Would Weighted-student Funding Enhance Intra-district Equity in Texas? A Simulation using DEA

Shawna Grosskopf
Department of Economics
Oregon State University
Corvallis, OR 97331

Kathy J. Hayes
Department of Economics
Southern Methodist University
Dallas, TX 75275

Lori L. Taylor
Bush School of Government & Public Service
and Department of Economics
1098 Allen Bldg, 4220 TAMU
College Station, TX 77843

William L. Weber
Department of Economics and Finance
Southeast Missouri State University
Cape Girardeau, MO 63701

August 18, 2014

Abstract

We use data envelopment analysis to model the educational production function, and then explore how a shift to weighted-student funding using the student weights embedded in the Texas School Finance Formula would alter the allocation of inputs and potential outputs. School outputs are measured as value-added reading and math scores on standard achievement tests. We find that if school districts allocated their resources efficiently, then they would not allocate their resources to campuses according to the funding model weights. Policies that promote greater efficiency would also enhance equity in educational outcomes.

1 Introduction

Researchers have long recognized that the cost of providing a standard quality education differs from one school district to another. Those differences could arise from differences in labor cost, differences in student need or economies of scale or scope.

Policymakers have responded to evidence about such differences in cost with school funding formula that provide additional resources to school districts with higher labor cost (as in Texas, Wyoming, or Florida) higher student needs (as in all states but Nevada, Montana and South Dakota) or a lack of economies of scale (as in Texas, Louisiana or Kansas).[1]

Arguably, the factors that drive differences in cost for school districts also drive differences in cost at the school level. Therefore, researchers have been increasingly interested in the distribution of resources within school districts (e.g. Baker 2011, Ladd 2008, and Miles and Roza 2006).

In this paper we simulate the efficiency and equity effects of a move toward weighted-student funding as a means of determining school budgets. Disparities in per pupil expenditures across schools within a school district might be justified if some schools have students who are more costly to educate. Rather than allocate school staff on a per pupil basis, weighted-student funding provides greater amounts of funding for schools that have students who are more difficult to educate (Miles and Roza 2006). Rather than focus on equity in school resources, we examine equity in school outcomes, which we measure as value-added test scores in reading and mathematics. In addition, we control for differences in school input prices and the fixed inputs that schools use in the production of value-added test scores.

We examine 2,709 schools residing in 175 school districts in three Texas metropolitan areas: Houston, Dallas, and San Antonio. The schools include 1,694

[1]For a more complete description of school funding formula, see Verstegen and Jordan (2009) which is the source of the information about formula weights for student need. See Baker and Duncombe (2009) for a discussion of scale adjustments. See Taylor and Fowler (2006) for a discussion of formula adjustments for higher labor cost.

elementary schools, 618 middle schools, 387 high schools, and 10 mixed schools. We use DEA (data envelopment analysis) to model the multi-output multi-input production process of schools. The method of DEA allows schools to produce different mixes of outputs using different relative amounts of inputs. Schools that maximize the various outputs given inputs are efficient and can only expand outputs if given access to larger amounts of inputs. We compare school output efficiency when schools take inputs as given with school output efficiency given a budget with which to hire inputs. Holding the budget constant, some schools might find that they can further expand outputs if they were to reallocate inputs; for instance, by increasing or decreasing teachers and staff relative to non-personnel inputs, such as computer software expenditures.

After measuring school output efficiency relative to their observed inputs and school budget we simulate the effects of a change in school budgets toward weighted-student funding. Under weighted-student funding some schools will gain resources and some schools will lose resources. We use DEA to simulate the potential outputs that could be produced under weighted-student funding.

We examine several indicators of inequality for actual outputs, potential outputs if schools were efficient, and potential outputs that could be produced under weighted-student funding. These indicators include Theil's inequality measure, the Gini coefficient, Brazer's coefficient of variation, the range, and the 95 to 5 percentile ratio.

2 Efficiency and Equity Issues in Schools

Many researchers have applied the tools of efficiency analysis to public education. Indeed, one of the first applications of DEA examined 167 elementary schools in the Houston Independent School District (Bessent, Bessent, Kennington, and Reagan 1982). Among the 167 elementary schools under analysis, 78 were found to be inefficient in producing achievement on the Iowa Test of Basic Skills. Numerous studies of educational efficiency have since been undertaken studying individual students (e.g. Cherchye, De Witte, Ooghe, and Nicaise 2010), schools (e.g. Gronberg et al. 2013) and school districts (e.g. Chakraborty, Biswas and Lewis 2001).

In a series of papers studying Texas school districts Grosskopf et al. (1997, 1999, 2001) accounted for school district differences in input prices and students' own human capital and measured district technical efficiency in the production of value added on a battery of achievement tests in mathematics, reading, and writing. In a post-estimation simulation exercise Grosskopf et al. (1997) found that policy reforms aimed at equalizing budgets between school districts would generate a distribution of student achievement and school outputs that exhibited greater inequality than the status quo. However, Grosskopf et al. (1999) found that student achievement gains could be achieved by relaxing various regulations governing input use. In addition, Grosskopf et al (2001) found that greater competition and citizen monitoring enhance efficiency in school outcomes.

An even larger group of researchers have examined the equity of the public education system. For example, Murray, Evans, and Schwab (1998) conclude that differences across states are a more important component of measured inequality than are differences in expenditures within states, whereas Taylor (2006) concludes that once regional differences in labor cost are taken into account the within-state differences in expenditures are equally important. Ruggiero, Miner, and Blanchard (2002) examined New York School districts and found that approximately 50% of measured inequities in educational spending between districts were due to differences in input prices and the exogenous influences of school demographics and socio-economic characteristics.

Several concepts of equity are relevant in school research and policy. Horizontal equity refers to the equal treatment of equals (Berne and Stiefel 1999, Ladd 2008). Common measures of horizontal equity include the Gini coefficient, range, McLoone index, mean absolute deviation, and coefficient of variation (Toutkoushian and Michael 2007). Vertical equity refers to the unequal treatment of students in different circumstances. Different outcomes can be a consequence of different administrative and pedagogical processes used to provide education. In addition, student needs, geographical cost of living differences, and local capacity can lead to differences in per pupil spending that might still be regarded as equitable.

Rice (2004) argued that the equity and efficiency movements both failed to achieve their goals and that linking the two goals by recognizing their interrelations might provide a more reasonable policy goal. For instance, if some schools with more difficult to educate students are to receive greater amounts of resources, the increased funding should be contingent on efficiency. In addition, school voucher programs might be targeted at minority and low income students in an effort to enhance school outcomes and promote greater equity.

In a recent paper Cherchye, De Witte, Ooghe, and Nicaise (2010) used DEA to evaluate efficiency and equity in Flemish private and public schools. After controlling for environmental variables such as socio-economic status and past student achievement in mathematics and reading, the authors found no evidence that private schools dominate public schools in efficiency and equity outcomes.

Miles and Roza (2006) compare and contrast staff allocation formulas with weighted-student funding. Weighted student funding incorporates all educational and student needs into a formula that drives funding. Students with different needs are weighted differently. Common categories include the number of students in special education, poverty, limited English proficiency, vocational education, grade level, and gifted education. In theory, the formula would be derived from a cost analysis with the amount of funding depending on the specific needs of the students that the school serves but ultimately the political process plays an important role in determining the weights (Ladd 2008). In practice, weighted-student funding as examined by Miles and Roza (2009) appears to be based on linear cost adjustment factors and ignores any potential interaction between outputs and the various categories, the level and mix of outputs, differences in input prices across schools, or resources from central administration for things like professional development or special program staff. Miles and Roza examined Houston and Cincinnati school districts which had staff-based allocation formulas in 1998-99 and then switched to weighted-student funding. Using data from 1999-2000 2002-2003 they found that Houston experienced a small decline in inequality as measured by the coefficient of variation for the weighted-student funding index. In addition, the move toward weighted-student funding was positively correlated with an increase in the number of schools within the district that were within 5% or 10% of the district average expenditures per pupil after implementation of weighted-student funding.

A complicating factor in implementing a weighted-student funding system is accounting for incentive effects associated with differential student weights. Cullen (2003) found that school administrators in Texas were more likely to classify students as having a learning disability when the school funding formula provided a greater weight for students with a disability. Using estimates derived from 1991-1992 to 1996-97 Texas school districts Cullen found that a 10% increase in revenue generated by a special education student lead to a 2.1% increase in the student disability rate. The results were most pronounced for learning disabilities that involved subjective judgments on the part of those people evaluating students. Furthermore, there was a greater tendency to classify students as disabled in school districts with a small number of campuses which enabled greater centralized decision-making.

