda$, then the rate by which the cost of debt increases under PC is sufficiently low and it is welfare enhancing to reduce prices because the positive impact on consumer surplus outweighs the negative impact on the firm's cost of capital (and entrepreneur's profit). The regulator then sets the lowest price possible to extract all rent (so that the entrepreneur's participation constraint is binding). In this case, under both types of regulatory regimes the entrepreneur's expected profit is equal to zero. However, the firm's cost of capital is higher than k_f under PC regulation and equal to k_f under COS regulation. Thus, we have the following: If the entrepreneur undertakes $E=0$ under PC regulation, then COS regulation is welfare superior. The reason is the same as in the previous case. That is, because the firm's cost of capital is higher under PC than under COS (instead of a higher rent under PC than under COS, as in the previous case), the regulator extracts less rent under PC than under COS. This rent differential allows the regulator to set a lower expected price under COS than the price under PC regulation. However, if the entrepreneur undertakes $E=\varepsilon$ under PC regulation, then there is a trade-off between a higher level of effort under PC regulation and a lower cost of capital under COS regulation. Whether the regulator will

be able to set a lower price under PC than the expected price under COS, will depend on the parameter values.

The optimal choice of effort under PC regulation when the cost of debt is higher than the risk-free return (or the entrepreneur's profit is positive) will depend on parameter values as described in rows two ($E = \varepsilon$), four ($E = 0$) and five ($E = \varepsilon$ and $E = 0$) of Table 3.

5. Conclusion

We have investigated the relationship between price regulation and the cost of capital in a two-period model in which the regulator faces moral hazard and the entrepreneur is capital constrained. In our model, the cost of debt is higher than or equal to the cost of equity. Thus, the entrepreneur chooses the minimum level of debt possible.

In contrast to the previous papers, our model fully explores the implications of the timing associated with the price-setting process. Thus, we assume that under COS regulation price is set *ex post* to firm's investment and financing decisions and uncertainty resolution so that the regulated revenue covers exactly the firm's operational and capital costs. Under PC regulation, we assume that the regulator sets an *ex ante* price cap before the firm's investment and financing decisions and uncertainty resolution. This modelling choice allows us to fully explore the contrast between the cost-plus and fixed price nature of regulatory contracts.

Thus, we have seen that when the cost of capital under PC is equal to the risk-free rate, PC regulation generates at least the same welfare as COS regulation. In particular, if the extent of moral hazard is significant, then PC regulation is welfare superior. However, when the cost of capital under PC regulation is higher than the risk-free rate (or the entrepreneur's profit is positive because the rate by which the cost of capital increases is sufficiently high), then we have the following: if the extent of moral hazard is insignificant, then COS regulation is welfare superior; if the extent of moral hazard is significant, then there is a trade-off between a higher cost efficiency under PC regulation and a lower cost of capital or lower economic rent under COS regulation.

In summary, this paper has provided a channel through which PC regulation affects the cost of capital of the regulated firm. Therefore, it has shown that any comparison between PC and COS regulatory regimes has to take into account the trade-off between higher cost of capital and less moral hazard.

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Appendix

Proof of Proposition 1:

Under COS regulation the regulator does not observe E and sets $\underline{P}$ when $C = \alpha c$ and $\overline{P}$ when $C = c$ such that the regulated price always covers operational and capital costs. Under such prices, the entrepreneur's profit is given by:

$$\pi = \begin{cases} \pi_H = (\underline{P} - \alpha c) - (1 + k_f) I - E, if \quad C = \alpha c \\ \pi_L = (\overline{P} - c) - (1 + k_f) I - E, if \quad C = c \end{cases}.$$

Fix $\underline{P}$ and $\overline{P}$ at any level. From the equation above, the entrepreneur never chooses $E = \varepsilon$, as he can always guarantee higher profits by choosing $E = 0$. It follows that the welfare maximising regulated *ex post* prices (that guarantee zero profits and maximise consumer surplus) are equal to $\underline{P} = (1 + k_f) I + \alpha c$ and $\overline{P} = (1 + k_f) I + c$. □

Proof of Proposition 2:

We begin this Proof by showing that there are only five cases to examine. Then, we proceed to state these cases. First, assume that the minimum price that satisfies the participation constraint is higher than or equal to $(1 + k_f)(I - H) + c$. Recall from Lemma 2 that in this case the entrepreneur chooses only one level of effort $E \in \{0, \varepsilon\}$ within this price range. Indeed, the decision of undertaking a positive effort depends only on the resource cost of undertaking such effort (ε) being less than or equal to the expected benefit in terms of lower expected operational costs ($(1 - \alpha)(p(\varepsilon) - p(0))c$). These cases are stated as Cases 1 and 2 below.

