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Inference for Aggregate Efficiency: Theory and Guidelines for Practitioners

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Abstract

We expand the recently developed framework for the inference for aggregate efficiency, by extending the existing theory and providing guidelines for practitioners. In particular, we develop the central limit theorems (CLTs) for aggregate input-oriented efficiency, analogous to the output-oriented framework established by Simar and Zelenyuk (2018). To further improve the finite sample performance of the developed CLTs, we propose a simple yet easy to implement method through using the bias-corrected individual efficiency estimate to improve the variance estimator. The extensive Monte-Carlo experiments confirmed the developed CLTs for aggregate input-oriented efficiency and also confirmed the better performance of our proposed method in the finite sample sizes. Finally, we use two well-known empirical data sets to illustrate the differences across the existing methods to facilitate the use by practitioners.

Keywords: Data Envelopment Analysis, Efficiency, Non-parametric Efficiency Estimators, Free Disposal Hull, Aggregate Efficiency

JEL Classification: C1, C3

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1 Introduction

Nonparametric efficiency estimators are widely used tools to benchmark the performance of economic agents (firms, hospitals, banks, farms, etc.). The basic idea of these estimators is to use a sample of observed input-output pairs to estimate the production possibility set, i.e., the set describing how economic agents could use inputs to produce outputs, and then benchmark each producer relative to the upper boundary or frontier of the set.[1]

The nonparametric efficiency estimators that we focus on here include data envelopment analysis (DEA) (Farrell, 1957; Charnes et al., 1978; Banker et al., 1984) which imposes convexity on the production set, and free disposal hull (FDH) (Deprins et al., 1984) which relaxes the convexity assumption and assumes only free disposability on inputs and outputs. Based on the different assumptions on the returns to scale of the frontier, DEA estimators are further divided into constant returns to scale (CRS-DEA) and variable returns to scale (VRS-DEA) among others. Currently, VRS-DEA, CRS-DEA and FDH are the three widely used tools in the literature. Their theoretical and statistical properties are well-documented (e.g., see Kneip et al., 2015, 2016).

Recently, Kneip et al. (2015) established the central limit theorems (CLTs) results for the simple mean (or unweighted) technical efficiency estimates, which were further extended to the context of aggregate (or weighted) efficiency (Simar and Zelenyuk, 2018), conditional efficiency (Daraio et al., 2018), sources of Malmquist productivity change (Simar and Wilson, 2019), overall and allocative efficiency (Simar and Wilson, 2020), Malmquist productivity indices (Kneip et al., 2021) and aggregate Malmquist productivity indices (Pham et al., 2023), to name a few.

It is worth noting that Simar and Zelenyuk (2018, 2020) only focused on aggregate output-oriented efficiency. In many contexts, researchers may need to use input-oriented efficiency, which is a similar yet different measure and which also implies a different aggregation weight. In principle, the CLT results for aggregate input-oriented efficiency are "analogous" to the output-oriented framework developed by Simar and Zelenyuk (2018) and so we make a modest contribution from a pure theoretical perspective. However, before the analogous theoretical framework is spelled out by someone, one can hardly hope it will be practically

[1] See Sickles and Zelenyuk (2019) for a comprehensive review and examples.

used by an applied researcher. Indeed, practitioners typically need to see the exact formulas spelled out precisely, and preferably without errors and typos, so it would be easy to make and could be checked by anyone, that can then propagate to the applied work, by undermining its validity and related policy implications. While theory can be obvious for some, the reality often is that the absence of a well-presented theoretical framework, even if "analogous", sometimes stalls its practical applications. Hence, providing such a framework is one of the goals of this paper. As importantly, we also provide the free tools (the programmed and tested in many scenarios *R* codes) to simplify the life of practitioners interested in implementing this framework. Furthermore, we also illustrate the use of this approach for real data sets.

Another important goal of this paper is to provide the practitioners with information about the performance of this approach in small samples, for different ranges of samples (10, 20, 50, 100, 200, 300, 500, 1000) and different dimensions of the models. Such information is very important for practitioners to understand the levels of accuracy one can expect (or hope) for the samples in hand and for the models they intend to use. For example, such information can suggest to a practitioner that for the sample they have (say about 300 observations) they need to use some adequate dimension reduction approaches to reduce the model dimension to no more than four, if they want to achieve the desired level of accuracy. Such information can hardly be obtained from just knowing that the theory is "analogous" or even from deriving the full theory itself. At best it can be obtained from extensive Monte Carlo (MC) simulations, for various scenarios, and we perform them in this work.

Interestingly, performing such simulations nudged us to discover further theoretical improvements that can be made to the current theory found in Simar and Zelenyuk (2018, 2020). These improvements are based on the adaptation and extension of ideas of data sharpening for DEA from Nguyen et al. (2022) and an alternative estimator of the variance from Simar et al. (2023), and we improve upon these two works as well. Again, these improvements are modest from the perspective of high-level mathematics, yet they turn out to provide substantial improvements for practitioners in terms of application of the theory for relatively small samples and especially for relatively high dimensions of the DEA models, as confirmed by the evidence from the simulations.

The rest of the paper is organized as follows. Section 2 briefly introduces the theoretical background on technical efficiency and aggregate efficiency, and then briefly summarizes the main theoretical results for aggregate input-oriented efficiency, adapted from Simar and Zelenyuk (2018) for aggregate output-oriented efficiency. Section 3 introduces the existing methods to improve the finite sample performance of CLT results for aggregate efficiency, including the variance correction method in Simar and Zelenyuk (2020) and a variant of the data sharpening method in Nguyen et al. (2022). Our proposed method is also introduced in Section 3. Section 4 conducts extensive MC experiments to compare the performance of all these methods. In Section 5 we use the well-known Philippines rice data set and Penn World Table data set as two examples to illustrate the differences between all these methods in the estimated variance as well as the estimated confidence intervals for the aggregate input-oriented and output-oriented (respectively) efficiency. Section 6 concludes and recommends future research directions. Additional results are provided in a separate, supplementary Appendix.

2 The Theoretical Background

2.1 The Production Economics Model

The economic agents or decision making units (DMUs) are assumed to use p number of inputs $x \in \mathbb{R}^p_+$ to produce q number of outputs $y \in \mathbb{R}^q_+$. Suppose the best-practice technology available to all DMUs is characterized by technology set Ψ, defined as

$$\Psi := \{(x, y) \mid x \text{ can produce } y\}. \tag{2.1}$$

That is, Ψ includes all the feasible combinations of inputs and outputs given the current technology and we assume the standard regularity assumptions of production theory hold, in particular:

Assumption 2.1. *The production set Ψ is closed.*

Assumption 2.2. *No free lunch; i.e., $(x, y) \notin \Psi$ if $x = 0,\ y \geqq 0,\ y \neq 0$.*

Assumption 2.3. $\forall\, (x, y) \in \Psi$, *(i) $\widetilde{x} \geqq x \Rightarrow (\widetilde{x}, y) \in \Psi$ and (ii) $\widetilde{y} \leqq y \Rightarrow (x, \widetilde{y}) \in \Psi$; i.e., inputs and outputs are strongly disposable.*

The *frontier* or *technology* contained in Ψ is defined as

$$\Psi^{\partial} := \{(x, y) \mid (x, y) \in \Psi,\ (x/\gamma, \gamma y) \notin \Psi \text{ for any } \gamma \in (1, \infty)\} . \tag{2.2}$$

Intuitively, the technology frontier represents the set of technically efficient input-output allocations.

The gap or the distance between an input-output allocation in Ψ and the frontier of Ψ is deemed as technical inefficiency, which can be measured in different ways, e.g., in different directions or with different types of measurements. Farrell-type measures of efficiency (input-oriented and output-oriented) appear to be the most popular in practice. Since the previous developments for the aggregate context were mainly done for the output orientation, here we will focus on the input orientation, where the Farrell input efficiency measure (Farrell, 1957) is defined as

$$\theta(x, y \mid \Psi) := \inf \{\theta > 0 \mid (\theta x, y) \in \Psi\} , \tag{2.3}$$

which calculates the proportion of inputs that can be scaled downward by the same scalar, holding outputs fixed. By construction in (2.3), $0 \leq \theta(x, y \mid \Psi) \leq 1$ for any $(x, y) \in \Psi$.

Another useful measure of efficiency is the cost efficiency (or the overall input efficiency). The cost efficiency for a DMU (x, y) facing input prices $w \in \mathbb{R}^p_+$ can be defined as

$$\mathcal{C}(x, y \mid w, \Psi) := \frac{C_{min}}{w^T x}, \tag{2.4}$$

where C_{min} is the minimum cost of producing a specific output vector of y given input prices w and production set Ψ, defined as

$$C_{min} := \min_x \left\{w^T x > 0 \mid (x, y) \in \Psi\right\} = w^T x^*, \tag{2.5}$$

where x^* is the argmin. Thus, cost efficiency can also be written as

$$\mathcal{C}(x, y \mid w, \Psi) = \frac{w^T x^*}{w^T x}. \tag{2.6}$$

By construction, $0 \leq \mathcal{C}(x, y \mid w, \Psi) \leq 1$.

The technical efficiency and the cost efficiency are well known to be closely related by Mahler's inequality, as

$$\mathcal{C}(x, y \mid w, \Psi) \leq \theta(x, y \mid \Psi). \tag{2.7}$$

This inequality can be used to define the input allocative efficiency (Färe et al., 1985), which is

$$\mathcal{A}(x, y \mid w, \Psi) = \frac{\mathcal{C}(x, y \mid w, \Psi)}{\theta(x, y \mid \Psi)}. \tag{2.8}$$

By construction, $0 \leq \mathcal{A}(x, y \mid w, \Psi) \leq 1$. Consequently, the technical, cost and allocative efficiency are related to each other in the following way,

$$\mathcal{C}(x, y \mid w, \Psi) = \theta(x, y \mid \Psi) \times \mathcal{A}(x, y \mid w, \Psi). \tag{2.9}$$

However, all the above introduced measures, Ψ, Ψ^∂, $\theta(x, y \mid \Psi)$, $\mathcal{C}(x, y \mid w, \Psi)$, and $\mathcal{A}(x, y \mid w, \Psi)$ are not observed and hence must be estimated from the data on input quantities $X_i \in \mathbb{R}^p_+$, output quantities $Y_i \in \mathbb{R}^q_+$ and input prices $w_i \in \mathbb{R}^p_+$, which we will denote as $\mathcal{S}_n = \{(X_i, Y_i, w_i)\}_{i=1}^n$. After introducing the aggregate efficiency, we will come back later to discuss the estimators.

2.2 Aggregate Efficiency

Researchers and policy makers are often more interested in the efficiency of a group of producers (such as the efficiency of an industry or a group within it), usually called *aggregate efficiency*. How to adequately aggregate the individual efficiency into a group efficiency is a question by itself that has been explored in the literature quite substantially, in the past seven decades.[2]

Here we will follow the approach of Färe and Zelenyuk (2003) who proposed using Koopmans aggregation theorem, in the output-oriented context and extended it further in Färe et al. (2004), and more recently in Mayer and Zelenyuk (2019). Specifically, their aggregate technical efficiency for a group of n producers is given by

$$\bar{\theta} := \sum_{i=1}^{n} \theta(X_i, Y_i \mid \Psi) S_i, \tag{2.10}$$

where S_i is the ith observation's cost weight, defined as

$$S_i = \frac{w_i^T X_i}{\sum_{i=1}^n w_i^T X_i}. \tag{2.11}$$

[2] For a recent review, see Zelenyuk (2020).

The aggregate cost efficiency is given by

$$\bar{\mathcal{C}} := \sum_{i=1}^{n} \mathcal{C}(X_i, Y_i \mid w_i, \Psi) S_i, \tag{2.12}$$

while the aggregate allocative efficiency is given by

$$\bar{\mathcal{A}} := \sum_{i=1}^{n} \mathcal{A}(X_i, Y_i \mid w_i, \Psi) S_i^a, \tag{2.13}$$

and where

$$S_i^a = \frac{w_i^T X_i^\partial}{\sum_{i=1}^n w_i^T X_i^\partial}, \tag{2.14}$$

and where $X_i^\partial = \theta(X_i, Y_i \mid \Psi) X_i$, i.e., X_i^∂ is the efficient input vector to produce the given output vector Y_i.

Similar to the relationship between the individual efficiency defined in (2.9), for aggregate technical, cost and allocative efficiency, we also have

$$\bar{\mathcal{C}} = \bar{\theta} \times \bar{\mathcal{A}}. \tag{2.15}$$

The above mentioned aggregate efficiency measures can also be expressed as ratios of simple means (Simar and Zelenyuk, 2018). Specifically,

$$\bar{\theta} = \sum_{i=1}^{n} \theta(X_i, Y_i \mid \Psi) S_i = \frac{(1/n)\sum_{i=1}^n w_i^T X_i^\partial}{(1/n)\sum_{i=1}^n w_i^T X_i}, \tag{2.16}$$

$$\bar{\mathcal{C}} = \sum_{i=1}^{n} \mathcal{C}(X_i, Y_i \mid w_i, \Psi) S_i = \frac{(1/n)\sum_{i=1}^n w_i^T X_i^*}{(1/n)\sum_{i=1}^n w_i^T X_i}, \tag{2.17}$$

and

$$\bar{\mathcal{A}} = \sum_{i=1}^{n} \mathcal{A}(X_i, Y_i \mid w_i, \Psi) S_i^a = \frac{(1/n)\sum_{i=1}^n w_i^T X_i^*}{(1/n)\sum_{i=1}^n w_i^T X_i^\partial}, \tag{2.18}$$

where $w_i^T X_i^*$ is the minimum cost in (2.5) for a DMU with an allocation (X_i, Y_i) facing input prices w_i and X_i^* is the argmin of the cost function. These expressions are useful as they can be used to develop the theoretical results for the aggregate efficiency by utilizing the statistical theory of the ratio of sample means. For the sake of conciseness, we will focus only on the aggregate input-oriented Farrell technical efficiency.

2.3 The Estimators of Individual Efficiency and their Asymptotic Properties

Given a random sample $\mathcal{S}_n = \{(X_i, Y_i)\}_{i=1}^n$, if one assumes the production technology set Ψ is convex and exhibits CRS, one could use the CRS-DEA estimator (Charnes et al., 1978), given by

$$\widehat{\theta}_{\text{CRS}}(x, y \mid \mathcal{S}_n) = \min_{\theta, s_1, \ldots, s_n} \left\{ \theta \mid y \leq \sum_{i=1}^n s_i Y_i,\ \theta x \geq \sum_{i=1}^n s_i X_i,\ \forall\ s_i \geq 0 \right\}. \tag{2.19}$$

Alternatively, if one assumes Ψ is convex but exhibits VRS, one could use the VRS-DEA estimator (Banker et al., 1984) given by

$$\widehat{\theta}_{\text{VRS}}(x, y \mid \mathcal{S}_n) = \min_{\theta, s_1, \ldots, s_n} \left\{ \theta \mid y \leq \sum_{i=1}^n s_i Y_i,\ \theta x \geq \sum_{i=1}^n s_i X_i,\ \sum_{i=1}^n s_i = 1,\ \forall\ s_i \geq 0 \right\}. \tag{2.20}$$

Alternatively, if one allows non-convexity and only assumes strong disposability of all inputs and all outputs, the FDH estimator proposed by Deprins et al. (1984) could be used, which for the input-oriented Farrell efficiency measurement is given by VRS-DEA with additional constraint $s_i \in \{0, 1\}$ added, or by computing

$$\widehat{\theta}_{\text{FDH}}(x, y \mid \mathcal{S}_n) = \min_{i \in \mathcal{D}_{x,y}} \max_{j=1,\ldots,p} \left(\frac{X_{ij}}{x_j} \right), \tag{2.21}$$

where for a vector a, a_j denotes its j-th component and $\mathcal{D}_{x,y} = \{i \mid (X_i, Y_i) \in \mathcal{S}_n, X_i \leq x, Y_i \geq y\}$.

The FDH, VRS, and CRS estimators of $\theta(x, y \mid \Psi)$ have well-developed statistical properties.[3] To summarize, all the estimators are consistent with a convergence rate n^κ where $\kappa = 2/(p+q)$, $2/(p+q+1)$, and $1/(p+q)$ for the CRS, VRS, and FDH estimators, respectively. Under an appropriate set of assumptions, all these estimators also have non-degenerate limiting distributions. Moreover, as recently pointed out by Kneip et al. (2015), all the estimators are biased, where the order of the bias is $O\left(n^{-\kappa}\right)$. If $\kappa \leq 1/2$, the bias term of their average does not vanish quickly enough (relative to the variance) to zero, and the usual CLT does not apply here. Kneip et al. (2015) also established new CLTs for the simple mean efficiency. The bias term for the simple mean (as well as individual efficiency) can be estimated using a generalized jackknife-type approach proposed by Kneip et al. (2015).

[3] See Kneip et al. (1998), Park et al. (2000), Kneip et al. (2008), Park et al. (2010), Kneip et al. (2015), and a recent review by Simar and Wilson (2015).

2.4 The Estimators of Aggregate Efficiency and their Asymptotics

In the conclusion section, Simar and Zelenyuk (2018) mentioned that "The same ideas can also be adapted to the case of aggregation of other measures based on the Farrell-type efficiency measures, such as input oriented technical, allocative, and cost efficiency measures; the input and output oriented scale efficiency measures; the input and output oriented Malmquist Productivity Indexes; the Hicks-Moorsteen Productivity Index, etc." In this section, we adapt the theoretical results in Simar and Zelenyuk (2018) to the case of aggregate input-oriented technical efficiency. The CLT theory turns out to be essentially the same as Simar and Zelenyuk (2018), and we restate it here for the sake of being self-contained. We will focus on the variant form of aggregate efficiency in (2.16), denoted as φ_n, namely

$$\varphi_n = \frac{n^{-1}\sum_{i=1}^{n} w_i^T X_i^\partial}{n^{-1}\sum_{i=1}^{n} w_i^T X_i}, \tag{2.22}$$

where $X_i^\partial = \theta(X_i, Y_i \mid \Psi)X_i$ is the hypothetical efficient input vector (given output vector Y_i) for the ith producer by projecting the ith producer to the frontier Ψ^∂ through the input orientation. To simplify the notation, denote $Z_i = w_i^T X_i$ and $Z_i^\partial = w_i^T X_i^\partial$. Now, our interest is the ratio of the true means $\varphi = \mu_1/\mu_2$, where $\mu_1 = E(Z_i^\partial)$ and $\mu_2 = E(Z_i)$. If the true efficiency $\theta(X_i, Y_i \mid \Psi)$ is observed, by the classical CLT for the ratio of means, we have

$$\sqrt{n}\,(\varphi_n - \varphi) \xrightarrow{\mathcal{L}} N(0, \sigma_\varphi^2), \tag{2.23}$$

where

$$\sigma_\varphi^2 = \varphi^2 \left[\frac{\sigma_1^2}{\mu_1^2} + \frac{\sigma_2^2}{\mu_2^2} - 2\frac{\sigma_{12}}{\mu_1\mu_2}\right], \tag{2.24}$$

and where $\sigma_1^2 = \mathrm{Var}(Z_i^\partial)$, $\sigma_2^2 = \mathrm{Var}(Z_i)$, and $\sigma_{12} = \mathrm{Cov}(Z_i^\partial, Z_i)$.

However, $\theta(X_i, Y_i \mid \Psi)$ is not observed, making φ_n unobserved, and the best we can hope for is an estimate of $\theta(X_i, Y_i \mid \Psi)$ from a random sample of data $\mathcal{S}_n = \{(X_i, Y_i)\}_{i=1}^n$, e.g., using FDH, or VRS-DEA or CRS-DEA estimators as discussed above in Subsection 2.3, to obtain $\widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n)$. Then an estimate of φ_n can be obtained by plugging $\widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n)$ into (2.22), i.e., via

$$\widehat{\varphi}_n = \frac{\widehat{\mu}_{1,n}}{\widehat{\mu}_{2,n}} = \frac{n^{-1}\sum_{i=1}^{n} \widehat{Z}_i^\partial}{n^{-1}\sum_{i=1}^{n} Z_i}, \tag{2.25}$$

where $\widehat{Z}_i^{\partial} = w_i^T \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n) X_i$. Through extending the theory of Kneip et al. (2015), we can establish the CLT for $\widehat{\varphi}_n$ by providing an estimate of the bias and variance, analogous to Simar and Zelenyuk (2018). These results are summarized as follows.

Notice that $\widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n)$ is a biased estimator of $\theta(X_i, Y_i \mid \Psi)$. Thus, $\widehat{\varphi}_n$ is also a biased estimator of φ_n. The bias of $\widehat{\varphi}_n$ comes from the bias of the numerator $\widehat{\mu}_{1,n}$. However, similar to Kneip et al. (2015) and Simar and Zelenyuk (2018), the bias of $\widehat{\mu}_{1,n}$ can be consistently estimated using the generalized jackknife method. The procedures described by Kneip et al. (2015) are as follows. Split the sample into two evenly sized subsamples (for simplicity, assuming n is even) $\mathcal{S}_{n/2}^{(1)}$ and $\mathcal{S}_{n/2}^{(2)}$, so that $\mathcal{S}_{n/2}^{(1)} \cap \mathcal{S}_{n/2}^{(2)} = \emptyset$ and $\mathcal{S}_{n/2}^{(1)} \cup \mathcal{S}_{n/2}^{(2)} = \mathcal{S}_n$. We estimate the simple mean of Z_i^{∂} for each subsample, denoted as $\widehat{\mu}_{1,n,m}^{(l)}$ where $l = 1, 2$. More specifically,

$$\widehat{\mu}_{1,n,m}^{(l)} = 2n^{-1} \sum_{\{i \mid (X_i, Y_i) \in \mathcal{S}_{n/2}^{(l)}\}} \widehat{Z}_i^{\partial} = 2n^{-1} \sum_{\{i \mid (X_i, Y_i) \in \mathcal{S}_{n/2}^{(l)}\}} \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_{n/2}^{(l)}) Z_i. \tag{2.26}$$

Then compute

$$\widehat{\mu}_{1,n,m} = \frac{1}{2}(\widehat{\mu}_{1,n,m}^{(1)} + \widehat{\mu}_{1,n,m}^{(2)}). \tag{2.27}$$

Repeating the above process for M times, we end up with the estimate of the bias term for $\widehat{\mu}_{1,n}$ given by

$$\widehat{B}_{\mu_1,n} = \frac{1}{M} \sum_{m=1}^{M} (2^{\kappa} - 1)^{-1} (\widehat{\mu}_{1,n,m} - \widehat{\mu}_{1,n}). \tag{2.28}$$

Thus, the bias term for $\widehat{\varphi}_n$ is given by

$$\widehat{B}_{\varphi,n} = \frac{\widehat{B}_{\mu_1,n}}{\widehat{\mu}_{2,n}}. \tag{2.29}$$

After obtaining the estimate of the bias term for $\widehat{\varphi}_n$, we have the following CLT for the aggregate input-oriented technical efficiency.

Theorem 1. *Under the appropriate set of assumptions described in Theorem 3.1, 3.2 or 3.3 of Kneip et al. (2015), for $\kappa \geq 2/5$ for CRS-DEA or VRS-DEA cases, or for $\kappa \geq 1/3$ for FDH case, as $n \to \infty$, we have*

$$\sqrt{n} \left(\widehat{\varphi}_n - \widehat{B}_{\varphi,n} - \varphi + R_{n,\kappa} \right) \xrightarrow{\mathcal{L}} N(0, \sigma_{\varphi}^2), \tag{2.30}$$

and if $\kappa < 1/2$, we have

$$\sqrt{n_\kappa}\left(\widehat{\varphi}_{n_\kappa} - \widehat{B}_{\varphi,n} - \varphi + R_{n,\kappa}\right) \xrightarrow{\mathcal{L}} N(0, \sigma^2_\varphi), \tag{2.31}$$

where $R_{n,\kappa} = o(n^{-\kappa})$ and $\widehat{\varphi}_{n_\kappa}$ is a random subsample version, with size $n_\kappa = \lfloor n^{2\kappa} \rfloor < n$, of $\widehat{\varphi}_n$.[4] *Formally,*

$$\widehat{\varphi}_{n_\kappa} = \frac{n_\kappa^{-1} \sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} \widehat{\theta}(X_i, Y_i|\mathcal{S}_n) Z_i}{n_\kappa^{-1} \sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} Z_i}, \tag{2.32}$$

where $\mathcal{S}_{n_\kappa}$ is a random subsample, with the sample size n_κ, of $\mathcal{S}_n$.[5]

Remark 1. *Although the aggregate efficiency φ_n in (2.22) is related to the cost, we cannot use the trick of dimension reduction we have for cost efficiency in (2.4) as described in Lemma 3.2 in Simar and Wilson (2020). Hence we keep the usual convergence rates defined by κ for aggregate input-oriented efficiency.*

In the Remark 1 of Simar and Zelenyuk (2018), if $\kappa = 2/5$ for CRS-DEA or VRS-DEA cases, or $\kappa = 1/3$ for FDH case, both (2.30) and (2.31) are applicable. However (2.31) is preferred due to a smaller remainder term. Further, σ^2_φ is not observed and must be estimated. To estimate σ^2_φ, Simar and Zelenyuk (2018) suggest plugging the corresponding empirical estimates of these components in σ^2_φ, i.e.,

$$\widehat{\sigma}^2_{\varphi,n} = \widehat{\varphi}^2_n \left[\frac{\widehat{\sigma}^2_{1,n}}{\widehat{\mu}^2_{1,n}} + \frac{\widehat{\sigma}^2_{2,n}}{\widehat{\mu}^2_{2,n}} - 2\frac{\widehat{\sigma}_{12,n}}{\widehat{\mu}_{1,n}\widehat{\mu}_{2,n}}\right], \tag{2.33}$$

where $\widehat{\sigma}^2_{1,n} = \widehat{\text{Var}}(\widehat{Z}^\partial_i)$, $\widehat{\sigma}^2_{2,n} = \widehat{\text{Var}}(Z_i)$, and $\widehat{\sigma}_{12,n} = \widehat{\text{Cov}}(\widehat{Z}^\partial_i, Z_i)$.

3 Improving the Approximation

For the simple mean efficiency (Kneip et al., 2015) and aggregate efficiency (Simar and Zelenyuk, 2018), their extensive MC experiments found that when the sample sizes increase, the coverages of the estimated confidence intervals based on the corresponding CLT results approximate the nominal coverage, which supports the established theoretical results. However, these authors also found that the estimated confidence intervals often under-cover the

[4] $\lfloor n^{2\kappa} \rfloor$ denotes the largest integer that is less or equal to $n^{2\kappa}$.

[5] Note that $\widehat{\theta}(X_i, Y_i|\mathcal{S}_n)$ is computed relative to all the data points in $\mathcal{S}_n$ rather than $\mathcal{S}_{n_\kappa}$.

true values, especially for small sample sizes and high dimensions measured by the number of inputs and outputs specified. This under-covering phenomenon in relatively small sample sizes was also observed for the aggregate input-oriented efficiency with MC results presented in Section 4. Some of the improvements were made through Simar and Zelenyuk (2020), Nguyen et al. (2022), and Simar et al. (2023) for the simple mean output-oriented efficiency, and through Simar and Zelenyuk (2020) for the aggregate output-oriented efficiency. In this section, we will adapt these methods to further improve the finite sample performance of the CLT results for aggregate input-oriented efficiency.

3.1 Improvements from Simar and Zelenyuk (2020)

To further improve the finite sample performance of the CLT results for the simple mean and aggregate efficiency, Simar and Zelenyuk (2020) proposed a simple method to correct the used statistics by adding a squared bias of mean efficiency into the variance estimator, thus making the variance estimate slightly larger than for the case where no squared bias was added (i.e., the original methods in Kneip et al., 2015 for simple mean efficiency and Simar and Zelenyuk, 2018 for aggregate efficiency). The extensive MC experiments confirmed the improved performance of their variance correction method over the original methods in Kneip et al. (2015) for simple mean efficiency and in Simar and Zelenyuk (2018) for aggregate efficiency. It is worth noting that Simar and Zelenyuk (2020) focused on the output (or revenue) orientation context and left the input (or cost) orientation to the readers. *Inter alia*, we will fill this gap.

