Efficiency Models for Education: with some applications

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Some of the Coauthors

- Rolf Färe is of course coauthor on theoretical and applied performance measurement research.
- Coauthors in the education/public sector work include Kathy Hayes (fellow Syracuse University classmate and SMU faculty), former student Bill Weber (SEMO) and former Hanushek student Lori Taylor (Texas A&M)
- Recent references:
- Grosskopf, Hayes and Taylor, Efficiency in Education: Research and Implications, *Applied Economic Perspectives and Policy*, 2014 36(2), 175-210.
- Grosskopf, Hayes and Taylor, Applied Efficiency Analysis in Education, *Economics and Business Letters*, 2014 3(1).

Basic Research Issues for Education:

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  Multi-stage network models?
- How to measure performance?
  Multiple output frontier methods?
- How to estimate efficiency?
  Parametric (SFA) and nonparametric estimators (DEA, robust conditional efficiency measures, m-frontier, etc)?

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Education involves multiple outputs or services with no obvious prices.

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- How do we account for the role of budgets?
  Cost Indirect output distance functions?
- And the role of bureaucrats, citizen-voters in the public school context?
- As a service, students, peers, environment, and previous human capital are also important factors.
  Environmental/fixed factors: conditional measures, two stage approaches?

Some Background: US perspective

- The U.S. spends more per pupil on primary and secondary education than any other OECD country except Luxembourg.
- U.S. students score no better than the OECD average in reading, math and science (PISA 2009).
- U.S. public schools are inefficient (see *A Nation at Risk*)
- Why?
  No carrot (no profit, but rents?)
  No stick (little or no competition)
  Regulatory distortion (state and recently federal regulations)

Outline: Some Familiar Models for Education and Efficiency

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  $f(x) = max\{y : y \in P(x)\}$
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- **Output Distance Function,**
  $D_o(x, y) = \min\{\theta : y/\theta \in P(x)\}$
  multiple output generalization of the production function,no prices required, Useful in constructing quantity, quality and productivity indexes

Less familiar models for measuring efficiency in education?

- **Directional Distance Function,**
  $\vec{D}(x, y; g) = \max\{\beta : (x - \beta g_x, y + \beta g_y) \in P(x)\}$
  allows for simultaneous expansion of outputs and contraction of inputs, or performance measurement with good and bad outputs (dropout rates?).

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- **Network Models**
  Specify links within the black production box, allowing for optimal allocation across subunits, multi-stage production, time, etc.

Plan

- Brief overview of some characteristics and applications of the familiar distance functions
- More detailed presentation of less familiar models
- Combine directional distance functions and cost indirect model in a network application to Texas schools

Some Applications of Input Distance Functions: $D_i(y,x) = \max\{\lambda : x/\lambda \in L(y)\}$

- Grosskopf and Hayes, 'Local Public Sector Bureaucrats and their input choices,' *Journal of Urban Economics*, 1993. Estimate technical and allocative efficiency of Illinois municipalities including instrumental variables: rural cities overcapitalize, urban cities overemploy.
- Grosskopf, Hayes, Taylor and Weber, 'On the determinants of school district efficiency: competition and monitoring,' *Journal of Urban Economics*, 2001. Estimate technical and allocative efficiency of Texas school districts; monitoring enhances technical and allocative efficiency, competition reduces allocative inefficiency.

Estimation of Allocative Efficiency using Distance Functions:

Compare observed relative prices to |slope| of tangent at $x^o/D_i(y,x^o)$ (relative shadow prices):

