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A unidimensional representation of multidimensional inequality, with an application to the Arab region*

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Abstract

This paper links the literature on multidimensional inequality with Alkire and Foster's (2011) counting approach to multidimensional poverty measurement. In doing so, it offers a multidimensional inequality framework applicable to any data used in applied multidimensional poverty analysis. The paper first introduces two new graphical tools: the *multidimensional complaint incidence curve* and the *cumulative multidimensional complaint incidence curve*. We then develop the dominance conditions associated with these two visual tools. These dominance conditions identify robust orderings of multidimensional inequality comparisons. It also provides the estimation and the statistical testing procedure linked with these dominance conditions. To show the applicability of the proposed approach, the paper offers an application of the theoretical conditions developed in the paper using two types of survey data for Arab countries. The empirical application shows that the method proposed in this paper also applies to the analysis of multidimensional inequality in developing countries with limited statistical information.

Keywords: Multidimensional inequality, Stochastic Dominance, Multidimensional complaint incidence curve, Cumulative multidimensional complaint curve
JEL Codes: D63, I31

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1 Introduction

This paper is motivated by the need for a stronger link between multidimensional poverty measurement and the growing theoretical literature on multidimensional inequality measurement. In particular, the literatures on multidimensional inequality and the counting approach to multidimensional poverty evolve in parallel. Since Alkire and Foster's (2011) seminal paper, the empirical literature measuring the non-income dimensions of poverty has flourished. Several empirical studies, including white papers, used their multidimensional poverty index, including the United Nations (see UNDP and OPHI, 2021 and 2022, among others). The numerous applications of the index speak to the importance of the multidimensional approach in the measurement of deprivations. An advantage of Alkire and Foster (2011) measurement approach resides in its usefulness for measuring poverty in contexts where only ordinal data on some dimensions of well-being are available. In addition, evidence from empirical studies suggests that a multidimensional poverty approach may reflect more severe and persistent poverty levels than traditional income-based poverty measures (Alkire and Fang, 2019). Thus the additional information provided by multidimensional poverty indices highlights the importance of investigating poverty and inequality by considering dimensions other than income.

The literature on multidimensional inequality is scant, but it is not new. The early work focusing on conceptualizing multidimensional inequality, such as Maasoumi (1986), Dardanoni (1996), and Tsui (1995, 1999), offered many significant contributions. More recently, the focus has been on the ethical principles associated with multidimensional inequality measures and the associated orderings (see, Decancq, 2012; Muller and Trannoy, 2012; Andreoli and Zoli, 2020; Fan, Henry, Pass, and Rivero, 2022). However, to the best of our best knowledge, there has yet to be a link between this theoretical literature on multidimensional inequality and the counting approach widely used in the literature on multidimensional poverty.

The paper aims to fill this gap in the literature by proposing a counting approach to multidimensional inequality comparisons. We use this counting approach in a framework where inequality is defined as an aggregation of individual complaints *a la* Temkin (1986). Assuming that the analyst adopts Temkin's (1986) definition of inequality as an aggregation of individual complaints, we first define an individual's complaint as the proportion of population that is *better-off* in each of the dimensions considered.[1] Thus, a multidimensional inequality index aggregates these complaints in this framework. The concept of complaint, as defined in this paper, is naturally linked to the survival function. A presence of a link between an individual's complaint and the survival function implies that any index within this framework will display an *aversion to positive k-rearrangement*, a fundamental axiom of multidimensional inequality (Decancq, 2012). An additional advantage of defining the multidimensional complaint as the value of the survival function at an individual's realization of the vector of attributes is that it allows us to offer a simple unidimensional representation of multidimensional inequality.

This paper also develops dominance conditions to identify robust rankings of multidimensional inequality. In doing so, we exploit the unidimensional representation of multidimensional inequality and introduce two curves. The first curve, the *multidimensional complaint incidence curve*, plays the same role as the Lorenz curve in the unidimensional inequality framework. It provides an overall intuitive portrait of inequality based on a multidimensional distribution. If the *multidimensional complaint incidence curves* of two distributions do not intersect, we have a multidimensional inequality ordering valid for all multidimensional inequality indices belonging to the class of indices considered in this paper. The second graphical tool is the *cumulative multidimensional complaint curve*. In situa-

[1] Cowell and Ebert (2004) also used Temkin's (1986) definition of inequality as an aggregation of individual complaints in another context.

tions where the *multidimensional complaint incidence curves* of two distributions intersect, we can check for non-intersection of their *cumulative multidimensional complaint curves* to identify robust orderings of multidimensional inequality. However, this latter focuses on a more restricted set of indices, the indices displaying a higher aversion to inequality than those considered by the *multidimensional complaint incidence curve.* Finally, we develop the estimation and statistical inference approaches associated with these dominance conditions.

The empirical section of the paper presents our findings from two applications of the proposed approach to study inequality in the Arab region. The presence of data deprivation issues in the Arab region provides an appropriate setting to demonstrate the empirical applicability and highlight the importance of this paper's contribution. Our method allows us to use information on well-being other than income and get a picture of inequality in this region that was previously unavailable due to data limitations. The first application uses data from the Harmonized Household Income and Expenditure Surveys (OAMDI, 2012, 2013, and 2015) provided by the Economic Research Forum for the Arab Countries, Iran, and Turkey (ERF). These surveys include information on three well-being dimensions: total household expenditures, education, and disability status. Unfortunately, these harmonized surveys are available for only a limited number of Arab countries: Egypt, Iraq, and Jordan. Indeed, the availability of household survey data on income and expenditure is an issue in the Arab region. Therefore researchers will inevitably need to use alternative data sources and new statistical methods to assess inequality (see, Atamanov, Tandon, Lopez-Acevedo, and Vergara Bahema, 2020). Despite the absence of regular extensive household surveys, the Arab Barometer is a reliable alternative source of information. The Arab Barometer data contains small regular surveys that monitor the state of public opinion in the Arab region. While these surveys are not designed to assess inequality and poverty, some survey waves include an interval income variable, an ordinal education variable, and other ordinal variables that measure the feeling of safety, autonomy, depression, and stress. These variables offer an adequate range of well-being dimensions that can be used to assess multidimensional inequality. In addition, the survey design is such that it is representative of the population. For all these reasons, our second application uses the 2018 wave of these surveys (the last available wave of the survey, including all these informations on well-being). Using this survey wave, we apply our measurement approach to compare multidimensional inequality in ten Arab countries: Algeria, Egypt, Iraq, Jordan, Lebanon, Libya, Morocco, Palestine, Sudan, and Tunisia.

The remainder of the paper is organized as follows. Section 2 presents our framework for measuring multidimensional inequality. Section 3 proposes dominance conditions for identifying robust orderings of multidimensional inequality. Section 4 offers the estimation and statistical inference approach. Section 5 offers our two empirical illustrations. Finally, Section 6 concludes and discusses future areas of investigation.

2 Theoretical Framework

Assume that we have a population (or a sample) of n individuals. Also, assume that a m-dimensional vector represents the well-being of each individual. These dimensions may relate to income, education, health, or other variables impacting well-being. We use an $n \times m$ matrix $\Xi = [x_{ij}]$ to represent the information on the well-being of this population. In this matrix, x_{ij} represents the achievement of individual i in dimension j.[2] Every row vector gives the list of individual i's achievements, and every column represents the distribution of attribute j in the population. These attributes can take the form of ordinal, cardinal, or ratio-scale variables.[3] This paper's objective is to develop a measurement framework

[2]It is helpful to consider Ξ as the realization of n draws from an actual data generating process.