Second, assume that the minimum price that satisfies the participation constraint is lower than $(1 + k_f)(I - H) + c$. We know from Lemma 2 that when price is lower than $(1 + k_f)(I - H) + c$ the decision of undertaking a positive effort depends on the resource cost of undertaking such effort (ε) being less than or equal to the sum of the expected benefit in terms of lower operational costs ($(1 - \alpha)(p(\varepsilon) - p(0))c$) plus the change in the total cost of debt associated with a positive effort ($[k_D^0(I - H, \bullet) - k_D^\varepsilon(I - H, \bullet)](I - H)$). Moreover, we know that $[k_D^0(I - H, \bullet) - k_D^\varepsilon(I - H, \bullet)](I - H)$ is positive and increases as price decreases. Thus, if the threshold to undertake a positive effort is reached at a specific price, the entrepreneur will undertake a positive effort for all prices lower or equal to this price. Thus, we have three cases to consider: In the first two cases the entrepreneur

chooses only one level of effort $E \in \{0, \varepsilon\}$ for all P_R such that the PR constraint is satisfied. In the third case, there is a price $\overline{P_R} < (1+k_f)(I-H)+c$ such that if $P_R > \overline{P_R}$ the entrepreneur undertakes $E = 0$ and if $P_R \leq \overline{P_R}$ the entrepreneur undertakes $E = \varepsilon$. These cases are stated below as Cases 3, 4 and 5.

Case 1: $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c$ and $(1+k_f)H + \varepsilon \geq (1-\alpha)p(\varepsilon)c$.

Lemma 2 states that if $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c$, then the entrepreneur undertakes $E = \varepsilon$ for all P_R such that the participation constraint is satisfied. If $(1+k_f)H + \varepsilon \geq (1-\alpha)p(\varepsilon)c$, then the minimum price that satisfies the participation constraint is $P_R \geq (1+k_f)(I-H)+c$ (we can find that by substituting P_R in $\pi_R(\varepsilon)$). We have seen that within the price range $[(1+k_f)(I-H)+c, P]$ we have $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -(1-\lambda) < 0$. Thus, the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to $(1+k_f)I + \varepsilon + (1-(1-\alpha)p(\varepsilon))c$ (we find this price by setting $\pi_R(\varepsilon) = 0$).

Case 2: $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c$, $(1+k_f)H + \varepsilon < (1-\alpha)p(\varepsilon)c$.

Lemma 2 states that if $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c$, then the entrepreneur undertakes $E = \varepsilon$ for all P_R such that the participation constraint is satisfied. If $(1+k_f)H + \varepsilon < (1-\alpha)p(\varepsilon)c$, then the minimum price that satisfies the participation constraint is $P_R < (1+k_f)(I-H)+c$ (we can find that by substituting P_R in $\pi_R(\varepsilon)$). We have seen that within the price range $[c, (1+k_f)(I-H)+c)$ we have that $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -\left(1 - \frac{\lambda}{p(\varepsilon)}\right)$. Thus, if $p(\varepsilon) < \lambda$, then the optimal price cap is $(1+k_f)(I-H)+c$. If $p(\varepsilon) \geq \lambda$, then the optimal price cap is the minimum price that satisfies the participation constraint $(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c$ (we find this price by setting $\pi_R(\varepsilon) = 0$).

Case 3: $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$ and $(1+k_f)H \geq (1-\alpha)p(0)c$.

Lemma 2 states that if $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$, then the entrepreneur undertakes $E = 0$ for all $P_R \geq (1+k_f)(I-H)+c$ such that the participation constraint is satisfied. If $(1+k_f)H \geq (1-\alpha)p(0)c$, then the minimum price that satisfies the participation constraint is $P_R \geq (1+k_f)(I-H)+c$ (we can

find that by substituting P_R in $\pi_R(0)$). We have seen that within the price range $[(1+k_f)(I-H)+c, P]$ we have $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -(1-\lambda) < 0$. Thus, the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to $(1+k_f)I + (1-(1-\alpha)p(0))c$ (we find this price by setting $\pi_R(0) = 0$).