For example, with two input case, If

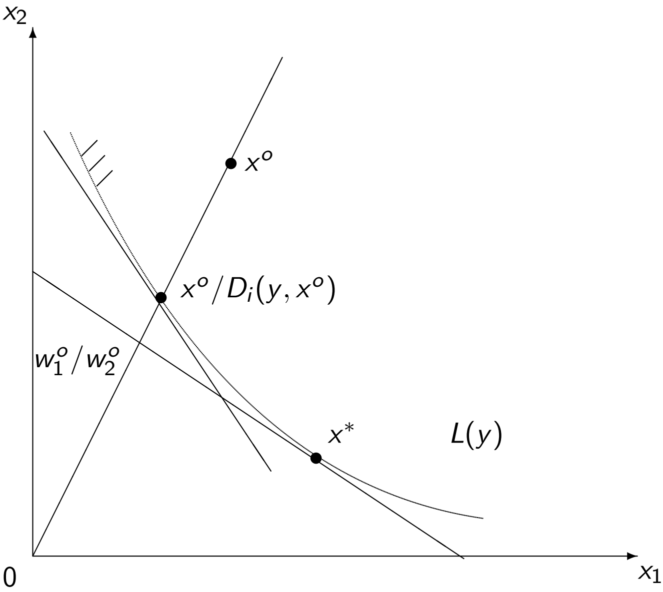
$$w_1^s/w_2^s > w_1^o/w_2^o \tag{1}$$

$\rightarrow$ too little x_1, too much x_2 at observed relative prices

Solve for relative shadow prices as dual Shephard's lemma:

$$w_1^s/w_2^s = \frac{\partial D_i(y,x)}{\partial x_1} / \frac{\partial D_i(y,x)}{\partial x_2} \tag{2}$$

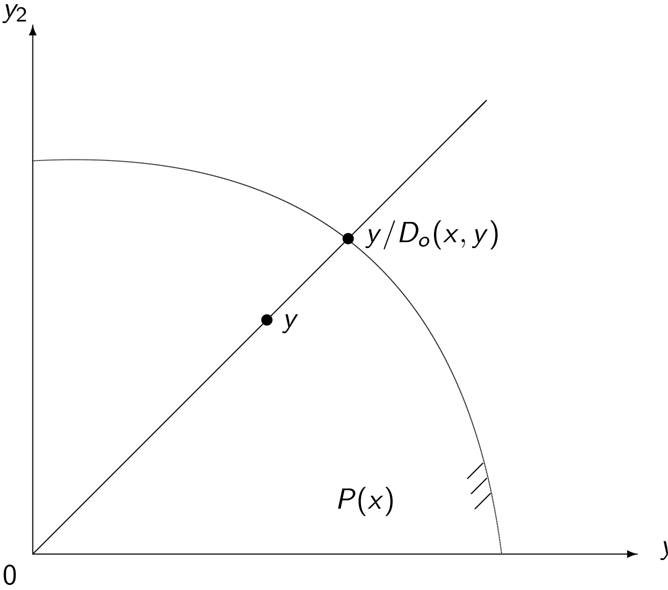
Allocative Efficiency with Input Distance Function



Estimation of Distance Functions:

- Parametric estimation facilitates solving for shadow prices.
- Translog function provides flexible functional form which facilitates imposition of homogeneity restrictions.
- See also Atkinson, Färe and Primont, Stochastic Estimation of Firm Inefficiency using Distance Functions, *Southern Economic Journal* 2003, 69, 596-611, for details including endogeneity and instrumental variables.

Output Distance Function



Some Applications of Output Distance Functions:

$$D_o(x, y) = \min\{\theta : y/\theta \in P(x)\}, x \in \Re^N_+$$

Useful in constructing index numbers as shown by Malmquist (1951) and Caves, Christensen and Diewert (1982).
Building block for productivity measurement, including Malmquist Index and Färe-Primont Index

- Productivity decomposes into various components, including catching up and shifts in the frontier

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Building block for productivity measurement, including Malmquist Index and Färe-Primont Index

- Productivity decomposes into various components, including catching up and shifts in the frontier
- No price information required, accommodates multiple outputs

Alternative Productivity Decomposition with Quality

Färe, Førsund, Grosskopf, Hayes and Heshmati, Measurement of productivity and quality in non-marketable services, *Quality Assurance in Education*, 2006.

Estimate a quality augmented output distance function $DQ_o(x, a, b, y)$, where a and b represent quality attributes of inputs and outputs.
Use $DQ(\cdot)$ to define a quality adjusted productivity index $MQ(t, t+1)$. Define an implicit quality index:

$$Q(t, t+1) = MQ(t, t+1)/M(t, t+1).$$

Rearranging we have

$$MQ(t, t+1) = Q(t, t+1) \cdot M(t, t+1)$$

Frontier Productivity of Universities

See Bonaccorsi, Daraio and Simar, Advanced indicators of productivity of universities: an application of robust nonparametric methods to Italian data, *Scientometrics* 66, 2006.