More specifically, for the aggregate input-oriented efficiency, following Simar and Zelenyuk (2020), we use a corrected version for the variance estimate,

$$\widetilde{\widehat{\sigma}}^2_{\varphi,n} = \widehat{\varphi}^2_n \left[\frac{\widetilde{\widehat{\sigma}}^2_{1,n}}{\widehat{\mu}^2_{1,n}} + \frac{\widehat{\sigma}^2_{2,n}}{\widehat{\mu}^2_{2,n}} - 2\frac{\widehat{\sigma}_{12,n}}{\widehat{\mu}_{1,n}\widehat{\mu}_{2,n}} \right], \tag{3.1}$$

where $\widetilde{\widehat{\sigma}}^2_{1,n} = \widehat{\sigma}^2_{1,n} + \widehat{B}^2_{\mu_1,n}$. That is, we add the estimated squared bias of mean efficiency for $\mu_{1,n}$ into its variance estimator $\widehat{\sigma}^2_{1,n}$, thus making the variance estimate slightly larger than for the case where no squared bias was added, i.e., the original method in (2.33).

3.2 Improvements from Nguyen et al. (2022)

Although the variance correction method in Simar and Zelenyuk (2020) increases the performance of the CLT results for finite sample sizes, it is also observed from the simulations that the coverages of the Simar and Zelenyuk (2020) method are often still smaller than the nominal coverages. Recently, Nguyen et al. (2022) proposed the data sharpening method to further improve finite sample approximation of CLT results for the simple mean output-oriented technical efficiency. This data sharpening method is inspired by Simar and Zelenyuk (2006) and Kneip et al. (2011). The two latter papers used the sharpening to derive simple consistent bootstrap approximations of distribution of efficiency scores.

The data sharpening method works by smoothing out the observations near the boundary, i.e., the so-called "spurious ones" and those with efficiency estimates close to one, where the magnitude of the closeness to one is determined based on the sample size and the convergence rate of the estimators. Nguyen et al. (2022) found that the Simar and Zelenyuk (2020) variance correction method combined with the data sharpening method results in a better performance compared to the initial approaches of Kneip et al. (2015) as well as Simar and Zelenyuk (2020). It is worth noting that Nguyen et al. (2022) only focused on the simple mean efficiency and did not investigate whether the data sharpening methods are also effective for the aggregate efficiency. *Inter alia*, we will also fill this gap.

Specifically, we adapt the idea of the data sharpening method in Nguyen et al. (2022) to the input-oriented technical efficiency as follows,

$$\widehat{\widehat{\theta}}(X_i, Y_i \mid \mathcal{S}_n) = \begin{cases} \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n), & \text{if } \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n) < 1 - \tau, \\ \\ \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n) \times \varepsilon_i, & \text{otherwise }, \end{cases} \tag{3.2}$$

where the sharpening parameter τ needs to be small enough, such that preserves the properties of the CLTs and provides robust and good performances in the MC experiments. Later, we will derive the theoretical range for τ. Further, ε_i is a random independent number drawn from a uniform distribution on the interval $[1-\tau, 1]$. It can be shown that $\widehat{\widehat{\theta}}(X_i, Y_i \mid \mathcal{S}_n) = \widehat{\theta}(\widetilde{X}_i, Y_i \mid \mathcal{S}_n)$, where

$$\widetilde{X}_i = \begin{cases} X_i, & \text{if } \widehat{\theta}(X_i, Y_i \mid \mathcal{S}_n) < 1 - \tau, \\ \\ X_i/\varepsilon_i, & \text{otherwise }. \end{cases} \tag{3.3}$$

Then the regular nonparametric efficiency estimators are applied for the sharpened sample points $\{(\widetilde{X}_i, Y_i)\}_{i=1}^n$, but with the reference set being the original sample $\mathcal{S}_n = \{(X_i, Y_i)\}_{i=1}^n$. It is worth noting that the data sharpening method will have an effect on the estimates of efficiency and on its bias and variance, yet it does not have an effect on other quantities, such as $\widehat{\mu}_2$ and $\widehat{\sigma}^2_{2,n}$.

Now, we will formally derive the theoretical range for τ that preserves the properties of the CLTs. What we need is to keep τ small enough so that the CLTs expressed in (2.30) and (2.31) are still valid for the sharpened estimates. For notational simplicity, in what follows, we denote $\widehat{\theta}_i = \widehat{\theta}(X_i, Y_i|\mathcal{S}_n)$ and $\widehat{\widehat{\theta}}_i = \widehat{\widehat{\theta}}(X_i, Y_i|\mathcal{S}_n)$. Our data sharpening method in (3.2) then can be expressed as,

$$\widehat{\widehat{\theta}}_i = \begin{cases} \widehat{\theta}_i & \text{if } \widehat{\theta}_i < 1-\tau, \\ \widehat{\theta}_i \varepsilon_i & \text{if } 1-\tau \le \widehat{\theta}_i \le 1, \end{cases} \tag{3.4}$$

where $\varepsilon_i = 1 - U_i$ and $U_i \sim \text{Unif}(0, \tau)$ is independent of $\widehat{\theta}_i$, so that $\text{Prob}(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i \ge 0) = 1$. The consequence of this is that

$$\widehat{\varphi}_n = \frac{n^{-1}\sum_{i=1}^n w_i^T \widehat{\theta}_i X_i}{n^{-1}\sum_{i=1}^n w_i^T X_i}, \tag{3.5}$$

$$\widehat{\varphi}_{n_\kappa} = \frac{n_\kappa^{-1}\sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} w_i^T \widehat{\theta}_i X_i}{n_\kappa^{-1}\sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} w_i^T X_i}, \tag{3.6}$$

in (2.30) and (2.31) will be respectively replaced by

$$\widehat{\widehat{\varphi}}_n = \frac{n^{-1}\sum_{i=1}^n w_i^T \widehat{\widehat{\theta}}_i X_i}{n^{-1}\sum_{i=1}^n w_i^T X_i}, \tag{3.7}$$

$$\widehat{\widehat{\varphi}}_{n_\kappa} = \frac{n_\kappa^{-1}\sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} w_i^T \widehat{\widehat{\theta}}_i X_i}{n_\kappa^{-1}\sum_{\{i|(X_i,Y_i)\in\mathcal{S}_{n_\kappa}\}} w_i^T X_i}. \tag{3.8}$$

So, to keep the CLTs in (2.30) and (2.31), we need in all the cases that $(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i) = o_p\big(n^{-\tilde{\kappa}}\big)$ where $\tilde{\kappa} = \min(\kappa, 1/2)$.

Now by Markov Inequality, we have that for all $a > 0$,

$$\text{Prob}\big(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i \ge a\big) \le \frac{\text{E}(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i)}{a}. \tag{3.9}$$

Since $\widehat{\theta}_i - \widehat{\widehat{\theta}}_i = \widehat{\theta}_i U_i \mathbb{1}(\widehat{\theta}_i \geq 1-\tau)$ where $\mathbb{1}(.)$ is the indicator function and given that U_i is independent of $\widehat{\widehat{\theta}}_i$, we have

$$\mathrm{E}\big(\widehat{\theta}_i U_i \mathbb{1}(\widehat{\theta}_i \geq 1-\tau)\big) = \mathrm{E}\big(\widehat{\theta}_i \mathbb{1}(\widehat{\theta}_i \geq 1-\tau)\big)\mathrm{E}(U_i) \tag{3.10}$$

$$= \frac{\tau}{2}\int_{1-\tau}^{1} \widehat{\theta} f(\widehat{\theta}) d\widehat{\theta} \tag{3.11}$$

$$= \frac{\tau}{2}\tau\bar{\theta} f(\bar{\theta}), \tag{3.12}$$

where $f(\widehat{\theta})$ is the density of $\widehat{\theta}_i$ and $\bar{\theta}$ is, by the Mean Value Theorem, some number in the interval $(1-\tau, 1)$. So we end up with

$$\mathrm{E}(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i) = c_\theta \tau^2, \tag{3.13}$$

for some finite $c_\theta > 0$. So by the Markov inequality in (3.9), we have for all $a > 0$,

$$\mathrm{Prob}\big(n^{\tilde{\kappa}}(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i) \geq a\big) \leq \frac{c_\theta \tau^2}{a n^{-\tilde{\kappa}}}. \tag{3.14}$$

It is clear that if $\tau = n^{-\gamma}$ for $\gamma > \tilde{\kappa}/2$ this probability converges to zero as $n \to \infty$, so that $(\widehat{\theta}_i - \widehat{\widehat{\theta}}_i) = o_p\big(n^{-\tilde{\kappa}}\big)$, as required.[6] Therefore, to keep the CLTs properties for aggregate input-oriented efficiency, we only need $\tau = n^{-\gamma}$ and $\gamma > \tilde{\kappa}/2$.

3.3 Our Proposed Method

More recently, Simar et al. (2023) proposed using the bias-corrected individual efficiency estimates instead of the original individual efficiency estimates, to obtain the variance estimator for the simple mean efficiency. The simulation results in Simar et al. (2023) showed the effectiveness of their proposed method on improving the finite sample approximation of CLT results for the simple mean efficiency. They also provided a theoretical justification. However, similar to Nguyen et al. (2022), Simar et al. (2023) only focused on the simple mean efficiency and did not investigate whether their methods are also useful for the aggregate efficiency. *Inter alia*, we will also fill this gap.

We propose adapting the method in Simar et al. (2023) to the case of aggregate efficiency (with a focus on input-oriented technical efficiency). Specifically, for the original variance

[6] Nguyen et al. (2022) use the same property for deriving a lower bound to γ, but without a formal proof using only an intuitive (and correct) argument. Moreover, the upper bound for γ is generally not needed for our purposes of data sharpening in the context of approximation of the CLTs.

estimator of aggregate efficiency in (2.33), we propose replacing $\widehat{\theta}(X_i, Y_i|\mathcal{S}_n)$ by $\widetilde{\theta}(X_i, Y_i|\mathcal{S}_n) = \widehat{\theta}(X_i, Y_i|\mathcal{S}_n) - \widehat{B}_i$ at every place (the relevant quantities are $\widehat{\varphi}_n$, $\widehat{\sigma}^2_{1,n}$, $\widehat{\mu}_{1,n}$ and $\widehat{\sigma}_{12,n}$), where $\widehat{B}_i$ is the estimated individual bias using the generalized jackknife method of Kneip et al. (2015). That is, we estimate σ^2_φ as follows:

$$\widetilde{\sigma}^2_{\varphi,n} = \widetilde{\varphi}^2_n \left[\frac{\widetilde{\sigma}^2_{1,n}}{\widetilde{\mu}^2_1} + \frac{\widehat{\sigma}^2_{2,n}}{\widehat{\mu}^2_{2,n}} - 2\frac{\widetilde{\sigma}_{12,n}}{\widetilde{\mu}_1 \widehat{\mu}_{2,n}} \right], \tag{3.15}$$

where

$$\widetilde{\varphi}_n = \widehat{\varphi}_n - \widehat{B}_{\varphi,n}, \tag{3.16}$$

$$\widetilde{\mu}_1 = \widehat{\mu}_{1,n} - \widehat{B}_{\mu_1,n}, \tag{3.17}$$

$$\widetilde{\sigma}^2_{1,n} = \widehat{\text{Var}}(\widetilde{\theta}(X_i, Y_i|\mathcal{S}_n)Z_i), \tag{3.18}$$

and

$$\widetilde{\sigma}_{12,n} = \widehat{\text{Cov}}(\widetilde{\theta}(X_i, Y_i|\mathcal{S}_n)Z_i, Z_i). \tag{3.19}$$

Note that, comparing (3.15) with (2.33), we not only use the bias-corrected individual efficiency to obtain estimates for σ^2_1 but also for φ, μ_1 and σ_{12}. Our proposed method here is different from the simple mean case in Simar et al. (2023), as the variance of aggregate efficiency is more complicated, involving several moments and the covariance term in a nonlinear relationship.

Compared with (2.33) and (3.1), (3.15) has the potential to improve, since (3.15) replaces the $\widehat{\theta}(X_i, Y_i|\mathcal{S}_n)$ in (2.33) by its bias-corrected estimate $\widetilde{\theta}(X_i, Y_i|\mathcal{S}_n)$ at every relevant place. To see this, we may view σ^2_φ as a function of only μ_1, σ^2_1 and σ_{12} since the replacement only has effects on these three terms in the variance estimator. Hence, the Taylor expansion of $\widehat{\sigma}^2_{\varphi,n}$ in (2.33) around σ^2_φ is given by

$$\widehat{\sigma}^2_{\varphi,n} = \sigma^2_\varphi + A_1(\widehat{\mu}_1 - \mu_1) + A_2(\widehat{\sigma}^2_1 - \sigma^2_1) + A_3(\widehat{\sigma}_{12} - \sigma_{12}) + R_n, \tag{3.20}$$

while the Taylor expansion of our proposed estimator $\widetilde{\sigma}^2_{\varphi,n}$ in (3.15) around σ^2_φ is given by

$$\widetilde{\sigma}^2_{\varphi,n} = \sigma^2_\varphi + A_1(\widetilde{\mu}_1 - \mu_1) + A_2(\widetilde{\sigma}^2_{1,n} - \sigma^2_1) + A_3(\widetilde{\sigma}_{12,n} - \sigma_{12}) + R_n, \tag{3.21}$$

where $A_1 = \frac{2\mu_1\sigma^2_2}{\mu^4_2} - \frac{2\sigma_{12}}{\mu^3_2}$, $A_2 = \frac{1}{\mu^2_2}$, and $A_3 = -\frac{2\mu_1}{\mu^3_2}$, while R_n is the remainder of a smaller order, i.e., $o(n^{-\kappa})$, and can be ignored for our purposes here.

According to Lemma 1 in Simar and Zelenyuk (2018), we have $\widehat{\sigma}_1^2 - \sigma_1^2 = o(n^{-\kappa/2})$, $\widehat{\sigma}_{12} - \sigma_{12} = o(n^{-\kappa/2})$, $\widehat{\mu}_1 - \mu_1 = O(n^{-\kappa})$. Thus, the first order error in (3.20) is given by

$$A_2(\widehat{\sigma}_1^2 - \sigma_1^2) + A_3(\widehat{\sigma}_{12} - \sigma_{12}) = o(n^{-\kappa/2}), \tag{3.22}$$

while the second order error is given by

$$A_1(\widehat{\mu}_1 - \mu_1) = O(n^{-\kappa}). \tag{3.23}$$

Similarly, for (3.21), we also have $\widetilde{\sigma}_1^2 - \sigma_1^2 = o(n^{-\kappa/2})$ and $\widetilde{\sigma}_{12} - \sigma_{12} = o(n^{-\kappa/2})$, but $\widetilde{\mu}_1 - \mu_1 = o(n^{-\kappa})$. Thus, the first order error in (3.21) is given by

$$A_2(\widetilde{\sigma}_{1,n}^2 - \sigma_1^2) + A_3(\widetilde{\sigma}_{12,n} - \sigma_{12}) = o(n^{-\kappa/2}), \tag{3.24}$$

while the second order error is given by

$$A_1(\widehat{\mu}_1 - \mu_1) = o(n^{-\kappa}). \tag{3.25}$$

That is, while using the bias-corrected individual efficiency estimate does not reduce the first order error (it is still of the order $o(n^{-\kappa/2})$), it turns out we are able to reduce the second order term from $O(n^{-\kappa})$ to $o(n^{-\kappa})$, which is an improvement.[7]

How important is this reduction in the theoretical order from $O(n^{-\kappa})$ to $o(n^{-\kappa})$ in practice? By and large, it depends on the actual sample: the actual data generating process it came from, its size and, as usual, the luck of the particular draw. The only way to sense it is to investigate it in various scenarios, as we do in the next section. We will see that, although (3.20) and (3.21) are asymptotically equivalent, the proposed method can be a substantial improvement for relatively small samples. Indeed, it is possible that for a particular sample the second order error $O(n^{-\kappa})$ is substantially larger than the first order error $o(n^{-\kappa/2})$, i.e., similarly as was pointed out and confirmed in simulations for a simpler case of the simple mean efficiency by Simar et al. (2023), and as we can see it in our simulations for the aggregate efficiency reported below.

[7] This theoretical justification is similar to that in Simar et al. (2023) and is adapted to the more complicated case of the weighted efficiency.

4 Monte-Carlo Evidence

4.1 Details on Monte-Carlo Simulations

Our Monte-Carlo experiments follow a similar strategy and framework as that found in Simar and Zelenyuk (2020). Formally,

$$y_i = \prod_{j=1}^{p} (x_{ji}^{\partial})^{\beta_j}, \tag{4.1}$$

where $1 \times p$ vector $x_i^{\partial} = (x_{1i}^{\partial}, x_{2i}^{\partial}, \ldots, x_{pi}^{\partial})$ and $x_{ji}^{\partial} \overset{\text{iid}}{\sim} \text{Unif}(0,1), \forall j \in 1, \ldots, p$. In addition, the ith true input-oriented efficiency score is generated from $\theta_i = 1/\lambda_i$, where $\lambda_i \sim |N(0, 0.42)| + 1$, i.e., we generate λ_i from the half normal distribution and then shift it up by 1 so that $\lambda_i \geq 1$. Upon obtaining the frontier points $\{(x_i^{\partial}, y_i)\}_{i=1}^n$, we then set the ith observed input as $x_i = x_i^{\partial}/\theta_i$, i.e., by projecting (x_i^{∂}, y_i) from the frontier into the production set (x_i, y_i). Thus, a simulated sample $\{(x_i, y_i)\}_{i=1}^n$ is created.[8] For each experiment, represented as one combination of (p, q, n), we perform $1,000$ trials of MC. For each trial, we check whether the estimated confidence intervals cover the true aggregate input efficiency. We then report the average of each trial's coverage over $1,000$ trials.

Table A.1 in the subsection A.1 of the separate Appendix A lists the values of β_j's and w_j's for each dimension p, where the β_j's are identical as in Simar and Zelenyuk (2020) and the values of w_j's are randomly chosen by the authors. The true aggregate input efficiency is obtained through simulating $10,000,000$ random realizations (x_i^{∂}, x_i) and then computing it using (2.22).

4.2 Simplified Nomenclature

In the simulations, we are going to thoroughly investigate the performance of our proposed method compared to the initial method in Simar and Zelenyuk (2018) and the variance correction method in Simar and Zelenyuk (2020). Further, we will also compare the above three methods for the cases—with and without the data sharpening method in Nguyen et al. (2022). Before presenting our MC results, to simplify the notation, we denote:

- Sol1: the original method in Simar and Zelenyuk (2018), i.e., the method in (2.33).

[8] Although not critical, the scenarios are calibrated so that the average efficiency is around 0.75, which is similar to the values from the empirical research.

- Sol2: the variance correction method in Simar and Zelenyuk (2020), i.e., the method in (3.1).
- Sol3: our proposed variance correction method, i.e., the method in (3.15).
- Sol4: Sol1 combined with the data sharpening in (3.3).
- Sol5: Sol2 combined with the data sharpening in (3.3).
- Sol6: Sol3 combined with the data sharpening in (3.3).

4.3 Monte-Carlo Results

4.3.1 General Remarks

As discussed earlier, to maintain the consistency of the density estimator after the data sharpening, the values of γ need to be no less than $\tilde{\kappa}/2$ where $\tilde{\kappa} = \min(\kappa, 1/2)$. In our simulations, we first consider various values of γ from 0.55κ, 0.65κ, 0.75κ, 0.85κ, 0.95κ, κ, 1.2κ and 1.5κ and then select the γ with the best performance. Interestingly, and unlike in Nguyen et al. (2022), for our context of input orientation we find that the level of γ near κ seems to yield the best performance. Consequently, in the following section we report the simulation results with $\gamma = \kappa$. The empirical coverages from the simulation results for 0.55κ, 0.65κ, 0.75κ, 0.85κ, 0.95κ, 1.2κ and 1.5κ are reported in the subsection of A.2 in the separate Appendix A.

4.3.2 Main Results

Figure 1 reports the empirical coverages for the aggregate input-oriented technical efficiency across dimensions and across finite sample sizes for Sol1–Sol6 when the nominal coverage is 0.95. The Monte Carlo simulations from Figure 1 confirm the results from Simar et al. (2023): across different sample sizes and across different dimensions, in increasing order of performance are Sol1, Sol2, Sol3 for the cases without data sharpening; and in increasing order of performance are Sol4, Sol5, Sol6 for the cases with data sharpening.[9] For instance,

[9] For the values of the empirical coverages, see Table A.2 in the separate Appendix A. Further, when $\kappa = 2/5$ for VRS-DEA, Simar and Zelenyuk (2018) finds that both Theorem 2 and Theorem 3 in Simar and Zelenyuk (2018) are applicable, while Theorem 3 is preferred. Thus, for $p = 3$ and $q = 1$, we only include the results from the preferred method.

when $n = 100$, $p = 4$, $q = 1$, the empirical coverage under Sol1, Sol2, and Sol3 is 0.487, 0.612, and 0.874, respectively, and under Sol4, Sol5, and Sol6 it is 0.772, 0.902, and 0.992, respectively. These results confirm that the proposed method in (3.15) further improves upon previous approaches for the approximation of the CLTs from Simar and Zelenyuk (2018) and their input oriented versions which we have presented here. In other words, for relatively small sample sizes, it is indeed useful to utilize the bias-corrected individual efficiency estimates (instead of the individual efficiency estimates) at every place they appear in the variance estimator of the aggregate efficiency.

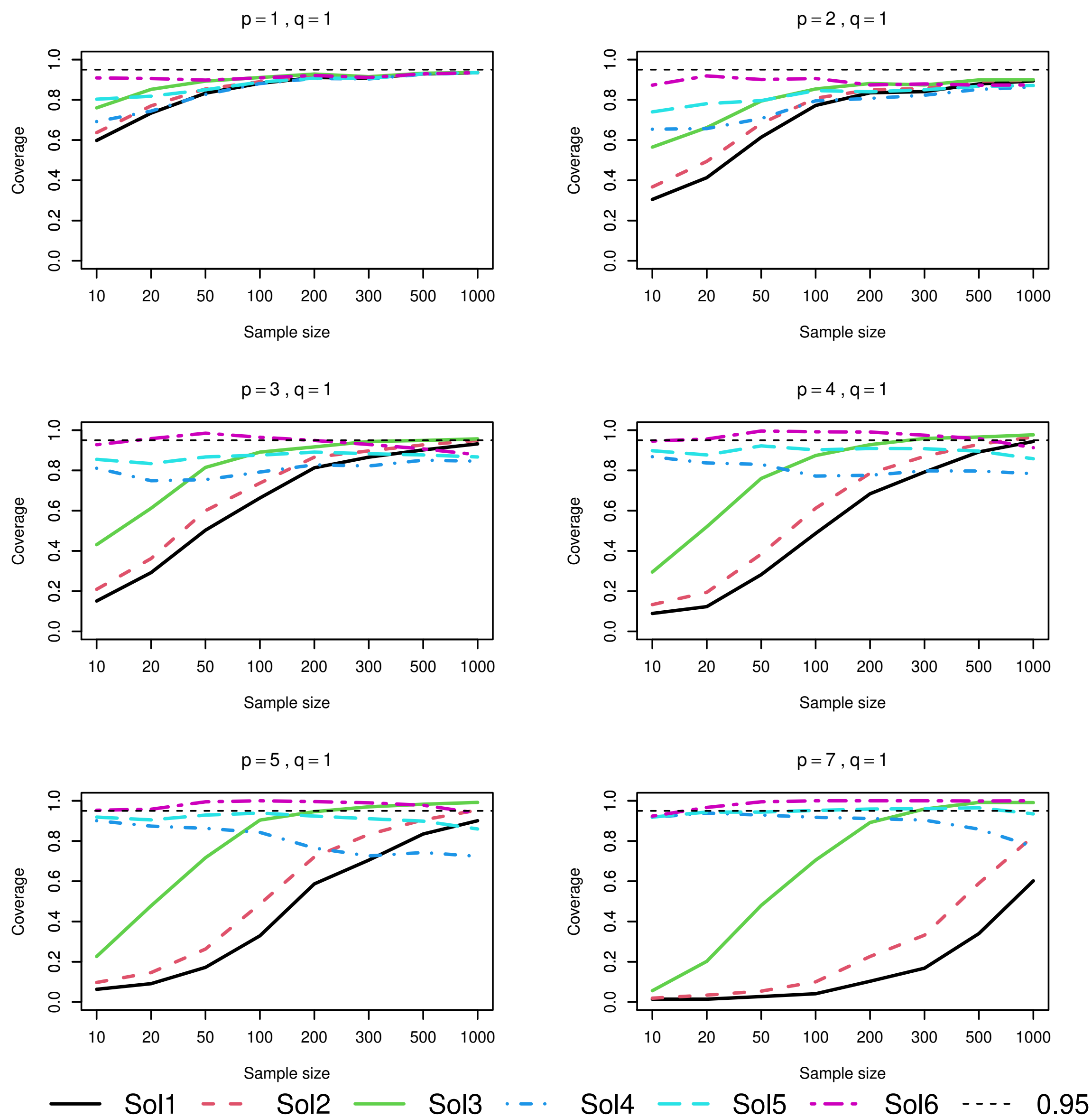
Comparing the results without and with the data sharpening (i.e., comparing Sol1 vs. Sol4, Sol2 vs. Sol5, Sol3 vs. Sol6), we see that the data sharpening methods (Sol4, Sol5 and Sol6), in general, have better performance when the sample size is smaller than 100. Further, when the sample size is smaller than 200, the improvements of Sol6 are even more substantial and persistent relative to all the other five methods. For instance, when $n = 200$, $p = 5$, $q = 1$, the empirical coverage under Sol1, Sol2, Sol3, Sol4, Sol5, and Sol6 is 0.587, 0.721, 0.946, 0.765, 0.924, and 0.996 respectively. The empirical coverage under Sol6 is 0.996, i.e., higher than all the other five methods and higher than the nominal coverage of 0.95.

However, it is also important to note that when the sample size is larger than 200, the data sharpening methods in this context of input-oriented aggregate efficiency may perform worse than the corresponding methods without data sharpening methods, implying that the improvements of data sharpening methods might be limited to relatively small sample sizes (e.g., up to about 200 for the dimensions we have considered here).

We also try various values of the tuning constant γ, where those results are reported in the subsection A.2 of the separate Appendix A. We find that the level of γ near κ seems to yield the best performance for our context (input-oriented aggregate efficiency).[10] This finding is different from Nguyen et al. (2022) who found the level of γ near 0.75κ seems to yield the best performance in a different though similar context (about the simple mean output-oriented efficiency). This suggests that the choice of the tuning constant seems to play a more important role than we had thought earlier, and hence this topic (selection of γ) may constitute a fruitful avenue for future research.

[10] We also present simulation results for the aggregate output-oriented efficiency in the Appendix B, which also shows that the level of $\gamma = \kappa$ seems to yield better performance.

Figure 1: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = \kappa$



Furthermore, it is also important to note that Sol6 may slightly overshoot sometimes (e.g, see Figure 1), especially when the dimensions are high. For instance, when $n = 100$, $p = 7$ and $q = 1$, the empirical coverage under Sol6 is 1.000 when the nominal coverage is 0.95 (see Figure 1). Such overshooting phenomenon is not unique for Sol6, and is also observed for the other methods. Importantly, and as pointed out by Simar et al. (2023) who observed similar phenomena in their simulations, the error coming from such overshooting might be more preferable than the error from undershooting (provided that the length of the confidence intervals is similar). This is because the overshooting means a more conservative approach (i.e., *not rejecting* the *correct*).

To conclude for this section, our general conclusion is that: when the sample sizes are relatively small (around 200 or less for the dimensions we have considered here), our proposed method combined with the data sharpening method (Sol6) provides a better performance; for the larger sample sizes, our proposed method without the data sharpening method (Sol3) provides a better or the same performance as other suitable methods known to date.

5 Empirical Illustrations

In this section, we use two real data sets to illustrate the differences between the six methods (Sol1–Sol6) discussed above in the estimates of variance as well as the confidence intervals for the aggregate efficiency. To provide different perspectives of the usefulness of our proposed method, we use VRS-DEA estimators in the input orientation for one data set in section 5.1 while using CRS-DEA estimators in the output orientation for another data set in section 5.2.

5.1 Inference for Aggregate Efficiency with Philippine Rice Data

The first illustration uses the same data set as Simar and Zelenyuk (2020) and Nguyen et al. (2022). This data set is a balanced panel data set, consisting of 43 rice producers in the Tarlac region of the Philippines over 1990–1997.[11] The rice producers are modeled to use three inputs (area planted x_1, labor used x_2 and fertilizers x_3) and their prices (w_1, w_2 and

[11] This data set was compiled by International Rice Research Institute and widely used in Coelli et al. (2005) for various illustrations of efficiency and productivity estimations. The data are available at `http://www.uq.edu.au/economics/cepa/crob2005/software/CROB2005.zip`.

w_3) to produce one output (rice y).