3 Method

Although past researchers have focused on equality in school budgets (e.g. Miles and Roza 2009, Berne and Stiefel 1994, Baker 2007, 2009, Toutkoushian and Michael 2007) our goal in this paper is to focus on equity in school outcomes. Specifically, we want to examine equity in actual reading and math test scores under the status quo allocation of resources versus the equity outcomes that might occur with a move toward weighted-student funding. Since numerous papers have found various levels of inefficiency in schools, we also want to control for any potential inefficiency that might be present.

One way to model a multi-output and multi-input school production process is to use a cost function as did Ruggiero, Miner, and Blanchard (2002). With a cost function approach any variation between the minimum cost of production and the actual cost of production would be attributed to inefficiency. However, with weighted-student funding schools would receive a budget and citizens, parents, and policy-makers would like school administrators to try and maximize outputs and would not necessarily have cost minimization as a goal. As mentioned by Rice (2004), schools that receive larger resources should be expected to make efficient use of those resources. In addition, Ladd (2008) argues that weighted-student funding "enhances equity defined in terms of outcomes" as long as "the weights correctly reflect differential needs" (Ladd 2008, p. 416). Therefore, we would like our method to provide an assessment of whether weighted-student funding as determined through the political process has enhanced equity in school outcomes.

One way to account for inefficiency and determine maximum potential outputs given a budget that administrators can use to hire inputs is to use distance functions. In this section we develop a distance function based indicator of school performance that accounts for the multiple inputs that schools use to produce various outputs. Our performance indicator builds on the directional output distance function developed by Chambers, Chung, and Färe (1996, 1998). This distance function is an outgrowth of Luenberger's (1992) benefit function that was used in consumer theory. Directional distance functions can be estimated using a linear programming method called DEA which was developed by Charnes, Cooper, and Rhodes (1978).

We illustrate our method of measuring efficiency and simulating a policy change to weighted-student funding graphically. A particular school within a school district uses variable inputs, $x \in R_+^N$, and fixed inputs, $F \in R_+^J$, to produce outputs, $y \in R_+^M$. Each school uses some technology that transforms the inputs into various outputs and we represent the technology by the output possibility set: $P(x, F) = \{y : (x, F) \text{ can produce } y\}$, that is, $P(x, F)$ gives the set of outputs that can be produced from the variable and fixed inputs. In our analysis of Texas schools we assume that the variable inputs, x, consist of school specific personnel (teachers) and non-personnel (maintenance) inputs. The fixed inputs, F, include a share of the central administration overhead expenses and the demographic characteristics of the student population.

Suppose the school faces input prices $w \in R_+^N$ with which to hire inputs x. In addition, the school faces some fixed costs and fixed inputs in producing the outputs. The cost of using x and F to produce y is $\sum w_n x_n \leq c$. Let the school have discretion over which variable inputs to use as long as the input choice satisfies the budget constraint. Then different choices of x will generate different output possibility sets. Figure 1 depicts the budget constraint facing the school district and the output possibility sets for three choices of variable inputs, x^A, x^B, and x^*. Each of these input choices satisfies the budget constraint. We let the set of outputs that can be produced given fixed inputs and the budget, c be represented by the budget-constrained output possibility set: $IP(w/c, F) = \{y : (x, F) \text{ can produce } y \text{ and } \sum w_n x_n \leq c\}$.

Each of the individual production possibility sets, $P(x, F)$, is a subset of the budget-constrained production possibility set, $IP(w/c, F)$. Inputs are efficiently allocated when, given the budget and input prices, the school is able to produce the maximum amounts of the two outputs. In Figure 1 the largest production possibility set occurs when the chosen inputs are x^*. Other choices of inputs, say x^A or x^B, are affordable, but yield smaller production possibility sets than x^*. Much of the school choice literature argues that rules and regulations constrain schools in what they can achieve and that giving schools greater discretion over inputs is one way to enhance school efficiency.

The two production possibility sets have various properties. First, when the

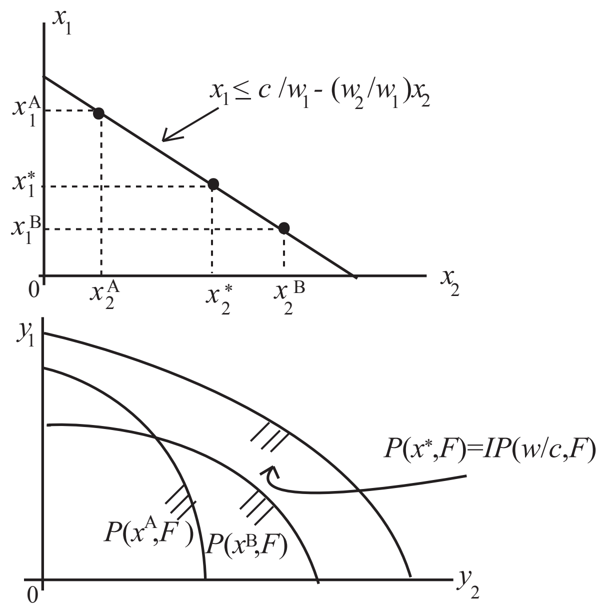

Figure 1: Campus Inefficiency

school has access to more inputs, (x, F), the set $P(x, F)$ expands. Second, when the school faces lower input prices or has a larger budget or is endowed with a larger amount of fixed input, the set $IP(w/c, F)$ expands. We make use of these properties in our simulation exercise that examines a policy of weighted-student funding.

We measure efficiency using the directional output distance function. Given inputs, the directional output distance function finds the maximum expansion in the various outputs that could be produced if a school were efficient. Outputs are expanded for the directional vector $g = (g_1, \ldots, g_M)$. For instance, when $g = (1, 1, \ldots, 1)$ the directional output distance function gives the maximum unit expansion in each of the M outputs. If instead, $g = (y_1, y_2, \ldots, y_M)$ the directional output distance function when multiplied by 100% gives the maximum percentage expansion in each of the M outputs. Formally, we can write this distance function as

$$\vec{D}_o(x, F, y; g) = \max\{\beta : (y + \beta g) \in P(x, F)\}. \tag{1}$$

When a school is efficient, $\vec{D}_o(x, F, y; g) = 0$ meaning that it is not possible to further expand outputs given inputs. Inefficient schools have $\vec{D}_o(x, F, y; g) > 0$ with larger values indicating greater inefficiency. Figure 2 illustrates how $\vec{D}_o(x, F, y; g)$ is estimated assuming a directional vector of $g = (1, 1)$ and two outputs: y_1=value-added on a reading test and y_2=value-added on a mathematics test. We observe

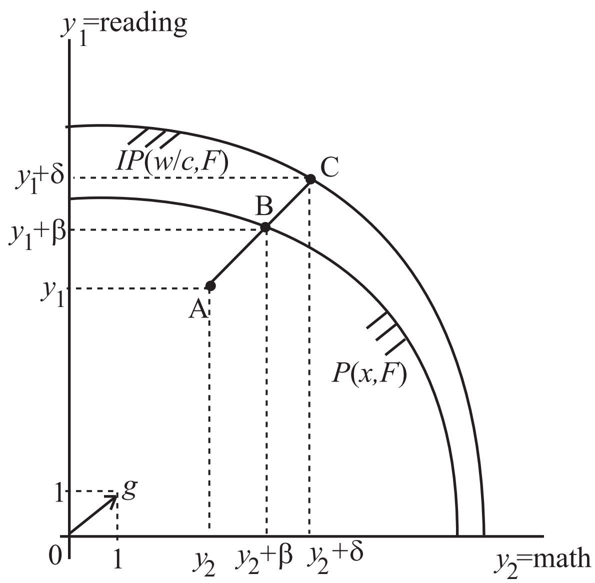

Figure 2: School Inefficiency

a particular school (campus) within a school district to produce at point A. The reference technology holding all inputs constant is $P(x, F)$. We seek the maximum expansion in the two outputs that is feasible given the technology. If campus A were to use its resources efficiently it could produce at B on the frontier of $P(x, F)$. Campus A's reading score could expand from y_1 to $y_1 + \beta$ and their math score could increase from y_2 to $y_2 + \beta$.

To measure efficiency relative to the budget-constrained production possibility set, $IP(w/c, F)$ we use the budget-constrained directional output distance function. Again, this function seeks the maximum expansion in outputs for a directional vector

g but in this case, the school can reallocate inputs, x, as long as those inputs satisfy the budget constraint. The budget-constrained directional output distance function takes the form

$$\vec{ID}_o(w/c, F, y; g) = \max \{\delta : (y + \delta g) \in IP(w/c, F)\}. \quad (2)$$

We again choose $g = (1, 1)$ and illustrate in Figure 2. Suppose that school A were able to reallocate their inputs, x, holding the budget with which to hire those inputs constant. Clearly, they could increase the two outputs to B by using the same inputs but reducing technical inefficiency. However, if school A were to reallocate its inputs so as to maximize the potential outputs it could expand its outputs even further: reading and math scores could be expanded to $y_1 + \delta$ and $y_2 + \delta$. A campus produces on the frontier of $IP(w/c, F)$ if $\delta = 0$ and is inefficient if $\delta > 0$.