Apply robust nonparametric methods—order-m frontiers—to estimate university performance with respect to scale and scope, as well as trade-offs between teaching vs research and academic research vs commercial applied work.

Cost Indirect Output Distance Function

$$
\begin{aligned}
ID_o(w/C, y) &= \min_{\theta,x}\{\theta : y/\theta \in IP(w/C)\} \quad (3)\\
&= \min_{\theta,x}\{\theta : y/\theta \text{ is feasible given budget } wx \leqq C\}
\end{aligned}
$$

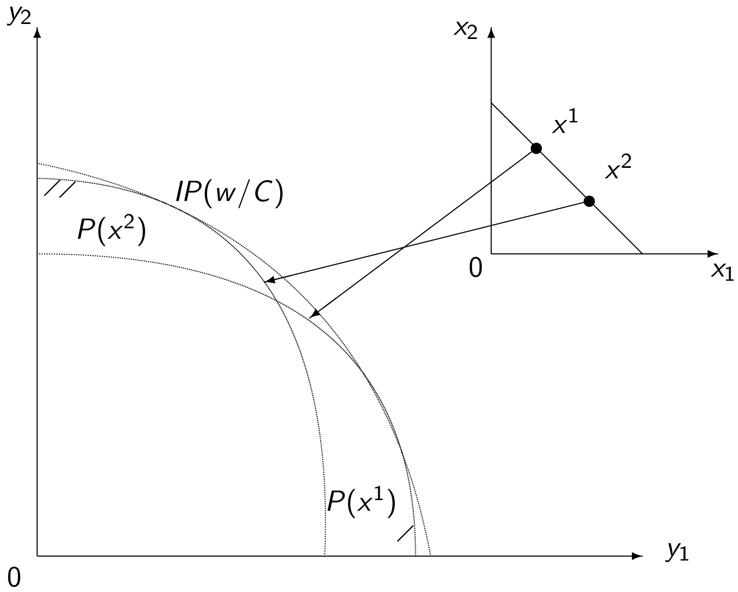
Due to Shephard (1970?)

Allows for budget constraint and reallocation of inputs, multiple outputs

captures allocative and technical efficiency

$IP(w/C)$ is the set of all direct output sets that satisfy the budget constraint $wx \leqq C$ where C is the given budget.

A Cost Indirect Output Set, $IP(w/C) = \{y : wx \leqq C\}$

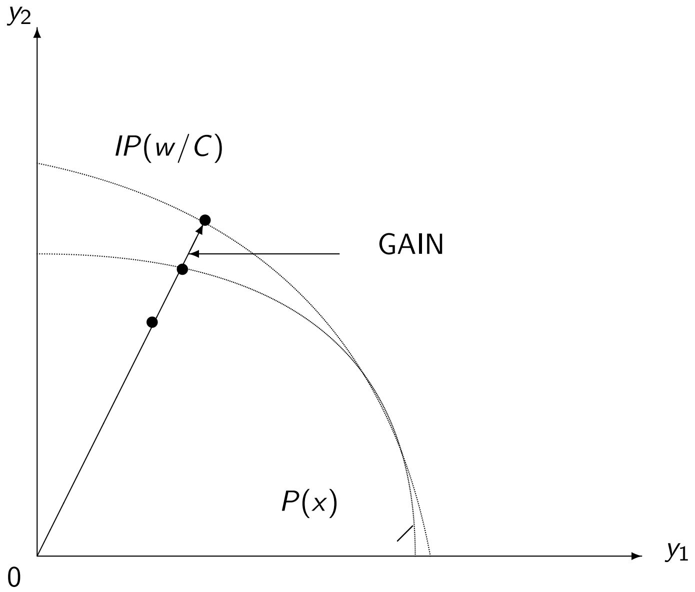


Application:

Grosskopf, Hayes, Taylor and Weber, ‘Anticipating the consequences of school reform: a new use of DEA,’ *Management Science*, 1999.

Simulate the introduction of site-based management, which would allow decentralization of public school budgets, by comparing maximum potential outputs given $P(x)$, i.e., the direct output set with inputs given, to maximum outputs given $IP(w/C)$, which allows reallocation of inputs subject to the budget for a sample of Texas school districts. The ratio gives the potential ‘Gain’ in outputs (3%). It also allows identification of the optimal input choices that satisfy the budget constraint; teachers and administrators would ‘lose’, teacher aides would ‘gain’. ‘Poorer’ school districts would realize smaller gains.

