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# Seven types of ambiguity

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### **Abstract**

Reducing ambiguity to a purely technical property of preferences misses much of the insight in Ellsberg's (1961) paper, as well as in more recent developments in the study of differential awareness. In this paper, syntactic approaches to ambiguity are used to illustrate the point that Reducing ambiguity to a purely technical property of preferences misses much of the insight in Ellsberg's (1961) paper, as well as in more recent developments in the study of differential awareness.

# 1 Introduction

The concept of ambiguity plays a central role in modern decision theory. Ellsberg's (1961) paper, 'Risk, Ambiguity and the Savage Axioms' in which he demonstrated the existence of preferences that could not be rationalized with respect to a probability distribution over outcomes, has received more than 10000 citations.

Although this was the first use of the term 'ambiguity' in its decision theoretic sense, the idea that preferences might not be expressible in terms of probability distributions is much older. Different versions of this idea were put forward by Knight and Keynes.

This history is well known to decision theorists. But the concept of ambiguity has a long, and almost entirely independent, history in literary criticism. Empson's (1930, 1947), *Seven Types of Ambiguity* is a classic in the field, widely seen as a foundational text for the New Criticism movement, which dominated literary studies in the mid-20th century.

Empson showed that, far from being a weakness in expression, ambiguity could play a central role in the effectiveness of poetry. His seven types began with the simple case of double meaning and ended with the idea that some texts required readers to supply their own meaning.

Even after the decline of New Criticism, Empson's work remains influential. His title has been reused several times, notably in a recent Australian novel (Perlman 2004) where the same scenes are viewed from seven different viewpoints.

This paper continues the tradition of homage in the choice of title. There is no attempt at a mechanical translation of Empson's categories. Rather the point of invoking his work is to stress the point that the concept of ambiguity is itself ambiguous, in decision theory, as in literature.

Reducing ambiguity to a purely technical property of preferences misses much of the insight in Ellsberg's (1961) paper, as well as in more recent developments in the study of differential awareness.

The seven types of ambiguity considered here are

1. Subjective and objective probabilities
2. Unknown probabilities and multiple priors
3. Coarse awareness
4. Restricted awareness
5. Awareness of unawareness
6. Ambiguous contracts
7. Deliberate ambiguity

Several of these categories are explicitly related to bounded and differential awareness. A central claim in this paper, developing the argument of Grant, Guerdjikova and Quiggin (2018) is that all forms of ambiguity may be traced to bounded and differential awareness.

Furthermore, once understood in this way, the relationship between the ordinary language, or syntactic, interpretation of ambiguity throws more light on decision theoretic concepts.

The paper is organized as follows. As background, Empson's seven types of ambiguity are summarized. Ellsberg's treatment of the problem is also discussed.

The focus then shifts to formal decision-theoretic concept of ambiguity. The starting point is the relationship between semantic (state-based) and syntactic (propositional) notions of ambiguity. In the Savage framework, there is a straightforward isomorphism between the two. This isomorphism extends to the most common decision-theoretic representation of ambiguity, in which agents are fully aware of the state space, but are not probabilistically sophisticated in the sense of Machina and Schmeidler (1992).

However, under conditions of bounded and differential awareness, semantic and syntactic representations differ in important ways.

## 2 Empson's seven types of ambiguity

### Seven Types of Ambiguity

As summarized by Wikipedia, Empson's seven types of ambiguity are:

1. The first type of ambiguity is the metaphor, that is, when two things are said to be alike which have different properties. This concept is similar to that of metaphysical conceit.

2. Two or more meanings are resolved into one. Empson characterizes this as using two different metaphors at once.

3. Two ideas that are connected through context can be given in one word simultaneously.

4. Two or more meanings that do not agree but combine to make clear a complicated state of mind in the author.

5. When the "author is discovering his idea in the act of writing..." Empson describes a simile that lies halfway between two statements made by the author.

6. When a statement says nothing and the readers are forced to invent a statement of their own, most likely in conflict with that of the author.

7. Two words that within context are opposites that expose a fundamental division in the author's mind.

The purpose of providing this list is not to suggest a mechanical translation of Empson's categorization of literary ambiguity into problems of interest in decision theory. Rather it is to emphasize that, in its original role as a description of language, ambiguity is at once powerful and hard to grasp; indeed, it is ambiguous.