There might be some campuses that produce outputs such that β=0 but $\delta > 0$. These schools are technically efficient but could expand outputs by reallocating their inputs, x. There might be other schools that are both technically efficient, β=0, and have also allocated inputs efficiently, $\delta = 0$. Appendix 1 shows how the two distance functions are estimated using DEA.

One of our objectives in this paper is to simulate how campus outputs would change if each school received a budget that had been determined by weighted-student funding. Such a funding formula would reallocate resources from some districts and campuses to other districts and campuses. Campuses that receive greater amounts of inputs would see their production possibilities expand, while campuses that receive smaller amounts of inputs would see their production possibilities contract. We define x_{wsf} as the inputs a campus would receive if a policy of weighted-student funding were adapted. Given the variable input prices that a campus faces we define $c_{wsf} = wx_{wsf}$ as the budget the campus would have available given weighted-student funding.

How will potential math and reading scores change under weighted-student funding? If $x_{wsf} \geq x$, then the production possibility set, $P(x, F) \subseteq P(x_{wsf}, F)$ and potential outputs can expand. On the other hand, if $x_{wsf} \leq x$ potential outputs will contract under weighted-student funding. Similarly, if $c_{wsf} \geq c$, the budget-constrained production possibility sets are such that $IP(w/c, F) \subseteq IP(w/c_{wsf}, F)$. In contrast, if $c_{wsf} \leq c$, the new budget-constrained production possibility set will be no larger than the status quo budget-constrained production possibility set.

Figure 3 illustrates two possible shifts in $IP(w/c, F)$ for a policy change consistent with a move to weighted-student funding. We observe a school operating inefficiently at point A. If that school were to produce efficiently it could expand reading and math scores to point B on the school's actual indirect production possibility frontier. If weighted-student funding is such that school A receives a larger budget ($c < c_{wsf}$) then the school's production possibilities would shift toward the northeast and the school could expand its two outputs to point C. In contrast, if school A receives a smaller budget under weighted-student funding the school's production possibilities would shift toward the southwest and the school would only be able to produce at point D.

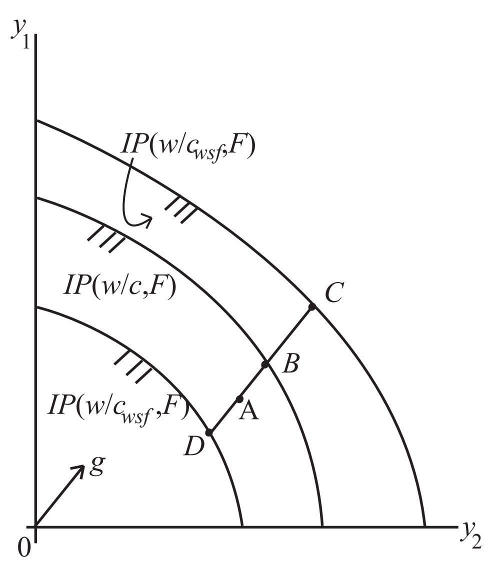

Figure 3: Output Gains or Losses under Weighted-student Funding

As with most public policies we anticipate winners and losers in a move toward weighted-student funding. If the schools that lose money ($c > c_{wsf}$) are inefficient, then the reduction in output will be somewhat tempered by any existing inefficiency. If the schools that gain money ($c < c_{wsf}$) are efficient under the status quo, then the expansion in the budget has real potential to help those schools achieve greater outputs. However, if the schools that lose money are efficient under the status quo, then the reduction in the budget will cause schools outputs to fall. Likewise, if the schools that gain money are inefficient under the status quo, then the expansion in the budget, while increasing potential outputs, will have no guarantee of increasing actual outputs.

4 Data

The data for our analysis come from administrative files and public records of the Texas Education Agency (TEA) and cover the three largest metropolitan areas in Texas-Dallas, Houston, and San Antonio-during the 2008-09 school year. These three metropolitan areas were chosen because they are the largest in Texas and among the largest in the nation. Nearly half of the public school students in Texas reside in one of these three metropolitan areas.

The unit of analysis is the school, and the analysis includes all traditional public schools with complete data that were located in one of the three metropolitan areas.[2] Table 1 provides descriptive statistics on the variables used in this analysis.

4.1 Outputs

We use two measures of school quality, both of which are based on the Texas Assessment of Knowledge and Skills, or TAKS. TAKS was a group of high stakes tests administered every year from 2003 through 2012. Student performance on TAKS

[2] Because they have access to a different educational technology, open enrollment charter schools have been excluded.

was used not only for federal accountability under the No Child Left Behind Act of 2001, but also for state accountability purposes. Students in the third and eighth grades had to pass TAKS to be promoted to the next grade, and students in the 11th grade had to pass TAKS in order to graduate. TAKS tests in mathematics and reading/language arts were administered annually in grades 3-11. Tests in other subjects such as science and history were also administered, but not in every grade level.

We measure school quality as the normalized gain score in reading and mathematics. The normalizations follow Reback (2008), Gronberg, Jansen and Taylor (2012), and Grosskopf et al. (forthcoming). They are designed to address concerns about reversion to the mean found in traditional gain scores.

In this normalization, we use test scores for student (i), grade (g), and time or year (t), denoted as S_{igt}. We measure each student's performance in each subject (reading or math) relative to the performance of all other students in the state at the same grade level with same past score:

$$Y_{igt} = \frac{S_{igt} - E(S_{igt}|S_{t,g-1,t-1})}{\left[E(S_{igt}^2|S_{i,g-1,t-1}) - E(S_{igt}|S_{i,g-1,t-1})^2\right]^{0.5}} \tag{3}$$

In calculating Y_{igt} we calculate the average test score at time t, grade g, for students scoring $S_{i,g-1,t-1}$ at time $t-1$, grade $g-1$. For example, we divide all fifth-grade students in the state into groups or bins based on their fourth-grade math test scores in 2008. We then calculate the average fifth grade math score and the standard deviation of the average fifth-grade math score for each bin. The average fifth-grade math score for each bin is the expected score for students in the bin. Our variable Y_{igt} measures the number of standard deviations from the expected score. This is a type of $z-$score, which has a mean of zero. These $z-$scores are averaged over all the students in each school to arrive at a school level measure of reading and math value added.[3]

[3] Students for whom the prior test score was missing are treated as one of the groups. This is

Because outputs with negative values are not tractable for analysis purposes, we further transform the $z-$scores into normal curve equivalent (NCE) scores. The normal curve equivalent, which is a standardization commonly used in the education literature, is defined as $50 + 21.06 \times z$. We multiply the NCE by the number of students at the school to obtain the aggregate school outputs.

4.2 Inputs

We use data on school and district expenditures to measure the campus-specific resources and the central administration resources in each school district. The data on actual expenditures come from administrative records provided by the TEA. Under the state's Public Education Information Management System, school districts are required to report the fund, function, object and financial unit (campus) for each dollar they spend, using standard definitions published by TEA. We use these data to calculate the level of personnel and non-personnel expenditures allocated to each campus. Expenditures not allocated to a specific campus are treated as overhead.

All measures (personnel expenditures at the campus level, non-personnel expenditures at the campus level, central administration personnel expenditures and central administration non-personnel expenditures) are aggregate amounts at the particular campus. However, the budget excludes food and student transportation expenditures. We exclude transportation expenditures on the grounds that they are unlikely to be explained by the same factors that explain student performance, and therefore that they add unnecessary noise to the analysis. We exclude food expenditures on similar grounds, and because it is not clear how to value the butter, eggs and other in-kind subsidies that school lunch programs may receive. We also exclude community service, debt service, capital outlays, facility acquisition and construction, and intergovernmental payments between school districts.

We note that the expenditures variables used in our analysis include all operat-

equivalent to assuming that all students with missing pre–test data had the state average pre–test score.

ing expenditures regardless of the sources of revenue. Charitable donations, federal grants, local tax revenues and state funding formula aid are all included. Our measures also include all types of operating expenditures in the designated categories. Thus, they include not only direct salary expenditures, but also contributions to the statewide teacher pension system, payments for group health and life insurance, and other outlays for employee benefits. The personnel expenditures measure includes payments for contracted workers as well as employees. The non-personnel expenditures measure includes payments for rents, utilities and supplies.