Table 1: Summary Statistics for the Pooled Data for Rice Producers

Variable	Min	Q1	Median	Mean	Q3	Max
y	0.090	2.790	5.110	6.540	8.920	31.100
x_1	0.200	1.000	1.750	2.144	3.400	7.000
x_2	8.000	49.750	87.500	108.342	152.000	437.000
x_3	10.000	67.725	142.200	189.235	254.350	1030.900
w_1	313.868	2905.218	4666.360	5289.332	6599.473	27787.784
w_2	22.960	53.268	68.856	70.767	84.716	219.556
w_3	6.903	12.832	14.078	14.885	16.190	32.396
cost	832.867	8815.604	16730.395	20871.644	27882.586	113983.620

Table 1 summarizes the variables used in the estimation for the pooled data, where $cost = w_1x_1 + w_2x_2 + w_3x_3$. Same as Simar and Zelenyuk (2020), we first obtain the results using only 43 observations for each year separately and then using all the 344 observations for the pooled data. Table 2 summarizes the results using VRS-DEA estimators in the input orientation. For Panel B with data sharpening methods, we let $\gamma = \kappa$ as indicated from MC simulations in the previous Section, and $\kappa = n^{-2/5}$ as we use VRS-DEA estimators and $p = 3$, $q = 1$.

First, for Panel A without data sharpening, for each year, in increasing width of confidence intervals (CIs) lie Sol1, Sol2 and Sol3; this result is due to the fact that, while Sol1–Sol3 yield the same aggregate efficiency estimates, the estimates of σ_φ are typically the smallest for Sol1 and the largest for Sol3, with Sol2 being between these two. For Panel B with data sharpening, for each year, in increasing width of CI lie Sol4, Sol5 and Sol6. Again, this result is due to the fact that, while Sol4–Sol6 yield the same aggregate efficiency estimates, the estimates of σ_φ are usually the smallest for Sol4 and the largest for Sol6. These results are consistent with the findings in the MC simulations that our proposed method yields slightly larger coverages than the previous methods, with and without the data sharpening method, suggesting a potential improvement of our proposed method for the finite sample sizes.

Second, the addition of the data sharpening method has two effects: (1) decreasing the estimates of aggregate efficiency; (2) decreasing the estimates of σ_φ. Due to the effect (i),

Table 2: VRS-DEA Estimates of Aggregate Input-Oriented Efficiency, Standard Deviations, and the 99% Confidence Intervals for Rice Producers in the Philippines

Panel A: Without Data Sharpening

				$\widehat{\sigma}_{\varphi}$			99% CI					
Year	$\widehat{\varphi}_{n_{\kappa}}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_{\kappa}} - \widehat{B}_{\varphi,n}$	Sol1	Sol2	Sol3	Sol1		Sol2		Sol3	
1990	0.8013	0.1708	0.6304	0.2124	0.2726	0.3476	0.5081	0.7528	0.4734	0.7874	0.4302	0.8307
1991	0.8157	0.1099	0.7058	0.2201	0.2460	0.3117	0.5791	0.8326	0.5641	0.8475	0.5263	0.8854
1992	0.8944	0.1034	0.7910	0.1317	0.1674	0.2586	0.7151	0.8668	0.6945	0.8874	0.6420	0.9399
1993	0.8586	0.0986	0.7600	0.2070	0.2293	0.2887	0.6408	0.8793	0.6280	0.8921	0.5938	0.9263
1994	0.8243	0.1751	0.6492	0.2663	0.3187	0.4500	0.4958	0.8026	0.4656	0.8327	0.3900	0.9083
1995	0.8665	0.1215	0.7451	0.2163	0.2481	0.3442	0.6204	0.8697	0.6021	0.8880	0.5468	0.9433
1996	0.8297	0.1273	0.7024	0.2466	0.2776	0.3540	0.5604	0.8445	0.5426	0.8623	0.4985	0.9063
1997	0.8166	0.2387	0.5779	0.2281	0.3302	0.4561	0.4465	0.7093	0.3877	0.7681	0.3152	0.8406
Pooled	0.6152	0.1867	0.4285	0.2654	0.3245	0.3445	0.3621	0.4949	0.3473	0.5097	0.3423	0.5147

Panel B: With Data Sharpening

				$\widehat{\sigma}_{\varphi}$			99% CI					
Year	$\widehat{\varphi}_{n_{\kappa}}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_{\kappa}} - \widehat{B}_{\varphi,n}$	Sol4	Sol5	Sol6	Sol4		Sol5		Sol6	
1990	0.7747	0.1686	0.6061	0.1861	0.2511	0.3191	0.4989	0.7133	0.4615	0.7507	0.4223	0.7899
1991	0.7349	0.1076	0.6274	0.1696	0.2009	0.2456	0.5296	0.7251	0.5117	0.7430	0.4859	0.7688
1992	0.8057	0.0955	0.7102	0.1164	0.1505	0.2258	0.6432	0.7773	0.6235	0.7970	0.5802	0.8403
1993	0.7637	0.0926	0.6711	0.1607	0.1855	0.2327	0.5785	0.7636	0.5642	0.7779	0.5370	0.8051
1994	0.7400	0.1704	0.5695	0.1971	0.2606	0.3645	0.4560	0.6831	0.4195	0.7196	0.3596	0.7795
1995	0.7913	0.1173	0.6741	0.1616	0.1997	0.2856	0.5810	0.7672	0.5590	0.7891	0.5095	0.8386
1996	0.7521	0.1204	0.6317	0.2000	0.2335	0.3043	0.5165	0.7469	0.4972	0.7662	0.4564	0.8069
1997	0.7537	0.2339	0.5198	0.1583	0.2824	0.3757	0.4287	0.6110	0.3572	0.6825	0.3034	0.7362
Pooled	0.6135	0.1866	0.4270	0.2521	0.3136	0.3290	0.3639	0.4900	0.3485	0.5054	0.3447	0.5093

the centering of the estimated CI with the data sharpening method will be slightly further away from 1 (toward 0) than the corresponding case without the data sharpening method, as the data sharpening method "smoothes out" the observations with estimated efficiency that equal 1 and within the neighborhood of 1. Meanwhile, due to the effect (ii), the data sharpening method yields a narrower CI than the corresponding case without the data sharpening method. Note that a narrower CI for the data sharpening method does not mean it will perform worse than the corresponding case without the data sharpening method, since the data sharpening method might have a more accurate estimate for the aggregate efficiency by smoothing out the spurious observations. In other words, the data sharpening method might have a more accurate estimate for the centering of CIs. This is mainly confirmed in our previous Monte Carlo evidence, where we find that: Sol6 persistently gave better coverage when the sample sizes are relatively small (up to 200), meanwhile when the sample size was relatively large (> 200), Sol3 usually performed better.

All in all, combining the above first and second discussions, for this specific empirical data set, we anticipate that Sol6 is likely to be more accurate for each year (with the sample size 43), while for the pooled sample (with the sample size 344), Sol3 is likely to be more accurate. And indeed our results show that for the pooled sample the CI for Sol3 ($[0.3423, 0.5147]$) is expected to have a better coverage than that for Sol6 ($[0.3447, 0.5093]$) as the CI of Sol6 is contained by the CI of Sol3. Although, and importantly, note that in the latter case, all the methods give very similar estimates of the CIs, supporting the theory that says they are all asymptotically equivalent. Meanwhile, the difference is fairly substantial among the methods for the individual years where the samples are relatively small, emphasizing the importance to select a method that promises higher accuracy. Thus, based on our discussions in the theoretical and simulation sections, we would recommend relying more on Sol6 for these relatively small samples.

5.2 Inference for Aggregate Efficiency with Penn World Table Data

Our second illustration uses the publicly available data from Penn World Table (PWT 10.0, Feenstra et al., 2015), which provides information on relative levels of aggregate input, output, income, etc, for many countries across the world from 1950 to 2019.[12] Following Färe et al. (1994), Kumar and Russell (2002), and Badunenko et al. (2008), we model the production of countries using labor and capital stock to produce GDP. The labor is measured using the number of persons engaged (in millions), i.e., the variable *emp* in PWT. The capital is measured using capital stock at current PPPs (in millions 2017 US$), i.e., the variable *cn* in PWT. The GDP is measured by output-side real GDP at current PPPs (in millions 2017 US$), i.e., the variable *cgdpo* in PWT. The output price is the same for all countries. To illustrate the evolution of the aggregate inefficiency over the latest 30 years (from 1990 to 2019), we will use the same sub-set of 84 countries/regions that was considered in Badunenko et al. (2008) and Pham et al. (2023).[13]

In previous DEA studies that utilized PWT data, output orientation is usually deployed (Färe et al., 1994; Ray and Desli, 1997; Kumar and Russell, 2002; and Badunenko et al., 2008) and so we focus on this case here. We use CRS-DEA estimators and $p = 2$, $q = 1$, thus $\kappa = n^{-2/3}$. When using data sharpening methods, we let $\gamma = \kappa$.[14] We obtain the results using only 84 observations for each year separately from 1990 to 2019 as the technology might have changed over time. Further, we focus on aggregate inefficiency while also presenting the results for simple mean inefficiency to compare.

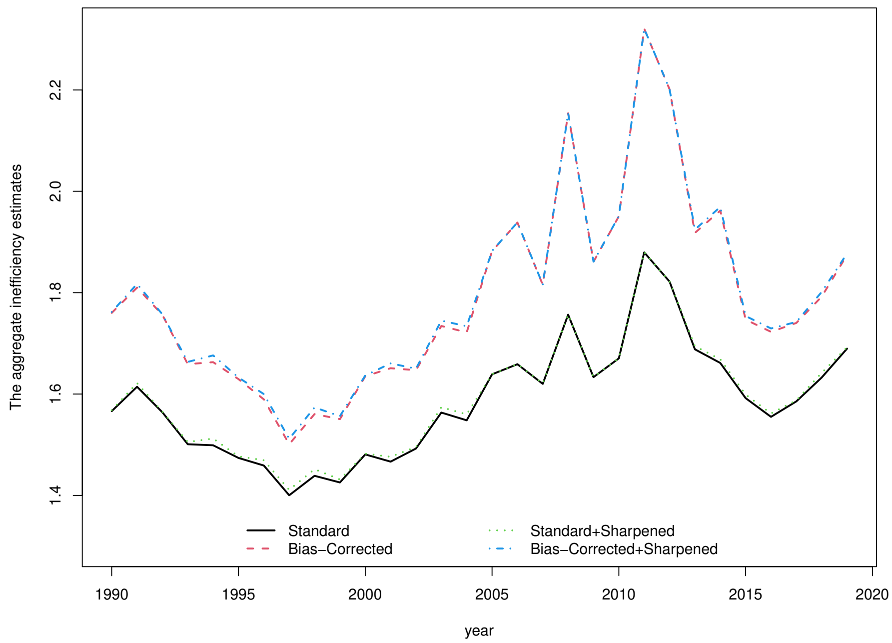
Figure 2a presents the dynamic changes of the aggregate output-oriented inefficiency over 1990–2019.[15] As expected, the "Standard" (i.e., the estimated aggregate inefficiency without bias correction) is substantially below the "Bias-Corrected" (i.e., the estimated aggregate inefficiency with bias correction). This is also true for the case using the data sharpening method, where the "Standard+Sharpened" (i.e., the estimated aggregate inefficiency without bias correction and with the data sharpening method) is below the "Bias-Corrected+Sharpened" (i.e., the estimated aggregate inefficiency with bias correction and with the data sharpening method). Comparing the cases with and without the data sharpening method, it is also expected that the "Standard" is below the "Standard+Sharpened" and

[12] The data are available at https://www.rug.nl/ggdc/productivity/pwt/?lang=en.

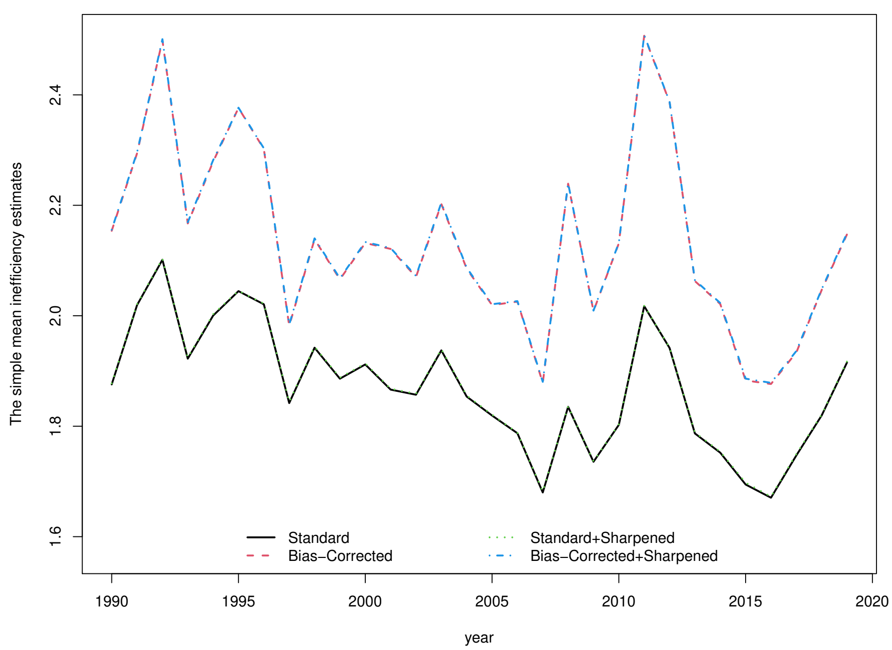
[13] The countries/regions included in our sample are listed in the separate appendix C.

[14] We also consider the case with $\gamma = 0.75\kappa$ as indicated from Simar et al. (2023) for the simple mean output-oriented inefficiency. These results are presented in the subsection C.2 of the separate appendix C, which are similar to those for $\gamma = \kappa$.

[15] Tables C.1 and C.2 in the separate appendix C present more details on the results of our estimations for aggregate inefficiency and simple mean inefficiency, respectively.



(a) For the Aggregate Output-Oriented Inefficiency



(b) For the Simple Mean Output-Oriented Inefficiency

Figure 2: The Estimated Aggregate/Simple Mean Output-Oriented Inefficiency for Countries/Regions from 1990 to 2019 when $\gamma = \kappa$

that the "Bias-Corrected" is below the "Bias-Corrected+Sharpened", as the data sharpening method "smoothes out" the observations that are spuriously equal to 1 and those within the neighborhood of 1.

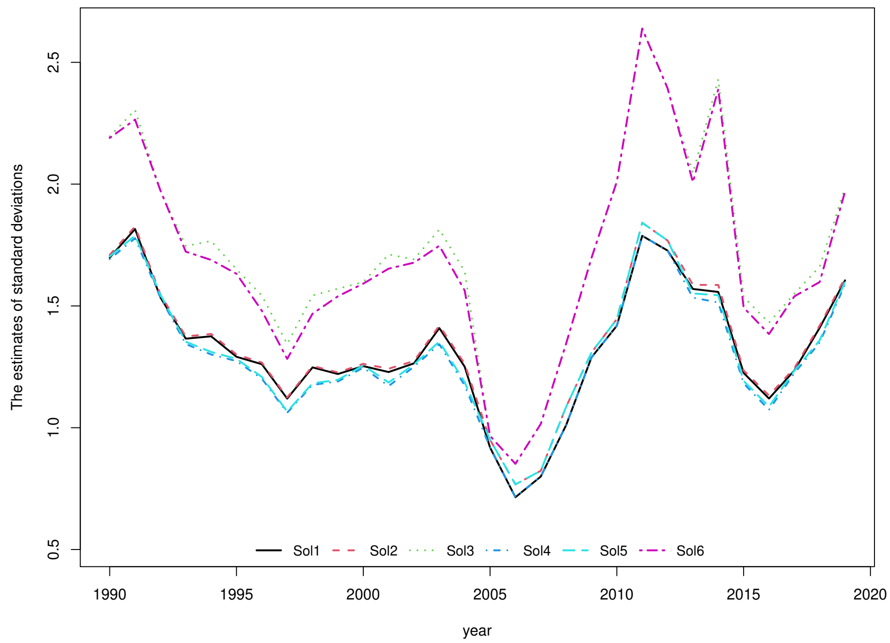
Notice that the average values of annual aggregate inefficiency estimates from 1990 to 2019 (i.e., we first calculate the aggregate inefficiency estimate for each year and then calculate average value of aggregate inefficiency estimates over these 30 years) are 1.7916 for "Bias-Corrected" and 1.7966 for "Bias-Corrected+Sharpened", implying that large inefficiency exists in these countries/regions covered in our sample and that they have the potential to nearly double their GDP without increasing inputs.

Further, Figure 2a shows that, for all these four cases, the aggregate inefficiency generally first decreases from 1990 to 1997, then increases until 2006, then it is very disruptive from 2007 to 2011 (during the global financial crisis), then again decreases until 2016 and increases again until 2019.

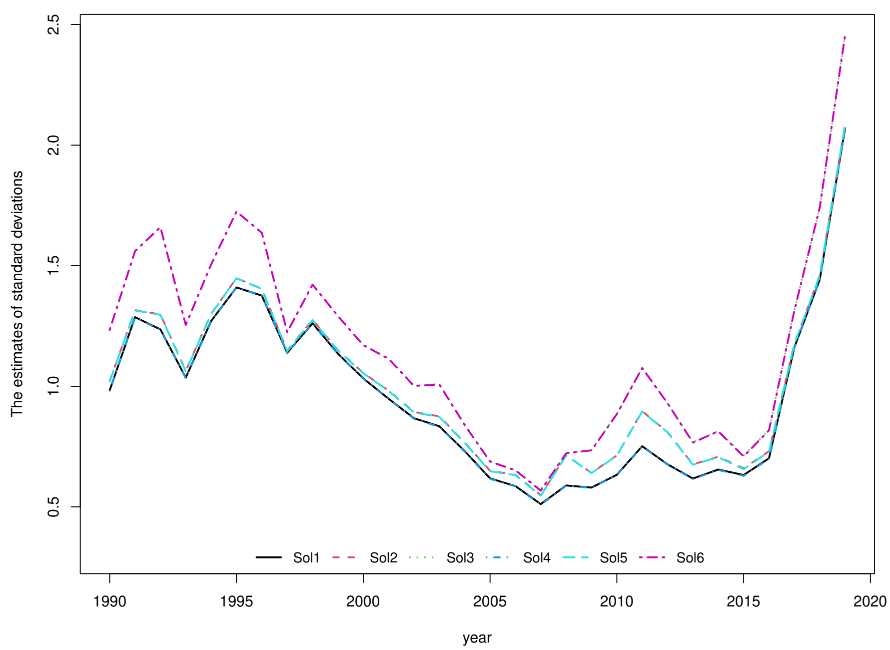
Figure 2b presents the corresponding results for the simple mean output-oriented inefficiency. The relationships among the "Standard", "Bias-Corrected", "Standard+Sharpened" and "Bias-Corrected+Sharpened" still hold as those for aggregate inefficiency. The average values of the annual simple mean inefficiency estimates from 1990 to 2019 are 2.1345 for "Bias-Corrected" and 2.1362 for "Bias-Corrected+Sharpened", which are slightly larger than the corresponding case for aggregate inefficiency (1.7916 and 1.7966, respectively).

Further, the dynamic changes of the simple mean inefficiency are substantially different from the aggregate inefficiency, especially before 2005. This confirms the expectations that it is useful for empirical researchers to report both unweighted and weighted inefficiency to check whether there is any substantial difference, potentially presenting a different big picture of the performance of a group of interest.

Figure 3a presents the estimates of the standard deviation σ_φ for the aggregate output-oriented inefficiency for Sol1–Sol6 over 1990–2019. It is clear that Sol3 has larger estimates for σ_φ than Sol1 and Sol2, and that Sol6 has larger estimates for σ_φ than Sol4 and Sol5, suggesting that the estimated confidence intervals using our proposed method (3.15) will be slightly wider than the previous methods, whether taken with or without the data sharpening method. Further, for each year the results for Sol3 and Sol6 are generally similar in terms of the estimates for σ_φ. The results for the standard deviations for the simple mean inefficiency

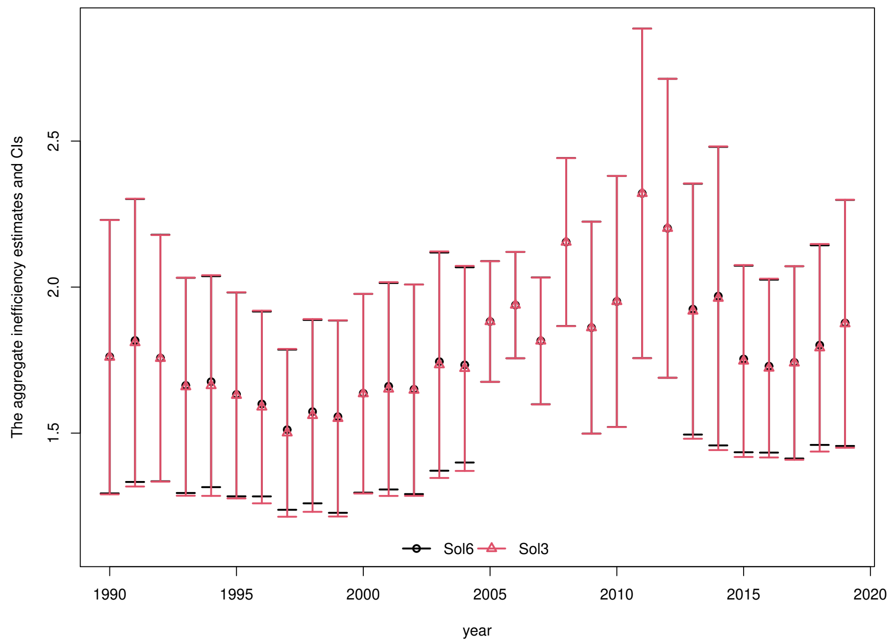


(a) For the Aggregate Output-Oriented Inefficiency

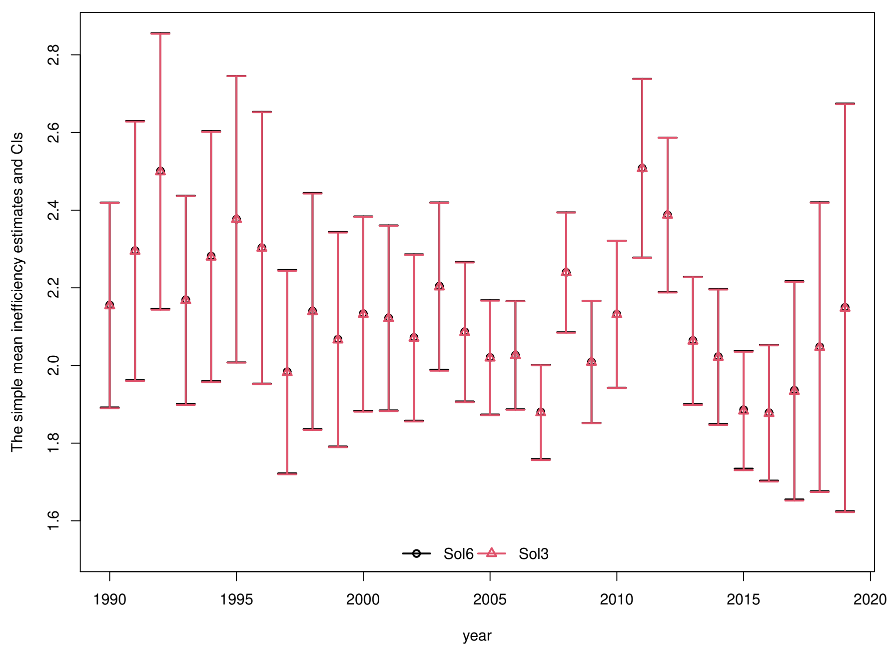


(b) For the Simple Mean Output-Oriented Inefficiency

Figure 3: The Estimated Standard Deviations for Aggregate/Simple Mean Output-Oriented Inefficiency for Countries/Regions from 1990 to 2019 when $\gamma = \kappa$



(a) For the Aggregate Output-Oriented Inefficiency



(b) For the Simple Mean Output-Oriented Inefficiency

Figure 4: CRS-DEA Estimates of Aggregate/Simple Mean Output-Oriented Inefficiency and their 95% Confidence Intervals for Countries/Regions from 1990 to 2019 when $\gamma = \kappa$

presented in Figure 3b are similar to those for aggregate inefficiency where Sol3 and Sol6 have slightly larger values than the remaining methods.

To compare Sol3 and Sol6 which could generate wider confidence intervals, we present the dynamics of aggregate inefficiency estimates and their CIs in Figure 4a. The center of the CIs is the corresponding bias-corrected aggregate output-oriented inefficiency. The results for Sol3 and Sol6 in Figure 4a are generally similar in terms of centers and boundaries of CIs, although the centers and lower boundaries for Sol6 are slightly higher than those for Sol3. Figure 4b presents the corresponding results for simple mean inefficiency, which are generally similar to Figure 4a for aggregate inefficiency.

As a conclusion for this empirical illustration section, we use two different real data sets and different estimators (input-oriented VRS vs. output-oriented CRS) to show the practical differences of the newly proposed method for approximating the new CLT theorems for aggregate efficiency.

6 Conclusions

The CLT results for the aggregate output-oriented efficiency estimated via DEA and FDH developed by Simar and Zelenyuk (2018) are useful recent advancements of the statistical theory for efficiency and productivity analyses, complementing with the CLT results for the individual and simple mean efficiency developed by Kneip et al. (2015). However, the CLT results for the aggregate input-oriented efficiency have not been developed yet, although they are analogous to the output-oriented framework. We fill this clear gap in the literature through establishing the CLT results for the aggregate input-oriented efficiency, developing the practical tools for the implementation of these CLTs, and testing them in extensive MCs.

Further, for relatively small sample sizes, the coverages of the estimated confidence intervals based on the CLT results for aggregate efficiency are well below the nominal coverage. This under-covering phenomenon in relatively small sample sizes was also observed for the simple mean efficiency. Some of the improvements were made through Simar and Zelenyuk (2020), Nguyen et al. (2022), and Simar et al. (2023) for the simple mean output-oriented efficiency, and through Simar and Zelenyuk (2020) for the aggregate output-oriented efficiency. However, whether the methods in Nguyen et al. (2022) and Simar et al. (2023) are effective to improve the finite sample performance of CLT results for aggregate input-oriented and output-oriented efficiency remains unknown. We fill the clear gap in the literature to thoroughly examine the performance of these methods in Simar and Zelenyuk (2020), Nguyen et al. (2022) and Simar et al. (2023) (compared with the developed CLT results in this paper) for aggregate input-oriented efficiency through extensive simulations, although we also provide MC evidence for aggregate output-oriented efficiency. Finally, we use two empirical illustrations to illustrate the differences between all the existing methods for aggregate input-oriented and output-oriented efficiency.

Specifically, to obtain the variance estimator for aggregate efficiency, we use the bias-corrected individual efficiency estimate to replace the individual efficiency estimate at every place. Although the logic of using the bias-corrected individual efficiency estimate to obtain the variance estimate is the same as the simple mean case in Simar et al. (2023), our proposed method here is slightly more complicated, as the variance of aggregate efficiency involves several first moments and also the covariance term. This newly proposed method involves no additional cost and preserves the developed CLT results for the aggregate input-oriented efficiency. We also provide a theoretical justification for the proposed improvement, similar to that in Simar et al. (2023): while the first order of the error is kept as the developed theoretical results for aggregate input-oriented efficiency in this paper, there might be some benefits to correct the second order of the error in the variance estimate.

In our extensive Monte-Carlo experiments, we compare the performance of our methods with the methods in Simar and Zelenyuk (2018) and Simar and Zelenyuk (2020) for both cases—with and without data sharpening methods in Nguyen et al. (2022). We find that: when the sample sizes are not large (≤ 200), our proposed method combined with the data sharpening method provides a better performance; and when the sample sizes are relatively large (> 200), our proposed method without the data sharpening method provides a better performance. In general, this holds for both small and large dimensions. Therefore, we are able to show that the methods in Nguyen et al. (2022) and Simar et al. (2023) are not only useful to improve the finite sample performance of CLT results for the simple mean efficiency, but also are useful for aggregate efficiency, although of course this is not a panacea.

Finally, this paper also shows that the selecting of the tuning parameter in the data sharpening methods plays an important role here, which was not discovered previously in Nguyen et al. (2022) and Simar et al. (2023), indicating that this might be a fruitful area for future research. We hope further research could hopefully bring more insights on this challenging topic. Our proposed method in this paper could be extended naturally to geometric means of the Malmquist index, technical efficiency change, scale efficiency change, technology change, and others.

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Inference for Aggregate Efficiency: Theory and Guidelines for Practitioners (Supplementary Appendix)

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Appendix A Additional Simulation Results for Aggregate Input-Oriented Efficiency

In this appendix, we present additional simulation results for aggregate input-oriented efficiency, that is not shown in the paper. Table A.1 lists the values of β_j's and w_j's for each dimension p, used in the simulation function in (4.1).