### 3 Syntactic and semantic representations

Feinberg (2000) and Halpern (2003) describe the interplay between syntactic (propositional) and semantic (set-theoretic) representations of uncertainty.

Syntactic representations of uncertainty are now part of the everyday mental equipment of most people in modern societies. For example, weather forecasts routinely state probabilities of rain on a given day.<sup>1</sup>

The dominant approach in formal decision theory, however, at least since the work of Savage (1954) has been semantic. The standard framework begins with prospects, considered as state-contingent vectors of outcomes. Under the Savage axioms, preferences over prospects may be represented by an expected utility (EU) functional, in which the expectation of a state-independent utility function over outcomes is taken with respect to a probability distribution over states. Even Savage, however, relies on verbal descriptions of states when he gives examples of decision-theoretic reasoning (for example, making an omelette when some eggs may be rotten).

In defending the syntactic approach, Feinberg [2000, p. 128] says

The syntactic formalism is the more fundamental and—intuitively—the more descriptive way to model economic situations that involve knowledge and belief . . . It is fine to use the semantic formalism, as long as what we say semantically has a fairly clear intuitive meaning—that it can be said in words. This amounts to saying that it can be stated syntactically.

Halpern (2018) points out several advantages of the syntactic approach. For our purposes, the most important is that

Syntax allows us to make finer distinctions than semantics; a set may be represented by the two different expressions, and an agent may not react to these expressions in the same way. Moreover, different agents may react differently to the same expression, that is, the expression may represent different sets according to different agents.

In ordinary usage, ‘ambiguity’ is a syntactic concept, referring primarily to language that may be interpreted in more than one way.

informational ambiguity, in the ordinary language sense that the available information is open to multiple interpretation, may be modeled using concepts from the literature on unawareness. When individuals are unaware

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<sup>1</sup>However, this is quite a recent development. Even in the late 1990s, it could be stated that ‘probability forecasts are still the exception rather than the rule’. It was widely argued that users of forecasts (especially the general public) would not accept—or will not be able to understand—probability forecasts.

of some possibilities relevant to the outcome of their decisions, there are multiple probability distributions that may be applicable, depending on whether or not these possibilities are realized.

## 4 Notation and terminology

### 4.1 Semantic representation

The standard semantic description of an individual decision problem consists of

- a finite<sup>2</sup> set of states  $S$ ;
  - a set of possible outcomes  $X$ , with an associated total order  $>$
  - a set of acts  $A : S \rightarrow X$ ;
- an information mapping  $\mathcal{I} : S \times N \rightarrow 2^S$  where  $\mathcal{I}(s)$  represents the information available to individual  $n$  at state  $s \in S$

### 4.2 Syntactic representation

The standard syntactic representation begins with a language  $L$  generated by a finite set of elementary propositions  $L^0$ , closed under (finite) conjunction and disjunction. In addition, the standard syntactic includes a knowledge operator  $k$  which satisfies the usual logical properties.

### 4.3 Equivalence of the standard representations

The truth table for  $L^0$ , determines a state-space  $S = 2^{L^0}$  which is sufficient to determine the truth value of all propositions in  $L$ . Conversely, each proposition  $p$  in  $L^0$  corresponds to an event  $E(p)$  in  $S$ , that is, a proper subset of  $S$  considered as an element of the discrete topology. We interpret  $s \in E(p)$  to mean ‘ $p$  is true at state  $s$ ’. The set of events  $\{E(p) : p \in L^0\}$  is a basis for the discrete topology on  $S$ . The knowledge operator  $s \in E(kp)$  is equivalent to  $\mathcal{I}(s) \subset E(p)$ , that is,  $p$  is true for every state  $s'$  that is possible given the information available at  $s$ .

Reversing the process, for any finite state space  $S$ , given a basis  $B$  for the discrete topology on  $S$ , we can label each element  $s \in B$  with a proposition  $p(s)$ . Closing the set of propositions (finite) conjunction and disjunction yields a language  $L(S)$  for which  $S$  represents the truth table.