Previous research has found that there is substantial difference in labor cost from one part of Texas to another. Therefore, transforming the personnel and non-personnel expenditures into effective input quantities requires use of a labor cost index. Following Gronberg, Jansen and Taylor (2011) and Grosskopf et al. (forthcoming), we estimate a hedonic wage model wherein teacher salaries are a function of teacher demographics and cost factors that are outside of school district control. Details of this model are described in Appendix 2. We use the hedonic model to predict the monthly wage each school would have to pay to hire a teacher with zero years of experience and a bachelor's degree, holding all other observable teacher characteristics constant at the statewide mean, and suppressing any charter school differentials. We then define the effective quantity of school personnel as the school's total expenditure on personnel, divided by the prevailing monthly wage. This approach treats compensation as a direct indicator of educator quality, and is consistent with work by Loeb and Page (2000) which indicates that there is a positive and statistically significant relationship between teacher salaries and teacher quality, once working conditions are taken into account.

There is no such evidence to suggest that there are systematic differences in the cost of non-personnel inputs. Therefore, we presume that the cost of non-personnel inputs is constant throughout the three metropolitan areas, and normalize it to one.

4.3 Other Environmental Factors

The model includes indicators for several environmental factors that influence the educational technology but which are not purchased inputs. To capture variations in costs that derive from variations in student needs, we include the percentages of students in each district who have high English proficiency (%HEP), percentage non-special education students (%nonspecialed) and percent with high socioeconomic status (%HighSES). Finally, we include the number of students on a particular campus as a fixed input for that campus, so that students are not reallocated (bussed) across campuses, say from one primary campus to another primary campus in that school district.

4.4 Weighted Student Funding Simulation

Under the Texas school funding model students with different characteristics generate additional revenues for a school district. For example, an economically disadvantaged student will generate 20% more revenue than a student who is not economically disadvantaged. A student who is in bilingual education programs would generate 10% more than a student who is not in bilingual education. Furthermore, the weights are additive, meaning that a student who is both economically disadvantaged and LEP would generate 30% more revenue than a student who is neither. Funding model weights are provided for students in compensatory education programs for economically disadvantaged students, special education programs, bilingual education programs, gifted education program, career and technology education programs and the high school program.

School districts are not obliged to use these formula weights when allocating resources to schools and most do not. However, we compare the level of performance given the status quo budget with the level of performance a school might achieve if each district allocated resources to schools according the state's school funding model, assuming no change in overhead expenses. Let $s = 1, \ldots, S$ represent a particular school (campus) within the district and let $i = 1, \ldots, P$ represent the number of programs. To obtain the amount school s would receive under weighted-student funding we calculate the school share of weighted-average daily attendance ($SWADA_s$) based on the school's programmatic enrollment:

$$SWADA_s = \frac{\sum_{i=1}^{P} DPP_i \times STUD_{si}}{\sum_{s=1}^{S} \sum_{i=1}^{P} DPP_i \times STUD_{si}} \tag{4}$$

where DPP_i is the district revenue per pupil for program i, and $STUD_{si}$ is enrollment at school s in program i. We then apply the school share of district weighted-average daily attendance, $SWADA_s$, to the district total spending on campus personnel and nonpersonnel to yield the school-level weighted-student funding. We note that DPP_i varies across school districts as each school district receives a basic amount of funding that is then adjusted for the size of the district, cost differences, and regulations such as hold-harmless provisions.

5 Performance Estimates

To estimate the distance functions we use data taken from the Texas Education Agency for the 2008-09 school year for 175 school districts in the Dallas (70 districts), Houston (66 districts) and San Antonio (39 districts) metropolitan areas. The 175 school districts include 387 high schools, 618 middle schools, 1694 elementary schools, and 10 mixed schools. Students in grades 3 to 11 take the Texas Assessment of Knowledge and Skills, a standardized achievement test. Each school is assumed to produce two outputs: value added on a statewide reading achievement test (y_1) and value added on a statewide mathematics achievement test. Each school uses variable inputs of personnel (x_1) and non-personnel expenditures (x_2). The personnel input is measured as the number of teacher units at the school. In addition, two kinds of school district central administration overhead expenses are allocated to each campus within the school district on a per pupil basis: central administration core operating overhead expenses(F_1) and central administration overhead payroll expenses (F_2). We also control for the number of students at each school (F_3), the percent of students at the school who have high English proficiency (F_4), the percent of students at the school who are deemed high socio-economic status (F_5), and the percent of students at the school who are not special education students (F_6).

Table 1 reports descriptive statistics for the 2,709 schools. The average school spends (wx) slightly more than $5 million on the two variable inputs of personnel and non-personnel maintenance and utilities and has an additional $1.36 million ($653 thousand and $483 thousand) in central administration overhead spending. Among the 802 average students, approximately 81% have high English proficiency and 91% are non-special education students, but only 44% are deemed high socio-economic status. Average reading and math NCE scores per pupil are 50.8 and 50.7.

Table 1: Descriptive Statistics for 2709 Schools

	mean	Std. Dev.	Minimum	Maximum
y_1=read	40738.5	27751.5	112.7	223181
y_1/student	50.8	5.6	4.3	71
y_2=math	40705.6	27944.9	220.0	212686
y_2/student	50.7	4.4	30.0	77
x_1=# of personnel	1149.4	726.6	21.4	6497
x_2=non-personnel exp.	476843.8	515593.7	0.0	5063815
F_1=personnel overhead	653633.7	560800.0	1711.9	8116015
F_2=non-personnel overhead	483186.8	703036.7	710.1	28610110
F_3=students	802.1	543.6	3.0	4572
F_4=% high English prof.	0.81	0.19	0.13	1
F_5=% high socio-econ. status	0.44	0.30	0	1
F_6=% non-special ed.	0.91	0.04	0.65	1
w_1	3937.0	109.3	3427.9	4093
w_2	1.0	1.0	1.0	1
$wx = c$	5027680.8	3345020.3	73308.0	30560288.3
wx/student	6457.9	1434.1	2810.8	25893

We solve four linear programming problems using DEA for each school (campus) within the district. Each model estimates either the directional output distance function or the budget-constrained directional output distance function for the directional vector $g = (1, 1)$. In model 1 we estimate the directional output distance function for each school and obtain an estimate of β. This estimate gives the simultaneous expansion in reading and math test scores given variable inputs x and fixed inputs F. In model 2 we estimate the budget-constrained output distance function for each school and obtain an estimate of δ. Here, δ gives the expansion in reading and math test scores given the school budget (c), input prices (w), and fixed inputs. In our last two models we switch to a system of weighted-student funding and simulate the expansion in reading and math scores that could result if schools were efficient. In model 3 we give each school variable inputs corresponding to weighted-student funding, x_{wsf}, but hold the fixed inputs (F) corresponding to central administration overhead and student demographic students constant. In model 4 we give each school a budget that is consistent with weighted-student funding, c_{wsf}, but hold the fixed inputs (F) constant and also hold variable input prices (w) constant.

Rather than pooling all schools together and assuming a common production technology we instead estimate each of the four models for the different types of schools: high schools, middle schools, elementary schools, and mixed schools. This amounts to assuming that different types of schools face different technologies. That is, a high school and an elementary school with identical amounts of variable and fixed inputs would have output possibility sets, $P(x, F)$, that are shaped and positioned differently. Similarly, a high school and an elementary school with the same budget, input prices, and fixed inputs would potentially have indirect output possibility sets, $IP(w/c, F)$, that are shaped and positioned differently. This assumption allows for the possibility that it might be easier or more difficult to educate an elementary school student than a high school student with the same demographic characteristics. However, we do assume that schools of the same type, say elementary schools, face the same technology regardless of the metropolitan area they reside in.

The model estimates derived from the linear programming problems that are stated in the Appendix 1 give the aggregate addition to reading and math test scores. However, for ease of exposition we report the estimates of inefficiency in Table 2 on a per student basis.

Elementary schools in the Dallas metropolitan area exhibit the most technical inefficiency ($\hat{\beta}$) followed by elementary schools in the Houston and San Antonio metropolitan areas. The mean estimate of inefficiency among all 1694 elementary schools is $\hat{\beta} = 8.81$ standardized points per student. This estimate indicates the per student amount that reading and math test scores could simultaneously increase if the average school were to become technically efficient in their use of existing resources. If schools were able to optimally reallocate their existing budgets by choosing the right mix of school personnel and non-personnel inputs they could increase reading and math scores by an additional 0.79 points ($\hat{\delta} - \hat{\beta}$) per student. In this case, elementary schools in San Antonio are the least inefficient, $\hat{\delta} = 9.92$, followed by schools in Dallas and then Houston.