A.1 Additional Simulation Results for the Main Results

Tables A.2–A.4 present additional results not shown in the paper for the case with $\gamma = \kappa$. Table A.2 presents the coverages of estimated confidence intervals for aggregate input-oriented efficency using the VRS-DEA estimator across different sample sizes and dimensions. Table A.3 presents the average value of estimated aggregate efficiency and bias using the VRS-DEA estimator, while Table A.4 presents the average value of the standard deviation for aggregate efficiency using the VRS-DEA estimator.

Table A.1: The Values of β_j and w_j

p	1	2	3	4	5	7
β_1	0.4	0.4	0.4	0.4	0.4	0.05
β_2		0.2	0.2	0.2	0.2	0.1
β_3			0.1	0.1	0.1	0.15
β_4				0.15	0.15	0.2
β_5					0.05	0.125
β_6						0.075
β_7						0.025
w_1	1	1	1	1	1	1
w_2		1.5	1.5	1.5	1.5	1.5
w_3			2	2	2	2
w_4				1	1	1
w_5					0.5	0.5
w_6						1
w_7						1

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
1	1	10	0.524	0.594	0.696	0.598	0.637	0.760	0.690	0.725	0.846
1	1	20	0.643	0.702	0.787	0.734	0.770	0.852	0.854	0.880	0.935
1	1	50	0.758	0.779	0.820	0.833	0.854	0.893	0.931	0.942	0.958
1	1	100	0.821	0.826	0.853	0.882	0.890	0.911	0.951	0.959	0.968
1	1	200	0.835	0.838	0.855	0.911	0.915	0.928	0.976	0.979	0.986
1	1	300	0.842	0.846	0.857	0.908	0.910	0.915	0.970	0.971	0.974
1	1	500	0.863	0.864	0.866	0.929	0.929	0.932	0.984	0.985	0.987
1	1	1000	0.881	0.881	0.883	0.935	0.935	0.936	0.978	0.978	0.979

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
1	1	10	0.615	0.706	0.855	0.692	0.803	0.909	0.806	0.913	0.962
1	1	20	0.669	0.730	0.842	0.744	0.818	0.906	0.869	0.918	0.964
1	1	50	0.760	0.777	0.831	0.827	0.850	0.898	0.933	0.947	0.966
1	1	100	0.822	0.830	0.848	0.881	0.888	0.909	0.956	0.961	0.969
1	1	200	0.831	0.835	0.849	0.906	0.908	0.921	0.973	0.976	0.982
1	1	300	0.842	0.844	0.853	0.904	0.906	0.911	0.971	0.972	0.976
1	1	500	0.859	0.860	0.865	0.927	0.929	0.929	0.983	0.984	0.984
1	1	1000	0.882	0.882	0.882	0.935	0.935	0.935	0.979	0.979	0.979

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
2	1	10	0.254	0.322	0.499	0.305	0.367	0.565	0.389	0.443	0.656
2	1	20	0.358	0.418	0.579	0.413	0.494	0.662	0.531	0.598	0.791
2	1	50	0.543	0.599	0.705	0.614	0.682	0.794	0.751	0.816	0.913
2	1	100	0.699	0.729	0.789	0.772	0.808	0.854	0.882	0.915	0.942
2	1	200	0.755	0.781	0.817	0.835	0.850	0.881	0.930	0.949	0.964
2	1	300	0.774	0.793	0.811	0.841	0.856	0.874	0.945	0.955	0.969
2	1	500	0.821	0.831	0.842	0.879	0.893	0.899	0.953	0.958	0.960
2	1	1000	0.834	0.839	0.847	0.894	0.897	0.900	0.972	0.974	0.975

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
2	1	10	0.588	0.669	0.834	0.654	0.740	0.873	0.792	0.882	0.932
2	1	20	0.579	0.690	0.862	0.657	0.781	0.919	0.792	0.894	0.975
2	1	50	0.626	0.718	0.837	0.707	0.796	0.901	0.823	0.900	0.964
2	1	100	0.736	0.778	0.840	0.795	0.846	0.906	0.908	0.947	0.972
2	1	200	0.725	0.754	0.798	0.807	0.840	0.874	0.917	0.940	0.961
2	1	300	0.744	0.768	0.794	0.823	0.850	0.879	0.927	0.938	0.952
2	1	500	0.778	0.787	0.794	0.852	0.867	0.873	0.940	0.947	0.955
2	1	1000	0.788	0.793	0.793	0.863	0.871	0.874	0.950	0.955	0.956

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
3	1	10	0.122	0.168	0.378	0.151	0.209	0.431	0.200	0.268	0.510
3	1	20	0.238	0.308	0.537	0.292	0.362	0.612	0.375	0.462	0.755
3	1	50	0.419	0.514	0.712	0.503	0.600	0.815	0.644	0.732	0.910
3	1	100	0.586	0.661	0.801	0.663	0.737	0.891	0.816	0.884	0.969
3	1	200	0.733	0.792	0.866	0.813	0.866	0.917	0.916	0.949	0.986
3	1	300	0.789	0.838	0.885	0.866	0.897	0.944	0.959	0.975	0.988
3	1	500	0.842	0.873	0.898	0.902	0.927	0.949	0.974	0.982	0.987
3	1	1000	0.867	0.886	0.899	0.932	0.949	0.957	0.982	0.986	0.989

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
3	1	10	0.732	0.779	0.884	0.811	0.855	0.928	0.908	0.949	0.967
3	1	20	0.674	0.755	0.924	0.749	0.834	0.958	0.858	0.943	0.982
3	1	50	0.677	0.800	0.943	0.754	0.867	0.985	0.862	0.965	0.998
3	1	100	0.704	0.816	0.920	0.792	0.876	0.965	0.888	0.961	0.997
3	1	200	0.747	0.828	0.893	0.828	0.891	0.949	0.926	0.971	0.988
3	1	300	0.736	0.806	0.857	0.822	0.883	0.929	0.932	0.967	0.983
3	1	500	0.765	0.809	0.846	0.851	0.877	0.907	0.938	0.964	0.978
3	1	1000	0.760	0.790	0.807	0.846	0.867	0.878	0.949	0.959	0.969

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
4	1	10	0.076	0.106	0.259	0.089	0.133	0.295	0.123	0.170	0.344
4	1	20	0.094	0.151	0.440	0.123	0.194	0.520	0.192	0.271	0.634
4	1	50	0.218	0.308	0.645	0.282	0.384	0.760	0.392	0.531	0.910
4	1	100	0.405	0.515	0.790	0.487	0.612	0.874	0.656	0.788	0.980
4	1	200	0.589	0.691	0.853	0.684	0.786	0.928	0.838	0.918	0.989
4	1	300	0.707	0.785	0.900	0.792	0.870	0.959	0.917	0.970	0.998
4	1	500	0.823	0.879	0.931	0.892	0.930	0.966	0.962	0.983	0.993
4	1	1000	0.892	0.927	0.951	0.944	0.966	0.977	0.982	0.989	0.995

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
4	1	10	0.796	0.828	0.903	0.868	0.898	0.946	0.945	0.971	0.982
4	1	20	0.756	0.804	0.928	0.837	0.877	0.956	0.922	0.956	0.981
4	1	50	0.759	0.864	0.990	0.830	0.922	0.996	0.931	0.986	0.999
4	1	100	0.689	0.832	0.969	0.772	0.902	0.992	0.885	0.978	1.000
4	1	200	0.680	0.820	0.956	0.776	0.909	0.991	0.894	0.984	0.999
4	1	300	0.703	0.835	0.933	0.798	0.908	0.975	0.914	0.978	0.996
4	1	500	0.702	0.805	0.886	0.797	0.896	0.958	0.922	0.973	0.989
4	1	1000	0.685	0.768	0.814	0.784	0.858	0.913	0.929	0.965	0.983

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
5	1	10	0.051	0.081	0.207	0.063	0.097	0.226	0.081	0.129	0.246
5	1	20	0.078	0.112	0.389	0.091	0.146	0.478	0.129	0.200	0.569
5	1	50	0.126	0.196	0.599	0.172	0.263	0.717	0.254	0.395	0.877
5	1	100	0.264	0.381	0.800	0.329	0.487	0.904	0.506	0.699	0.985
5	1	200	0.473	0.616	0.876	0.587	0.721	0.946	0.752	0.882	0.999
5	1	300	0.607	0.725	0.915	0.705	0.833	0.970	0.864	0.943	0.999
5	1	500	0.726	0.840	0.944	0.835	0.905	0.983	0.936	0.982	0.998
5	1	1000	0.827	0.899	0.959	0.901	0.954	0.992	0.981	0.999	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
5	1	10	0.844	0.854	0.907	0.901	0.919	0.953	0.954	0.972	0.987
5	1	20	0.805	0.841	0.935	0.874	0.905	0.958	0.960	0.975	0.988
5	1	50	0.789	0.865	0.988	0.863	0.929	0.995	0.945	0.985	0.999
5	1	100	0.754	0.872	0.999	0.843	0.939	1.000	0.926	0.992	1.000
5	1	200	0.676	0.841	0.981	0.765	0.924	0.996	0.897	0.984	1.000
5	1	300	0.648	0.808	0.970	0.726	0.911	0.990	0.894	0.982	1.000
5	1	500	0.633	0.803	0.945	0.743	0.899	0.978	0.892	0.979	1.000
5	1	1000	0.624	0.764	0.867	0.724	0.860	0.938	0.883	0.964	0.990

Table A.2: Coverages of Estimated Confidence Intervals for Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
7	1	10	0.014	0.017	0.048	0.014	0.019	0.056	0.016	0.023	0.065
7	1	20	0.009	0.023	0.169	0.014	0.034	0.202	0.028	0.048	0.244
7	1	50	0.020	0.038	0.355	0.027	0.054	0.480	0.051	0.093	0.650
7	1	100	0.025	0.066	0.527	0.041	0.100	0.705	0.080	0.185	0.902
7	1	200	0.078	0.137	0.733	0.103	0.226	0.892	0.205	0.422	0.992
7	1	300	0.112	0.221	0.829	0.168	0.332	0.959	0.317	0.618	0.999
7	1	500	0.250	0.432	0.914	0.340	0.590	0.992	0.563	0.839	1.000
7	1	1000	0.491	0.682	0.960	0.602	0.824	0.991	0.819	0.963	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
7	1	10	0.854	0.856	0.863	0.918	0.919	0.924	0.988	0.989	0.988
7	1	20	0.863	0.870	0.910	0.939	0.944	0.967	0.993	0.997	0.997
7	1	50	0.868	0.885	0.978	0.929	0.944	0.995	0.988	0.994	0.998
7	1	100	0.870	0.904	0.993	0.918	0.952	1.000	0.983	0.993	1.000
7	1	200	0.836	0.917	1.000	0.912	0.959	1.000	0.971	0.996	1.000
7	1	300	0.849	0.928	1.000	0.904	0.961	1.000	0.960	0.997	1.000
7	1	500	0.765	0.917	0.997	0.859	0.965	0.999	0.950	0.995	1.000
7	1	1000	0.669	0.871	0.993	0.772	0.935	1.000	0.908	0.992	1.000

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator

p	q	n	$\widehat{\varphi}_n$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_n - \widehat{B}_{\varphi,n}$	φ_n	φ
Without Data Sharpening							
1	1	10	0.8886	0.0813	0.8073	0.7496	0.7490
1	1	20	0.8440	0.0720	0.7720	0.7505	0.7490
1	1	50	0.8051	0.0490	0.7561	0.7500	0.7490
1	1	100	0.7839	0.0330	0.7509	0.7494	0.7490
1	1	200	0.7707	0.0221	0.7486	0.7494	0.7490
1	1	300	0.7646	0.0169	0.7477	0.7484	0.7490
1	1	500	0.7602	0.0121	0.7481	0.7488	0.7490
1	1	1000	0.7561	0.0075	0.7486	0.7491	0.7490
With Data Sharpening							
1	1	10	0.8113	0.0776	0.7337	0.7496	0.7490
1	1	20	0.8113	0.0710	0.7403	0.7505	0.7490
1	1	50	0.7955	0.0488	0.7467	0.7500	0.7490
1	1	100	0.7803	0.0330	0.7473	0.7494	0.7490
1	1	200	0.7693	0.0221	0.7473	0.7494	0.7490
1	1	300	0.7638	0.0169	0.7469	0.7484	0.7490
1	1	500	0.7598	0.0121	0.7477	0.7488	0.7490
1	1	1000	0.7559	0.0075	0.7484	0.7491	0.7490

NOTE: $\widehat{\varphi}_n$: Aggregate efficiency constructed using n observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator (continued)

p	q	n	$\widehat{\varphi}_n$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_n - \widehat{B}_{\varphi,n}$	φ_n	φ
Without Data Sharpening							
2	1	10	0.9439	0.0773	0.8665	0.7496	0.7490
2	1	20	0.9084	0.0896	0.8188	0.7496	0.7490
2	1	50	0.8607	0.0859	0.7748	0.7492	0.7490
2	1	100	0.8316	0.0711	0.7605	0.7503	0.7490
2	1	200	0.8078	0.0556	0.7522	0.7493	0.7490
2	1	300	0.7972	0.0468	0.7504	0.7491	0.7490
2	1	500	0.7858	0.0375	0.7482	0.7487	0.7490
2	1	1000	0.7745	0.0271	0.7475	0.7489	0.7490
With Data Sharpening							
2	1	10	0.8006	0.0677	0.7329	0.7496	0.7490
2	1	20	0.8198	0.0845	0.7354	0.7496	0.7490
2	1	50	0.8223	0.0844	0.7379	0.7492	0.7490
2	1	100	0.8126	0.0706	0.7420	0.7503	0.7490
2	1	200	0.7986	0.0554	0.7432	0.7493	0.7490
2	1	300	0.7912	0.0468	0.7445	0.7491	0.7490
2	1	500	0.7823	0.0375	0.7448	0.7487	0.7490
2	1	1000	0.7729	0.0270	0.7458	0.7489	0.7490

NOTE: $\widehat{\varphi}_n$: Aggregate efficiency constructed using n observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator (continued)

p	q	n	$\widehat{\varphi}_{n_\kappa}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_\kappa} - \widehat{B}_{\varphi,n}$	φ_n	φ
Without Data Sharpening							
3	1	10	0.9744	0.0548	0.9196	0.7498	0.7490
3	1	20	0.9457	0.0828	0.8629	0.7518	0.7490
3	1	50	0.9059	0.0986	0.8073	0.7486	0.7490
3	1	100	0.8743	0.0944	0.7799	0.7491	0.7490
3	1	200	0.8479	0.0841	0.7638	0.7494	0.7490
3	1	300	0.8341	0.0770	0.7571	0.7492	0.7490
3	1	500	0.8189	0.0662	0.7527	0.7493	0.7490
3	1	1000	0.8012	0.0531	0.7482	0.7493	0.7490
With Data Sharpening							
3	1	10	0.7831	0.0443	0.7388	0.7498	0.7490
3	1	20	0.8078	0.0727	0.7351	0.7518	0.7490
3	1	50	0.8262	0.0934	0.7328	0.7486	0.7490
3	1	100	0.8259	0.0919	0.7340	0.7491	0.7490
3	1	200	0.8199	0.0829	0.7369	0.7494	0.7490
3	1	300	0.8141	0.0762	0.7379	0.7492	0.7490
3	1	500	0.8059	0.0659	0.7400	0.7493	0.7490
3	1	1000	0.7941	0.0529	0.7412	0.7493	0.7490

NOTE: $\widehat{\varphi}_{n_\kappa}$: Aggregate efficiency constructed using n_κ observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator (continued)

p	q	n	$\widehat{\varphi}_{n_\kappa}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_\kappa} - \widehat{B}_{\varphi,n}$	φ_n	φ
			Without Data Sharpening				
4	1	10	0.9873	0.0348	0.9526	0.7545	0.7490
4	1	20	0.9704	0.0621	0.9083	0.7499	0.7490
4	1	50	0.9389	0.0927	0.8462	0.7504	0.7490
4	1	100	0.9125	0.1050	0.8075	0.7481	0.7490
4	1	200	0.8861	0.1037	0.7823	0.7495	0.7490
4	1	300	0.8706	0.0996	0.7709	0.7491	0.7490
4	1	500	0.8530	0.0925	0.7605	0.7490	0.7490
4	1	1000	0.8313	0.0802	0.7511	0.7487	0.7490
			With Data Sharpening				
4	1	10	0.7589	0.0266	0.7323	0.7545	0.7490
4	1	20	0.7924	0.0510	0.7414	0.7499	0.7490
4	1	50	0.8182	0.0832	0.7350	0.7504	0.7490
4	1	100	0.8272	0.0986	0.7287	0.7481	0.7490
4	1	200	0.8292	0.1000	0.7292	0.7495	0.7490
4	1	300	0.8273	0.0971	0.7302	0.7491	0.7490
4	1	500	0.8220	0.0910	0.7311	0.7490	0.7490
4	1	1000	0.8123	0.0794	0.7328	0.7487	0.7490

NOTE: $\widehat{\varphi}_{n_\kappa}$: Aggregate efficiency constructed using n_κ observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator (continued)

p	q	n	$\widehat{\varphi}_{n_\kappa}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_\kappa} - \widehat{B}_{\varphi,n}$	φ_n	φ
Without Data Sharpening							
5	1	10	0.9933	0.0248	0.9685	0.7506	0.7490
5	1	20	0.9823	0.0509	0.9313	0.7501	0.7490
5	1	50	0.9591	0.0834	0.8757	0.7501	0.7490
5	1	100	0.9365	0.1025	0.8340	0.7490	0.7490
5	1	200	0.9120	0.1102	0.8018	0.7493	0.7490
5	1	300	0.8976	0.1105	0.7871	0.7490	0.7490
5	1	500	0.8818	0.1080	0.7738	0.7490	0.7490
5	1	1000	0.8591	0.0998	0.7594	0.7490	0.7490
With Data Sharpening							
5	1	10	0.7380	0.0183	0.7198	0.7506	0.7490
5	1	20	0.7732	0.0404	0.7328	0.7501	0.7490
5	1	50	0.8036	0.0711	0.7325	0.7501	0.7490
5	1	100	0.8186	0.0922	0.7264	0.7490	0.7490
5	1	200	0.8244	0.1028	0.7215	0.7493	0.7490
5	1	300	0.8259	0.1048	0.7211	0.7490	0.7490
5	1	500	0.8271	0.1040	0.7230	0.7490	0.7490
5	1	1000	0.8224	0.0975	0.7249	0.7490	0.7490

NOTE: $\widehat{\varphi}_{n_\kappa}$: Aggregate efficiency constructed using n_κ observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.3: Aggregate Input-Oriented Efficiency and Bias using VRS-DEA Estimator (continued)

p	q	n	$\widehat{\varphi}_{n_\kappa}$	$\widehat{B}_{\varphi,n}$	$\widehat{\varphi}_{n_\kappa} - \widehat{B}_{\varphi,n}$	φ_n	φ
Without Data Sharpening							
7	1	10	0.9988	0.0056	0.9932	0.7513	0.7490
7	1	20	0.9969	0.0176	0.9793	0.7494	0.7490
7	1	50	0.9873	0.0455	0.9418	0.7502	0.7490
7	1	100	0.9758	0.0702	0.9057	0.7497	0.7490
7	1	200	0.9589	0.0917	0.8671	0.7497	0.7490
7	1	300	0.9500	0.1013	0.8486	0.7489	0.7490
7	1	500	0.9347	0.1110	0.8236	0.7489	0.7490
7	1	1000	0.9153	0.1179	0.7974	0.7487	0.7490
With Data Sharpening							
7	1	10	0.7021	0.0039	0.6982	0.7513	0.7490
7	1	20	0.7431	0.0130	0.7301	0.7494	0.7490
7	1	50	0.7802	0.0361	0.7441	0.7502	0.7490
7	1	100	0.8003	0.0580	0.7423	0.7497	0.7490
7	1	200	0.8147	0.0791	0.7355	0.7497	0.7490
7	1	300	0.8211	0.0893	0.7318	0.7489	0.7490
7	1	500	0.8245	0.1004	0.7241	0.7489	0.7490
7	1	1000	0.8292	0.1097	0.7196	0.7487	0.7490

NOTE: $\widehat{\varphi}_{n_\kappa}$: Aggregate efficiency constructed using n_κ observations; $\widehat{B}_{\varphi,n}$: Bias constructed using n observations; φ_n: Sample aggregate of true efficiency; φ: Population aggregate of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
1	1	10	0.1509	0.1736	0.2474	0.1221	0.1477	0.2103	0.1462	0.1641
1	1	20	0.1699	0.1861	0.2339	0.1404	0.1592	0.2031	0.1553	0.1641
1	1	50	0.1751	0.1825	0.2061	0.1623	0.1703	0.1928	0.1602	0.1641
1	1	100	0.1739	0.1772	0.1881	0.1683	0.1718	0.1823	0.1629	0.1641
1	1	200	0.1701	0.1716	0.1776	0.1678	0.1693	0.1752	0.1630	0.1641
1	1	300	0.1692	0.1701	0.1734	0.1679	0.1688	0.1720	0.1638	0.1641
1	1	500	0.1679	0.1683	0.1701	0.1672	0.1677	0.1694	0.1641	0.1641
1	1	1000	0.1658	0.1660	0.1665	0.1656	0.1657	0.1662	0.1635	0.1641

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
2	1	10	0.1041	0.1316	0.2394	0.1218	0.1436	0.2217	0.1428	0.1538
2	1	20	0.1398	0.1679	0.2565	0.1160	0.1456	0.2211	0.1479	0.1538
2	1	50	0.1597	0.1823	0.2328	0.1308	0.1568	0.2026	0.1501	0.1538
2	1	100	0.1645	0.1797	0.2080	0.1458	0.1625	0.1885	0.1526	0.1538
2	1	200	0.1644	0.1738	0.1880	0.1538	0.1637	0.1768	0.1531	0.1538
2	1	300	0.1639	0.1706	0.1791	0.1565	0.1634	0.1712	0.1536	0.1538
2	1	500	0.1621	0.1664	0.1705	0.1575	0.1620	0.1657	0.1537	0.1538
2	1	1000	0.1597	0.1620	0.1629	0.1574	0.1597	0.1605	0.1537	0.1538

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
3	1	10	0.0627	0.0844	0.1869	0.1317	0.1435	0.2045	0.1382	0.1503
3	1	20	0.1086	0.1385	0.2582	0.1195	0.1425	0.2322	0.1450	0.1503
3	1	50	0.1446	0.1761	0.2639	0.1161	0.1501	0.2278	0.1477	0.1503
3	1	100	0.1571	0.1839	0.2421	0.1266	0.1571	0.2103	0.1501	0.1503
3	1	200	0.1607	0.1817	0.2161	0.1372	0.1606	0.1921	0.1495	0.1503
3	1	300	0.1617	0.1792	0.2035	0.1430	0.1622	0.1843	0.1499	0.1503
3	1	500	0.1615	0.1746	0.1889	0.1478	0.1620	0.1747	0.1500	0.1503
3	1	1000	0.1602	0.1688	0.1749	0.1518	0.1608	0.1662	0.1503	0.1503

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
4	1	10	0.0344	0.0494	0.1246	0.1403	0.1467	0.1852	0.1402	0.1484
4	1	20	0.0749	0.0987	0.2265	0.1246	0.1375	0.2176	0.1419	0.1484
4	1	50	0.1208	0.1534	0.2760	0.1150	0.1430	0.2429	0.1458	0.1484
4	1	100	0.1402	0.1758	0.2722	0.1138	0.1512	0.2365	0.1473	0.1484
4	1	200	0.1513	0.1837	0.2499	0.1205	0.1570	0.2173	0.1479	0.1484
4	1	300	0.1557	0.1850	0.2355	0.1269	0.1600	0.2061	0.1483	0.1484
4	1	500	0.1584	0.1836	0.2184	0.1339	0.1620	0.1936	0.1481	0.1484
4	1	1000	0.1598	0.1788	0.1986	0.1419	0.1627	0.1804	0.1484	0.1484

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
5	1	10	0.0209	0.0327	0.0886	0.1535	0.1579	0.1846	0.1387	0.1475
5	1	20	0.0557	0.0767	0.2009	0.1347	0.1439	0.2121	0.1437	0.1475
5	1	50	0.1010	0.1321	0.2763	0.1221	0.1426	0.2467	0.1456	0.1475
5	1	100	0.1252	0.1625	0.2885	0.1155	0.1484	0.2530	0.1474	0.1475
5	1	200	0.1409	0.1793	0.2763	0.1147	0.1544	0.2405	0.1472	0.1475
5	1	300	0.1470	0.1841	0.2638	0.1169	0.1572	0.2294	0.1475	0.1475
5	1	500	0.1525	0.1870	0.2473	0.1217	0.1603	0.2156	0.1476	0.1475
5	1	1000	0.1568	0.1860	0.2248	0.1298	0.1624	0.1981	0.1474	0.1475

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Aggregate Input-Oriented Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ_φ
7	1	10	0.0035	0.0066	0.0199	0.1726	0.1734	0.1782	0.1359	0.1459
7	1	20	0.0166	0.0245	0.0837	0.1537	0.1557	0.1805	0.1399	0.1459
7	1	50	0.0555	0.0728	0.2195	0.1317	0.1379	0.2159	0.1441	0.1459
7	1	100	0.0807	0.1076	0.2707	0.1227	0.1365	0.2441	0.1451	0.1459
7	1	200	0.1028	0.1382	0.2926	0.1165	0.1412	0.2587	0.1453	0.1459
7	1	300	0.1136	0.1525	0.2947	0.1138	0.1449	0.2597	0.1459	0.1459
7	1	500	0.1243	0.1669	0.2898	0.1112	0.1499	0.2543	0.1456	0.1459
7	1	1000	0.1365	0.1804	0.2767	0.1114	0.1564	0.2419	0.1458	0.1459

NOTE: σ_s: Sample standard deviation of true efficiency; σ_φ: Population standard deviation of true efficiency.

A.2 Simulation Results for Various Values of Tuning Parameter

Figures A.1–A.7 present empirical coverages for the aggregate input-oriented efficiency for $\gamma = 0.55\kappa$, 0.65κ, 0.75κ, 0.85κ, 0.95κ, 1.2κ, and 1.5κ respectively.