[3]A ratio-scale variable is a cardinal variable with a well-defined 0. Income is an obvious example.

compatible with all these variables simutaneously.

Let $X_1, \ldots, X_m$ be random variables representing these m dimensions of well-being. Assume that there are d discrete and c continuous random variables with $d + c = m$. Let $\mathcal{D} \subset \{1, \ldots, m\}$ be the set of subscripts corresponding to discrete variables, and let $\mathcal{C} = \{1, \ldots, m\} \setminus \mathcal{D}$. Without loss of generality, we can assume that the first d variables are discrete and the remaining $c = m - d$ are continuous. The actual data-generating process is a joint density function (with respect to a certain measure) of $X = (X_1, \ldots, X_m)$ and is denoted by $f(x)$.

2.1 Measuring inequality

In this paper, we take an approach similar to Temkin (1986) and define inequality as an aggregation of individual "complaints". In his philosophical reflection on inequality in welfare, Temkin asks the following question: "how bad is inequality in a situation from the standpoint of particular individuals in that situation?" (Temkin, 1986, p. 102). Thus, the analyst can consider that an individual has a complaint if her welfare is not as high as another equally-deserving reference person. When defining the concept of reference person, Temkin considers that an individual's complaint may be relative to the average welfare, the best-off person, or all those *better-off*.

In this paper, we will define the individual's complaint with respect to all those who are *better-off*. Our choice for the reference person is motivated by the multidimensional context. Indeed, linking a complaint's size to cardinal differences may raise measurement issues because some dimensions may be ordinal. As a result, using the average welfare to measure complaints is not applicable because the average is not a well-defined statistic for ordinal variables. Also, the cardinal size of the distance in welfare is difficult to assess. Thus, using standard measures of dispersion, such as the variance, and inequality indices, and deriving tests, such as Lorenz dominance tests or stochastic dominance tests, can lead to arbitrary rankings of distributions (Zheng, 2008).

Previous contributions in the literature on poverty and health inequality in the presence of multiple categorical variables rely on a counting approach and a dichotomization of the ordinal variables (Alkire and Foster, 2011; Makdissi and Yazbeck, 2014). While counting dichotomized variables provides a setting in which one can use multiple categorical variables to compute inequalities, by adopting this approach, the researcher has to forgo the variation within each dimension. In addition, the analyst has to select a threshold in each dimension to perform this dichotomization. An alternative solution in the context of unidimensional inequality consists of using population shares in each category (see Allison and Foster, 2004; Abul Naga and Yalcin, 2008; Cowell and Flachaire, 2017; and Makdissi and Yazbeck, 2017). Specifically, Cowell and Flachaire (2017) build their measurement framework on the status of a person in a society, which they define as the position of the person in the unidimensional distribution of an ordinal attribute.

In this paper, we also take an approach based on population shares and extend it to a multidimensional framework. We define an individual's complaint as the proportion of the population that is *better-off*. However, since there is more than one dimension, some people may be better-off in some dimensions but worse-off in others. We adopt an intersection approach and define the individual's complaint as the proportion of individuals in a population who are *better-off* than the individual in all the dimensions. Let $S(x)$ be the survival function which we define formally as

$$S(x) = \Pr[X \geq x] = \sum_{t_1 > x_1} \cdots \sum_{t_d > x_d} \int_{x_{d+1}}^{\infty} \cdots \int_{x_m}^{\infty} f(t) \, \mathrm{d}t_{d+1} \cdots \mathrm{d}t_m. \tag{1}$$

It is important to note that we are using a definition of the survival function definition that allows for equality rather than just strict inequality. The survival function as defined in equation (1) allows us to define individual complaints as follows.

Definition 1 INDIVIDUAL'S COMPLAINT: *An individual's complaint associated with an attribute vector x is given by $S(x)$.*

Note that technically, Definition 1 defines all those who are *better-off* as those who are at least as well-off in all dimensions. Nevertheless, we will keep the terminology "*better-off*".

If μ is the measure induced by S (or f), then the Kendall survival function will be given by

$$K_S(p) = \Pr[S(X) \geq p] = \mu\left(\{x \in \Re^m | S(x) \geq p\}\right) \tag{2}$$

A useful tool to measure multidimensional complaints is the inverse of this Kendall survival function

$$\psi_S(p) = \sup\left\{q \,|K_S(q) \geq p\right\}. \tag{3}$$

It gives the value of the survival function associated with the $(100-p)$th quantile in the distribution of complaints. In other words, it gives the level of complaints at the $(100-p)$th quantile in the distribution of complaints. In this context, one can interpret p as social ranks. For this reason, we call this function $\psi_S(p)$,[4] the *multidimensional complaint incidence curve* since it gives the "expected complaint" for each social rank p.

The empirical intuition underlying the *multidimensional complaint incidence curve* is relatively straightforward from a counting approach perspective. Assume that we have a survey of N observations. For each observation, $i \in \{1, 2, \ldots, N\}$, $S(x_i)$ corresponds to the proportion of the population that is *better-off* in all the dimensions. Then, one can rank the observations according to the value of their $S(x_i)$, from the highest to the lowest. Assume that r_i is the rank of observation i and that there is no tie between the values of $S(x)$. The socioeconomic rank of i is then $p_i = r_i/N$. The multidimensional complaint incidence curve, $\psi_S(p)$, is obtained by mapping the values of $S(x_i)$ in coordinate with p_i in

[4] We chose ψ for the multidimensional complaint curve because $\psi\acute{o}\gamma o\zeta$ (psogos) means criticism in ancient Greek.

ordinate.

In many empirical cases, all attributes may be discrete. Applying Definition 1 will not generate enough variability between individuals. In these cases, we shall use an alternative definition.

Remark 1 Individual's complaint in discrete cases: *When all attributes are discrete,* $S_D(x) = \Pr[X \geq x] - \Pr[X = x]$ *is the individual complaint associated with an attribute vector* x*.*

The multidimensional complaint incidence curve, $\psi_S(p)$, gives an interesting unidimensional graphical representation of multidimensional inequality. For illustrative purposes, let us consider the "smooth" versions of the multidimensional complaint curve where all variables are continuous (see Figure 1). There are two possible extreme cases for the function $\psi_S(p)$. The first extreme case is a situation where all dimensions are perfectly correlated. For instance, assume that the attributes are income and health and that both are continuous. If the income gradient in health is everywhere positive, then we have maximum multidimensional inequality. This maximum multidimensional inequality case is represented by the straight line $\psi_S^{\max}(p)$. The second case is a situation of absence of multidimensional inequality. The absence of inequality happens if the income gradient in health is everywhere negative. Thus, although there is inequality in the unidimensional distribution of each attribute, the perfect negative correlation implies that there is no multidimensional inequality. This situation is represented by the straight line $\psi_S^{\min}(p)$. The curve $\psi_S(p)$ that appears in Figure 1 is an intermediate case with some multidimensional inequality. It displays the expected complaint at each social rank p. The value of $p_I := \inf\{p|\psi_S(p) = 0\}$ gives the proportion of the population that is affected by multidimensional inequality, which is the intersection of the curve $\psi_S(p)$ with the horizontal axis. The remaining $1 - p_I$ of the population is not affected by multidimensional inequality (they have no complaints).