Dallas also has the most technical inefficiency among middle schools, high schools, and mixed schools. However, relative to elementary schools, middle schools and high schools exhibit less technical inefficiency on average, $\hat{\beta} = 4.51$ for middle schools and $\hat{\beta} = 4.91$ among high schools.

Next we turn to our estimates of what potential inefficiency might be if districts allocated resources internally to schools consistent with weighted-student funding. To do the simulation we first estimate the budget the school would receive as described by Equation (4). Let this budget equal c_{wsf}. Then, given the simulated budget and the actual input prices, w, actual fixed inputs, F, and actual outputs, y, we re-estimate the budget-constrained directional output distance function. This problem is described in Appendix 1. Schools which receive a larger budget will see their production possibility frontiers shift outward. Those schools will have the potential to increase outputs which will show up as increased inefficiency in our sim-

Table 2: Potential Test Score Gains from Enhanced Efficiency under the Status Quo (SQ) and Weighted Student Funding (WSF). $\hat{\beta}$ represents the gain from reducing technical inefficiency and $\hat{\delta}$ represents the gain from reducing technical inefficiency and allocative inefficiency.

		All	Dallas	Houston	San Antonio
			Elementary Schools		
# of schools		1694	663	732	299
$\hat{\beta}$/stud.	SQ	8.81	9.19	8.57	8.55
$\hat{\delta}$/stud.	SQ	9.60	9.90	9.20	9.92
$\hat{\beta}$/stud.	WSF	8.38	8.85	7.95	8.41
$\hat{\delta}$/stud.	WSF	8.89	9.33	8.38	9.17
			Middle Schools		
# of schools		618	221	293	104
$\hat{\beta}$/stud.	SQ	4.51	4.84	4.25	4.56
$\hat{\delta}$/stud.	SQ	5.18	5.52	4.86	5.37
$\hat{\beta}$/stud.	WSF	3.97	4.49	3.66	3.70
$\hat{\delta}$/stud.	WSF	4.54	5.05	4.11	4.65
			High Schools		
# of schools		387	151	169	67
$\hat{\beta}$/stud.	SQ	4.91	4.98	4.84	4.90
$\hat{\delta}$/stud.	SQ	5.55	5.85	5.28	5.53
$\hat{\beta}$/stud.	WSF	2.84	3.32	2.55	2.47
$\hat{\delta}$/stud.	WSF	4.79	5.33	4.29	4.85
			Mixed Schools		
# of schools		10	4	3	3
$\hat{\beta}$/stud.	SQ	0.74	1.42	0	0.58
$\hat{\delta}$/stud.	SQ	0.89	1.76	0	0.61
$\hat{\beta}$/stud.	WSF	-7.78	-4.11	-20.48	0.03
$\hat{\delta}$/stud.	WSF	-4.94	-1.69	-15.40	1.20
			All Schools		
# of schools		2709	1039	1197	473
$\hat{\beta}$/stud.	SQ	7.24	7.62	6.96	7.11
$\hat{\delta}$/stud.	SQ	7.98	8.35	7.56	8.24
$\hat{\beta}$/stud.	WSF	6.52	7.07	6.06	6.48
$\hat{\delta}$/stud.	WSF	7.26	7.80	6.70	7.51

ulation estimates. Schools which receive a smaller budget will see their production possibility frontier shift inward resulting in a contraction in potential outputs which will show up as a decline in inefficiency.

We also estimated the amount of variable inputs, x, that schools would receive under weighted-student funding. Here, we assume that the share of the budget allocated to nonpersonnel expenditures remains constant. For instance, if a school allocated 45% of their actual budget to nonpersonnel expenditures then we assume they allocate 45% of the budget they receive under weighted-student funding to nonpersonnel expenditures. That is $s = \frac{w_2 x_2}{c} = \frac{w_2 x_{2,wsf}}{c_w sf}$ so that $x_{2,wsf} = \frac{c_{wsf}}{w_2}$. Given the share allocated to nonpersonnel expenditures we calculate the quantity of personnel as $x_{1,wsf} = \frac{c_{wsf} - w_2 x_{2,wsf}}{w_1}$. We use simulated quantities of the variable inputs, $x_{1,wsf}$ and $x_{2,wsf}$, along with the actual fixed inputs, F, and the actual outputs, y, in calculating the directional output distance function. This problem is described in Appendix 1.

Average technical inefficiency ($\hat{\beta}$) and overall inefficiency ($\hat{\delta}$) are lower for the pooled sample of 2709 schools under weighted-student funding than they are under the status quo for each type of school in each metropolitan area. This finding indicates that on average, a movement toward weighted-student funding will result in the average school receiving resources such that potential outputs will shrink.

Our linear programming method results in a distribution of inefficiencies that in general is not normally distributed. We test the null hypothesis that various statistics or distribution functions of the estimates of δ for the status quo and for δ under weighted-student funding are equal using a series of nonparametric tests. Table 3 reports the results of the Anova-F test which does assume normality, and the nonparametric Kruskal-Wallis, Median, van der Waerden, Savage Scores tests as well as the Kolmogorv-Smirnov test and Qi Li's (1996) T-test which are nonparametric distributional tests of the kernel densities of inefficiency. Kneip, Simar, and Wilson (2013) show that standard central limit theorems do not hold for means of inefficiency scores estimated via data envelopment analysis. Therefore, although we report all of the various tests, we focus on the results of the Kolmogorov-Smirnov test and Qi Li's T-test which are valid tests of differences in the empirical distribution functions. For Qi Li's T-test we bootstrap the results 500 times following Pagan and Ullah (1999). Both tests reject the null hypothesis of equal distributions of inefficiency under the status quo and the simulated policy of weighted-student funding for elementary schools, middle schools, and high schools. The T-test also rejects the null for mixed schools but the Kolmogorov-Smirnov test does not. We conclude that there is at least some evidence that a move toward weighted-student funding will result in a leftward shift in the empirical distribution functions of inefficiencies. Such a result implies that potential outputs will shrink under a policy of weighted-student funding that uses the state's funding formula weights.

Table 3: Will Weighted-student Funding Change Potential Outputs? Nonparametric Tests

	Elementary	Middle	High Schools	Mixed Schools
Anova-F	5.25	5.35	30.48	4.56
(Prob>F)	(0.02)	(0.02)	(0.01)	(0.05)
Kruskal-Wallis	2.48	0.76	18.17	1.17
(Prob>X^2)	(0.12)	(0.38)	(0.01)	(0.28)
Median	1.89	0	4.96	0.54
(Prob>X^2)	(0.17)	(1)	(0.03)	(0.46)
van der Waerden	5.79	4.05	26.95	1.35
(Prob>X^2)	(0.02)	(0.04)	(0.01)	(0.24)
Savage Scores	3.59	0.36	9.44	0.33
(Prob>X^2)	(0.06)	(0.55)	(0.01)	(0.56)
Kolmogorov-Smirnov	1.48	2.42	3.85	1.12
(Prob>Ks_a)	(0.03)	(0.01)	(0.01)	(0.16)
Li's T-test	3.51	3.89	2.13	1.96
(Prob>T)	(0.01)	(0.01)	(0.03)	(0.05)

6 The Effects of Weighted Student Funding on Inequality

Next, we turn our attention to an examination of various measures of vertical equity for the status quo outputs and for the potential outputs that might occur if schools were able to realize greater efficiency under weighted-student funding. Table 4 reports five different measures of inequality in various school resources and outcomes. We report Brazer's coefficient of variation (CV) which equals the interquartile range as a proportion of the median, the Gini coefficient which ranges from 0 (perfect equality) to 1 (perfect inequality), the Theil inequality index, the range, and the ratio of the 95 percentile value to the 5 percentile value. All variables are measured in per pupil terms. The amount of variable spending (wx) per student exhibits greater inequality than the outputs per student. For instance, Brazer's CV is approximately 1.5 time greater for spending per student than it is for the reading test score per student and 1.88 times greater than it is for the math test score per student. Similarly, the Gini coefficient is twice as large for spending per student as it is for the reading score per student and 2.6 times larger for spending per student than for the math score per student. We also find greater inequality in the actual level of reading scores (y_1) than in math scores (y_2).

As Table 4 illustrates policies that reduce school inefficiency tend to enhance equality. For instance, comparing actual reading scores (y_1) with potential reading scores ($y_1 + \beta$) every measure of inequality is reduced if technical inefficiency is reduced and output is expanded. The same is true for math scores except for the CV which increases slightly from 0.113 to 0114. When comparing actual reading scores with potential reading scores ($y_1 + \delta$) if technical efficiency is reduced and if school resources are allocated efficiently we find all measures of inequality are reduced. The same is true for math scores. This pattern suggests that inefficiency is an important source of outcomes' inequality.