Case 4: $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(\varepsilon)}[(1-\alpha)p(0)c - (1+k_f)H]$ and $(1+k_f)H < (1-\alpha)p(0)c$.

Lemma 2 states that if $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))cQ + \frac{(p(\varepsilon)-p(0))}{p(\varepsilon)}[(1-\alpha)p(0)cQ - (1+k_f)H]$, then the entrepreneur undertakes $E = 0$ for all $P_R \geq c$ such that the participation constraint is satisfied. If $(1+k_f)H < (1-\alpha)p(0)c$, then the minimum price that satisfies the participation constraint is $P_R < (1+k_f)(I-H)+c$ (we can find that by substituting P_R in $\pi_R(0)$). We have seen that within the price range $[c, (1+k_f)(I-H)+c)$ we have that $\frac{\partial W_{PC}(I-H,\bullet)}{\partial P_R} = -\left(1 - \frac{\lambda}{p(0)}\right)$. Thus, if $p(0) < \lambda$, then the optimal price cap is $(1+k_f)(I-H)+c$. If $p(0) \geq \lambda$, then the optimal price cap is the minimum price that satisfies the participation constraint, which is equal to $(1+k_f)(I-(1-p(0))H) + (1-(1-\alpha)p^2(0))c$ (we find this price by setting $\pi_R(0) = 0$).

Case 5: $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$, $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(0)}[(1-\alpha)p(\varepsilon)c - (1+k_f)H - \varepsilon]$ and $(1+k_f)H < (1-\alpha)p(0)c$.

Lemma 2 states that if $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$ and $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(0)}[(1-\alpha)p(\varepsilon)c - (1+k_f)H - \varepsilon]$, then the entrepreneur undertakes two levels of effort within the price range $[(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c, (1+k_f)(I-H)+c)$. More specifically, there is a price $\overline{P_R} \in [(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c, (1+k_f)(I-H)+c)$ such that if $P_R > \overline{P_R}$ the entrepreneur undertakes $E = 0$ and if $P_R \leq \overline{P_R}$ the entrepreneur undertakes $E = \varepsilon$. If $(1-\alpha)p(0)c > (1+k_f)H$, then the minimum price that satisfies the participation constraint is $P_R < (1+k_f)(I-H)+c$ (we can find that by substituting P_R in $\pi_R(0)$).

We have seen in Cases 2 and 4 that if $p(\varepsilon) < \lambda$ and $p(0) < \lambda$, then the optimal price cap is $(1+k_f)(I-H)+c$. Thus, if $p(0) < p(\varepsilon) < \lambda$, then the optimal price cap is $(1+k_f)(I-H)+c$. We have

also seen that if $p(\varepsilon) \geq \lambda$ and $p(0) \geq \lambda$, then the optimal price cap is the minimum price that satisfies the participation constraint. Thus, if $\lambda \leq p(0) < p(\varepsilon)$, then the optimal price cap is $(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c$.

It is easy to see that if we have $p(0) < \lambda \leq p(\varepsilon)$, then the optimal price cap will be $(1+k_f)(I-H)+c$ or $(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c$, as $\frac{\partial W_{PC}(D(0),\bullet)}{\partial P_R} > 0$ and $\frac{\partial W_{PC}(D(\varepsilon),\bullet)}{\partial P_R} < 0$. By calculating (12) - (11), where $E = \varepsilon$ in (12) and $E = 0$ in (11), we find that $(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c$ is welfare superior if:

$$p(\varepsilon)\varepsilon < (1-\alpha)(p^2(\varepsilon) - \lambda p(0))c + (1+k_f)(\lambda - p(\varepsilon))H \quad (13)$$

Otherwise, $(1+k_f)(I-H)+c$ is the optimal price cap. □

Proof of Proposition 3:

We proceed to prove Proposition 3 using the cases obtained in Proposition 2.

Case 1: $\varepsilon \leq (1-\alpha)(p(\varepsilon) - p(0))c$ and $(1+k_f)H + \varepsilon \geq (1-\alpha)p(\varepsilon)c$.