Applying Cost Indirect Output Distance Function



More Applications of $ID_o(y, w/C)$

Grosskopf, Hayes, Taylor and Weber, Budget Constrained frontier measures of fiscal equality and efficiency in schooling, *Review of Economics and Statistics*, 1997.

Estimate parametric cost indirect output distance function for Texas school districts in 1989. Simulate school finance reforms: compare potential output to observed output when: 1) level below–average districts up to average budget, 2) equalize per pupil expenditure at mean for all districts, adjust input prices and expenditures to yield mean values. All reforms yield distribution of outcome equality that is less equal than the status quo.

Directional Distance Functions

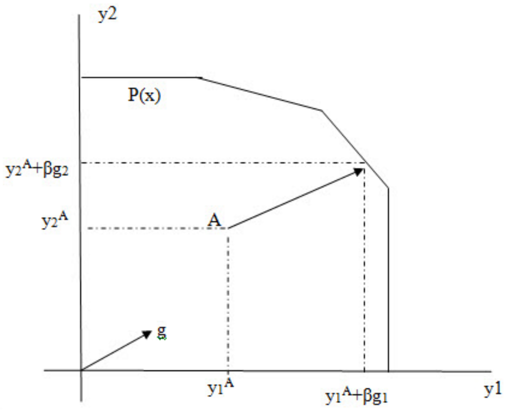
$\vec{D}_T(x, y; g) = \max_\beta\{\beta : (y + \beta g_y, x - \beta g_x) \in T\}$
where g is the direction in which the input-output bundle is projected to the frontier.

- In contrast to usual Shephard distance functions, this has an additive structure.
- This function satisfies translation rather than homogeneity.
- This implies that parameterization of the function should be quadratic rather than the translog—translog is appropriate for homogeneous functions.
- On parameterization, see Chambers, Färe, Grosskopf and Vardanyan, Generalized Quadratic Revenue Functions,' *Journal of Econometrics* 173, 2013.
- Endogeneity issues under current debate, including endogenizing the direction vector

Applications of the Directional Distance Functions

- Partitioning output vector allows for simultaneous expansion of good outputs (test scores, graduation) and contraction of bad outputs (dropouts) in assessing performance
- Can be used to estimate shadow prices of non marketed goods and bads
- Has been used extensively in environmental applications
- Has been extended to case of robust conditional estimator

Directional (good) Output Distance Function



Network Models

Networks are very flexible models, which allow for specification of links across observations, multi-stage processes, links across time. Examples:

- dynamic productivity: endogenize investment which links production across time periods
- specification of intermediate or multi-stage production in a static setting
- simulate permit trading or cap and trade, which requires reallocation across observations to meet an environmental target
- model reallocation across bank branches

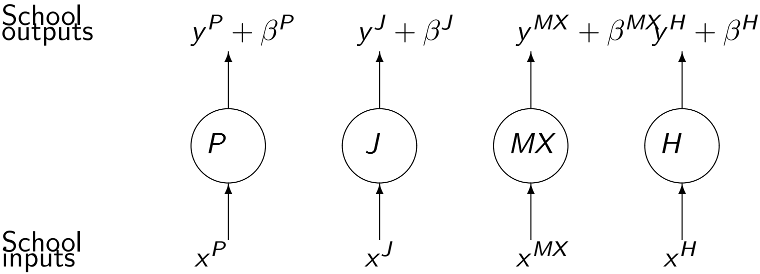
To date, most applications have employed activity analysis (Data Envelopment Analysis).

Network Applied to Texas Schools

Grosskopf, Hayes, Taylor and Weber, Centralized or Decentralized Control of Resources? A Network Model, *Journal of Productivity Analysis*, 2014.

- No Child Left Behind changed the focus from school districts to performance and accountability of individual schools.
- We explicitly model the school district as a collection of campuses, i.e., a network which is connected through the school district budget.
- We use a cost indirect directional output distance function to model the network and solve for the optimal allocation of school district resources.
- We apply this to 70 school districts in the Dallas Metropolitan area and solve for the optimal allocation of resources within the network using activity analysis (or Data Envelopment Analysis).
- We begin with a simple school district set up where there is no connection across schools.