Once individual complaints are defined, the analyst needs to choose a method to aggregate these complaints at the social level. Temkin (1986) proposes three methods of aggregation. The first consists of using the maximin principle and focusing on the worst-off. The second is an additive approach in which one adds up the complaints of all individuals. Finally, a third approach (a generalization that incorporates the first two approaches as specific cases) would be to use a weighted additive approach. Let us define a aggregation framework that is compatible with all three approaches. We assume that a sympathetic ethical observer evaluates multidimensional inequality and aggregates all individual complaints. This sympathetic ethical observer weights each complaint with a function $\omega(p)$ with $\omega(p) \geq 0$ for all $p \in [0,1]$ and $\int_0^1 \omega(p)\mathrm{d}p > 0$. In this context, multidimensional inequality is a weighted average of individual complaints (multiplied by some constant):[5]

$$I(S) = \int_0^1 \omega(p)\psi_S(p)\mathrm{d}p. \tag{4}$$

Equation (4) is compatible with the three aggregation methods proposed by Temkin (1986). If one opts for the maximin principle and defines the worst-off group as those individuals at social ranks $p \in [0,z]$ then the weight function would be such that $\omega(p) \geq 0$ if $p \in [0,z]$, and $\omega(p) = 0$ elsewhere. If one prefers the additive approach, then $\omega(p) = 1$ (or any other constant) for all $p \in [0,1]$. The weighted additive approach would be compatible with weight functions such as $\omega(p) \geq 0$ for all $p \in [0,1]$.

Let $\mathscr{C}$ represent the set of all continuous functions on $[0,1]$. Then, we can formally define the set of rank-dependent multidimensional inequality indices.

Definition 2 Set of rank dependent multidimensional inequality indices:

$$\Upsilon := \left\{ I(S) \,\middle|\, \omega(p) \in \mathscr{C} \ \wedge \ \omega(p) \geq 0 \ \wedge \ \int_0^1 \omega(p)\mathrm{d}p > 0 \right\}.$$

[5]One may consider restricting $\int_0^1 \omega(p)\mathrm{d}p = 1$. However, this would be less general and may preclude the analyst from rescaling the index to make it lie in some desirable interval, like between 0 and 1. A quick inspection of Figure 1 indicates that the maximum average complaint equals 1/2. In this context, if one wants an index that takes values between 0 and 1, it would be necessary to multiply this average by 2.

The rank-dependent multidimensional inequality indices described in Definition 2 rely on the population shares that are *better-off* than x. Moreover, these population shares are robust statistics for ratio-scale, cardinal, and ordinal variables. Thus, by adopting such an approach, the analyst overcomes the potential of arbitrary rankings raised by Zheng (2008).

2.2 Properties

We propose an axiomatic framework for multidimensional inequality measurement. In order to describe these axioms, we always compare an initial matrix of achievement Ξ^0 and the survival function of its associated data-generating process, S_0, to an alternative matrix of achievement Ξ^1 and the survival function of its associated data-generating process S_1.

Definition 3 REPLICATION: *A matrix of attributes Ξ^1 is said to be a replication of Ξ^0 if*

$$\Xi^1 = \begin{pmatrix} \Xi^0 \\ \Xi^0 \\ \vdots \\ \Xi^0 \end{pmatrix}$$

When Ξ^1 is a replication of Ξ^0, the only difference between the two populations is their size. In this case, it is reasonable to assume that the data-generating processes are the same (i.e., $S_1 = S_0$) since Ξ^1 has the same underlying relative structure as Ξ^0 except for the number of observations. Hence, the index of multidimensional inequality should take a similar value in these two cases.

Axiom 1 REPLICATION INVARIANCE: *If Ξ^1 is obtained from Ξ^0 by a replication, then* $I(S_1) = I(S_0)$

This first axiom allows meaningful comparisons of populations of different sizes. Moreover, since the multidimensional inequality measure we are proposing in this paper builds on the data-generating process's multivariate survival function, this axiom is automatically verified.

We impose an additional property, symmetry, to guarantee that the index is invariant to the person's identity and only depends on the list of attributes allocated to this person.

Definition 4 PERMUTATION: *A matrix of attribute Ξ^1 is obtained from Ξ^0 by a permutation if $\Xi^1 = \Pi\Xi^0$, where Π is some $n \times n$ permutation matrix.*[6]

When Ξ^1 is a permutation of Ξ^0, the only difference is the order in which each observation appears. In this case, it is reasonable to assume that the data-generating processes are the same (i.e., $S_1 = S_0$) since Ξ^1 has the same underlying relative structure as Ξ^0 except for the arbitrary order of the observations. Once again, the index of multidimensional inequality should take the same value in these two cases.

Axiom 2 SYMMETRY: *If Ξ^1 is obtained from Ξ^0 by a permutation, then $I(S_1) = I(S_0)$*

This axiom implies that the inequality index does not vary with a switch of all attributes between two individuals. Once again, given that we are using the multivariate survival function, this axiom is automatically obeyed.

One desirable property of a multidimensional inequality index is that it takes its minimum value when the set of all row vectors x_i is an antichain (the case of absence of multidimensional inequality) and a maximum value when it is a chain (the perfect correlation case).[7] To have this property, the index should potentially take at least two different values.

Axiom 3 NONTRIVIALITY: *$I(S)$ takes at least two distinct values.*

Since we assume that $\int_0^1 \omega(p)\mathrm{d}p > 0$, this implies $\omega(p) > 0$ for some measurable space on $[0, 1]$. In turn, this guarantees that the index obeys the axiom of nontriviality.

[6] A permutation matrix is a square matrix with a single 1 in each row and each column and 0 elsewhere.

[7] A chain is a set in which each pair of vectors is comparable, and an antichain is a set in which each pair is incomparable.

In addition to the Axiom of *Nontriviality*, one needs to define the proper concept of inequality aversion for the index to have the above desirable property. In the context of unidimensional inequality, additional ethical axioms focus on principles of transfers between individuals. However, in a multidimensional framework, these axioms may lead to counterintuitive results. To understand why it is the case consider two individuals with vectors of attribute $x_i^0 = (x_1^\ell, x_2^h)$ and $x_{i'}^0 = (x_1^h, x_2^\ell)$, where $x_1^\ell < x_1^h$ and $x_2^\ell < x_2^h$. Then, if we perform a *progressive* transfer in the first dimension alone, equalizing the two individuals at $\overline{x}_1$ in that dimension, we end up with $x_i^1 = (\overline{x}_1, x_2^h)$ and $x_{i'}^1 = (\overline{x}_1, x_2^\ell)$. Intuitively, many would consider starting from a situation in which none of the two individuals is *better-off* than the other in the two dimensions and ending with a situation in which one individual, i, is strictly better than the other, constitutes an increase in inequality. However, the unidimensional concept of *progressive* transfer taken alone would consider this change a strict social improvement.