Figure A.1: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 0.55\kappa$

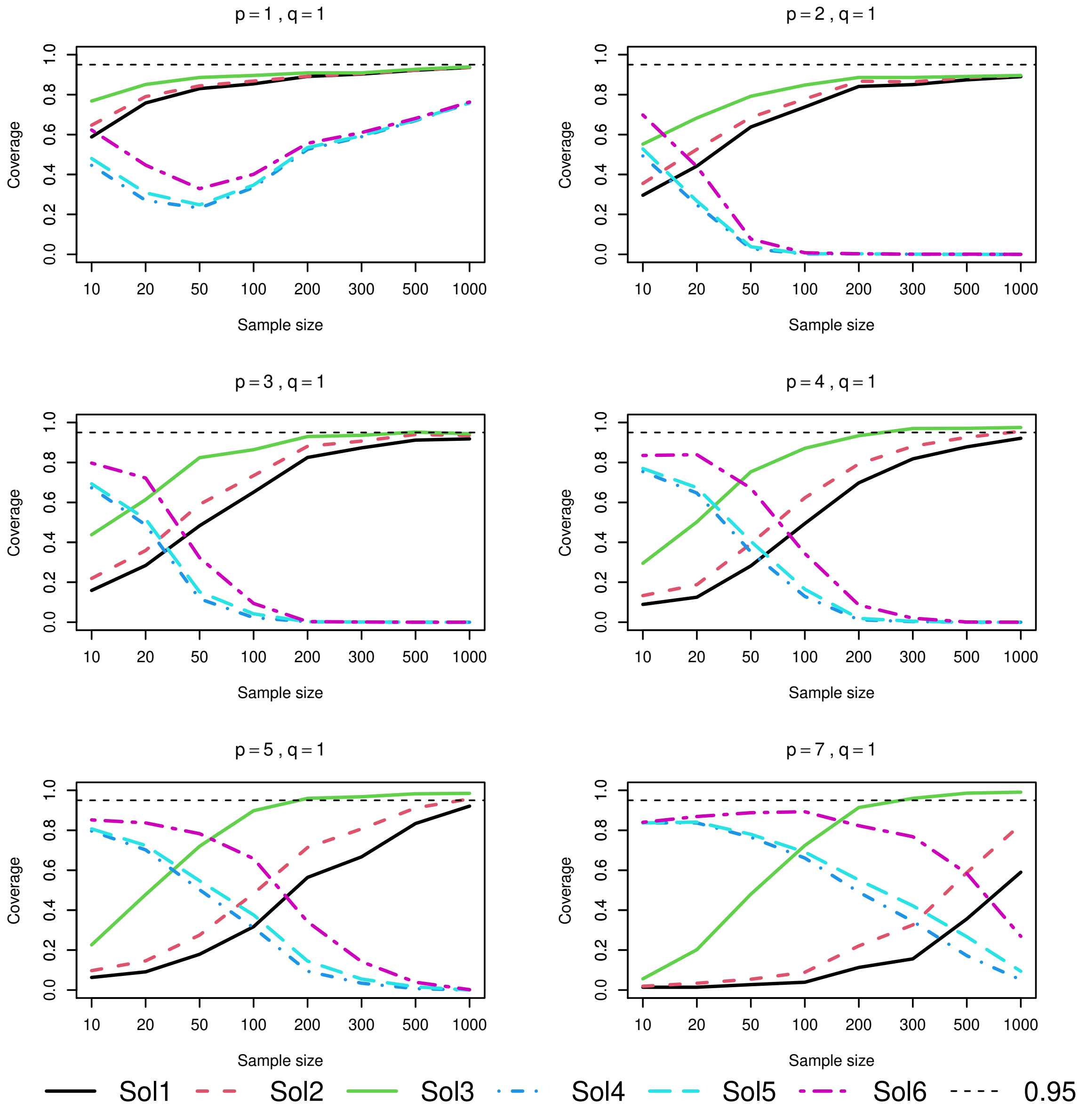


Figure A.2: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 0.65\kappa$

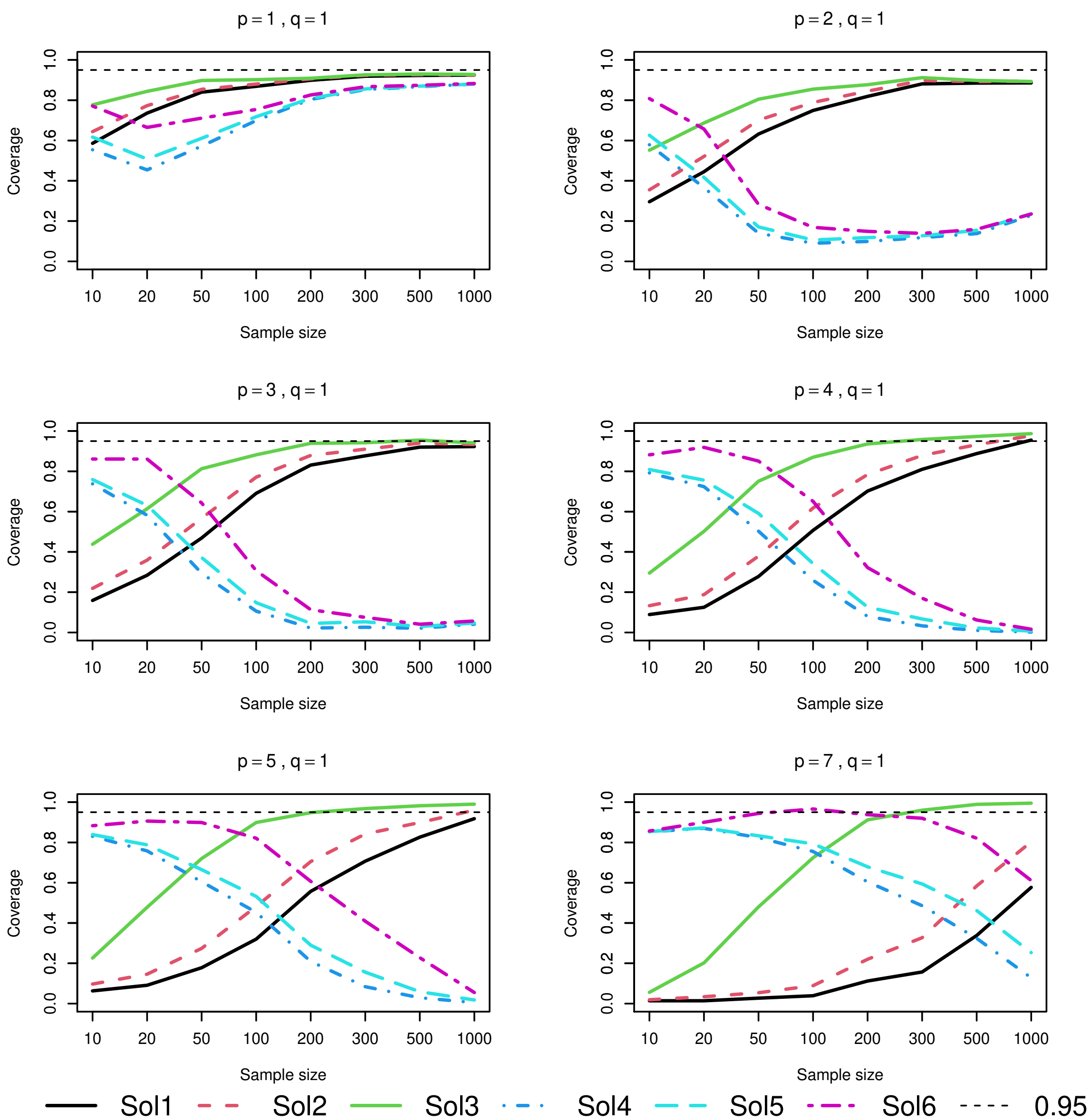


Figure A.3: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 0.75\kappa$

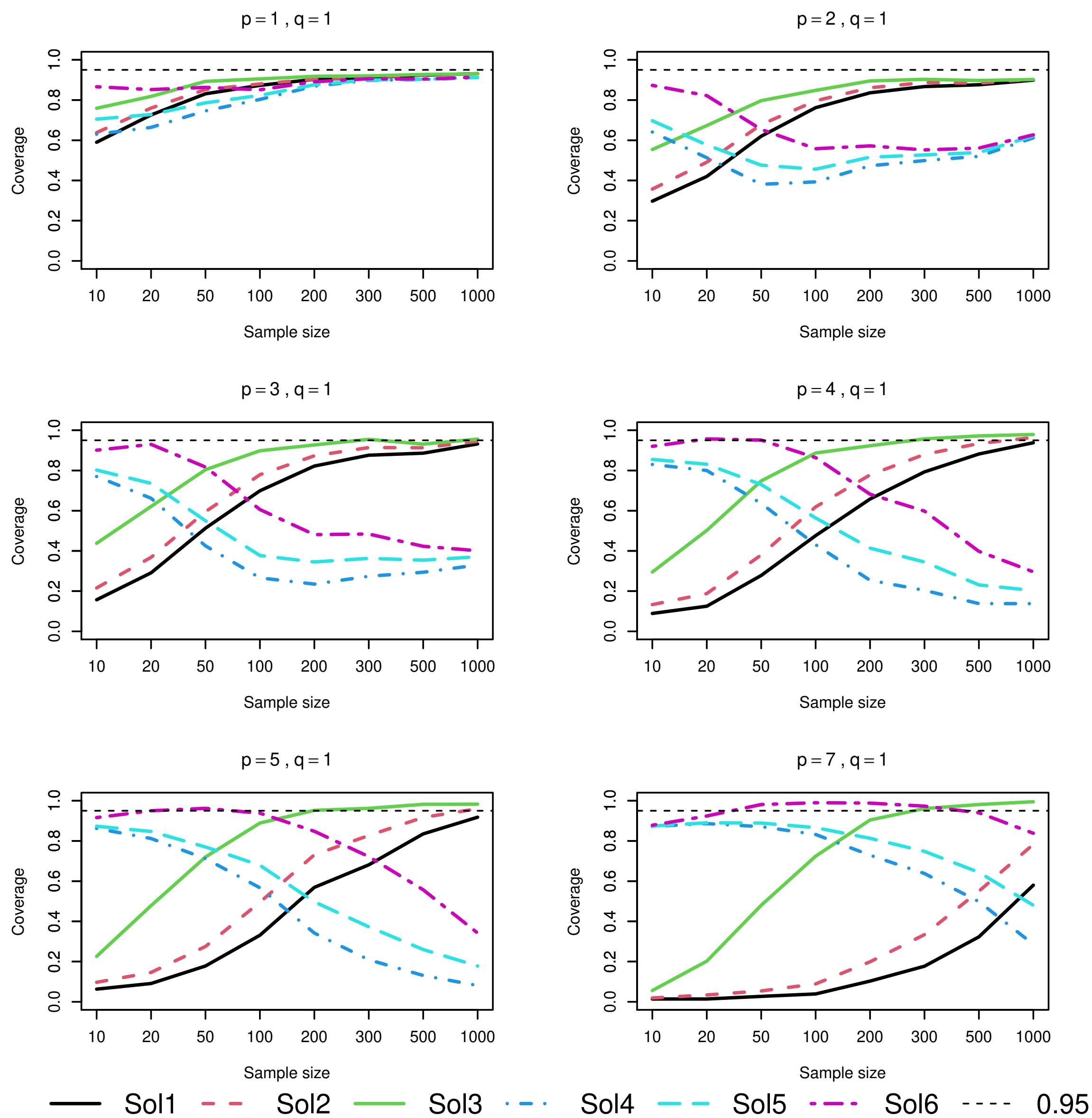


Figure A.4: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 0.85\kappa$

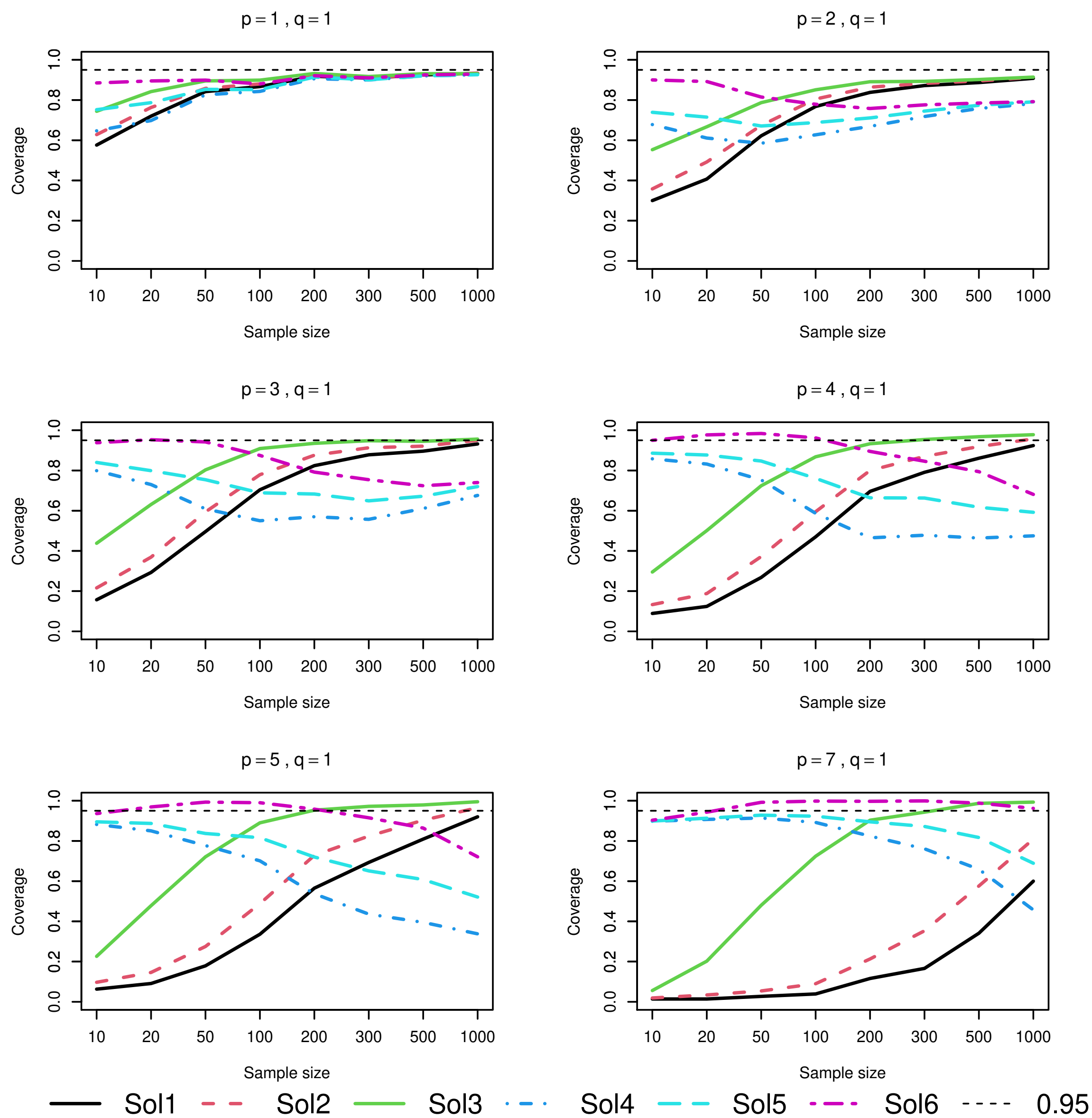


Figure A.5: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 0.95\kappa$

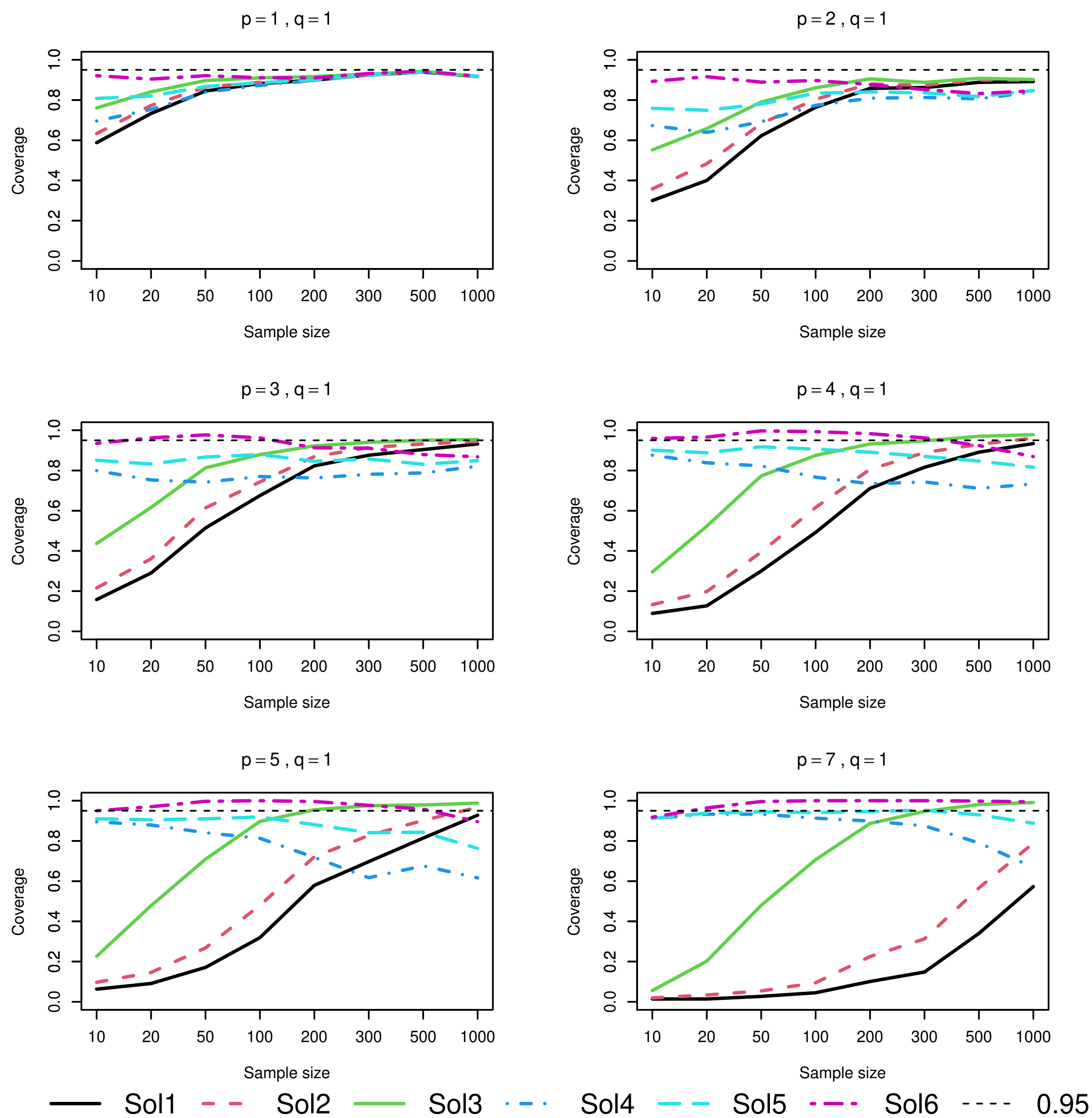


Figure A.6: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 1.2\kappa$

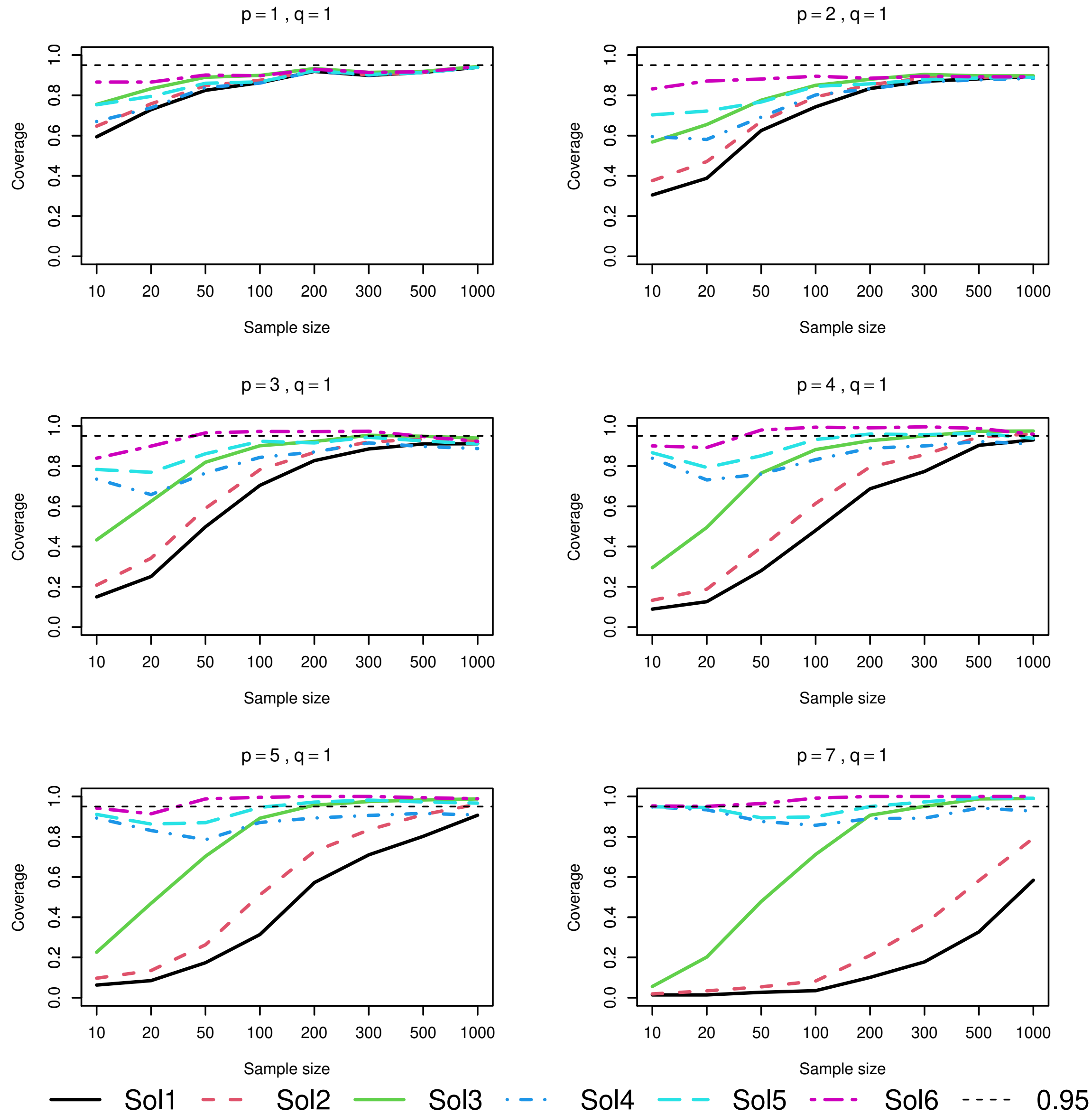
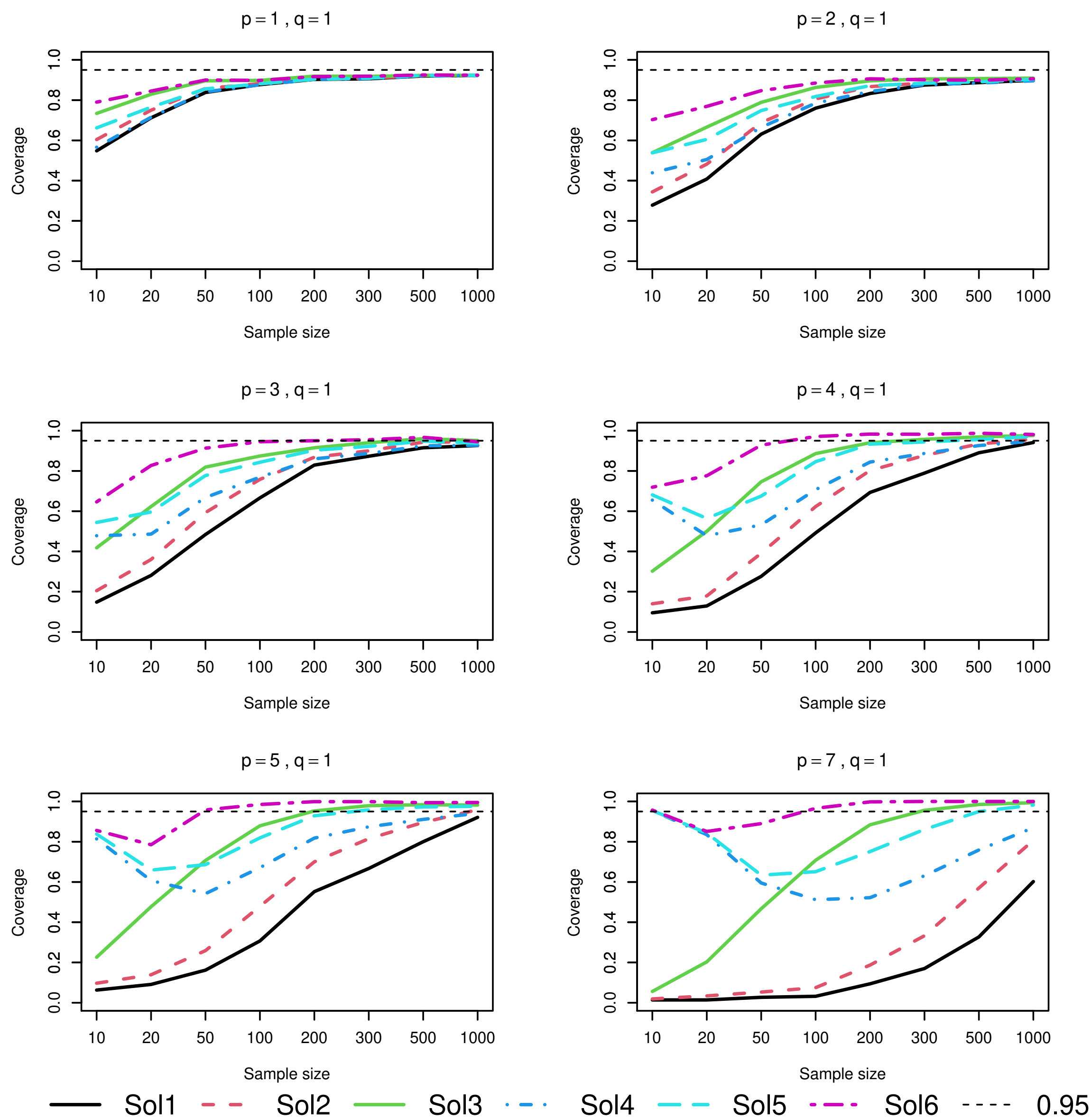


Figure A.7: Empirical Coverages for the Aggregate Input-Oriented Efficiency when $\gamma = 1.5\kappa$



Appendix B Simulation Results for Aggregate Output-Oriented Efficiency

In this appendix, we present the simulation results for the aggregate output-oriented technical efficiency.

B.1 Details on Monte-Carlo Simulations

Our Monte-Carlo experiments closely follow Simar and Zelenyuk (2020). Formally,

$$y_i^{\partial}(x_i) = \prod_{j=1}^{p} x_{ji}^{\beta_j}, \tag{B.1}$$

where $1 \times p$ vector $x_i = (x_{1i}, x_{2i}, \ldots, x_{pi})$ and $x_{ji} \overset{\text{iid}}{\sim} \text{Unif}(0,1)$, $\forall\, j \in 1, \ldots, p$. In addition, we generate the true inefficiency scores from $\lambda_i \sim |N(0,1)| + 1$ (i.e., we generate the true inefficiency from the half normal distribution and then shift it up by 1). The observed output is $y_i = y^{\partial}(x_i)/\lambda_i$, i.e., by projecting $y^{\partial}(x_i)$ from the frontier into the production set. Thus a simulated sample $\mathcal{S}_n = \{(x_i, y_i)\}_{i=1}^n$ is created. The values of β_j's are set the same as Simar and Zelenyuk (2020) and the output price is set to be 1.

B.2 Data Sharpening Methods

Let $\mathcal{S}_n = \{(X_i, Y_i)\}_{i=1}^n$ denote a random sample of input-output pairs, and $\widehat{\lambda}(X_i, Y_i \mid \mathcal{S}_n) \geq 1$ is the estimated output-oriented Farrell technical efficiency. Denote $\widehat{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n) = 1/\widehat{\lambda}(X_i, Y_i \mid \mathcal{S}_n) \leq 1$. We adapt the data sharpening method in Nguyen et al. (2022) as follows.

$$\widetilde{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n) = \begin{cases} \widehat{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n), & \text{if } \widehat{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n) < 1 - \tau, \\ \\ \widehat{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n) \times \varepsilon_i, & \text{otherwise}, \end{cases} \tag{B.2}$$

where $\tau = n^{-\gamma}$, $\gamma \geq \kappa/2$, and $\kappa = 2/(p+q+1)$ for the VRS-DEA estimator. Further, ε_i is a random independent draw from a uniform distribution on the interval $[1-\tau, 1]$. It can be shown that $\widetilde{\lambda}_I(X_i, Y_i \mid \mathcal{S}_n) = 1/\widehat{\lambda}(X_i, \widetilde{Y}_i \mid \mathcal{S}_n)$, where

$$\widetilde{Y}_i = \begin{cases} Y_i, & \text{if } \widehat{\lambda}(X_i, Y_i \mid \mathcal{S}_n) > 1/(1-\tau), \\ \\ \varepsilon_i Y_i, & \text{otherwise}. \end{cases} \tag{B.3}$$

The observations after the data sharpening are $(X_i, \widetilde{Y}_i)$. Then the regular nonparametric efficiency estimators are applied for the sharpened sample points $\{(X_i, \widetilde{Y}_i)\}_{i=1}^n$, but with the reference set being the original sample $\mathcal{S}_n = \{(X_i, Y_i)\}_{i=1}^n$.

B.3 Our Proposed Method

Similar to the case for aggregate input-oriented efficiency, we use bias-corrected individual efficiency estimate to improve the variance estimator for the aggregate output-oriented efficiency.

B.4 Simulation Results

Figures B.1–B.8 present empirical coverages for the aggregate output-oriented efficiency for $\gamma = 0.55\kappa$, 0.65κ, 0.75κ, 0.85κ, 0.95κ, κ, 1.2κ and 1.5κ, respectively. The notations for Sol1–Sol6 are the same as for the aggregate input-oriented efficiency. These figures show that the level of $\gamma = \kappa$ seems to yield better performance for the aggregate output-oriented technical efficiency.