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<sup>2</sup>We will assume that the set of states of the world relevant to lifetime consumption decisions is finite, since the world itself is finite. Further, we will assign the discrete topology so that every set of states is an event and vice versa. The analysis may be extended to compact state spaces, but this adds little insight. As Feinberg (2000) shows, infinite state spaces in general are more problematic.

## 5 Bounded awareness

An agent is fully aware if they have complete preferences over the set of all possible acts  $\mathbf{A} = X^S$ . However, this requirement is impossibly demanding for human agents and can, at best, serve as a benchmark. In reality, agents will be aware of only a limited set of acts, states and possible outcomes. The language used by agent  $i$  to describe their awareness may be denoted  $L^i$

Two special cases of bounded awareness are of particular interest. The first is the case where the agent's language  $L^i$  generates  $S^i = 2^{L^i}$  where  $S^i$  is a proper partition of  $S$ . Thus, any event  $E^i$  in  $S^i$  is an event in  $S$  but not vice versa.

The second case is that of restricted awareness<sup>3</sup>. In a semantic representation of restricted awareness, the state space  $S^i$  is a proper subset of  $S$ , so that some states  $s$  are not conceived of as possible. In syntactic terms, the proposition  $\neg p(s)$  is implicitly assumed to be true.

## 6 The seven types of ambiguity

### 6.1 Knightian uncertainty

The use of the term 'ambiguity' in decision theory is normally attributed to Ellsberg. However Knight (1921) refers repeatedly to the 'fatal ambiguity' of the term 'risk', which motivated his own distinction between 'risk' and 'uncertainty'.

What exactly Knight meant by this distinction is unclear. It is difficult to the language used to discuss probability in the 1920s into modern terms. As Langlois and Cosgel (1991) observe, there is an ever-present danger of 'Whig history', in which modern concepts are projected back onto people who thought in quite different terms Knight's own idiosyncratic style adds to the confusion<sup>4</sup>.

However, the most natural interpretation is that Knight was distinguishing a frequentist version of objective probability, suitable for setting an insurance premium, from a subjective assessment, of the kind made by an entrepreneur deciding to launch a new venture.

However, as the example of life insurance shows, this distinction is problematic. For most of the period in which life insurance policies have been offered, age-specific mortality rates have been declining. Conversely, 'life expectancy at birth', an estimator based on the assumption of constant age-specific mortality, has been biased downwards, relative to the average

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<sup>3</sup>There is no standard terminology here. The first writer to draw this distinction was Li (2008, 2009) who used the term 'pure unawareness' to refer to coarse awareness, and 'partial unawareness' to refer to restricted awareness.

<sup>4</sup>The same points may be made with respect to Keynes (1921).



longevity of any given birth cohort.<sup>5</sup> Given that life insurance policies are typically long-term contracts, they involve uncertainty about future mortality rates as well as risk associated with the outcomes for any individual.

## 6.2 Unknown probabilities and multiple priors

Although earlier writers including Knight, Keynes and even Adam Smith had raised doubts concerning the feasibility of assigning probabilities to events, Ellsberg (1961) was the first to give a convincing formal argument. This is because the provision of such an argument required a formal theory of subjective probability, of the kind proposed by Savage (1954)<sup>6</sup>.

The now-standard usage is quite different from that in Ellsberg's (1961) original article. Ellsberg treated ambiguity, not as a property of preferences or relative likelihoods, but as a property of the information on which judgments of relative likelihoods might be based.

Responses from confessed violators [of the EU axioms] indicate that the difference is not to be found in terms of the two factors commonly used to determine a choice situation, the relative desirability of the possible pay-offs and the relative likelihood of the events affecting them, but in a third dimension of the problem of choice: the nature of one's information concerning the relative likelihood of events. What is at issue might be called the **ambiguity** of this information, a quality depending on the amount, type, reliability and 'unanimity' of information, and giving rise to one's degree of 'confidence' an estimate of relative likelihoods. (emphasis added)

The most common decision-theoretic interpretation of ambiguity in this sense is the 'multiple priors' model, beginning with Gilboa and Schmeidler (1989). Gilboa and Schmeidler propose a maxmin interpretation, in which prospects are evaluated with respect to the least favorable prior under consideration. An important difficulty with this model is the question of how priors should be updated in response to new information. A variety of proposals have been put forward, including full (or generalized) Bayesian updating, maximum likelihood updating, or Dempster-Shafer updating. However most of these proposals but most involve drawbacks such as violations of dynamic consistency (Hanany, Klibanoff and Marom 2011).