The literature on multidimensional inequality introduced a concept that captures the fundamental idea of aversion to multidimensional inequality. This concept is based on elementary rearrangement to address the multidimensional nature of inequality.[8] In the 2-dimension case, an elementary rearrangement runs along these lines. Consider the two same individuals i and i' with vectors of attribute $x_i^0 = (x_1^\ell, x_2^h)$ and $x_{i'}^0 = (x_1^h, x_2^\ell)$. If we perform a switch of the second attribute between the two individuals to get $x_i^1 = (x_1^\ell, x_2^\ell)$ and $x_{i'}^1 = (x_1^h, x_2^h)$, inequality should not go down. If we want to frame the analysis in terms of the underlying data-generating process, a positive rearrangement consists in marginally changing the probability density, $f_0(x)$, at $x_i^0, x_{i'}^0, x_i^1, x_{i'}^1$ such that $f_1(x) = f_0(x) - \delta$ for $x \in \{x_i^0, x_{i'}^0\}$, $f_1(x) = f_0(x) + \delta$ for $x \in \{x_i^1, x_{i'}^1\}$, and $f_1(x) = f_0(x)$ for $x \notin \{x_i^0, x_{i'}^0, x_i^1, x_{i'}^1\}$. This rearrangement is such that $S_1(x) = S_0(x)$ everywhere except within the rectangle

[8] See, Atkinson and Bourguignon, 1982; Tsui, 1999; Bourguignon and Chakravarty, 2003; D'Agostino and Dardanoni, 2009; Alkire and Foster, 2011; Decancq, 2012; Gravel and Moyes, 2012.

formed by $x_i^0, x_{i'}^0, x_i^1$, and $x_{i'}^1$ where $S_1(x) \geq S_0(x)$.[9]

Decancq (2012) generalizes the concept of positive rearrangement to the multivariate case. Following Decancq (2012) we denote by $x <_k y$ the relation in which $x_j \leq y_j$ for all $j \in \{1, \ldots, m\}$ and $x_j < y_j$ for k values in $\{1, \ldots, m\}$. Let us assume that we have two different vectors $x^\ell = (x_1^\ell, \ldots, x_m^\ell)$ and $x^h = (x_1^h \ldots, x_m^h)$ such that $x^\ell <_k x^h$ and $2 \leq k \leq m$. Let $B^k(x^\ell, x^h) = \prod_{j=1}^m \left[x_j^\ell, x_j^h\right]$ be a k-dimensional hyperbox with 2^k vertices. Let $\mathscr{B}^k(x^\ell, x^h)$ be the set of all vertices, $\mathscr{B}_o^k(x^\ell, x^h) \subset \mathscr{B}^k(x^\ell, x^h)$, the set of all odd-numbered vertices such that $x_j = x_j^\ell \neq x_j^h$ and $\mathscr{B}_e^k(x^\ell, x^h) \subset \mathscr{B}^k(x^\ell, x^h)$, the set of all vertices with an even number of entries such that $x_j = x_j^\ell \neq x_j^h$. Decancq proposes a generalization of the positive rearrangement, which rearranges probability mass on the vertices of the hyperbox.

Definition 5 Positive k-rearrangement: *Let $f_1(x)$ and $f_0(x)$ be the probability distribution underlying Ξ^1 and Ξ^0. Ξ^1 is obtained from Ξ^0 by a positive k-rearrangement if there exists x^ℓ and x^h with $x^\ell <_k x^h$ and $\delta > 0$ such that*

$$f_1(x) = \begin{cases} f_0(x) + \delta & \text{if } x \in \mathscr{B}_e^k(x^\ell, x^h) \\ f_0(x) - \delta & \text{if } x \in \mathscr{B}_e^k(x^\ell, x^h) \\ f_0(x) & \text{if } x \notin \mathscr{B}^k(x^\ell, x^h) \end{cases}$$

A positive k-rearrangement plays the same role as the positive rearrangement in the bivariate case. This rearrangement is such that $S_1(x) = S_0(x)$ everywhere except within the hyperbox $B^k(x^\ell, x^h)$ where $S_1(x) \geq S_0(x)$.

Axiom 4 Weak rearrangement: *If Ξ^1 is obtained from Ξ^0 by a positive k-rearrangement, then $I(S_1) \geq I(S_0)$.*

Axiom 4 guarantees that the index takes its minimum value when the set of all row vectors x_i is an antichain and its maximum value when it is a chain.

[9] $S_1(x) = S_0(x) + \delta$ for all x in the rectangle except on the line segments $(x_i^1, x_i^0]$ and $(x_i^1, x_{i'}^0]$ where $S_1(x) = S_0(x)$

In the context of income inequality, Kolm (1976) introduced the principle of transfer sensitivity (for a unidimensional setting). The idea underlying this principle is that a regressive transfer is more than compensated by an equivalent progressive transfer happening in a lower part of the income distribution. This principle captures the idea of higher inequality aversion. We can adapt a similar concept to the context of the multidimensional inequality framework.

Definition 6 Composite rearrangement: *Let $f_0(x)$, $f_{1_a}(x)$, $f_{1_b}(x)$, and $f_1(x) = 0.5 f_{1_a} + 0.5 f_{1_b}(x)$ be the probability distribution underlying Ξ^0, Ξ^{1_a}, Ξ^{1_b}, and Ξ^1. Let $x^{\ell_b} <_k x^{h_b} <_k x^{\ell_a} <_k x^{h_a}$. If Ξ^{1_a} is obtained from Ξ^0 by a positive k-rearrangement on the $\mathscr{B}^k(x^{\ell_a}, x^{h_a})$, Ξ^0 is obtained from Ξ^{1_b} by a positive rearrangement on the $\mathscr{B}^k(x^{\ell_b}, x^{h_b})$, and $\int_0^1 \left[\psi_{S_{1_a}}(p) - \psi_{S_0}(p)\right] \mathrm{d}p = \int_0^1 \left[\psi_{S_0}(p) - \psi_{S_{1_b}}(p)\right] \mathrm{d}p$, we say that Ξ^1 is obtained from Ξ^0 by a composite rearrangement.*

Axiom 5 Weak rearrangement sensitivity: *If Ξ^1 is obtained from Ξ^0 by a composite rearrangement, then $I(S_1) \leq I(S_0)$.*

Theorem 1 *A rank dependent multidimensional inequality index $I(S)$ satisfies replication invariance, symmetry, nontriviality and weak rearrangement. In addition, if $\mathrm{d}\omega(p)/\mathrm{d}p \leq 0$, it also satisfies weak rearrangement sensitivity.*

Proof. See appendix. ■

Note that by imposing weak rearrangement sensitivity, we provide a formalized version of the weighted average approach proposed by Temkin (1986). Indeed, Axiom 5 is compatible with Temkin's philosophical view that "the larger someone's complaint is, the more weight is attached to it" (Temkin, 1986, p. 113).

3 Robust Orderings

In addition to its intuitive graphical interpretation, the *multidimensional complaint incidence curve*, $\psi_S(p)$, can be used to identify a first partial ordering of rank-dependent multidimensional inequality indices.

Theorem 2 *$I(S_1) \leq I(S_0)$ for all $I(S) \in \Upsilon$ if and only if*

$$\psi_{S_1}(p) \leq \psi_{S_2}(p) \text{ for all } p \in [0,1].$$

Proof. See appendix. ■

Theorem 2 offers a simple graphical condition for identifying robust multidimensional inequality orderings. It implies that if two *multidimensional complaint incidence curves* do not intersect, the one below the other displays less multidimensional inequality for any indices belonging to the set Υ.