Figure B.1: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 0.55\kappa$

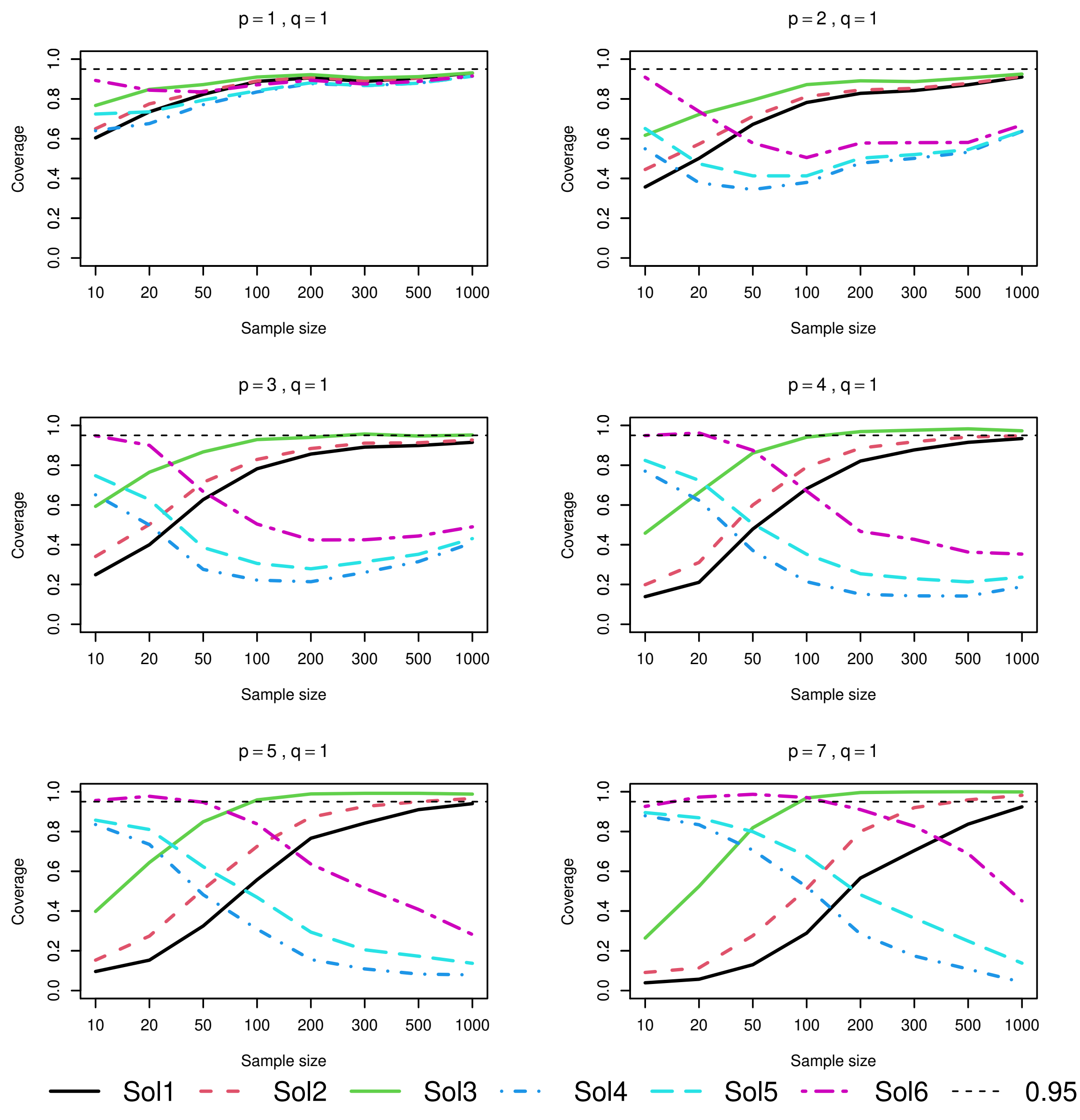


Figure B.2: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 0.65\kappa$

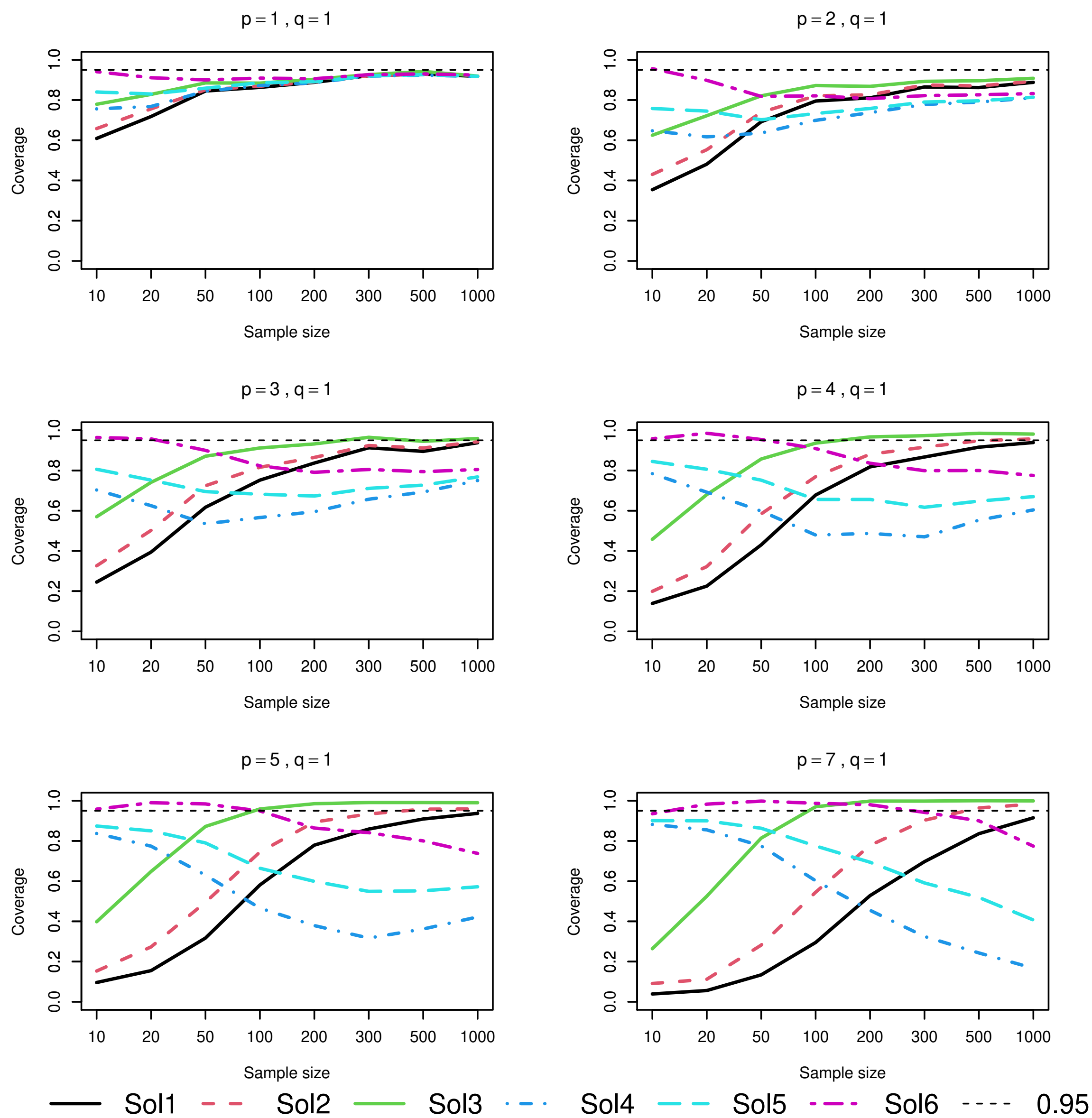


Figure B.3: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 0.75\kappa$

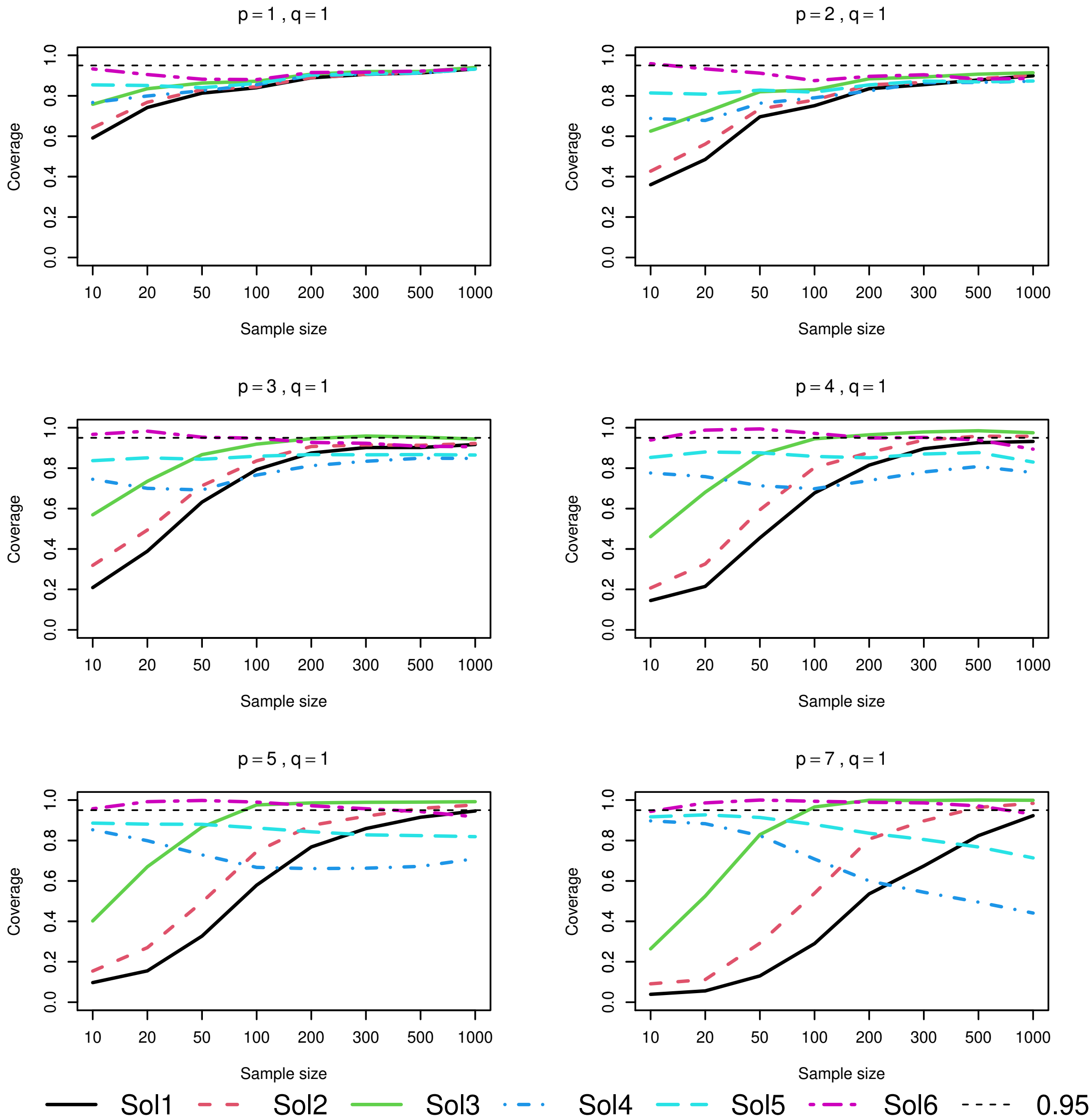


Figure B.4: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 0.85\kappa$

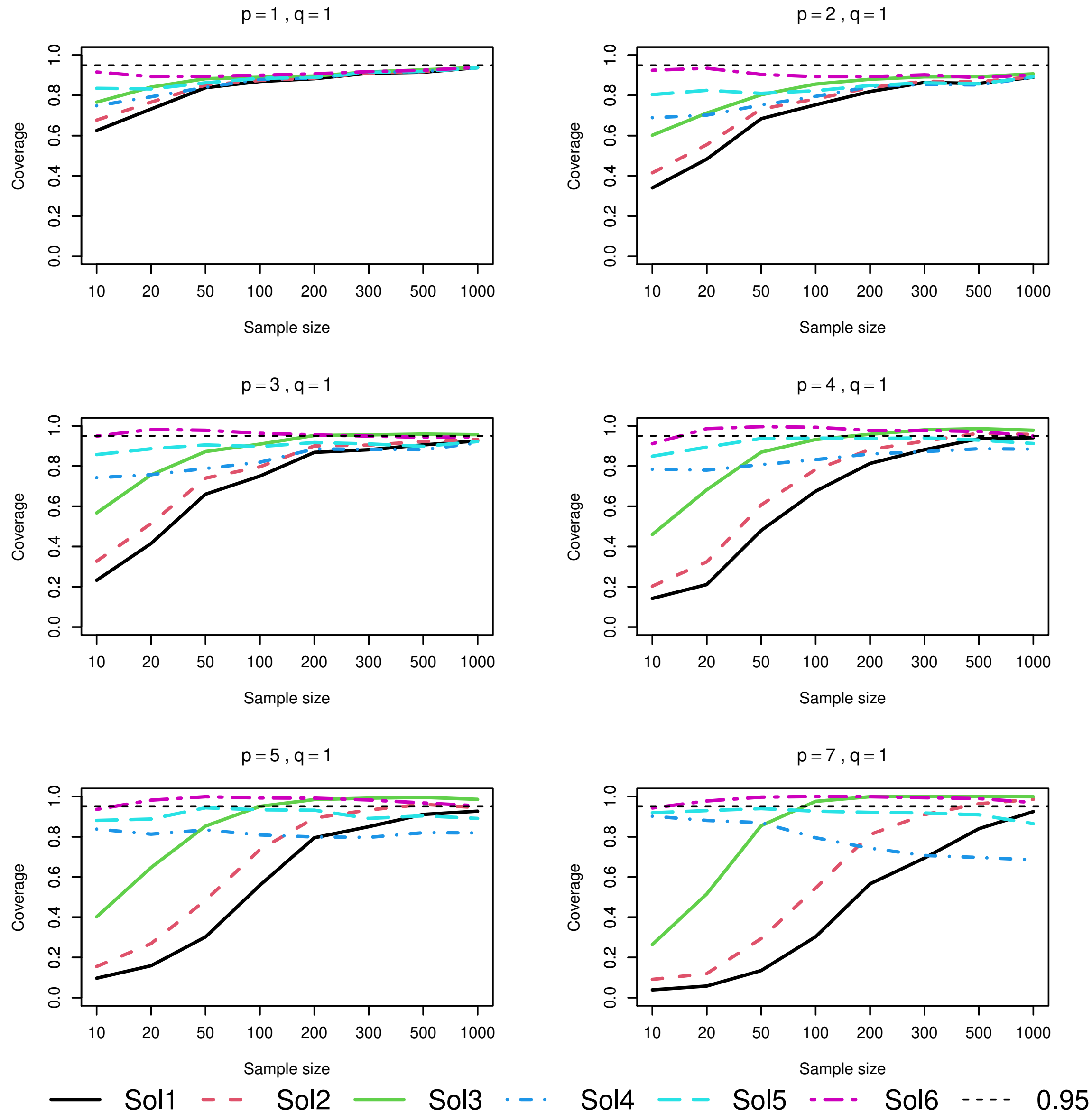


Figure B.5: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 0.95\kappa$

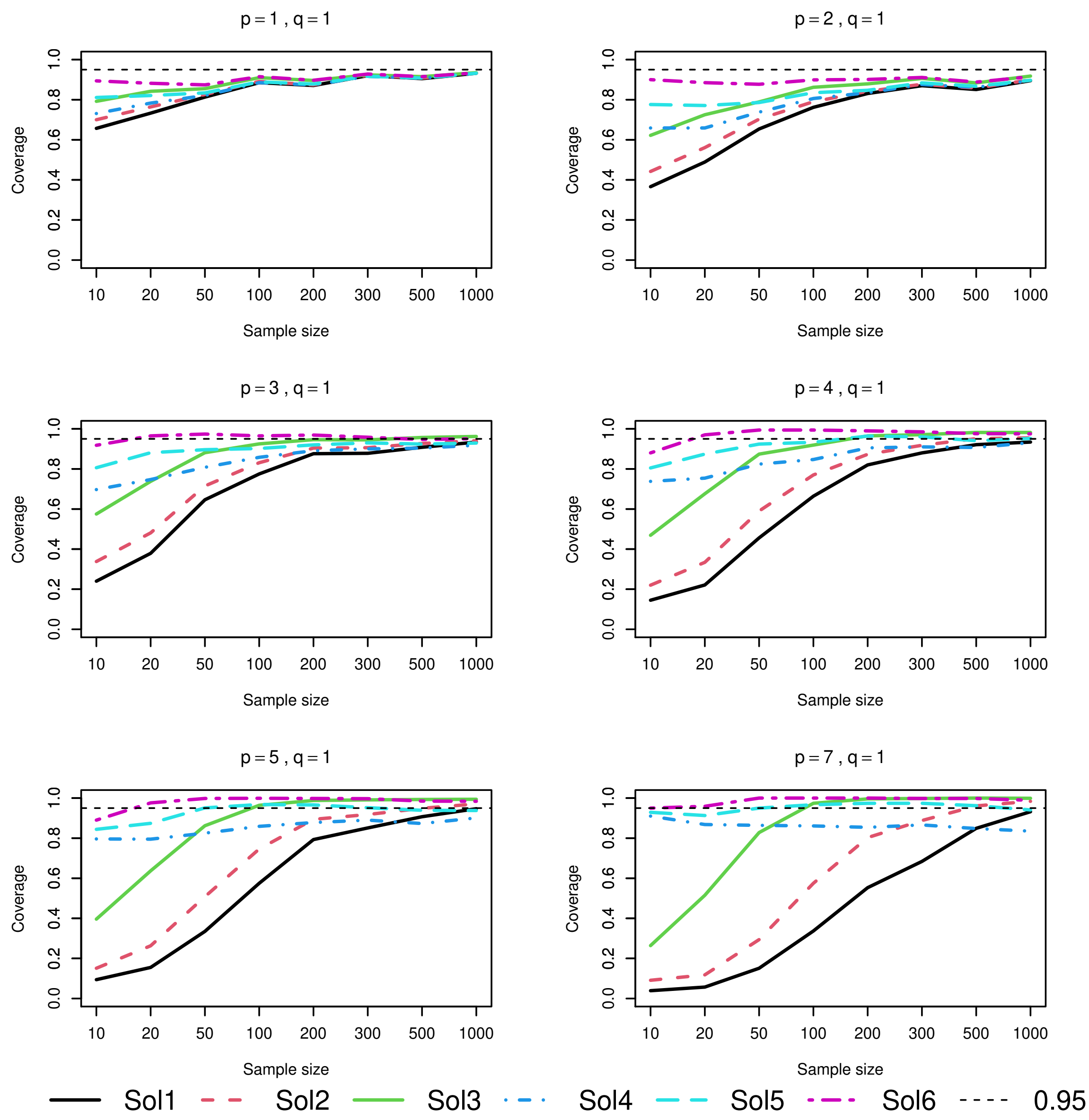


Figure B.6: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = \kappa$

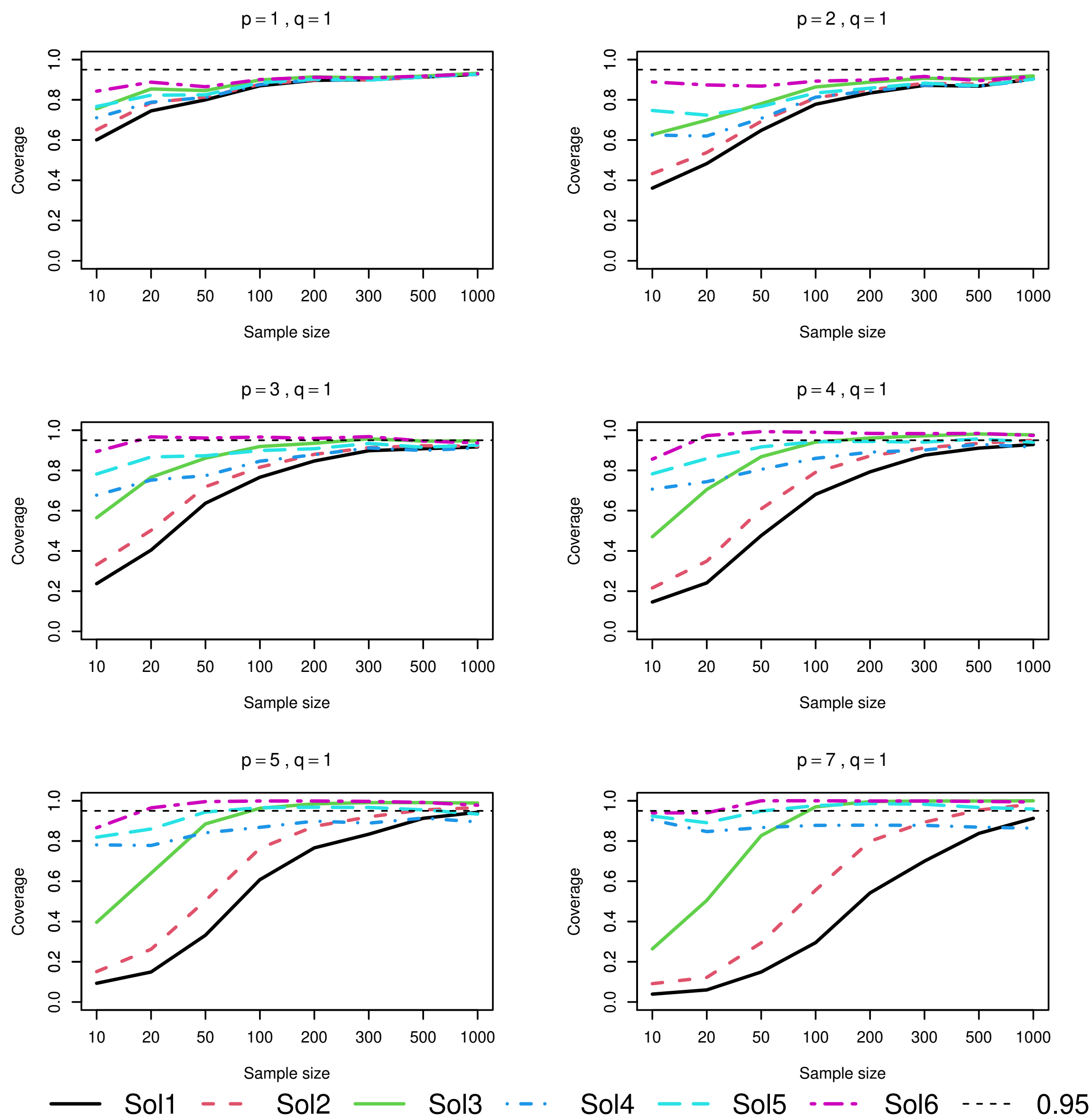


Figure B.7: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 1.2\kappa$

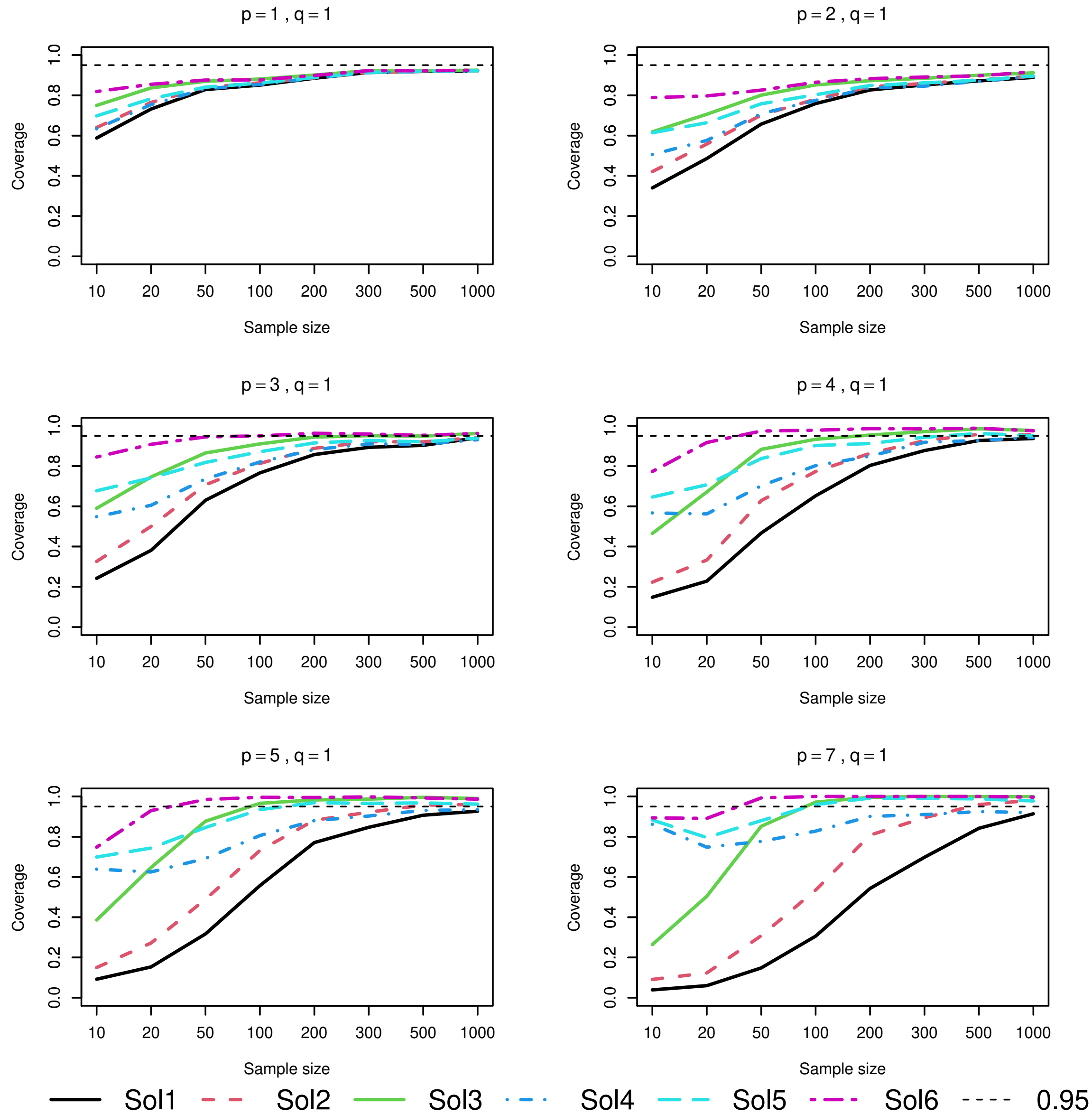
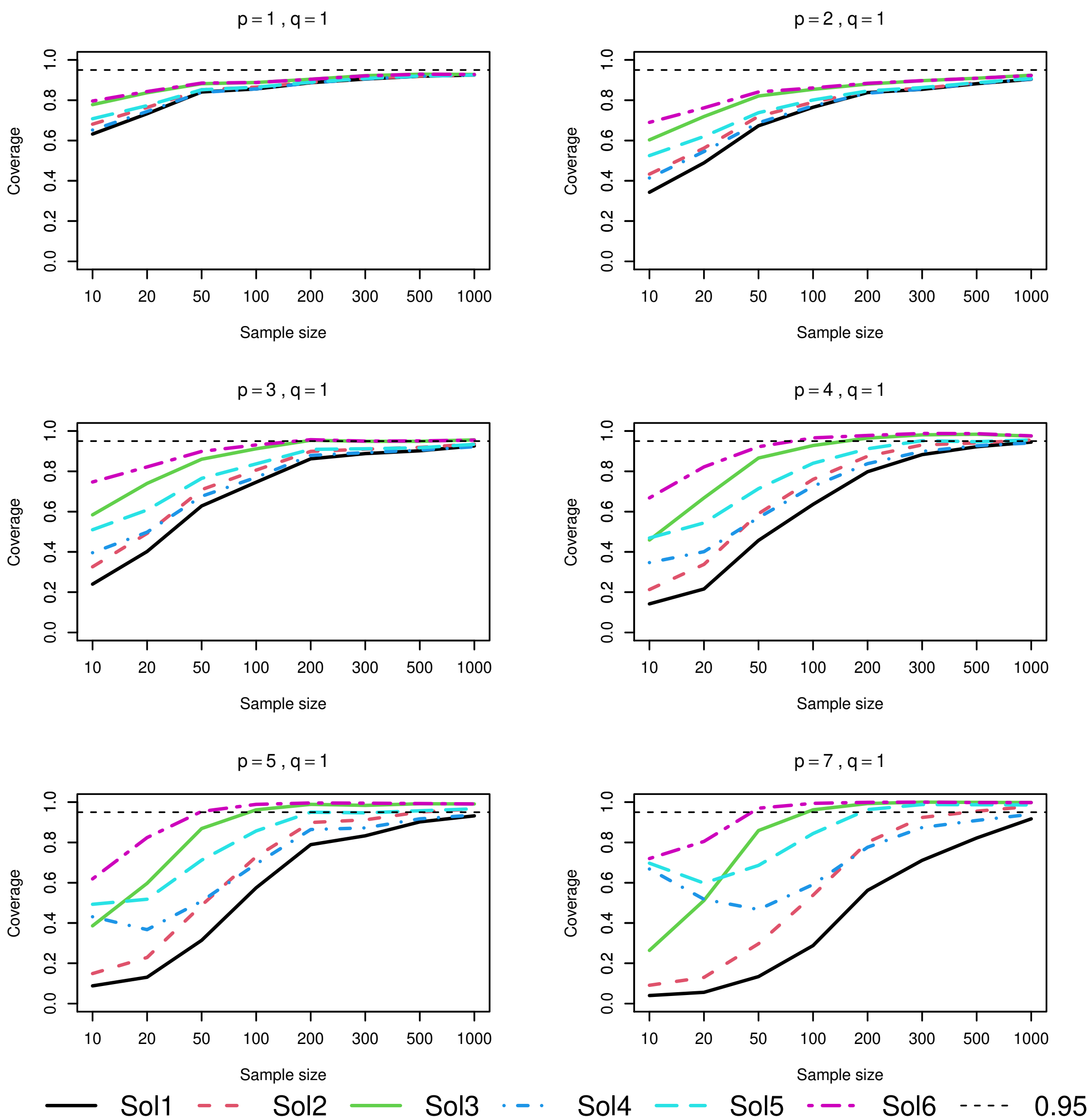


Figure B.8: Empirical Coverages for the Aggregate Output-Oriented Efficiency when $\gamma = 1.5\kappa$



Appendix C More Details on the Empirical Illustration with Penn World Table

In this appendix, we present the additional results not shown in the paper for the empirical illustration with Penn World Table.