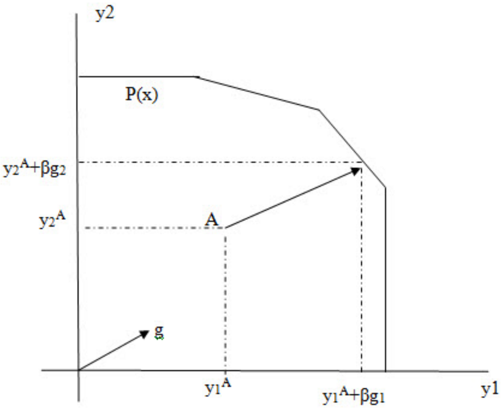
Stylized School District—no Network



(P=Primary, J=Middle, MX=Mixed, H=High School)

Estimation: Benchmark Directional Output Distance Function–No Network

$$\vec{D}_o(x, y; g) = \max\{\beta : (y + \beta g) \in P(x)\} \tag{4}$$



Stylized Setup for Network Model of a School District

- Allow for reallocation of school district resources across primary (P), middle (J), mixed (MX)and secondary schools (H) within a school district (D).

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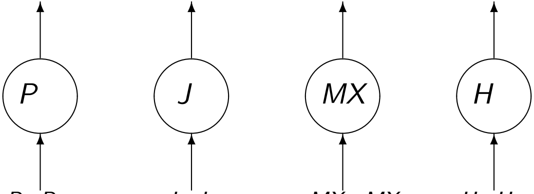
- Objective is to maximize potential output gains of the school district.
- Network model solves for the maximum potential output as well as optimal degree of centralization within districts, optimal allocation of resources across campuses and optimal allocation of resources across input types that satisfy the school district budget.

Stylized School District as a Network

Obj: Max

Potential Outputs $(y^P + \beta^P) + (y^J + \beta^J) + (y^{MX} + \beta^{MX}) + (y^H + \beta^H)$



District Budget $w^P x^P + \quad w^J x^J + \quad w^{MX} x^{MX} + \quad w^H x^H + O \leq \overline{B}$

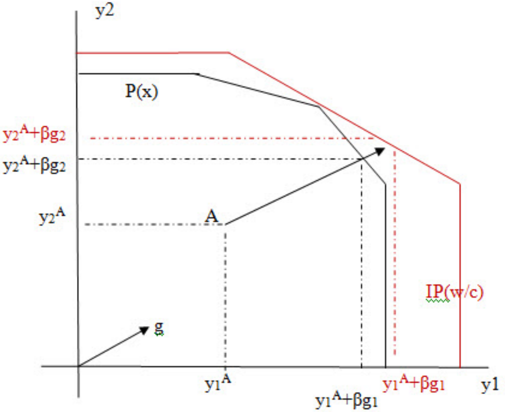
(P=Primary,J=Middle,MX=Mixed, H=Secondary School)

Est: Dir. Dist. Fn. with budget constraint

input prices=w, budget=B, direction vector, g=1

$$IP(w/B) = \{y : x \text{ can produce } y \text{ and } wx \leq B\} \tag{5}$$

$$\overrightarrow{ID}_o(w/B, y; g) = \max_{\beta, x}\{\beta : (y + \beta g) \in IP(w/B)\} \tag{6}$$



Remarks

- School district maximizes the sum of the potential gains (the optimal β's for which we solve) in school outputs
- By removing technical efficiency only in the benchmark model (all inputs fixed)
- By removing technical inefficiency and allowing for reallocation of resources in the network model,
- Both models subject to the output, technology, and input constraints
- Network model solves for the optimal allocation of variable inputs, two types of overhead (personnel and non-personnel) across schools for each school district.

More Remarks

- To estimate our model we use mathematical programming techniques familiar from DEA.
- Reference technologies include all schools in the sample of the associated type.
- Included in the network model is a reallocation constraint of District budget across schools in each district.