Table 4: Inequality Measures for 2709 Schools-All Variables per Student

Variable	CV	Gini	Theil	Range	95 to 5 pct. Ratio
wx	0.212	0.139	0.021	23082	1.76
wx_{wsf}	0.188	0.086	0.012	8429	1.68
		Reading scores			
y_1	0.142	0.061	0.0062	66.70	1.43
$y_1 + \beta$	0.134	0.048	0.0046	66.70	1.36
$y_1 + \delta$	0.130	0.051	0.0041	46.26	1.34
$y_1 + \beta$(wsf)	0.132	0.056	0.0055	48.63	1.40
$y_1 + \delta$(wsf)	0.127	0.052	0.0045	45.61	1.36
		Math scores			
y_2	0.113	0.053	0.0037	47.39	1.32
$y_2 + \beta$	0.114	0.046	0.0034	38.81	1.31
$y_2 + \delta$	0.113	0.048	0.0037	47.38	1.32
$y_2 + \beta$(wsf)	0.112	0.050	0.0043	49.95	1.35
$y_2 + \delta$(wsf)	0.107	0.045	0.0034	42.38	1.31

Table 5: Inequality Measures for Houston Independent School District-210 Schools, All Variables per Student

Variable	CV	Gini	Theil	Range	95 to 5 pct. Ratio
$/stud	0.165	0.0947	0.0239	20744	1.54
$/stud(wsf)	0.170	0.0703	0.0079	4448	1.53
		Reading scores			
y_1	0.148	0.0647	0.0065	28.66	1.47
$y_1 + \beta$	0.122	0.0498	0.0040	26.52	1.33
$y_1 + \delta$	0.125	0.0498	0.0040	26.10	1.33
$y_1 + \beta$(wsf)	0.127	0.0520	0.0044	31.97	1.35
$y_1 + \delta$(wsf)	0.121	0.0510	0.0043	31.58	1.34
		Math scores			
y_2	0.142	0.0567	0.0051	29.34	1.38
$y_2 + \beta$	0.136	0.0539	0.0047	25.84	1.39
$y_2 + \delta$	0.130	0.0531	0.0045	25.84	1.36
$y_2 + \beta$(wsf)	0.115	0.0493	0.0042	31.25	1.34
$y_2 + \delta$(wsf)	0.122	0.050	0.0043	31.51	1.34

Table 6: Inequality Measures for Dallas School District-xxx Schools, All Variables per Student

Variable	CV	Gini	Theil	Range	95 to 5 pct. Ratio
\$/stud	0.217	0.1183	0.0407	20313	1.84
\$/stud(wsf)	0.066	0.0367	0.0036	3214	1.24
		Reading scores			
y_1	0.157	0.0710	0.0104	36.13	1.57
$y_1 + \beta$	0.138	0.0567	0.0063	34.57	1.39
$y_1 + \delta$	0.148	0.0579	0.0064	34.57	1.39
$y_1 + \beta$(wsf)	0.150	0.0583	0.0067	35.53	1.40
$y_1 + \delta$(wsf)	0.131	0.0541	0.0057	32.14	1.37
		Math scores			
y_2	0.133	0.0552	0.0078	39.51	1.36
$y_2 + \beta$	0.134	0.0526	0.0069	31.90	1.34
$y_2 + \delta$	0.140	0.0534	0.0059	31.90	1.32
$y_2 + \beta$(wsf)	0.132	0.0518	0.0053	30.86	1.36
$y_2 + \delta$(wsf)	0.122	0.0453	0.0039	23.48	1.27

Tables 5, 6, and 7 examine spending per student and performance for three large school districts in the three metro areas: the Houston Independent School District (ISD), the Dallas ISD, and the San Antonio ISD. The Dallas ISD has the greatest inequality in spending per student except for the Range which is greatest for the Houston ISD. The San Antonio ISD exhibits the least inequality for all five measures for spending per student. However, our simulation indicates that if the Dallas ISD moved to a system of weighted-student funding it would have the lowest level of inequality in spending per student except for the Range which would be lowest at the San Antonio ISD. Dallas also has the highest inequality in reading test scores and San Antonio has the lowest inequality in both reading and math scores.

Reducing technical inefficiency would reduce inequality in reading scores Houston and Dallas, but increase inequality for the CV and Gini in reading scores in San Antonio. The same pattern holds for math scores except for an increase from 0.133 to 0.134 for the CV in Dallas and an increase in inequality for all five mea-

Table 7: Inequality Measures for San Antonio Independent School District-85 schools, All Variables per Student

Variable	CV	Gini	Theil	Range	95 to 5 pct. Ratio
$/stud	0.146	0.063	0.0072	4508.59	1.48
$/stud(wsf)	0.117	0.053	0.0047	3099.17	1.39
	Reading scores				
y_1	0.146	0.0648	0.0070	32.07	1.46
$y_1 + \beta$	0.197	0.0670	0.0069	25.62	1.44
$y_1 + \delta$	0.196	0.0646	0.0066	23.65	1.43
$y_1 + \beta$(wsf)	0.226	0.0818	0.0119	37.80	1.66
$y_1 + \delta$(wsf)	0.179	0.0591	0.0054	22.73	1.40
	Math scores				
y_2	0.094	0.0424	0.0031	22.27	1.28
$y_2 + \beta$	0.175	0.0546	0.0046	24.04	1.37
$y_2 + \delta$	0.146	0.0503	0.0039	20.84	1.31
$y_2 + \beta$(wsf)	0.154	0.0653	0.0072	26.91	1.53
$y_2 + \delta$(wsf)	0.113	0.0411	0.0026	17.43	1.25

sures of inequality in San Antonio. Relative to actual test scores, reducing both technical inefficiency and allocative inefficiency would reduce inequality in reading scores and math scores in Houston, would reduce inequality in reading scores in Dallas, and reduce inequality in math scores in Dallas for all measures except the CV. In San Antonio, reducing both technical inefficiency and allocative inefficiency would increase inequality the CV measure of inequality, but reduce the other 4 measures of inequality. For math scores, reducing would increase inequality except when inequality is measured by the Range, which would decrease.

Next, we turn to examining inequality in reading and math scores with a move toward weighted-student funding Comparing actual reading (y_1) and math scores (y_2) with simulated reading ($y_1 + \delta_{wsf}$) and math scores ($y_2 + \delta_{wsf}$) if both technical and allocative inefficiencies are reduced we see a decline in all five measures of inequality for reading and math scores in Houston and Dallas, and a decline in inequality for reading and math scores except for the CV in San Antonio.

We further examine the effects of the simulated move toward weighted-student funding in Table 8 and Figure 4. Table 8 reports the number of winners and losers under weighted-student funding. Comparing the actual indirect production possibility set, $IP(w/c, F)$, with the indirect production possibility set that would exist under weighted-student funding, $IP(w/c_{wsf}, F)$, we see that among all 2,709 schools, 668 schools (24.6%) would see their potential outputs expand, 1,351 schools (49.9%) would see their potential outputs contract, and 690 schools (25.5%) would see no change in their potential outputs. Elementary schools have both the largest number of schools which would gain under weighted-student funding, 445 (26.3%), but also have the largest number of schools that would lose under weighted-student funding, 944 (55.7%). The average gain for all 2,709 schools is 0.81 points on the reading and math tests while the average loss is 1.84 points. For elementary, middle, and high schools the potential gains average between 0.79 to 0.82 points, while the potential loss in the two outputs averages 1.66 for elementary schools, 2.24 for middle schools, and 2.07 for high schools.