Same as those in Badunenko et al. (2008), the countries included in our sample are: Albania, Argentina, Armenia, Australia, Austria, Azerbaijan, Belarus, Belgium Bolivia (Plurinational State of), Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Croatia, Czech Republic, Denmark, Dominican Republic, Ecuador, Estonia, Finland, France, Germany, Greece, Guatemala, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Kazakhstan, Kenya, Republic of Korea, Kyrgyzstan, Latvia, Lithuania, North Macedonia, Madagascar, Malawi, Malaysia, Mauritius, Mexico, Republic of Moldova, Morocco, Netherlands, New Zealand, Nigeria, Norway, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Russian, Federation, Sierra Leone, Singapore, Slovakia, Slovenia, Spain, Sri Lanka, Sweden, Switzerland ,Syrian Arab Republic, Taiwan, Tajikistan, Thailand, Turkey, Ukraine, United Kingdom, Uruguay, United States, Venezuela (Bolivarian Republic of), Zambia, and Zimbabwe.

C.1 Estimation Results When $\gamma = \kappa$

Tables C.1 and C.2 summarize the estimation results for the CRS-DEA estimates of aggregate and simple mean output-oriented inefficiency, respectively, when $\gamma = \kappa$.

Table C.1: CRS-DEA Estimates of Aggregate Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = \kappa$

Panel A: Without Data Sharpening

				$\widehat{\sigma}_\omega$			95% CI					
Year	$\widehat{\omega}_n$	$\widehat{B}_{\omega,n}$	$\widehat{\omega}_n - \widehat{B}_{\omega,n}$	Sol1	Sol2	Sol3	Sol1		Sol2		Sol3	
1990	1.5662	-0.1938	1.7600	1.6981	1.7092	2.1960	1.3969	2.1232	1.3945	2.1255	1.2904	2.2296
1991	1.6143	-0.1956	1.8100	1.8156	1.8261	2.3043	1.4217	2.1982	1.4195	2.2005	1.3172	2.3027
1992	1.5639	-0.1921	1.7561	1.5349	1.5469	1.9743	1.4278	2.0843	1.4253	2.0869	1.3339	2.1783
1993	1.5009	-0.1580	1.6590	1.3657	1.3748	1.7457	1.3669	1.9510	1.3650	1.9530	1.2856	2.0323
1994	1.4989	-0.1640	1.6629	1.3753	1.3850	1.7667	1.3688	1.9570	1.3667	1.9591	1.2851	2.0407
1995	1.4740	-0.1555	1.6294	1.2912	1.3005	1.6506	1.3533	1.9056	1.3513	1.9076	1.2765	1.9824
1996	1.4589	-0.1305	1.5895	1.2608	1.2675	1.5425	1.3198	1.8591	1.3184	1.8605	1.2596	1.9193
1997	1.4003	-0.1007	1.5010	1.1196	1.1241	1.3436	1.2615	1.7404	1.2606	1.7414	1.2136	1.7883
1998	1.4387	-0.1218	1.5605	1.2477	1.2536	1.5431	1.2937	1.8273	1.2924	1.8286	1.2305	1.8905
1999	1.4255	-0.1249	1.5504	1.2205	1.2269	1.5709	1.2894	1.8115	1.2881	1.8128	1.2145	1.8864
2000	1.4809	-0.1542	1.6351	1.2529	1.2624	1.5982	1.3671	1.9030	1.3651	1.9050	1.2933	1.9769
2001	1.4666	-0.1844	1.6509	1.2286	1.2423	1.7112	1.3882	1.9137	1.3853	1.9166	1.2850	2.0169
2002	1.4925	-0.1545	1.6471	1.2642	1.2736	1.6913	1.3767	1.9174	1.3747	1.9194	1.2854	2.0087
2003	1.5634	-0.1709	1.7343	1.4091	1.4194	1.8136	1.4330	2.0356	1.4308	2.0378	1.3465	2.1221
2004	1.5481	-0.1735	1.7216	1.2504	1.2624	1.6415	1.4542	1.9890	1.4516	1.9916	1.3706	2.0726
2005	1.6389	-0.2431	1.8820	0.9188	0.9504	0.9661	1.6855	2.0785	1.6788	2.0853	1.6754	2.0886
2006	1.6589	-0.2795	1.9384	0.7146	0.7673	0.8516	1.7856	2.0912	1.7743	2.1025	1.7562	2.1205
2007	1.6201	-0.1953	1.8154	0.7995	0.8230	1.0151	1.6444	1.9864	1.6394	1.9914	1.5983	2.0325
2008	1.7566	-0.3975	2.1540	1.0105	1.0858	1.3452	1.9379	2.3701	1.9218	2.3862	1.8664	2.4417
2009	1.6333	-0.2276	1.8609	1.2886	1.3086	1.6969	1.5854	2.1365	1.5811	2.1408	1.4981	2.2238
2010	1.6704	-0.2803	1.9507	1.4187	1.4461	2.0104	1.6473	2.2541	1.6414	2.2599	1.5207	2.3806
2011	1.8795	-0.4413	2.3208	1.7878	1.8414	2.6386	1.9384	2.7031	1.9270	2.7146	1.7565	2.8850
2012	1.8219	-0.3794	2.2013	1.7273	1.7685	2.3937	1.8319	2.5707	1.8231	2.5795	1.6894	2.7132
2013	1.6882	-0.2297	1.9180	1.5697	1.5864	2.0443	1.5823	2.2536	1.5787	2.2572	1.4808	2.3551
2014	1.6613	-0.3006	1.9619	1.5574	1.5861	2.4313	1.6288	2.2949	1.6227	2.3011	1.4420	2.4818
2015	1.5919	-0.1550	1.7469	1.2251	1.2349	1.5364	1.4849	2.0089	1.4828	2.0110	1.4183	2.0754
2016	1.5550	-0.1677	1.7227	1.1199	1.1324	1.4313	1.4832	1.9622	1.4805	1.9649	1.4166	2.0288
2017	1.5855	-0.1547	1.7403	1.2348	1.2445	1.5500	1.4762	2.0043	1.4741	2.0064	1.4088	2.0717
2018	1.6326	-0.1596	1.7922	1.4105	1.4195	1.6615	1.4906	2.0938	1.4886	2.0957	1.4369	2.1475
2019	1.6896	-0.1850	1.8746	1.6050	1.6156	1.9857	1.5313	2.2178	1.5291	2.2201	1.4499	2.2992

Table C.1: CRS-DEA Estimates of Aggregate Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = \kappa$ (continued)

Panel B: With Data Sharpening

				$\widehat{\sigma}_\omega$			95% CI					
Year	$\widehat{\omega}_n$	$\widehat{B}_{\omega,n}$	$\widehat{\omega}_n - \widehat{B}_{\omega,n}$	Sol4	Sol5	Sol6	Sol4		Sol5		Sol6	
1990	1.5680	-0.1938	1.7619	1.6919	1.7030	2.1896	1.4001	2.1237	1.3977	2.1261	1.2936	2.2301
1991	1.6216	-0.1956	1.8172	1.7780	1.7887	2.2651	1.4370	2.1974	1.4347	2.1997	1.3328	2.3016
1992	1.5651	-0.1921	1.7573	1.5344	1.5464	1.9738	1.4291	2.0854	1.4266	2.0879	1.3352	2.1793
1993	1.5054	-0.1580	1.6634	1.3436	1.3528	1.7228	1.3761	1.9507	1.3741	1.9527	1.2950	2.0318
1994	1.5122	-0.1640	1.6762	1.3008	1.3111	1.6889	1.3980	1.9544	1.3958	1.9566	1.3150	2.0374
1995	1.4771	-0.1555	1.6326	1.2735	1.2829	1.6321	1.3603	1.9049	1.3583	1.9070	1.2836	1.9816
1996	1.4693	-0.1305	1.5999	1.2015	1.2086	1.4801	1.3429	1.8568	1.3414	1.8583	1.2833	1.9164
1997	1.4110	-0.1007	1.5117	1.0616	1.0664	1.2825	1.2847	1.7388	1.2837	1.7398	1.2375	1.7860
1998	1.4517	-0.1218	1.5736	1.1760	1.1823	1.4682	1.3221	1.8251	1.3207	1.8264	1.2596	1.8876
1999	1.4314	-0.1249	1.5564	1.1898	1.1964	1.5395	1.3019	1.8108	1.3005	1.8122	1.2271	1.8856
2000	1.4823	-0.1542	1.6366	1.2465	1.2560	1.5915	1.3700	1.9031	1.3680	1.9051	1.2962	1.9769
2001	1.4761	-0.1844	1.6605	1.1711	1.1855	1.6530	1.4101	1.9109	1.4070	1.9140	1.3070	2.0140
2002	1.4955	-0.1545	1.6500	1.2505	1.2600	1.6777	1.3826	1.9175	1.3806	1.9195	1.2913	2.0088
2003	1.5740	-0.1709	1.7449	1.3427	1.3536	1.7469	1.4577	2.0320	1.4554	2.0343	1.3713	2.1184
2004	1.5599	-0.1737	1.7337	1.1735	1.1863	1.5633	1.4827	1.9846	1.4800	1.9874	1.3994	2.0680
2005	1.6393	-0.2431	1.8824	0.9193	0.9509	0.9665	1.6858	2.0790	1.6790	2.0857	1.6757	2.0891
2006	1.6590	-0.2795	1.9385	0.7147	0.7674	0.8517	1.7857	2.0913	1.7744	2.1026	1.7564	2.1206
2007	1.6208	-0.1953	1.8161	0.7998	0.8233	1.0155	1.6451	1.9871	1.6400	1.9922	1.5989	2.0333
2008	1.7568	-0.3975	2.1543	1.0106	1.0859	1.3453	1.9382	2.3704	1.9221	2.3865	1.8666	2.4420
2009	1.6336	-0.2276	1.8612	1.2887	1.3086	1.6970	1.5856	2.1368	1.5814	2.1411	1.4983	2.2241
2010	1.6707	-0.2803	1.9509	1.4187	1.4461	2.0104	1.6475	2.2543	1.6417	2.2602	1.5210	2.3809
2011	1.8798	-0.4413	2.3211	1.7878	1.8414	2.6386	1.9387	2.7034	1.9273	2.7149	1.7568	2.8853
2012	1.8220	-0.3794	2.2015	1.7273	1.7685	2.3937	1.8321	2.5709	1.8233	2.5797	1.6896	2.7134
2013	1.6945	-0.2299	1.9245	1.5340	1.5512	2.0079	1.5964	2.2525	1.5927	2.2562	1.4951	2.3539
2014	1.6685	-0.3006	1.9691	1.5143	1.5438	2.3901	1.6453	2.2929	1.6390	2.2992	1.4580	2.4802
2015	1.5989	-0.1550	1.7539	1.1817	1.1918	1.4934	1.5012	2.0066	1.4990	2.0088	1.4345	2.0733
2016	1.5617	-0.1677	1.7294	1.0736	1.0866	1.3846	1.4998	1.9590	1.4970	1.9618	1.4333	2.0255
2017	1.5875	-0.1547	1.7422	1.2242	1.2339	1.5393	1.4804	2.0040	1.4784	2.0061	1.4131	2.0714
2018	1.6418	-0.1596	1.8014	1.3495	1.3589	1.5982	1.5128	2.0900	1.5108	2.0920	1.4596	2.1432
2019	1.6923	-0.1850	1.8773	1.5896	1.6003	1.9701	1.5374	2.2172	1.5351	2.2195	1.4560	2.2986

Note: $\widehat{\omega}_n$ is the aggregate output efficiency estimate, $\widehat{B}_{\omega,n}$ is the estimated bias and $\widehat{\sigma}_\varphi$ is the standard deviation estimate.

Table C.2: CRS-DEA Estimates of Simple Mean Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = \kappa$

Panel A: Without Data Sharpening

				$\widehat{\sigma}_\lambda$			95% CI					
Year	$\widehat{\lambda}_n$	$\widehat{B}_{\lambda,n}$	$\widehat{\lambda}_n - \widehat{B}_{\lambda,n}$	Sol1	Sol2	Sol3	Sol1		Sol2		Sol3	
1990	1.8756	-0.2782	2.1538	0.9833	1.0219	1.2356	1.9436	2.3641	1.9353	2.3724	1.8896	2.4181
1991	2.0193	-0.2749	2.2942	1.2867	1.3157	1.5616	2.0190	2.5693	2.0128	2.5755	1.9602	2.6281
1992	2.1014	-0.3975	2.4989	1.2357	1.2980	1.6613	2.2347	2.7632	2.2213	2.7765	2.1436	2.8542
1993	1.9223	-0.2449	2.1672	1.0357	1.0642	1.2563	1.9457	2.3887	1.9396	2.3948	1.8985	2.4358
1994	2.0003	-0.2788	2.2791	1.2705	1.3008	1.5071	2.0074	2.5508	2.0009	2.5573	1.9568	2.6014
1995	2.0446	-0.3316	2.3762	1.4099	1.4483	1.7249	2.0747	2.6777	2.0665	2.6860	2.0074	2.7451
1996	2.0208	-0.2813	2.3021	1.3759	1.4044	1.6370	2.0078	2.5963	2.0017	2.6024	1.9520	2.6521
1997	1.8419	-0.1399	1.9818	1.1390	1.1476	1.2259	1.7382	2.2253	1.7364	2.2272	1.7196	2.2439
1998	1.9422	-0.1963	2.1385	1.2610	1.2762	1.4228	1.8688	2.4082	1.8656	2.4114	1.8342	2.4428
1999	1.8861	-0.1798	2.0658	1.1352	1.1493	1.2928	1.8231	2.3086	1.8201	2.3116	1.7894	2.3423
2000	1.9118	-0.2199	2.1318	1.0325	1.0557	1.1723	1.9110	2.3526	1.9060	2.3575	1.8811	2.3825
2001	1.8663	-0.2547	2.1210	0.9482	0.9818	1.1155	1.9182	2.3237	1.9110	2.3309	1.8824	2.3595
2002	1.8570	-0.2133	2.0703	0.8674	0.8933	1.0032	1.8848	2.2558	1.8793	2.2613	1.8558	2.2848
2003	1.9374	-0.2651	2.2024	0.8342	0.8753	1.0098	2.0241	2.3808	2.0153	2.3896	1.9865	2.4184
2004	1.8537	-0.2315	2.0852	0.7313	0.7670	0.8419	1.9288	2.2415	1.9211	2.2492	1.9051	2.2652
2005	1.8193	-0.2000	2.0193	0.6181	0.6497	0.6902	1.8872	2.1515	1.8804	2.1583	1.8718	2.1669
2006	1.7873	-0.2383	2.0256	0.5861	0.6327	0.6530	1.9003	2.1509	1.8903	2.1609	1.8860	2.1653
2007	1.6800	-0.1984	1.8784	0.5114	0.5486	0.5701	1.7691	1.9878	1.7611	1.9957	1.7565	2.0003
2008	1.8351	-0.4040	2.2391	0.5887	0.7140	0.7235	2.1132	2.3650	2.0864	2.3918	2.0844	2.3938
2009	1.7356	-0.2728	2.0084	0.5799	0.6409	0.7356	1.8844	2.1325	1.8714	2.1455	1.8511	2.1657
2010	1.8025	-0.3287	2.1312	0.6330	0.7133	0.8858	1.9958	2.2666	1.9787	2.2837	1.9418	2.3206
2011	2.0174	-0.4896	2.5070	0.7513	0.8968	1.0772	2.3463	2.6677	2.3152	2.6988	2.2767	2.7374
2012	1.9417	-0.4453	2.3870	0.6763	0.8098	0.9308	2.2424	2.5316	2.2138	2.5602	2.1879	2.5860
2013	1.7869	-0.2758	2.0628	0.6174	0.6762	0.7688	1.9307	2.1948	1.9181	2.2074	1.8984	2.2272
2014	1.7520	-0.2692	2.0212	0.6549	0.7081	0.8153	1.8812	2.1613	1.8698	2.1726	1.8469	2.1956
2015	1.6945	-0.1884	1.8829	0.6321	0.6595	0.7132	1.7477	2.0180	1.7418	2.0239	1.7303	2.0354
2016	1.6705	-0.2059	1.8765	0.7012	0.7308	0.8197	1.7265	2.0264	1.7202	2.0327	1.7012	2.0517
2017	1.7471	-0.1864	1.9335	1.1611	1.1760	1.3158	1.6852	2.1818	1.6820	2.1850	1.6521	2.2149
2018	1.8196	-0.2271	2.0467	1.4414	1.4592	1.7410	1.7384	2.3550	1.7346	2.3588	1.6744	2.4190
2019	1.9156	-0.2320	2.1476	2.0696	2.0826	2.4558	1.7051	2.5902	1.7023	2.5930	1.6225	2.6728

Table C.2: CRS-DEA Estimates of Simple Mean Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = \kappa$ (continued)

Panel B: With Data Sharpening

				$\widehat{\sigma}_\lambda$			95% CI					
Year	$\widehat{\lambda}_n$	$\widehat{B}_{\lambda,n}$	$\widehat{\lambda}_n - \widehat{B}_{\lambda,n}$	Sol4	Sol5	Sol6	Sol4		Sol5		Sol6	
1990	1.8776	-0.2782	2.1558	0.9815	1.0202	1.2338	1.9459	2.3657	1.9377	2.3740	1.8920	2.4197
1991	2.0210	-0.2749	2.2958	1.2853	1.3144	1.5602	2.0210	2.5707	2.0148	2.5769	1.9622	2.6295
1992	2.1033	-0.3975	2.5008	1.2340	1.2964	1.6596	2.2370	2.7647	2.2236	2.7781	2.1459	2.8557
1993	1.9243	-0.2449	2.1692	1.0339	1.0625	1.2544	1.9481	2.3903	1.9420	2.3964	1.9010	2.4375
1994	2.0028	-0.2788	2.2817	1.2686	1.2988	1.5049	2.0104	2.5529	2.0039	2.5594	1.9598	2.6035
1995	2.0451	-0.3316	2.3767	1.4095	1.4480	1.7245	2.0753	2.6781	2.0671	2.6864	2.0079	2.7455
1996	2.0220	-0.2813	2.3033	1.3750	1.4035	1.6360	2.0092	2.5973	2.0031	2.6034	1.9534	2.6531
1997	1.8442	-0.1399	1.9840	1.1374	1.1459	1.2241	1.7408	2.2272	1.7390	2.2291	1.7222	2.2458
1998	1.9438	-0.1963	2.1402	1.2598	1.2750	1.4215	1.8708	2.4096	1.8675	2.4128	1.8362	2.4442
1999	1.8879	-0.1798	2.0677	1.1338	1.1479	1.2913	1.8252	2.3101	1.8222	2.3132	1.7915	2.3438
2000	1.9137	-0.2199	2.1336	1.0309	1.0541	1.1706	1.9131	2.3541	1.9082	2.3590	1.8833	2.3839
2001	1.8679	-0.2547	2.1226	0.9467	0.9804	1.1139	1.9201	2.3250	1.9129	2.3322	1.8844	2.3608
2002	1.8588	-0.2133	2.0721	0.8657	0.8916	1.0013	1.8870	2.2572	1.8814	2.2628	1.8580	2.2862
2003	1.9395	-0.2651	2.2046	0.8318	0.8730	1.0073	2.0267	2.3825	2.0179	2.3913	1.9892	2.4200
2004	1.8555	-0.2315	2.0870	0.7292	0.7651	0.8396	1.9310	2.2429	1.9234	2.2506	1.9074	2.2665
2005	1.8209	-0.2000	2.0209	0.6161	0.6477	0.6879	1.8892	2.1527	1.8824	2.1594	1.8738	2.1680
2006	1.7882	-0.2383	2.0265	0.5849	0.6316	0.6516	1.9014	2.1516	1.8915	2.1616	1.8872	2.1659
2007	1.6819	-0.1984	1.8804	0.5089	0.5463	0.5672	1.7715	1.9892	1.7635	1.9972	1.7591	2.0017
2008	1.8360	-0.4040	2.2399	0.5876	0.7130	0.7221	2.1143	2.3656	2.0874	2.3924	2.0855	2.3944
2009	1.7365	-0.2728	2.0093	0.5788	0.6399	0.7344	1.8856	2.1331	1.8725	2.1462	1.8523	2.1664
2010	1.8034	-0.3287	2.1321	0.6319	0.7123	0.8846	1.9970	2.2672	1.9798	2.2844	1.9430	2.3213
2011	2.0185	-0.4896	2.5081	0.7499	0.8956	1.0757	2.3477	2.6684	2.3165	2.6996	2.2780	2.7381
2012	1.9425	-0.4453	2.3878	0.6752	0.8089	0.9296	2.2434	2.5322	2.2148	2.5608	2.1890	2.5866
2013	1.7886	-0.2759	2.0644	0.6154	0.6744	0.7666	1.9328	2.1960	1.9202	2.2087	1.9005	2.2284
2014	1.7539	-0.2692	2.0231	0.6529	0.7062	0.8130	1.8834	2.1627	1.8720	2.1741	1.8492	2.1969
2015	1.6977	-0.1884	1.8861	0.6285	0.6561	0.7092	1.7517	2.0205	1.7458	2.0264	1.7345	2.0378
2016	1.6726	-0.2059	1.8785	0.6993	0.7290	0.8175	1.7290	2.0280	1.7226	2.0344	1.7037	2.0533
2017	1.7497	-0.1864	1.9361	1.1595	1.1743	1.3140	1.6882	2.1841	1.6850	2.1873	1.6551	2.2171
2018	1.8211	-0.2271	2.0482	1.4406	1.4584	1.7402	1.7401	2.3563	1.7363	2.3601	1.6761	2.4203
2019	1.9177	-0.2320	2.1498	2.0687	2.0816	2.4549	1.7074	2.5921	1.7046	2.5949	1.6248	2.6747

Note: $\widehat{\lambda}_n$ is the simple mean output efficiency estimate, $\widehat{B}_{\lambda,n}$ is the estimated bias and $\widehat{\sigma}_\lambda$ is the standard deviation estimate.

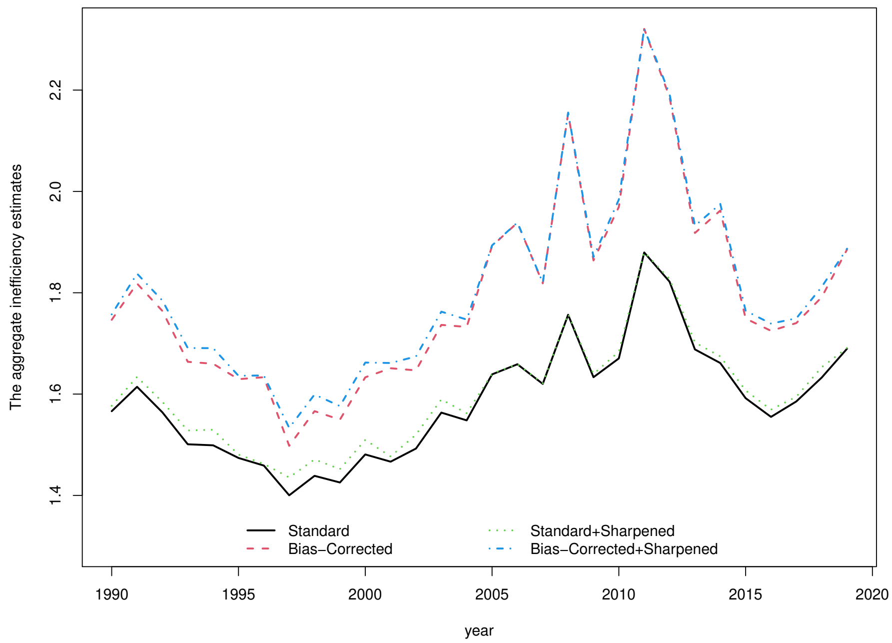
C.2 Estimation Results When $\gamma = 0.75\kappa$

In this subsection, we present the corresponding results for the case when $\gamma = 0.95\kappa$.

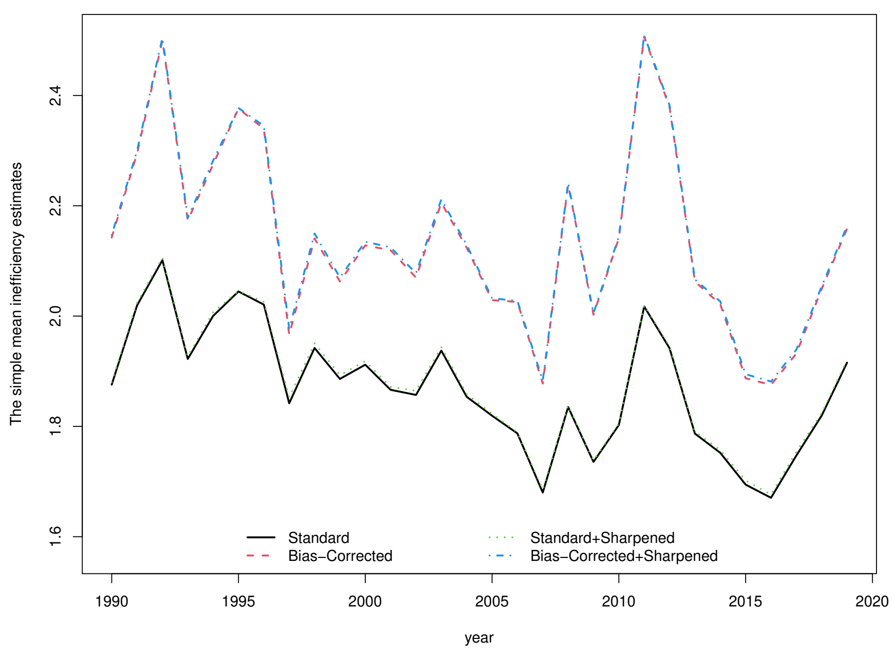
Figures C.1a and C.1b present the dynamic changes of aggregate and simple mean inefficiency, respectively, over 1990–2019. Figures C.2a and C.2b present the estimates of the standard deviations for aggregate and simple mean inefficiency, respectively, for Sol1–Sol6 over 1990–2019. Figures C.3a and C.3b present the dynamics of CIs constructed using Sol3 and Sol6 for aggregate and simple mean inefficiency, respectively, over 1990–2019.

Tables C.3 and C.4 summarize the estimation results of the CRS-DEA estimates of aggregate and simple mean output-oriented inefficiency, respectively, when $\gamma = 0.75\kappa$.

From these figures, we can see that the results for $\gamma = 0.75\kappa$ are similar to those for $\gamma = \kappa$.