The orderings obtained by the condition in Theorem 2 may be incomplete. It is, however, possible to increase the power of ordering by restricting the analysis to rank-dependent multidimensional inequality indices obeying weak rearrangement sensitivity.

Definition 7 Set of rank dependent multidimensional inequality indices obeying weak rearrangement sensitivity:

$$\Upsilon_R := \left\{ I(S) \in \Upsilon \;\middle|\; \frac{\mathrm{d}\omega(p)}{\mathrm{d}p} \leq 0 \right\}.$$

To identify robust orderings of multidimensional inequality that remain valid for all indices belonging to Υ_R, we rely on another curve, the *cumulative multidimensional complaint curve*, $\Psi_S(p)$.

$$\Psi_S(p) = \int_0^p \psi_S(u)\mathrm{d}u. \tag{5}$$

Figure 2 illustrates this concept of the *cumulative multidimensional complaint curve.* Since we rank individuals from the highest to the lowest level of complaints, $\Psi_S(p)$ has a concave shape. In the extreme case of the absence of multidimensional inequality, *cumulative multidimensional complaint curve* is equal to the horizontal axis:

$$\Psi_S^{\min}(p) = 0 \text{ for all } p \in [0,1]. \tag{6}$$

For the other extreme case where all dimensions are perfectly correlated, the *cumulative multidimensional complaint curve* is

$$\Psi_S^{\max}(p) = \frac{p(2-p)}{2}. \tag{7}$$

Note that, in general, the value of the *cumulative multidimensional complaint curve* at $p = 1$, $\Psi_S(1)$ gives the unweighted average of complaints and corresponds to the value of inequality if the analyst uses the ethical weight function $\omega(p) = 1$ for all $p \in [0,1]$:

$$\overline{\psi}_S = \int_0^1 \psi_S(p)\mathrm{d}p \tag{8}$$

The cases of non-intersecting *cumulative multidimensional complaint curves* of complaints identify robust partial orderings of multidimensional inequality for indices obeying weak rearrangement sensitivity. Formally,

Theorem 3 *$I(S_1) \leq I(S_0)$ for all $I(S) \in \Upsilon_R$ if and only if*

$$\Psi_{S_1}(p) \leq \Psi_{S_2}(p) \textit{ for all } p \in [0,1]. \tag{9}$$

Proof. See appendix. ■

4 Statistical inference

If we denote by $X_1, \ldots, X_n$ an i.i.d. sample from a m-dimensional attribute vectors where $X_i = (X_{1,i}, \ldots, X_{m,i})$, then an empirical counterpart of the individual complaint, $\widehat{S}(x)$, at the point $(x_1, \ldots, x_m)$, is given by

$$\widehat{S}(x) = \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{m} \mathbb{1}\{X_{k,i} \geqslant x_k\} - \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{m} \mathbb{1}\{X_{k,i} = x_k\}. \tag{10}$$

An estimator of $K_S(p)$, as defined in equation (2), is given by

$$\widehat{K}_S(p) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\{\widehat{S}(X_i) \geqslant p\}. \tag{11}$$

The generalized inverse of $\widehat{K}_S$ gives us a natural estimator of ψ_S:

$$\widehat{\psi}_S = \sup\left\{ q \,\middle|\, \widehat{K}_S(q) \geq p \right\}. \tag{12}$$

A numerical integration can easily yield an estimator of $\Psi_S(p)$.

Assume we have two i.i.d. samples $X_1, \ldots, X_{n_1}$ and $Y_1, \ldots, Y_{n_2}$ with S_1 and S_2 being the individual complaints for both variables X and Y. If ψ_{S_1}, Ψ_{S_1} (respectively ψ_{S_2}, Ψ_{S_2}) are the functions corresponding to S_1 (and S_2 respectively), then stochastic dominance tests associated with the conditions in Theorems 2 and 3 can be conducted as follows. For Theorem 2, we aim to test $H_0 : \psi_{S_1}(p) \leqslant \psi_{S_2}(p)\ \forall p \in [0,1]$ v.s $H_1 : \psi_{S_1}(p) > \psi_{S_2}(p)$ for some $p \in [0,1]$. To do so, the following statistic could be computed

$$\widehat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p [\widehat{\psi}_{S_1}(p) - \widehat{\psi}_{S_2}(p)] \tag{13}$$

The bootstrap can then be used by creating B samples of size n_1 and n_2 by independent sampling with replacement of the indices of the observations in the original samples and then computing bootstrap estimators $\widehat{\psi}^b_{S_1}$ and $\widehat{\psi}^b_{S_2}$ for $b = 1, \ldots, B$. The distribution of the recentered bootstrap statistic

$$\widehat{\tau}_b = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p [\widehat{\psi}^b_{S_1}(p) - \widehat{\psi}^b_{S_2}(p) - \widehat{\psi}_{S_1}(p) + \widehat{\psi}_{S_2}(p)] \tag{14}$$

could be used to determine the critical values for the test. The p-values are then obtained using

$$p = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}\left[\tau_b > \tau\right] \tag{15}$$

It is important to note that in the test above, the null hypothesis is a null of dominance, and the alternative is non-dominance. The obvious question is, why not test for a null of non-dominance to establish strict dominance? There are two reasons for not doing this as it is done in most empirical settings. First, the null of non-dominance over all the support of a curve is rarely rejected (see Davidson and Duclos, 2013). One can easily understand this issue in our context. If we refer to Figure 1, at $p = 1$, $\psi_S^{\min}(1) = \psi_S^{\max}(1) = 0$. This implies that at $p = 1$, all $\psi_S(1) = 0$. This mathematical result alone makes it theoretically impossible to reject a null of non-dominance $H_0 : \exists p \in [0, 1]$ such that $\psi_{S_1}(p) \geq \psi_{S_2}(p)$ v.s $H_1 : \psi_{S_1}(p) < \psi_{S_2}(p)\ \forall p \in [0, 1]$. This is a direct consequence of the fact that $\psi_{S_1}(1) = \psi_{S_2}(1) = 0$ for any pair of multidimensional complaint incidence curves. Another reason for not using a null of non-dominance is that, if one refers to Theorem 2, only non-strict dominance is required to establish robust orderings of the distributions. For these reasons, empirical researchers usually test for both

$$H_0^A : \ \psi_{S_1}(p) \leqslant \psi_{S_2}(p) \quad \forall p \in [0, 1]$$

$$H_0^B : \ \psi_{S_2}(p) \leqslant \psi_{S_1}(p) \quad \forall p \in [0, 1]$$

against their respective alternatives and use the decision rules depicted in Table 1. One can adapt a similar procedure for Ψ_S in testing the dominance condition in Theorem 3.