Table 8: Winners and Losers under Weighted Student Funding

		N=2709 All Schools	N=1694 Elementary	N=618 Middle	N=387 High Schools	N=10 Mixed
# of winners	$\hat{\beta} < \hat{\beta}_{wsf}$	967	657	251	58	1
# of losers	$\hat{\beta} > \hat{\beta}_{wsf}$	1419	866	267	280	6
# no change	$\hat{\beta} = \hat{\beta}_{wsf}$	323	171	100	49	3
Average gain		1.23	1.28	1.04	1.46	1.28
Average loss		2.12	1.81	2.24	3.16	14.42
# of winners	$\hat{\delta} < \hat{\delta}_{wsf}$	668	445	148	74	1
# of losers	$\hat{\delta} > \hat{\delta}_{wsf}$	1351	944	231	170	6
# no change	$\hat{\delta} = \hat{\delta}_{wsf}$	690	305	239	143	3
Average gain		0.81	0.82	0.81	0.79	1.89
Average loss		1.84	1.66	2.24	2.07	10.02

To further illustrate Figure 4 plots the budget-constrained output distance function estimates (δ) for the status quo budget (vertical axis) against the distance function estimates for the simulated budget under weighted-student funding (horizontal axis). There were 229 out of 2,709 schools that produced on the frontier of $IP(w/c, F)$ for the status quo budget and those schools lie along the horizontal axis. Out of the 229 frontier schools 41 of those schools are to the right of the origin and under weighted-student funding would receive a budget that would allow them to expand their production of math and reading test scores if they could use the new funds efficiently. The 103 schools to the left of the origin would see their budgets shrink under weighted-student funding, and, given their initial efficient level of production, the smaller budget would cause a decline in math and reading test scores. The remaining 85 frontier schools would receive the same budget under weighted-student funding. The 2,480 schools that are inefficient given the status quo budget lie above the horizontal axis. To compare the potential performance of the inefficient schools under the status quo budget and their potential performance under weighted-student funding we draw a 45^o line from the origin as a reference. To the right of the 45^o line lie 627 schools which would experience an increase in their budget (and production possibilities) under weighted-student funding. Along the 45^o line (but excluding the origin) lie 605 inefficient schools which would receive the same budget under weighted-student funding as they do under the status quo. To the left of the 45^o line lie the remaining 1,248 schools which would receive a smaller budget under weighted-student funding.

Our simulation predicts one thing with relative certainty. To the extent that school resources matter in the production of value added test scores, the efficient schools under the status quo allocation of inputs and associated budget which lie to the left of the origin (103 schools) would see their production possibilities shrink. We are less certain about the following: for the 41 efficient schools that receive more inputs and a larger budget under weighted-student funding outputs can possibly expand if the resources are used efficiently. Given that these schools were efficient to begin with, it seems reasonable to think that these schools can efficiently use the extra resources to expand output.

For the 2,480 schools that were inefficient under the status quo several possibilities emerge with a change to weighted-student funding. First, those schools remain inefficient and the change toward weighted-student funding only redistributes inputs with no change in value-added test scores. Second, those schools that were inefficient under the status quo somehow become efficient under weighted-student funding policy. For the 627 schools that lie to the right of the 45^o line a change toward weighted-student funding **and** greater efficiency would result in enhanced test scores. This possibility seems much less certain. Now, consider the inefficient schools that lie between the vertical axis and the 45^o degree line. These 1,168 schools would see their production possibilities shrink under weighted-student funding, but they were inefficient for the status quo budget. In fact, their inefficiency was great enough so that even though their budgets shrink under weighted-student funding, enhanced efficiency could more than offset their smaller budgets. Perhaps the declining budgets and resources would refocus administrator and teacher efforts on getting the most out of the now smaller set of inputs. Finally, 80 schools are inefficient given their status quo budgets and lie to the left of the vertical axis. These 80 schools would see their production possibilities shrink to the extent that even if they were to become fully efficient they would still experience a decline in value-added test scores.

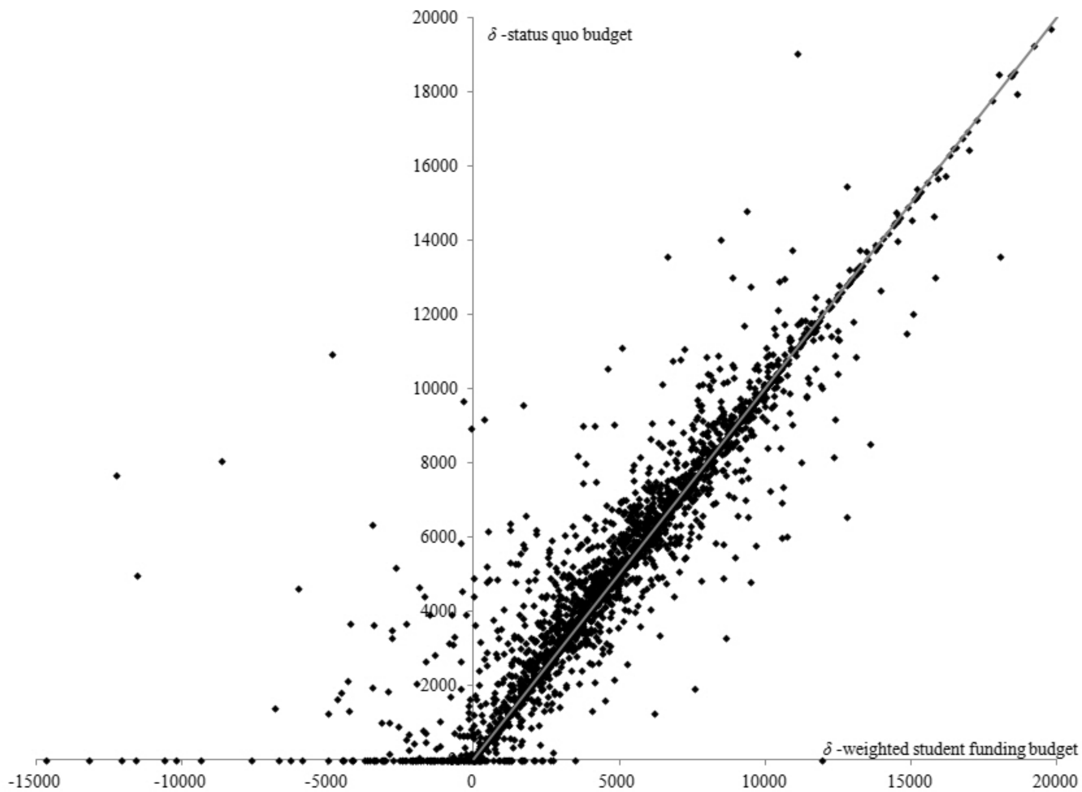

Figure 4: Budget Inefficiency Under the Status Quo versus Weighted-student Funding

7 Conclusions

Policy makers have long sought to foster equity in public school funding. Although equalizing per pupil expenditures was once the goal, differences in the marginal cost of educating students has caused researchers to shift their focus toward equalizing school outcomes. One such policy under consideration is weighted-student funding where school funding formulas would take account of differences in the costs of educating different students. Under weighted-student funding schools would receive larger allocations of money for students with disabilities, students who come from disadvantaged socio-economic backgrounds, or students who do not speak English as their native language.

We simulate a policy move toward weighted-student funding for three Texas metropolitan areas-Dallas, Houston, and San Antonio-which together account for approximately half of all students in Texas. Our model compares potential value added on reading and math scores with what might be achieved by the various schools if they were to adopt the best-practice technology from the sample of observed schools. In addition, our simulation indicates that weighted-student funding would generate both winners and losers. Under weighted-student funding 668 schools would gain resources be able to increase value added test scores by 0.81 points if they use those resources efficiently. However, 135 schools would lose resources and be subject to a potential loss of 1.84 points.

We offer several caveats of our study. Fist, although our simulation shows that a move to weighted-student funding could enhance equity in outcomes as measured by value-added test scores, schools also produce other outputs that we have not accounted for such as socialization, preparation for the job market, and extracurricular activities. Second, changes in the school funding formula would likely provide school administrators an incentive to reclassify some students as having a disability. Third, our simulation showing enhanced equity in educational outcomes is predicated on schools reducing various technical and allocative inefficiencies. If inefficient schools that receive enhanced funding cannot reduce existing inefficiencies the new funding formula will be for nought.