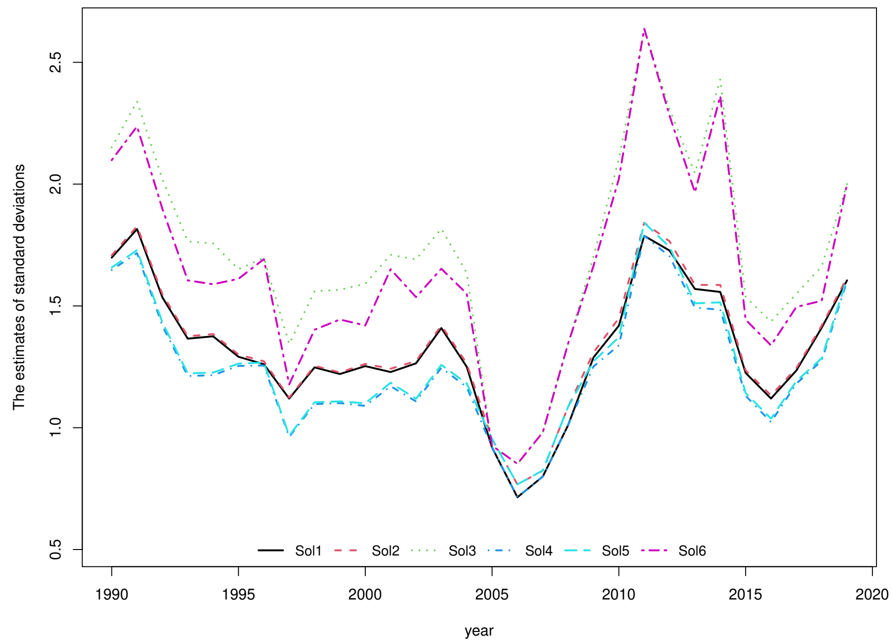


(a) For the Aggregate Output-Oriented Inefficiency

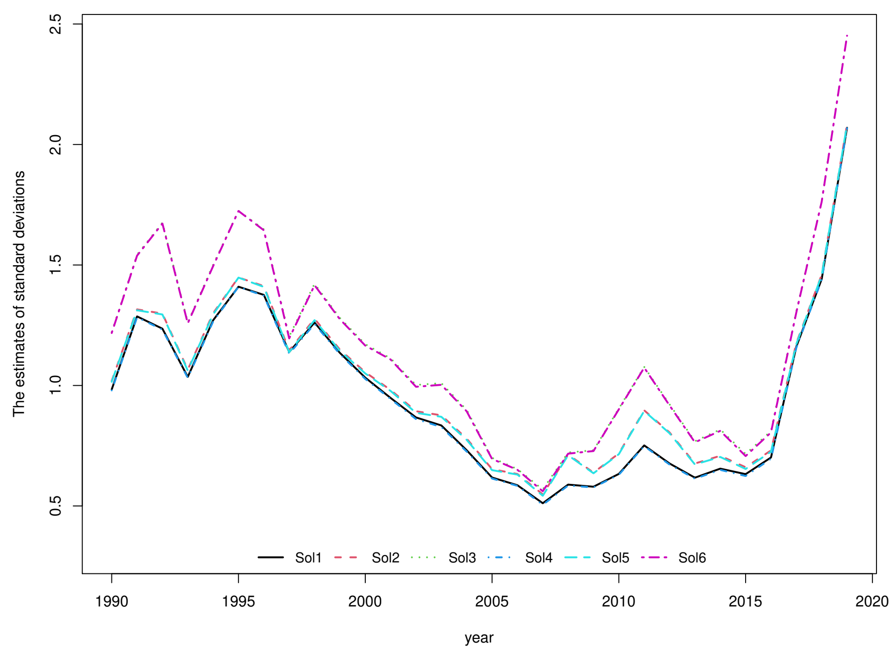


(b) For the Simple Mean Output-Oriented Inefficiency

Figure C.1: The Estimated Aggregate/Simple Mean Output-Oriented Inefficiency for Countries/Regions from 1990 to 2019 when $\gamma = 0.75\kappa$

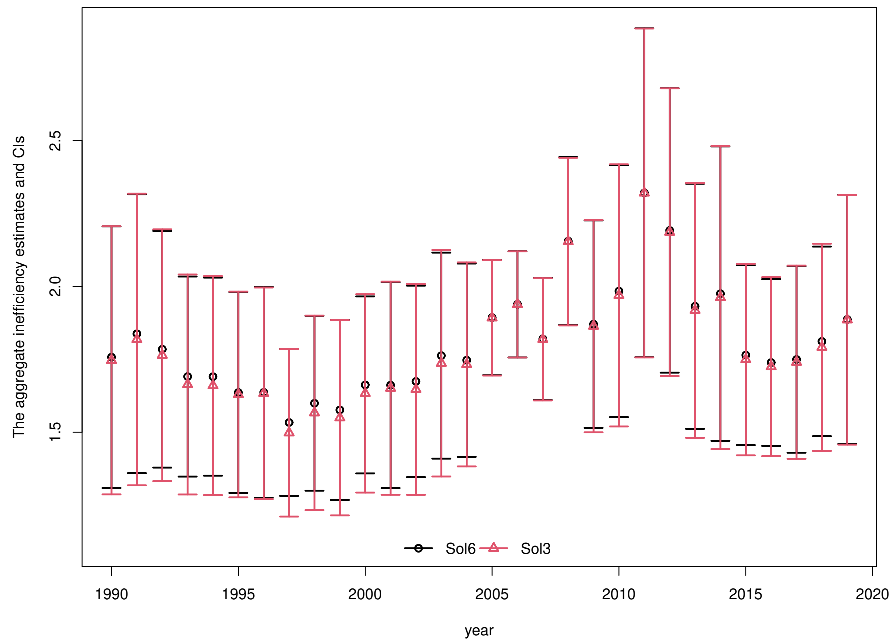


(a) For the Aggregate Output-Oriented Inefficiency

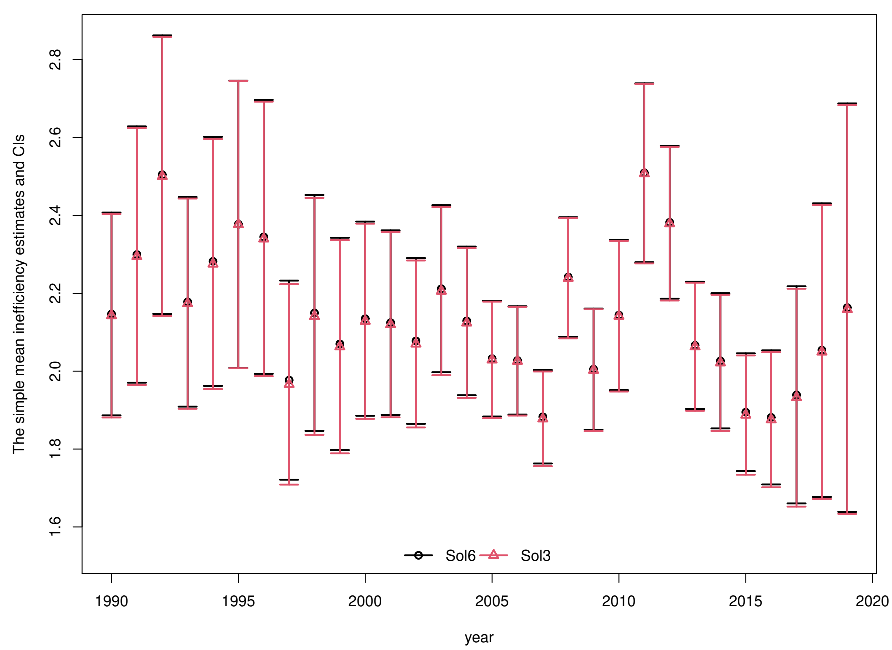


(b) For the Simple Mean Output-Oriented Inefficiency

Figure C.2: The Estimated Standard Deviations for Aggregate/Simple Mean Output-Oriented Inefficiency for Countries/Regions from 1990 to 2019 when $\gamma = 0.75\kappa$



(a) For the Aggregate Output-Oriented Inefficiency



(b) For the Simple Mean Output-Oriented Inefficiency

Figure C.3: CRS-DEA Estimates of Aggregate/Simple Mean Output-Oriented Inefficiency and their 95% Confidence Intervals for Countries/Regions from 1990 to 2019 when $\gamma = 0.75\kappa$

Table C.3: CRS-DEA Estimates of Aggregate Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = 0.75\kappa$

Panel A: Without Data Sharpening

				$\widehat{\sigma}_\omega$			95% CI					
Year	$\widehat{\omega}_n$	$\widehat{B}_{\omega,n}$	$\widehat{\omega}_n - \widehat{B}_{\omega,n}$	Sol1	Sol2	Sol3	Sol1		Sol2		Sol3	
1990	1.5662	-0.1804	1.7466	1.6981	1.7077	2.1509	1.3834	2.1097	1.3814	2.1118	1.2866	2.2065
1991	1.6143	-0.2038	1.8181	1.8156	1.8270	2.3398	1.4299	2.2064	1.4274	2.2088	1.3178	2.3185
1992	1.5639	-0.2001	1.7640	1.5349	1.5479	2.0198	1.4358	2.0923	1.4330	2.0950	1.3321	2.1959
1993	1.5009	-0.1628	1.6637	1.3657	1.3753	1.7646	1.3717	1.9558	1.3696	1.9578	1.2864	2.0411
1994	1.4989	-0.1610	1.6599	1.3753	1.3846	1.7566	1.3658	1.9540	1.3638	1.9560	1.2842	2.0355
1995	1.4740	-0.1555	1.6294	1.2912	1.3005	1.6506	1.3533	1.9056	1.3513	1.9076	1.2765	1.9824
1996	1.4589	-0.1744	1.6333	1.2608	1.2728	1.6976	1.3637	1.9029	1.3611	1.9055	1.2703	1.9964
1997	1.4003	-0.0976	1.4979	1.1196	1.1238	1.3436	1.2585	1.7373	1.2575	1.7382	1.2105	1.7852
1998	1.4387	-0.1277	1.5664	1.2477	1.2542	1.5602	1.2996	1.8332	1.2982	1.8346	1.2327	1.9000
1999	1.4255	-0.1240	1.5495	1.2205	1.2268	1.5663	1.2885	1.8105	1.2872	1.8119	1.2146	1.8845
2000	1.4809	-0.1522	1.6331	1.2529	1.2621	1.5911	1.3652	1.9010	1.3632	1.9030	1.2928	1.9734
2001	1.4666	-0.1844	1.6510	1.2286	1.2423	1.7099	1.3883	1.9137	1.3853	1.9167	1.2853	2.0167
2002	1.4925	-0.1543	1.6468	1.2642	1.2736	1.6910	1.3765	1.9172	1.3745	1.9192	1.2852	2.0084
2003	1.5634	-0.1731	1.7366	1.4091	1.4197	1.8171	1.4353	2.0379	1.4330	2.0402	1.3480	2.1252
2004	1.5481	-0.1844	1.7325	1.2504	1.2639	1.6365	1.4651	1.9999	1.4622	2.0028	1.3826	2.0825
2005	1.6389	-0.2533	1.8922	0.9188	0.9531	0.9253	1.6958	2.0887	1.6884	2.0961	1.6944	2.0901
2006	1.6589	-0.2795	1.9384	0.7146	0.7673	0.8516	1.7856	2.0912	1.7743	2.1025	1.7562	2.1205
2007	1.6201	-0.1985	1.8186	0.7995	0.8238	0.9804	1.6476	1.9895	1.6424	1.9947	1.6089	2.0282
2008	1.7566	-0.3974	2.1540	1.0105	1.0858	1.3452	1.9379	2.3700	1.9218	2.3862	1.8663	2.4416
2009	1.6333	-0.2305	1.8638	1.2886	1.3091	1.7031	1.5882	2.1394	1.5839	2.1437	1.4996	2.2280
2010	1.6704	-0.2990	1.9694	1.4187	1.4498	2.1029	1.6660	2.2728	1.6593	2.2794	1.5197	2.4191
2011	1.8795	-0.4413	2.3208	1.7878	1.8414	2.6386	1.9384	2.7031	1.9270	2.7146	1.7565	2.8850
2012	1.8219	-0.3644	2.1862	1.7273	1.7653	2.3077	1.8168	2.5556	1.8087	2.5637	1.6927	2.6797
2013	1.6882	-0.2297	1.9180	1.5697	1.5864	2.0443	1.5823	2.2536	1.5787	2.2572	1.4808	2.3551
2014	1.6613	-0.3009	1.9622	1.5574	1.5862	2.4316	1.6292	2.2953	1.6230	2.3014	1.4422	2.4822
2015	1.5919	-0.1573	1.7492	1.2251	1.2352	1.5366	1.4872	2.0112	1.4850	2.0133	1.4206	2.0778
2016	1.5550	-0.1698	1.7248	1.1199	1.1327	1.4356	1.4853	1.9643	1.4826	1.9670	1.4178	2.0318
2017	1.5855	-0.1547	1.7402	1.2348	1.2444	1.5499	1.4761	2.0043	1.4741	2.0063	1.4088	2.0716
2018	1.6326	-0.1586	1.7912	1.4105	1.4194	1.6620	1.4896	2.0928	1.4877	2.0947	1.4358	2.1466
2019	1.6896	-0.1958	1.8854	1.6050	1.6169	2.0017	1.5422	2.2286	1.5397	2.2312	1.4574	2.3135

Table C.3: CRS-DEA Estimates of Aggregate Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = 0.75\kappa$ (continued)

Panel B: With Data Sharpening

				$\widehat{\sigma}_\omega$			95% CI					
Year	$\widehat{\omega}_n$	$\widehat{B}_{\omega,n}$	$\widehat{\omega}_n - \widehat{B}_{\omega,n}$	Sol4	Sol5	Sol6	Sol4		Sol5		Sol6	
1990	1.5769	-0.1804	1.7574	1.6479	1.6577	2.0985	1.4050	2.1098	1.4028	2.1119	1.3086	2.2061
1991	1.6339	-0.2038	1.8378	1.7168	1.7288	2.2368	1.4706	2.2049	1.4681	2.2075	1.3594	2.3161
1992	1.5845	-0.2001	1.7846	1.4180	1.4321	1.8985	1.4813	2.0878	1.4783	2.0908	1.3786	2.1906
1993	1.5282	-0.1628	1.6909	1.2123	1.2232	1.6047	1.4317	1.9502	1.4293	1.9525	1.3478	2.0341
1994	1.5296	-0.1610	1.6906	1.2157	1.2263	1.5889	1.4307	1.9506	1.4284	1.9529	1.3508	2.0304
1995	1.4807	-0.1555	1.6362	1.2537	1.2633	1.6114	1.3681	1.9043	1.3660	1.9063	1.2916	1.9808
1996	1.4623	-0.1745	1.6367	1.2555	1.2675	1.6921	1.3683	1.9052	1.3657	1.9078	1.2749	1.9986
1997	1.4354	-0.0979	1.5334	0.9623	0.9673	1.1781	1.3276	1.7392	1.3265	1.7402	1.2814	1.7853
1998	1.4711	-0.1281	1.5992	1.0969	1.1044	1.4023	1.3646	1.8338	1.3630	1.8354	1.2993	1.8991
1999	1.4520	-0.1243	1.5763	1.1011	1.1081	1.4442	1.3408	1.8118	1.3393	1.8133	1.2674	1.8851
2000	1.5095	-0.1527	1.6622	1.0896	1.1002	1.4198	1.4292	1.8952	1.4269	1.8975	1.3586	1.9659
2001	1.4769	-0.1844	1.6613	1.1702	1.1847	1.6507	1.4111	1.9116	1.4080	1.9147	1.3083	2.0143
2002	1.5199	-0.1544	1.6743	1.1080	1.1187	1.5362	1.4373	1.9112	1.4350	1.9135	1.3458	2.0028
2003	1.5895	-0.1732	1.7627	1.2461	1.2580	1.6526	1.4962	2.0292	1.4937	2.0317	1.4093	2.1161
2004	1.5623	-0.1848	1.7471	1.1670	1.1816	1.5506	1.4975	1.9967	1.4944	1.9998	1.4155	2.0787
2005	1.6401	-0.2534	1.8934	0.9201	0.9543	0.9262	1.6967	2.0902	1.6894	2.0975	1.6954	2.0915
2006	1.6592	-0.2795	1.9387	0.7149	0.7676	0.8519	1.7858	2.0915	1.7745	2.1028	1.7565	2.1208
2007	1.6214	-0.1985	1.8199	0.8001	0.8244	0.9811	1.6488	1.9910	1.6436	1.9962	1.6101	2.0297
2008	1.7582	-0.3975	2.1557	1.0110	1.0863	1.3459	1.9395	2.3719	1.9234	2.3880	1.8679	2.4435
2009	1.6397	-0.2310	1.8707	1.2513	1.2725	1.6640	1.6031	2.1383	1.5986	2.1428	1.5149	2.2265
2010	1.6838	-0.3002	1.9840	1.3383	1.3716	2.0210	1.6977	2.2702	1.6906	2.2773	1.5518	2.4161
2011	1.8802	-0.4413	2.3214	1.7878	1.8414	2.6386	1.9391	2.7037	1.9276	2.7152	1.7572	2.8857
2012	1.8273	-0.3649	2.1922	1.7036	1.7422	2.2812	1.8279	2.5565	1.8196	2.5648	1.7043	2.6800
2013	1.7019	-0.2302	1.9320	1.4931	1.5107	1.9661	1.6127	2.2513	1.6090	2.2551	1.5116	2.3525
2014	1.6744	-0.3010	1.9754	1.4843	1.5146	2.3616	1.6580	2.2928	1.6515	2.2993	1.4704	2.4804
2015	1.6072	-0.1573	1.7644	1.1313	1.1422	1.4434	1.5225	2.0063	1.5202	2.0087	1.4558	2.0731
2016	1.5693	-0.1698	1.7391	1.0232	1.0372	1.3381	1.5203	1.9579	1.5173	1.9609	1.4530	2.0253
2017	1.5948	-0.1547	1.7495	1.1810	1.1911	1.4959	1.4970	2.0021	1.4948	2.0042	1.4296	2.0694
2018	1.6530	-0.1586	1.8116	1.2754	1.2852	1.5211	1.5388	2.0843	1.5367	2.0864	1.4863	2.1369
2019	1.6912	-0.1958	1.8870	1.6023	1.6142	1.9990	1.5444	2.2297	1.5418	2.2322	1.4595	2.3145

Note: $\widehat{\omega}_n$ is the aggregate output efficiency estimate, $\widehat{B}_{\omega,n}$ is the estimated bias and $\widehat{\sigma}_\varphi$ is the standard deviation estimate.

Table C.4: CRS-DEA Estimates of Simple Mean Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = 0.75\kappa$

Panel A: Without Data Sharpening

				$\widehat{\sigma}_\lambda$			95% CI					
Year	$\widehat{\lambda}_n$	$\widehat{B}_{\lambda,n}$	$\widehat{\lambda}_n - \widehat{B}_{\lambda,n}$	Sol1	Sol2	Sol3	Sol1		Sol2		Sol3	
1990	1.8756	-0.2667	2.1423	0.9833	1.0188	1.2216	1.9321	2.3526	1.9245	2.3602	1.8811	2.4036
1991	2.0193	-0.2752	2.2945	1.2867	1.3158	1.5426	2.0193	2.5696	2.0131	2.5759	1.9646	2.6244
1992	2.1014	-0.3986	2.5000	1.2357	1.2984	1.6760	2.2358	2.7643	2.2224	2.7777	2.1416	2.8585
1993	1.9223	-0.2511	2.1734	1.0357	1.0657	1.2620	1.9519	2.3949	1.9455	2.4013	1.9035	2.4433
1994	2.0003	-0.2747	2.2750	1.2705	1.2999	1.5010	2.0033	2.5467	1.9970	2.5530	1.9540	2.5960
1995	2.0446	-0.3316	2.3762	1.4099	1.4483	1.7249	2.0747	2.6777	2.0665	2.6860	2.0074	2.7451
1996	2.0208	-0.3188	2.3396	1.3759	1.4123	1.6479	2.0453	2.6338	2.0375	2.6416	1.9872	2.6920
1997	1.8419	-0.1241	1.9660	1.1390	1.1457	1.2032	1.7225	2.2096	1.7210	2.2111	1.7087	2.2233
1998	1.9422	-0.1985	2.1407	1.2610	1.2766	1.4223	1.8710	2.4103	1.8677	2.4137	1.8365	2.4448
1999	1.8861	-0.1767	2.0627	1.1352	1.1488	1.2801	1.8200	2.3055	1.8170	2.3084	1.7890	2.3365
2000	1.9118	-0.2164	2.1282	1.0325	1.0550	1.1714	1.9074	2.3490	1.9026	2.3538	1.8777	2.3788
2001	1.8663	-0.2531	2.1194	0.9482	0.9814	1.1126	1.9167	2.3222	1.9096	2.3293	1.8815	2.3574
2002	1.8570	-0.2128	2.0698	0.8674	0.8932	1.0022	1.8843	2.2553	1.8788	2.2608	1.8555	2.2841
2003	1.9374	-0.2681	2.2055	0.8342	0.8762	1.0097	2.0271	2.3839	2.0181	2.3928	1.9895	2.4214
2004	1.8537	-0.2702	2.1238	0.7313	0.7796	0.8988	1.9674	2.2802	1.9571	2.2905	1.9316	2.3160
2005	1.8193	-0.2095	2.0289	0.6181	0.6527	0.6998	1.8967	2.1611	1.8893	2.1685	1.8792	2.1786
2006	1.7873	-0.2383	2.0256	0.5861	0.6327	0.6530	1.9003	2.1509	1.8903	2.1609	1.8860	2.1653
2007	1.6800	-0.1976	1.8776	0.5114	0.5483	0.5686	1.7682	1.9870	1.7603	1.9948	1.7560	1.9992
2008	1.8351	-0.4034	2.2386	0.5887	0.7137	0.7221	2.1127	2.3645	2.0859	2.3912	2.0841	2.3930
2009	1.7356	-0.2668	2.0025	0.5799	0.6384	0.7312	1.8785	2.1265	1.8660	2.1390	1.8461	2.1588
2010	1.8025	-0.3386	2.1411	0.6330	0.7179	0.9045	2.0057	2.2765	1.9876	2.2946	1.9477	2.3345
2011	2.0174	-0.4896	2.5070	0.7513	0.8968	1.0772	2.3463	2.6677	2.3152	2.6988	2.2767	2.7374
2012	1.9417	-0.4368	2.3786	0.6763	0.8052	0.9219	2.2339	2.5232	2.2064	2.5507	2.1814	2.5757
2013	1.7869	-0.2758	2.0628	0.6174	0.6762	0.7688	1.9307	2.1948	1.9181	2.2074	1.8984	2.2272
2014	1.7520	-0.2693	2.0213	0.6549	0.7081	0.8174	1.8813	2.1614	1.8699	2.1728	1.8465	2.1961
2015	1.6945	-0.1928	1.8873	0.6321	0.6608	0.7159	1.7521	2.0224	1.7459	2.0286	1.7342	2.0404
2016	1.6705	-0.2047	1.8753	0.7012	0.7305	0.8112	1.7253	2.0252	1.7190	2.0315	1.7018	2.0487
2017	1.7471	-0.1849	1.9320	1.1611	1.1757	1.3082	1.6837	2.1803	1.6805	2.1834	1.6522	2.2117
2018	1.8196	-0.2297	2.0493	1.4414	1.4596	1.7652	1.7410	2.3575	1.7371	2.3614	1.6718	2.4268
2019	1.9156	-0.2428	2.1584	2.0696	2.0838	2.4537	1.7158	2.6010	1.7128	2.6040	1.6337	2.6831

Table C.4: CRS-DEA Estimates of Simple Mean Output-Oriented Inefficiency, Standard Deviations, and the 95% Confidence Intervals for Countries/Regions when $\gamma = 0.75\kappa$ (continued)

Panel B: With Data Sharpening

				$\widehat{\sigma}_\lambda$			95% CI					
Year	$\widehat{\lambda}_n$	$\widehat{B}_{\lambda,n}$	$\widehat{\lambda}_n - \widehat{B}_{\lambda,n}$	Sol4	Sol5	Sol6	Sol4		Sol5		Sol6	
1990	1.8801	-0.2668	2.1468	0.9796	1.0152	1.2177	1.9374	2.3563	1.9297	2.3640	1.8864	2.4072
1991	2.0242	-0.2753	2.2995	1.2830	1.3122	1.5387	2.0251	2.5738	2.0189	2.5801	1.9704	2.6285
1992	2.1059	-0.3987	2.5046	1.2318	1.2947	1.6720	2.2412	2.7680	2.2277	2.7815	2.1470	2.8622
1993	1.9269	-0.2511	2.1780	1.0317	1.0619	1.2579	1.9573	2.3986	1.9509	2.4051	1.9090	2.4470
1994	2.0071	-0.2748	2.2820	1.2655	1.2950	1.4954	2.0113	2.5526	2.0050	2.5589	1.9622	2.6018
1995	2.0456	-0.3316	2.3772	1.4091	1.4476	1.7241	2.0759	2.6786	2.0677	2.6868	2.0086	2.7459
1996	2.0258	-0.3190	2.3449	1.3724	1.4090	1.6440	2.0514	2.6383	2.0435	2.6462	1.9933	2.6964
1997	1.8525	-0.1245	1.9770	1.1318	1.1386	1.1953	1.7349	2.2190	1.7335	2.2205	1.7213	2.2326
1998	1.9506	-0.1989	2.1495	1.2553	1.2710	1.4161	1.8811	2.4180	1.8777	2.4213	1.8467	2.4524
1999	1.8931	-0.1769	2.0700	1.1301	1.1439	1.2747	1.8283	2.3117	1.8254	2.3146	1.7974	2.3426
2000	1.9182	-0.2166	2.1348	1.0272	1.0498	1.1656	1.9151	2.3545	1.9103	2.3593	1.8855	2.3840
2001	1.8714	-0.2533	2.1246	0.9438	0.9772	1.1078	1.9228	2.3265	1.9157	2.3336	1.8877	2.3615
2002	1.8646	-0.2130	2.0776	0.8606	0.8865	0.9947	1.8936	2.2616	1.8880	2.2672	1.8649	2.2903
2003	1.9434	-0.2682	2.2117	0.8277	0.8701	1.0028	2.0347	2.3887	2.0256	2.3977	1.9972	2.4261
2004	1.8588	-0.2703	2.1290	0.7256	0.7743	0.8927	1.9739	2.2842	1.9635	2.2946	1.9381	2.3199
2005	1.8227	-0.2096	2.0322	0.6139	0.6487	0.6952	1.9010	2.1635	1.8935	2.1710	1.8836	2.1809
2006	1.7892	-0.2383	2.0276	0.5835	0.6303	0.6500	1.9028	2.1524	1.8928	2.1624	1.8886	2.1666
2007	1.6852	-0.1977	1.8829	0.5051	0.5424	0.5611	1.7749	1.9910	1.7669	1.9989	1.7629	2.0029
2008	1.8382	-0.4036	2.2418	0.5848	0.7105	0.7172	2.1167	2.3668	2.0898	2.3937	2.0884	2.3951
2009	1.7381	-0.2669	2.0050	0.5769	0.6357	0.7280	1.8817	2.1284	1.8691	2.1410	1.8494	2.1607
2010	1.8053	-0.3387	2.1440	0.6297	0.7150	0.9011	2.0093	2.2786	1.9911	2.2969	1.9513	2.3367
2011	2.0197	-0.4896	2.5093	0.7483	0.8943	1.0740	2.3493	2.6694	2.3181	2.7006	2.2797	2.7390
2012	1.9452	-0.4371	2.3823	0.6720	0.8016	0.9171	2.2385	2.5260	2.2108	2.5537	2.1861	2.5784
2013	1.7905	-0.2759	2.0664	0.6132	0.6724	0.7641	1.9352	2.1975	1.9226	2.2102	1.9030	2.2298
2014	1.7571	-0.2695	2.0266	0.6497	0.7034	0.8117	1.8876	2.1655	1.8762	2.1770	1.8530	2.2002
2015	1.7018	-0.1928	1.8946	0.6244	0.6535	0.7071	1.7610	2.0281	1.7548	2.0343	1.7434	2.0458
2016	1.6765	-0.2048	1.8813	0.6958	0.7254	0.8050	1.7325	2.0301	1.7262	2.0364	1.7091	2.0534
2017	1.7542	-0.1850	1.9391	1.1568	1.1715	1.3034	1.6918	2.1865	1.6886	2.1897	1.6604	2.2179
2018	1.8242	-0.2298	2.0540	1.4390	1.4572	1.7626	1.7462	2.3617	1.7423	2.3656	1.6770	2.4309
2019	1.9203	-0.2429	2.1632	2.0676	2.0818	2.4515	1.7210	2.6053	1.7180	2.6084	1.6389	2.6874

Note: $\widehat{\lambda}_n$ is the simple mean output efficiency estimate, $\widehat{B}_{\lambda,n}$ is the estimated bias and $\widehat{\sigma}_\lambda$ is the standard deviation estimate.

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