The asymptotic distributions for $\widehat{\psi}_S$ and $\widehat{\Psi}_S$ follow by basic finite-dimensional law of large numbers and central limit theorems as long as the support of X is finite. We can compute the asymptotic variances using the bootstrap. We assume finite support for this paper. This assumption is relevant if one is ready to consider all variables as discrete. For example, one can assume that total expenditure is a discrete random variable with a large finite support. In the case where X is absolutely continuous, empirical process methods derived for Kendall processes (as in Barbe, Genest, Ghoudi, and Rémillard, 1996; and van der Vaart and Wellner, 2007) can be used to derive the asymptotic distribution of ψ_S and

Ψ_S under additional smoothness conditions. In the case where X is absolutely continuous, it is immediate that ψ_S is a function of the copula of X and does not depend on its margins. The literature has not yet studied the mixed case where X has a subvector whose distribution is absolutely continuous with respect to Lebesgue's measure and another subvector whose distribution is discrete. However, results in Genest, Nešlehová, and Rémillard (2017) on multilinear empirical copula processes constitute a promising avenue for deriving the asymptotic distributions of the nonparametric estimators $\widehat{\psi}_S$ and $\widehat{\Psi}_S$. Note that alternative semiparametric estimators of $\widehat{\psi}_S$ and $\widehat{\Psi}_S$ (where a parametric copula family is combined with nonparametric marginal distribution estimators) could be derived using the methods in Gunawan, Khaled, and Kohn (2020).

5 Empirical application: Multidimensional inequality in the Arab World

This section presents two empirical applications. In the first application, we use data from the Harmonized Household Income and Expenditure Surveys (HHIES) compiled by the ERF. The data covers three countries: Egypt, Iraq, and Jordan. The HHIES are the typical household living-standard surveys used in the inequality and poverty analysis. These surveys have all the information necessary for unidimensional and multidimensional inequality comparison. Nevertheless, the number of countries for which the data is available is limited. In addition, the frequency of these surveys is relatively low in the Arab region.

In the second empirical application, we use data for ten Arab countries from Wave V of the Arab Barometer that covers the year 2018 (there is a more recent wave, but it does not include the income variable. These ten countries are Algeria, Egypt, Iraq, Jordan, Lebanon, Libya, Morocco, Palestine, Sudan, and Tunisia.[10]

[10]This survey includes data for Yemen. However, the income in Yemen is continuous, and the variable consists of discrete categories for all other countries. We have decided not to use Yemen for two reasons. First, for comparability with other countries, Second, the proportion of non-response to this continuous income question is much higher than for the discrete version in the other countries.

5.1 Multidimensional inequality using the Harmonized Household Income and Expenditure Surveys

In this section, we use the three following harmonized Household Income and Expenditure Surveys: (1) the Income, Expenditure & Consumption Survey,Egypt (2015);[11] (2) the Household Socio-Economic Survey, Iraq (2012); and (3) the Household Expenditure and Income Survey, Jordan (2013). For our multidimensional inequality analysis, we use three variables, one of which (total expenditures) is continuous. We discard individuals for whom there are missing crucial information and assume that they are missing at random. The final sample sizes are 11,988 for Egypt, 25,105 for Iraq, and 4,850 for Jordan. Table 2 provides information on the type variables and their categories when applicable.

To build the figures and perform the dominance tests associated with Theorems 2 and 3, we use 299 bootstrap replications. We first test Theorems 2 and 3, by conducting some visual checks. Figure 3 displays the comparisons of multidimensional complaint incidence curves, $\psi_S(p)$, for these three countries. A visual inspection of these figures suggests that Iraq has less multidimensional inequality than Egypt and Jordan. We then test the theorems formally in Table 3 using the p-values associated with testing the condition in Theorem 2. The p-values confirm that the multidimensional complaint incidence curve of Iraq is everywhere below Egypt's and Jordan's incidence curves.[12] As for Egypt's and Jordan's multidimensional inequality comparisons, there is no clear dominance because the curves for Egypt and Jordan intersect in Figure 3. As mentioned, earlier when there is no dominance depicted using Theorem 2, we restrict our testing procedure to indices who *obey weak rearrangement sensitivity* thus, test Theorem 3. Table 3 displays the p-values associated with testing the condition in Theorem 3 for the Egypt vs. Jordan comparison confirming the absence of clear dominance. Table 4 summarizes the results for the rankings obtained

[11] There is a 2017/2018 version, but we use the year closer to years of surveys for Iraq and Jordan.

[12] We use a critical level of 0.05 for the dominance tests.

using the aforementioned dominance tests. On the one hand Table 4 shows that Iraq has less multidimensional inequality than Egypt and Jordan for all indices in Υ. On the other hand it shows that comparisons of multidimensional inequality between Egypt and Jordan are not robust neither under the condition in Theorem 2 nor in Theorem 3.

5.2 Multidimensional inequality using the Arab Barometer illustration

We expand our comparisons to include a larger number of Arab countries, and show that the proposed approach can be used with surveys where data on income is categorical. We base our multidimensional inequality analysis on six discrete variables as detailed in Table 5. We discard individuals for whom there are missing crucial information and assume that they are missing at random. The final sample sizes are 750 for Algeria, 1,008 for Egypt, 1,200 for Iraq, 1,172 for Jordan, 1,149 for Lebanon, 892 for Libya, 663 for Morocco, 1,176 for Palestine, 644 for Sudan, and 1,045 for Tunisia. Despite the relatively small sample sizes of the Arab Barometer surveys compared to the HHIES, the method proposed in this paper allows us to rank multidimensional inequality in many cases.

To build the figures and perform the dominance tests associated with Theorems 2 and 3, we use 299 bootstrap replications. We first test Theorems 2 and 3, by conducting some visual checks and compare them with the visual checks from the previous empirical application. Figure 4 displays the comparisons of multidimensional complaint incidence curves, $\psi_S(p)$, for the countries that appear in both surveys, namely Egypt, Iraq, and Jordan. A visual inspection of these figures suggests that Iraq has less multidimensional inequality than Egypt and Jordan. To confirm the visual inspection's results, we test the theorems formally in Table 3 using the p-values associated with testing the condition in Theorem 2. Table 6 displays all the p-values of the tests associated with Theorem 2. For the countries covered in both surveys, the p-values confirm the intuition of the visual inspection. As mentioned, earlier there is no dominance depicted when using Theorem 2. Therefore, we restrict our testing procedure to indices who *obey weak rearrangement sensitivity* thus, test Theorem 3. Table 7 displays the p-values associated with these additional tests. As we will discuss later, the results from Table 7 is a clear case that higlights the practical use of the dominance condition of Theorem 3. Indeed, in the empirical application, the practitioner can obtain five additional multidimensional inequality rankings by restricting indices to those belonging to Υ_R.

Among the multi-country comparisons we are conducting, the comparison of multi-dimensional inequality between Lebanon and Libya presents an interesting case from a methodological perspective. From the visual inspection of the multidimensional complaint incidence curves in the left panel of Figure 5, it may be tempting to conclude that Libya has less multidimensional inequality than Lebanon for all indices in Υ. In this case, the two statistical tests to confirm the accuracy of the visual check are

$$H_0^A : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0, 1]$$

and

$$H_0^B : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0, 1].$$

The p-value associated with testing H_0^A is 0.0502, and the one associated with testing H_0^B is 0.9766 (see, Table 6). An analyst following the decision rules in Table 1 would conclude that $\psi_{S_{\text{Lebanon}}}(p) = \psi_{S_{\text{Libya}}}(p) \;\; \forall p \in [0, 1]$. In other words, the analyst will conclude that a robust ranking cannot be obtained at a critical level of 0.05 because the statistical tests cannot distinguish the two multidimensional complaint incidence curves at this significance level since $0.0502 > 0.05$. Nevertheless, the visual check of the right panel suggests that Libya's cumulative multidimensional complaint curve is everywhere below Lebanon's curve. This example highlights the importance of combining information from the visual checks and the statistical tests when deciding whether to test for a more restrictive set of inequality indices. The p-values of the statistical tests the associated with Theorem 3 reported in

Table 7 confirm the intuition obtained from the visual inspection. Thus we can conclude that there is more multidimensional inequality in Lebanon than in Libya for all indices belonging to Υ_R.