Full Model Specification

We can now state the programming problem for a given school district d':

$$\max_{\beta,z,x,O_v} \sum_{pd=1}^{Pd} \beta_{pd} + \sum_{jd=1}^{Jd} \beta_{jd} + \sum_{hd=1}^{Hd} \beta_{hd} + \sum_{mxd=1}^{MXd} \beta_{mxd} \qquad (7)$$

subject to: **The output constraints**

$$\begin{aligned}
y_m^{pd'} + \beta_{pd} \cdot 1 &\leqq \textstyle\sum_{kp=1}^{KP} z_{kp} y_m^{kp}, & m = (\text{math,read}), \forall pd \\
y_m^{jd'} + \beta_{jd} \cdot 1 &\leqq \textstyle\sum_{kj=1}^{KJ} z_{kj} y_m^{kj}, & m = (\text{math,read}), \forall jd \\
y_m^{hd'} + \beta_{hd} \cdot 1 &\leqq \textstyle\sum_{kh=1}^{KH} z_{kh} y_m^{kh}, & m = (\text{math,read}), \forall hd \\
y_m^{mxd'} + \beta_{mxd} \cdot 1 &\leqq \textstyle\sum_{kmx=1}^{KMX} z_{kmx} y_m^{kmx}, & m = (\text{math,read}), \forall mxd
\end{aligned}$$

The Variable Input Constraints

$$
\begin{aligned}
x_n^{pd} &\geqq \sum_{kp=1}^{KP} z_{kp} x_n^{kp}, \quad && n = (\text{teachers, non-personnel}), \forall pd \\
x_n^{jd} &\geqq \sum_{kj=1}^{KJ} z_{kj} x_n^{kj}, \quad && n = (\text{teachers, non-personnel}), \forall jd \\
x_n^{hd} &\geqq \sum_{kh=1}^{KH} z_{kh} x_n^{kh}, \quad && n = (\text{teachers, non-personnel}), \forall hd \\
x_n^{mxd} &\geqq \sum_{kmx=1}^{KMX} z_{kmx} x_n^{kmx}, \quad && n = (\text{teachers, non-personnel}), \forall mxd
\end{aligned}
$$

Fixed Input Constraints

$$xf_n^{pd'} \geqq \sum_{kp=1}^{KP} z_{kp} xf_n^{kp}, \quad n = (\text{stud}, \%\text{HEP}, \%\text{nospeced}, \%\text{HSES}), \forall pd$$

$$xf_n^{jd'} \geqq \sum_{kj=1}^{KJ} z_{kj} xf_n^{kj}, \quad n = (\text{stud}, \%\text{HEP}, \%\text{nospeced}, \%\text{HSES}), \forall jd$$

$$xf_n^{hd'} \geqq \sum_{kh=1}^{KH} z_{kh} xf_n^{kh}, \quad n = (\text{stud}, \%\text{HEP}, \%\text{nosped}, \%\text{HSES}), \forall hd$$

$$xf_n^{mxd'} \geqq \sum_{kmx=1}^{KMX} z_{kmx} xf_n^{kmx}, \quad n = (\text{stud}, \%\text{HEP}, \%\text{nosped}, \%\text{HSES}), \forall mxd$$

Overhead Constraints

$$
\begin{aligned}
O_v^{pd} &\geqq \sum_{kp=1}^{KP} z_{kp} O_v^{kp}, && v = 1, 2, pd = 1, \ldots, Pd \\
O_v^{jd} &\geqq \sum_{kj=1}^{KJ} z_{kj} O_v^{kj}, && v = 1, 2, jd = 1, \ldots, Jd \\
O_v^{hd} &\geqq \sum_{kh=1}^{KH} z_{kh} O_v^{kh}, && v = 1, 2, hd = 1, \ldots, Hd \\
O_v^{mxd} &\geqq \sum_{kmx=1}^{KMX} z_{kmx} O_v^{kmx}, && v = 1, 2, mxd = 1, \ldots, MXd
\end{aligned}
$$

The District d' Budget Constraint

$$\sum_{pd=1}^{Pd}(\sum_{n=1}^{2} w_n^{pd} x_n^{pd} + \sum_{v=1}^{2} O_v^{pd})$$
$$+\sum_{jd=1}^{Jd}(\sum_{n=1}^{2} w_n^{jd} x_n^{jd} + \sum_{v=1}^{2} O_v^{jd})$$
$$+\sum_{hd=1}^{Hd}(\sum_{n=1}^{2} w_n^{hd} x_n^{hd} + \sum_{v=1}^{2} O_v^{hd})$$
$$+\sum_{mxd=1}^{MXd}(\sum_{n=1}^{2} w_n^{mxd} x_n^{mxd} + \sum_{v=1}^{2} O_v^{mxd})$$
$$\leq B^{d'}$$