Grant, Guerdjikova and Quiggin (2018) argue that ambiguity represented by multiple priors should be interpreted in terms of bounded awareness. They

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<sup>5</sup>This has ceased to be true for the United States, where age-specific mortality rates have increased in recent years, first for specific subgroups such as white women, and more recently for the population as a whole.

<sup>6</sup>Earlier work by de Finetti (1931) received only limited attention at the time

represent a boundedly rational decision-maker as one who is unaware of at least some propositions in  $P$ . For simplicity, consider the case when an agent is aware of a proposition  $p$ , but not of a related proposition  $q$ . In this situation, the proposition  $p$  is ambiguous since it may mean either  $p \wedge q$  or  $p \wedge \neg q$ . From the agent's viewpoint, her information about  $p$  is incomplete, since it is open to multiple interpretations.

This interpretation allows a new approach to the problem of updating multiple priors, based on a syntactic representation. In this representation, standard Bayesian updating corresponds to learning the truth value of a proposition of which the decision-maker is already aware. Updating may also arise from increased awareness, represented as awareness of new elementary propositions  $q$ . When information is received with no change in awareness, each element of the set of priors is updated in the standard Bayesian fashion as in Ghirardato, Maccheroni, and Marinacci (2008). By contrast, an increase in awareness is represented by an expansion of the state space to which the decision maker has access, and by a corresponding contraction in the set of priors under consideration, that is, a reduction in ambiguity

As the decisionmaker approaches full awareness, the set of priors contracts to a singleton  $\{\pi^*\}$ , and decisions become unambiguous. Relative to  $\pi^*$  the set of priors at any time  $t$  may be represented as a set of conditional probability distributions, one for each configuration of the truth values of propositions of which the decision-maker is unaware.

### 6.3 Coarse awareness

Coarse awareness arises when decisionmakers fail to distinguish between distinct states that differ in relevant ways. In syntactic terms, a language displays coarse awareness with respect to states  $s, s'$  if there is no proposition  $p$  such that  $p$  is true for  $s$  but not for  $s'$ <sup>7</sup>. Converted into semantic terms, the decisionmaker's partition of  $S$ , generated by the truth table for  $L$  is coarser than the maximally refined partition given by the discrete topology.

The implications of coarse awareness for ambiguity arise in relation to acts that are not measurable with respect to the decisionmaker's partition of  $S$ . One way to treat this possibility, considered by Guerdjikova and Quiggin (2016, 2019) is to assume that decisionmakers are unaware of such acts. The other, considered by Heifetz, Meier and Schipper (2006) and Grant and Quiggin (2013) among others, is to suppose that DMs treat the act as a relation yielding some subset of the possible outcomes associated with a given event.

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<sup>7</sup>Note that the symmetric case applies to  $\neg p$

In the first case, decisionmakers have unambiguous preferences over a restricted set of acts. The results broadly speaking are the same as if the set of acts were exogenously restricted. As Guerdjikova and Quiggin (2016, 2019) show, this may or may not make agents worse off in models of survival in financial markets. Agents are more likely to survive, since their beliefs are more likely to accurate with respect to a coarse state space. On the other hand, they will typically forgo beneficial opportunities for risk sharing.

## 6.4 Restricted awareness

Restricted awareness arises when a state  $s$  is not considered by agent  $i$ , that is, when there is no proposition  $p_i(s)$  such that  $p_i(s)$  is true if and only if state  $s$  prevails. More precisely,  $\nexists p \in L^i, E(p) = \{s\}$ .

If agents are unaware of an event  $E$ , then the complementary event is implicitly treated as encompassing all possible outcomes, and therefore as having probability 1. More generally, if agents hold well-defined subjective probabilities over the events of which they are aware the probability of any event containing  $E$  will be understood and the probability of the complementary event understated.

From the point of view of a fully aware interlocutor, statements made by agent with restricted awareness are ambiguous. Consider a proposition  $q \in L$  such that  $s \in E(q)$ . The corresponding proposition  $q \in L^i$  may be interpreted by the fully aware interlocutor either as being equivalent to  $q \in L$  or as meaning  $q \wedge \neg p(s)$ .