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Appendix 1

The linear programming problems that define the DEA reference technology and estimate the directional output distance function relative to the technologies $P(x, F)$ and $IP(w/c, F)$ are presented in this appendix. We assume there are $k = 1, \ldots, K$ observations of schools. Each school produces M outputs, $y_k = (y_{k1}, \ldots, y_{kM})$ using N variable inputs, $x_k = (x_{k1}, \ldots, x_{kN})$, and J fixed inputs, $F = (F_{k1}, \ldots, F_{kJ})$. We let $z = (z_1, \ldots, z_K)$ represent a vector of intensity variables that are used to form linear combinations of the inputs and outputs of all observed schools. These linear combinations of inputs and outputs "envelop" the data and form the best-practice technology. The output possibility set for school "o" is given as

$$
\begin{aligned}
P(x_o, F_o) = \{y :& \\
& \sum_{k=1}^{K} z_k y_{km}, \geq y_m, m = 1, \ldots, M, \\
& \sum_{k=1}^{K} z_k x_{kn} \leq x_{on}, n = 1, \ldots, N, \\
& \sum_{k=1}^{K} z_k F_{kj} \leq F_{oj}, j = 1, \ldots, J, \\
& z_k \geq 0, k = 1, \ldots, K\}. \qquad (5)
\end{aligned}
$$

In words, the DEA output possibility set consists of the outputs that can be produced by the variable and fixed inputs such that no more output can be produced using no less of the fixed and variable inputs than a linear combination of all the observed school outputs and inputs. To measure performance relative to $P(x, F)$ we use the directional output distance function. We choose a directional vector $g = (1, \ldots, 1)$ to scale outputs to the frontier of $P(x, F)$. The directional output distance function for school "o" is estimated using DEA as

$$
\begin{aligned}
\vec{D}_o(x_o, F_o, y_o; 1) = \max_{z,\beta} \beta \\
\sum_{k=1}^{K} z_k y_{km} \geq & y_{om} + \beta, m = 1, \ldots, M, \\
\sum_{k=1}^{K} z_k x_{kn} \leq & x_{on}, n = 1, \ldots, N, \\
\sum_{k=1}^{K} z_k F_{kj} \leq & F_{oj}, j = 1, \ldots, J, \\
z_k \geq & 0, k = 1, \ldots, K\}.
\end{aligned} \tag{6}
$$

On the right-hand side of (6) are the observed outputs and inputs of school "o" and on the left-hand side of (6) is the best-practice DEA technology comprising all observed schools outputs and inputs.

The budget-constrained output possibility set gives the set of outputs that can be produced given fixed inputs F and the budget c that a school has to purchase variable inputs x at input prices $w = (w_1, \ldots, w_N)$. Here, the school can choose the variable inputs as long as the choice satisfies the budget constraint: $\sum_{n=1}^{N} w_n x_n \leq c$.

The DEA budget-constrained output possibility set for school "o" takes the form:

$$\begin{aligned} IP(w_o/c_o, F_o) = \{ y : & \\ & \sum_{k=1}^{K} z_k y_{km}, \geq y_m, m = 1, \ldots, M, \\ & \sum_{k=1}^{K} z_k x_{kn} \leq x_n, n = 1, \ldots, N, \\ & \sum_{k=1}^{K} z_k F_{kj} \leq F_{oj}, j = 1, \ldots, J, \\ & \sum_{n=1}^{N} w_{on} x_n \leq c_o \\ & z_k \geq 0, k = 1, \ldots, K \}. \end{aligned} \tag{7}$$

We note that in (5) the observed variable inputs for school "o" enter on the right-hand side but in (7) the variable inputs, x_n $n = 1, ..., N$, can be chosen so long as they satisfy the budget constraint.

The budget-constrained directional distance function with directional vector $g = (1, \ldots, 1)$ takes the form:

$$\begin{aligned} \overrightarrow{ID}_o(w_o/c_o, F_o, y_o; 1) = \max_{z,\delta} \ & \delta \\ \sum_{k=1}^{K} z_k y_{km} \geq & y_{om} + \delta, m = 1, \ldots, M, \\ \sum_{k=1}^{K} z_k x_{kn} \leq & x_n, n = 1, \ldots, N, \\ \sum_{k=1}^{K} z_k F_{kj} \leq & F_{oj}, j = 1, \ldots, J, \\ \sum_{n=1}^{N} w_{on} x_n \leq & c_o \\ z_k \geq & 0, k = 1, \ldots, K \}. \end{aligned} \tag{8}$$

We recalculate the problems given in (6) and (8) under weighted-student funding. In (8) we change the budget, c, to c_{wsf} as calculated in (4).

Appendix 2: The Labor Cost Index

The hedonic model is a very simple one, wherein wages are a function of labor market characteristics, job characteristics, observable teacher characteristics, and unobservable teacher characteristics. Formally, the specification can be expressed as:

$$\ln(W_{idjt}) = \alpha_i + D_{dt}\delta + T_{it}\gamma + \mu_j + \epsilon_{idjt} \tag{9}$$

where the subscripts i, d, j, and t stand for individuals, districts, labor markets, and time, respectively, W_{idjt} is the teacher's full-time-equivalent monthly salary, D_{dt} is a vector of job and labor market characteristics that could give rise to compensating differentials, T_{it} is a vector of individual characteristics that vary over time, the μ_j are labor market fixed effects and the α_i are individual teacher fixed effects. Any time-invariant differences in teacher quality will be captured by the fixed effects.

The data on teacher salaries and individual teacher characteristics come from the Texas Education Agency (TEA) and Texas' State Board for Educator Certification (SBEC). The measure of teacher salaries that is used in this analysis is the total full-time equivalent monthly salary, excluding supplements for athletics coaching. The hedonic model includes controls for teacher experience (the log of years of experience, and the square of log experience) and indicators for the teacher's gender, race (black, Hispanic or Asian/Indian), educational attainment (no degree, master's degree or doctorate), teaching assignment (math, science, special education, health and physical education or language arts) and certification status (certified in any subject, and specifically certified in mathematics, science, special education or bilingual education). Only teachers with complete data who worked at least half time for a charter school or traditional Texas school district during the analysis period are included in the analysis. The hedonic wage analysis covers the five year period from 2004–05 through 2008–09.

The job characteristics used in this analysis allow for teachers to expect a compensating differential based on student demographics, school size, school type or district size. The student demographics used in this analysis are the percentage of students in the district who are economically disadvantaged, limited English proficient, black and Hispanic. We measure school size as the log of average campus enrollment in the district. There are three indicators for school type (elementary schools, middle schools, high schools). The analysis also includes four indicators for school district size: one indicator variable for very small districts (those with less than 800 students in average daily attendance), one for small districts (those with at least 800, but less than 1,600 students), one for midsized school districts (those with at least 1,600 but less than 5,000 students) and one for very large school districts (those with more than 50,000 students in average daily attendance.

In addition to the metropolitan area fixed effects, we include three indicators for local labor market conditions outside of education. We updated the National Center for Education Statistics' Comparable Wage Index to measure the prevailing wage for college graduates in each school district (Taylor and Fowler, 2006). We include the Department of Housing and Urban Development's estimate of Fair Market Rents (in logs) and the Bureau of Labor Statistics measure of the metropolitan area unemployment rate.

Table 9 presents coefficient estimates and robust standard errors for the hedonic wage model.

Table 9: Hedonic Model of Labor Cost

	Baseline Coefficient	Charter Interaction Term
OE Charter School		-0.092**
		(0.010)
Years of Experience (log)	0.003**	0.085**
	(0.001)	(0.010)
Log experience squared	0.037**	-0.027**
	(0.000)	(0.004)
No Degree	0.004**	-0.070**
	(0.001)	(0.016)
MA	0.026**	-0.002
	(0.000)	(0.009)
Ph.D.	(0.037**	0.009
	(0.004)	(0.040)
Math certified	0.005**	0.006
	(0.001)	(0.016)
Science certified	0.004**	0.034*
	(0.002)**	(0.016)
Bilingual/ESL certified	0.004**	0.051**
	(0.001)	(0.010)
Special Education certified	0.004**	-0.006
	(0.001)	(0.015)
Certified teacher	0.005**	0.020**
	(0.001)	(0.007)
New Hire	-0.005**	0.009*
	(0.000)	(0.004)
Mathematics	0.001**	-0.002
	(0.000)	(0.008)
Science	0.000	-0.009
	(0.000)	(0.008)
Special Education	0.002**	0.015
	(0.000)	(0.015)
Health and P.E.	0.005**	-0.029**
	(0.000)	(0.009)
Language Arts	-0.000	0.002
	(0.000)	(0.006)
Coach	-0.030**	0.000
	(0.001)	(0.014)
% Economically Disadvantaged Students	0.003**	
	(0.001)	
% LEP students	(0.006**	
	(0.001)	

	Baseline Coefficient	Charter Interaction Term
% Special Education students	0.001	
	(0.003)	
Campus enrollment	0.005**	
	(0.000)	
Very small district	-0.099**	
	(0.002)	
Small district	-0.089**	
	(0.002)	
Mid-sized district	-0.046**	
	(0.001)	
Big district	0.015**	
	(0.001)	
Elementary school	-0.008**	
	(0.001)	
Middle school	-0.003*	
	(0.001)	
Secondary school	-0.002	
	(0.001)	
Comparable wage index	-0.139**	
	(0.007)	
Fair market rent (log)	0.035**	
	(0.001)	
Unemployment rate	0.001**	
	(0.000)	
Observations	1,183,902	
Number of teachers	352,755	
R-squared	0.84	

Note: Robust standard errors in parentheses. The model also includes individual teacher fixed effects, metropolitan area fixed effects, and year fixed effects. The asterisks indicate a coefficient that is significant at the * 5%; **significant at 1%. Source: Authors' calculations from PEIMS.