Remarks

- There will be similar problems for each school district, $d = 1, \ldots, D$
- The reference technology for each school type in each district includes all of the schools of the same type in the sample
- The benchmark model (no network) does not allow for reallocation of budget, thus removes only technical inefficiency.
- The network model solves for optimal levels of variable inputs, overhead O_v and intensity variables z which will maximize the sum of the potential gains in reading and math summed over all schools in the district
- There are KP=663 primary schools, KJ=221 middle schools, KH=151 high schools and KMX=4 mixed schools in a total of D=70 Dallas area school districts operating in 2008-2009.

Data: variable specification

- Outputs: normalized, student-level gains in mathematics and reading (i.e., value added approach)
- Variable Inputs: teachers, non personnel expenditures
- Fixed Inputs: student enrollment,%high English proficiency, % high SES, % non special ed

Remarks: Dallas School Districts

- Average school district has over 10,000 students, with 10 primary schools, 3 middle and 2 high schools
- The smallest has 201 students, the largest 151,610
- Budgets range from 1.7 million to 1.3 billion dollars, average per pupil spending is $ 8829
- On average 79% of the budget is allocated to school campuses, 21% remaining in central administration on average
- Maximum central administration share is 59%, minimum 9%

Static and Optimal Network Results

	Primary Schools KP=663	Middle Schools KJ=221	High Schools KH=151	Mixed Schools KMX=4	District D=70
Static Model, all inputs fixed					
$\beta/students$	7.14 (4.96)	2.29 (2.46)	3.63 (3.66)	0 (0)	5.63 (3.03)
Network Model, reallocation of inputs					
$\beta/students$	11.56 (5.81)	3.52 (2.51)	5.28 (3.78)	0 (0)	9.29 (3.03)

Remarks

- As expected, allowing for reallocation of resources increases potential gains in outcomes
- Average potential gain in static case is 5.63 points per student, with reallocation that increases to 9.29 additional points on average
- Largest average gains at the primary level

Notes

- Optimal reallocation increases teachers for primary but not middle or high school on average
- Optimal reallocation reduces non-personnel spending at every level on average
- Optimal reallocation reduces central administration for primary but not middle or high schools on average

School District Ratios: Actual vs Optimal Shares

		Mean	St. Dev.	Min	Max
Obs School Share	$\sum(w_1 x_1 + w_2 x_2)/B$	0.79	.08	0.41	0.91
Opt School Share	$\sum(w_1 x_1^* + w_2 x_2^*)/B$	0.84	.02	0.78	0.93
Obs Overhead Share	$(O_1 + O_2)/B$	0.21	.08	0.09	0.59
Opt Overhead Share	$(O_1^* + O_2^*)/B$	0.16	.02	0.07	0.22

School District Ratios

In general, allocated shares to schools increase with reallocation, and non-allocated (central administration) decreases. The reallocation reduces the range and standard deviation of these shares in general, suggesting that reduced variation is efficient.

Future Efforts

Address:

- Robustness–we have tried several formulations, qualitative results are similar
- Bootstrapping?
- Does school district size matter?
- Currently extending to effect on equity of outcomes of reallocation.

Output Specification

Normalized student level gains in math and reading standardized tests following Reback (2007) and as required by the state as part of their accountability system to satisfy No Child Left Behind. Each student's performance in math and reading is compared to the average score in the current year of all other students in the state at the same grade level who had the same score from the previous year. This average is used as the expected score for each student. The difference between the actual and expected score is then normalized to yield the number of standard deviations from the expected score and transformed into Normal Curve Equivalent scores.

$$Y_{igt} = \frac{S_{igt} - E(S_{igt}|S_{i,g-1,t-1})}{[E(S_{igt}^2|S_{i,g-1,t-1}) - E(S_{igt|S_{i},g-1,t-1})^2]^{0.5}} \tag{8}$$