More generally, restricted awareness implies that probability beliefs are erroneous. It follows from the arguments of Blume and Easley (1992) that, agents with restricted awareness will be ‘selected out’ of financial markets

### 6.4.1 Restricted awareness is not zero probability

Restricted awareness is not the same as attaching zero probability to an event. This point can be made in various ways. Li (2008) gives a formal analysis (Li uses the term ‘partial unawareness’). We will consider less formal arguments.

First, we may approach the question through introspection. There are many events we can conceive, but to which we attach zero probability (for example, a discovery that  $2+2=5$ ). On the other hand, there are many events we fail to consider, but can recognize as having positive probability when they occur, or simply when the possibility is pointed out to us. For example, even people who took the prospect of a global pandemic seriously mostly did not conceive of possible responses such as lockdowns and border closures until they were already imminent.

Second, we may consider responses to offers to bet against the event in question. If the event is indeed one of zero probability, willingness to bet

will be bounded only by concerns such as the probability that the other party will default. But an agent offered a bet against an event they have previously not considered, must thereby become aware of the event and, in general, will consider the event possible.

Third, we may consider off-equilibrium strategies in extensive form games. All such actions have, by definition, zero probability in equilibrium. But, as Heifetz et al observe, the strategy chosen by one player will depend on their awareness of the strategies available to others. This choice of strategy in turn informs other players about the awareness or unawareness of their opponents.

## 6.5 Awareness of unawareness

The question of whether agents can be aware of their own bounded awareness has been a long-standing problem. Introspection suggests that the answer is obvious: we are all boundedly aware, and we know it. But formal representations of awareness of unawareness have proved problematic.

This in turn creates problems for the discussion of ambiguity. If ambiguity is derived from bounded awareness, agents may be unaware that propositions are ambiguous.

Modica and Rusticchini (1994, 1999) present definitions of awareness and unawareness using a model logical approach. They show that, assuming partitional information structures, agents cannot be aware of their own unawareness.

Grant and Quiggin (2012) support this conclusion, using an argument based on finiteness. They argue, however, that agents who have been regularly surprised in the past can conclude, on the basis of inductive reasoning, that they will be surprised in the future. This conclusion justifies heuristic constraints on decisions.

Grant and Quiggin (2014) present a formal model of choice under surprise, in which the propositions which agents use to describe states of nature are ambiguous and subject to surprise. Optimisation on the basis of the ambiguous model is constrained by the need to avoid or minimize unfavorable surprises.

## 6.6 Ambiguous contracts

The interplay between ordinary language interpretations of ‘ambiguity’ and the technical interpretation standard in decision theory is particularly striking in relation to contracts. In the legal studies literature, as in ordinary language, the term ‘ambiguity’ is used to describe statements which are open to multiple interpretations. The key concern is which interpretation should be preferred in construing the provisions of contracts. .

Under the assumption of common full awareness, and assuming risk aversion, contracts should specify an outcome conditional on every possible state of nature, subject only to the transactions cost of specifying distinct outcomes for states that differ only marginally. Chung (1991) has pointed out that simple contracts are typically efficient when both parties are risk averse.

Grant, Kline and Quiggin (2012, 2014, 2018) have examined the problem of contracting in the presence of syntactic ambiguity. In these papers, parties to a contract have coarse awareness, and their representations of the world are not mutually measurable. Hence, *ex post*, they may disagree as to which state of the world has occurred, and therefore as to what actions are required by the contract. Such disagreement leads to disputes. Agents may prefer simpler, less ambiguous, contracts when facing potential disputes even though this implies forgoing opportunities for beneficial risk sharing.

Grant, Kline and Quiggin (2014) consider the contractual specification of damages that apply when one party is unable (or finds it undesirable) to fulfill their contractual obligations. In this context, they consider ‘liquidated damages’ contracts which specify a constant payment for the case of default. optimality of liquidated damages contracts in a setting of contractual ambiguity and potential for disputes. When parties are ambiguity averse enough, they will optimally choose liquidated damages contracts and sacrifice risk sharing opportunities.