Table 8 summarizes the rankings provided by the dominance tests on the conditions in Theorems 2 and 3 for the aforementioned comparisons and that of all other countries included in the Arab Barometer survey. Many robust rankings are obtained for all indices in Υ at the 0.05 level (many even at the 0.01 level).

We divide the resulting robust rankings for all indices in Υ, in two groups based on the extent of their multidimensional inequality. The first group contains, Iraq and Palestine. These two countries have less multidimensional inequality than Egypt, Jordan, Lebanon, Morocco, Sudan, and Tunisia. The second group is at the other end of the distribution and contains Egypt, Lebanon, Morocco, and Sudan. These are the countries with the most multidimensional inequality. None of these countries dominate any other country surveyed by the Arab Barometer. As for remaining robust comparisons for Algeria, Jordan, Libya, and Tunisia. The results indicate that Algeria has less inequality than Egypt, Jordan, Lebanon, Morocco, and Sudan. Also, Libya has less inequality than Egypt, Jordan, Morocco, and Tunisia. Furthermore,Tunisia has less inequality than Egypt, Morocco, and Sudan. Finally, Jordan has less inequality than Tunisia. If we consider the robust rankings for all indices in Υ_R, we can add five additional rankings. For all these indices, Iraq has less inequality than Algeria, Jordan has less inequality than Lebanon and Morocco, and Libya has less inequality than Lebanon and Sudan. It is important to note both empirical applications provide consistent conclusions. The three countries included in both applications have the same rankings. Nevertheless, it is essential to keep in mind that this is not a generalizable result. Indeed, the rankings may change when the number of dimensions considered for the index changes.

6 Conclusion

In this paper, we link the literature on multidimensional inequality with Alkire and Foster's (2011) counting approach to multidimensional poverty measurement. In doing so, we offer a multidimensional inequality framework applicable to any data used in applied multidimensional poverty analysis. We first introduce two new graphical tools: the *multidimensional complaint incidence curve* and the *cumulative multidimensional complaint incidence curve*. We then develop the dominance conditions associated with these two visual tools. These dominance conditions identify robust orderings of multidimensional inequality comparisons. We also provide the estimation and the statistical testing procedure linked with these dominance conditions. To show the applicability of the proposed approach, we test the theoretical conditions developed in the paper using two types of survey data. Finally, our empirical illustration shows that the method proposed in this paper also applies to the analysis of multidimensional inequality in developing countries with limited statistical information.

In future research, it would be interesting to characterize a specific parametric class of indices axiomatically. This theoretical result would allow researchers to complete the picture of multidimensional inequality. Furthermore, since the methodology we propose applies to data that is more available for developing countries, it would be interesting from an empirical perspective to extend the empirical application of this paper to other regions of the World to build and monitor a global profile of multidimensional inequality.

References

[1] Abul Naga, R. H. and T. Yalcin (2008), Inequality measurement for ordered response health data, *Journal of Health Economics* 27, 1614-1625.

[2] Alkire, S. and Y. Fang (2019), Dynamics of Multidimensional Poverty and Unidimensional Income Poverty: An Evidence of Stability Analysis from China, *Social Indicators Research*, 142, 25-64.

[3] Alkire, S. and J. Foster (2011), Counting and multidimensional poverty measurement, *Journal of Public Economics*, 95, 476-487.

[4] Allison, R. A. and J. E. Foster (2004), Measuring health inequality using qualitative data, *Journal of Health Economics*, 23, 505-524.

[5] Andreoli, F., and C. Zoli (2020), From unidimensional tomultidimensional inequality: a review, *Metron*, 78, 5-42.

[6] Atamanov, A., S. Tandon, G. Lopez-Acevedo and M.A. Vergara Bahema (2020), Measuring Monetary Poverty in the Middle East and North Africa (MENA) Region. Data Gaps and Different Options to Address Them, Policy Research Working Paper 9259, World Bank Group, Poverty and Equity Global Practice.

[7] Atkinson, A.B. (2003), Multidimensional deprivation. Contrasting social welfare and counting approaches, *Journal of Economic Inequality*, 1, 51-65.

[8] Atkinson, A. B. and F. Bourguignon (1982), The comparison of multi-dimensioned distributions of economic status, *Review of Economic Studies*, 49, 183-201.

[9] Barbe, P., C. Genest, K. Ghoudi, and B. Rémillard (1996), On Kendall's process, *Journal of Multivariate Analysis*, 58, 197-229.

[10] Bourguignon, F. and S. R. Chakravarty (2003), The measurement of multidimensional poverty,*The Journal of Economic Inequality*,1, 25-49.

[11] Cowell, F. and U. Ebert (2004), Complaints and inequality, *Social Choice and Welfare*, 23, 71-89.

[12] Cowell, F. and E. Flachaire (2017), Inequality with ordinal data, *Economica*, 84, 290-321.

[13] D'Agostino, M. and V. Dardanoni (2009), The measurement of rank mobility, *Journal of Economic Theory*, 144, 1783-1803.

[14] Dardanoni,V. (1996), On multidimensional inequality measurement, in Dagum, C., Lemmi,A. (eds.), *Research on Economic Inequality: Income Distribution, Social Welfare, Inequality and Poverty*, 6, 201-205.

[15] Davidson, R. and J.-Y. Duclos, (2013), Testing for restricted stochastic dominance, *Econometric Review*, 32, 84-125.

[16] Decancq, K. (2012), Elementary multivariate rearrangements and stochastic dominance on a Fréchet class, *Journal of Economic Theory*, 147, 1450-1459.

[17] Fan, Y., M. Henry, B. Pass, and J.A. Rivero (2022), Lorenz map, inequality ordering and curves based on multidimensional rearrangements. arXiv preprint arXiv:2203.09000. https://arxiv.org/abs/2203.09000

[18] Genest, C., J.G. Nešlehová, and B. Rémillard (2017), Asymptotic behavior of the empirical multilinear copula process under broad conditions, *Journal of Multivariate Analysis*, 159, 82-110.

[19] Gravel, N. and P. Moyes (2012), Ethically robust comparisons of bidimensional distributions with an ordinal attribute, *Journal of Economic Theory*, 147, 1384-1426.

[20] Gunawan, D., M.A. Khaled, and R. Kohn (2020), Mixed marginal copula modeling, *Journal of Business and Economic Statistics*, 38, 137-147.

[21] Kolm, S.-C. (1976), Unequal inequalities I, *Journal of economic Theory*, 12, 416-442.

[22] Makdissi, P. and M. Yazbeck (2014), Measuring Socioeconomic Health Inequalities in Presence of Multiple Categorical Information, *Journal of Health Economics*, 34, 84-95.

[23] Makdissi, P. and M. Yazbeck (2017), Robust rankings of socioeconomic health inequality using a categorical variable, *Health Economics*, 26, 1132-1145.

[24] Muller, C., A. Trannoy (2012), Multidimensional inequality comparisons: A compensation perspective, *Journal of Economic Theory* 147, 1427-1449.