We know from Proposition 2 that under these market conditions the optimal price cap is $P_R = (1+k_f)I + \varepsilon + (1-(1-\alpha)p(\varepsilon))c$. By taking the difference between (10) ($E = \varepsilon$) and (8) we find that the PC always generates at least the same welfare as the COS since $\varepsilon \leq (1-\alpha)(p(\varepsilon) - p(0))c$.

Case 2: $\varepsilon \leq (1-\alpha)(p(\varepsilon) - p(0))c$ and $(1+k_f)H + \varepsilon < (1-\alpha)p(\varepsilon)c$.

We know from Proposition 2 that if $p(\varepsilon) < \lambda$ the optimal price cap is $P_R = (1+k_f)(I-H)+c$. By taking the difference between (11) ($E = \varepsilon$) and (8) we find that the PC will generate at least the same welfare as the COS if $\lambda[(1-\alpha)p(\varepsilon)c - (1+k_f)H - \varepsilon] \geq [(1-\alpha)p(0)c - (1+k_f)H]$. We know from Proposition 2 that if $p(\varepsilon) \geq \lambda$ the optimal price cap is $(1+k_f)(I-(1-p(\varepsilon))H) + p(\varepsilon)\varepsilon + (1-(1-\alpha)p^2(\varepsilon))c$. By taking the difference between (12) ($E = \varepsilon$) and (8) we find that the PC will generate at least the same welfare as the COS if $p(\varepsilon)\varepsilon \leq (1-\alpha)(p^2(\varepsilon) - p(0))c + (1-p(\varepsilon))(1+k_f)H$.

Case 3: $\varepsilon > (1-\alpha)(p(\varepsilon) - p(0))c$ and $(1+k_f)H \geq (1-\alpha)p(0)c$.

We know from Proposition 2 that under these market conditions the optimal price cap is $P_R = (1+k_f)I + (1-(1-\alpha)p(0))c$. By taking the difference between (10) ($E=0$) and (8) we find that the PC will generate the same welfare as the COS.

Case 4: $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(\varepsilon)}[(1-\alpha)p(0)c-(1+k_f)H]$ and $(1+k_f)H < (1-\alpha)p(0)c$.

We know from Proposition 2 that if $p(0) < \lambda$ the optimal price cap is $P_R = (1+k_f)(I-H)+c$. By taking the difference between (11) ($E=0$) and (8) we find that the PC is welfare inferior as we never have $\lambda[(1-\alpha)p(0)c-(1+k_f)H] \geq [(1-\alpha)p(0)c-(1+k_f)H]$. We know from Proposition 2 that if $p(0) \geq \lambda$ the optimal price cap is $(1+k_f)(I-(1-p(0))H)+(1-(1-\alpha)p^2(0))c$. By taking the difference between (12) ($E=0$) and (8) we find that the PC is welfare inferior as we never have $(1-\alpha)p(0)c \leq (1+k_f)H$.

Case 5: $\varepsilon > (1-\alpha)(p(\varepsilon)-p(0))c$, $\varepsilon \leq (1-\alpha)(p(\varepsilon)-p(0))c + \frac{(p(\varepsilon)-p(0))}{p(0)}[(1-\alpha)p(\varepsilon)c-(1+k_f)H-\varepsilon]$ and $(1+k_f)H < (1-\alpha)p(0)c$.

We know from Proposition 2 that if $p(\varepsilon) < \lambda$ the optimal price cap is $P_R = (1+k_f)(I-H)+c$ (See Proof of Case 4). If $p(0) \geq \lambda$, the optimal price cap is $(1+k_f)(I-(1-p(\varepsilon))H)+p(\varepsilon)\varepsilon+(1-(1-\alpha)p^2(\varepsilon))c$ (See Proof of Case 2). If $p(0) < \lambda \leq p(\varepsilon)$, then the optimal price cap is $P_R = (1+k_f)(I-H)+c$ or $(1+k_f)(I-(1-p(\varepsilon))H)+p(\varepsilon)\varepsilon+(1-(1-\alpha)p^2(\varepsilon))c$. If the former is the optimal price cap then see Proof of Case 4. If the latter is the optimal price cap then see Proof of Case 2. □