## 6.7 Deliberate ambiguity

As Kellner and Le Quement (2018) observe

On the face of it, the phenomenon of ambiguous language is puzzling because it appears to gratuitously decrease the precision of transmitted information.

However, Kellner and Le Quement find that ambiguous language on the contrary increases the payoffs achievable by both parties. Blume and Board use the term ‘intentional vagueness’ in describing a similar result

This paper analyzes communication with a language that is vague in the sense that identical messages do not always result in identical interpretations. It is shown that strategic agents frequently add to this vagueness by being intentionally vague, i.e. they deliberately choose less precise messages than they have to among the ones available to them in equilibrium. Having to communicate with a vague language can be welfare enhancing because it mitigates conflict. In equilibria that satisfy a dynamic stability condition intentional vagueness increases with the degree of conflict between sender and receiver.

The central point in these papers is that the sender and receiver have both conflicting interests in relation to the action to be taken by the receiver and a shared interest in adjusting this action to the state of nature, known only to the sender. In a model of unambiguous communication, conflict reduces the willingness of the sender to transmit information. In some cases, this leads to the pooling solution, where no information is sent, and the receiver takes their ex ante optimal action. However, if the receiver responds to ambiguity by adjusting their action in the direction desired by the sender, it may be beneficial to send ambiguous information.

Kellner and Le Quement model ambiguous communications by proposing that the sender's message is made conditional on the colour of a ball drawn from an Ellsberg urn, the composition of which is not known either by the sender or the receiver. This is plausible enough as regards the receiver, but problematic as regards the sender, who presumably has access to the urn.

The idea of the Ellsberg urn may be treated as metaphorical. It is, after all, hard to imagine that receivers would treat as a serious a message generated by such an arbitrary randomiser. But if the urn is metaphorical it would seem preferable to replace it with a more plausible source of ambiguity. The most promising way of doing this is to adopt a syntactic approach, recognising that some terms in the language available to senders are ambiguous. A preference for terms of this kind, as opposed to clear and unambiguous language has been a notorious feature of bureaucratic communication ever since Dickens satirized the Circumlocution Office in *Little Dorrit*. Repeated attempts to promote clearer language, such as Gowers' (1948,2014), *Plain Words* have done little, if anything to improve the situation. A syntactic model of deliberate ambiguity might be of assistance here.

### 6.7.1 Fedspeak

A commonly used example is that of the US Federal Reserve, and more particularly the language used by long-serving Reserve Chairman in discussing the future course of monetary policy, referred to as 'Fedspeak'. However, it is unclear whether a complex model of communication is needed here. As described by Greenspan, Fedspeak was simply

a language of purposeful obfuscation to avoid certain questions coming up, which you know you can't answer, and saying—'I will not answer,' or basically, 'no comment,' is, in fact, an answer.

Moreover, some statements presented as examples of 'Fedspeak' seem to involve nothing more than the presentation of relatively complex concepts to an audience with limited understanding of the issues. For example, Wikipedia cites the following from Greenspan (2005)

Risk takers have been encouraged by a perceived increase in economic stability to reach out to more distant time horizons. But long periods of relative stability often engender unrealistic expectations of it[s] permanence and, at times, may lead to financial excess and economic stress.

which is a cautiously worded statement of the main result of Borio and Lowe, borne out in spectacular fashion by the Global Financial Crisis which occurred a few years later.

Nevertheless, it seems likely that ambiguous statements from central banks are more than mere obfuscation. Central bankers want to achieve their policy goals (for example, maximizing the value of a Taylor-style objective function) while minimizing the disruptive effects of large adjustments to their main policy instrument, typically a short-term interest rate. Statements that lead agents to anticipate (for example) a tightening of monetary policy, without committing to such a tightening or even signalling the probability of such an event may serve this purpose.

## 7 Concluding comments

In this paper, it has been argued that the semantic interpretation of ‘ambiguity’ to mean the absence of well-defined subjective probabilities is restrictive and unhelpful. Whereas the usual interpretation implies full awareness of the state space, a correct understanding of ambiguity must be linked to bounded awareness.

The use of syntactic as well as semantic approaches helps to illustrate the central point of the paper: that ‘ambiguity’ is an ambiguous concept, but nonetheless a powerful one.

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