[25] OAMDI (2012), Iraq Household Socio-Economic Survey (IHSES), http://erf.org.eg/data-portal/, Egypt: Economic Research Forum (ERF).

[26] OAMDI (2013), Household Expenditure and Income Survey (HEIS), 2013, http://erf.org.eg/data-portal/, Egypt: Economic Research Forum (ERF).

[27] OAMDI (2015), Household Income, Expenditure and Consumption Survey (HIECS), 2015, http://erf.org.eg/data-portal/, Egypt: Economic Research Forum (ERF).

[28] Temkin, L. S. (1986), Inequality, *Philosophy & Public Affairs*, 15, 99-121.

[29] Tsui, K.-Y. (1995), Multidimensional generalizations of the relative and absolute inequality indices: The Atkinson–Kolm–Sen approach, *Journal of Economic Theory*, 67, 251-265.

[30] Tsui, K.-Y. (1999), Multidimensional inequality and multidimensional generalized entropy measures: An axiomatic derivation, *Social Choice and Welfare*, 16, 145-157.

[31] UNDP and OPHI (2021), *Global Multidimensional Poverty Index 2021. Unmasking disparities by ethnicity, caste and gender.*

[32] UNDP and OPHI (2022), *Global Multidimensional Poverty Index 2021. Unpacking deprivation bundles to reduce multidimensional poverty.*

[33] Van Der Vaart, A.W. and J.A. Wellner (2007), Empirical processes indexed by estimated functions. *IMS Lecture Notes-Monograph Series*, 55, 234-252.

[34] Zheng, B. (2008), Measuring inequality with ordinal data: a note, in Bishop, J. and B. Zheng (Eds.), *Inequality and Opportunity: Papers from the Second ECINEQ Society Meeting (Research on Economic Inequality, Vol. 16)*, Emerald Group Publishing Limited, 177-188.

A Proofs

Proof of Theorem 1.

- The index being constructed from the survival function insures that it obeys replication invariance and symmetry.

- Since $\int_0^1 \omega(p)\mathrm{d}p > 0$, this implies that $\omega(p) > 0$ over at least some interval on $[0, 1]$. This insures nontriviality.

- Weak rearrangement is then a consequence of Proposition 3 in Decancq (2012). This proposition implies that the survival function $S_1(x) \leq S_0(x)$ for all $x \in \Re^m$. This implies that $K_{S_1}(p) \leq K_{S_0}(p)$ for all $p \in [0, 1]$, $\psi_{S_1}(p) \leq \psi_{S_0}(p)$ for all $p \in [0, 1]$ and $I(S_1) \leq I(S_0)$.

- $\int_0^1 [\psi_{S_{12}}(p) - \psi_{S_0}(p)]\,\mathrm{d}p = \int_0^1 [\psi_{S_0}(p) - \psi_{S_{11}}(p)]\,\mathrm{d}p$ and $\mathrm{d}\omega(p)/\mathrm{d}p \leq 0$ implies weak rearrangement sensitivity.

Proof of Theorem 2. Sufficiency is a direct implication of $\omega(p) \geq 0$. In order to prove for necessity, consider the following weight function that assigns all the social weight on the social positions $p \in [p_0 + p_0 + \varepsilon]$:

$$\omega_{\varepsilon 1}(p) = \begin{cases} 0 & \text{if } p \in [0, p_0] \\ \frac{1}{\varepsilon} & \text{if } p \in [p_0, p_0 + \varepsilon] \\ 0 & \text{if } p \in [p_0 + \varepsilon, 1] \end{cases} \tag{16}$$

If $\psi_{S_1}(p) > \psi_{S_2}(p)$ on the interval $[p_0 + p_0 + \varepsilon]$ then $I(S_1) > I(S_0)$ for $\omega_{\varepsilon 1}(p)$. Hence it cannot be that $\psi_{S_1}(p) > \psi_{S_2}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, for any p_0 and any $\varepsilon > 0$. This proves the necessity of the condition.

Proof of Theorem 3. In order to prove Theorem 3, we first need to integrate equation (2) by parts:

$$I(S) = \omega(p)\Psi_S(p)|_0^1 - \int_0^1 \frac{\mathrm{d}\omega(p)}{\mathrm{d}p}\Psi_S(p)\mathrm{d}p \tag{17}$$

Since by definition $GL(0) = 0$, the difference $I(S_1) - I(S_0)$ can be written as

$$I(S_1) - I(S_0) = \omega(1)\left[\Psi_{S_1}(1) - \Psi_{S_0}(1)\right] - \int_0^1 \frac{\mathrm{d}\omega(p)}{\mathrm{d}p}\left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right]\mathrm{d}p \tag{18}$$

To prove sufficiency, we only need to remember that $\omega(p) \geq 0$ and $\frac{\mathrm{d}\omega(p)}{\mathrm{d}p} \leq 0$. We can then conclude that, if we have $\Psi_{S_1}(p) - \Psi_{S_0}(p) \leq 0$ for all $p \in [0, 1]$, we must have $I(S_1) - I(S_0) \leq 0$.

In order to establish necessity, we need to consider two particular cases. First consider the ethical weight function $\omega(p) = 1$ for all $p \in [0, 1]$. Since, for this particular index, $\frac{\mathrm{d}\omega(p)}{\mathrm{d}p} = 0$, equation (18) becomes:

$$I(S_1) - I(S_0) = \omega(1)\left[\Psi_{S_1}(1) - \Psi_{S_0}(1)\right]. \tag{19}$$

Hence it cannot be that $\Psi_{S_1}(1) > \Psi_{S_0}(1)$. Now consider the following weight function:

$$\omega_{\varepsilon 2}(p) = \begin{cases} \frac{1}{p_0+0.5\varepsilon} & \text{if } p \in [0, p_0] \\ \frac{p_0+\varepsilon-p}{p_0+0.5\varepsilon} & \text{if } p \in [p_0, p_0+\varepsilon] \\ 0 & \text{if } p \in [p_0+\varepsilon, 1] \end{cases} \tag{20}$$

For this particular index, equation (18) becomes:

$$I(S_1) - I(S_0) = \int_{p_0}^{p_0+\varepsilon} \frac{1}{p_0 + 0.5\varepsilon}\left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right]\mathrm{d}p. \tag{21}$$

If $\Psi_{S_1}(p) > \Psi_{S_0}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, then $I(S_1) > I(S_0)$ for this index. Since the ethical weight function in (20) is not everywhere differentiable, it does not belong to Υ. However, there is a sequence of ethical functions $\{\omega_n(p)\}$ belonging to Υ such that $\lim_{n\to\infty} \omega_n(p) = \omega_{\varepsilon 2}(p)$. Applying Lebesgue's dominated convergence theorem leads to

$$\lim_{n\to\infty} \int_0^1 \omega_n(p)[\psi_{S_1}(p) - \psi_{S_2}(p)]\mathrm{d}p = \int_{p_0}^{p_0+\varepsilon} \frac{1}{p_0 + 0.5\varepsilon}\left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right]\mathrm{d}p. \tag{22}$$

Hence it cannot be that $\Psi_{S_1}(p) > \Psi_{S_0}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, for any p_0 and any $\varepsilon > 0$. This proves the necessity of the condition.

B Figures

Figure 1: Multidimensional complaint incidence curve, $\psi_S(p)$

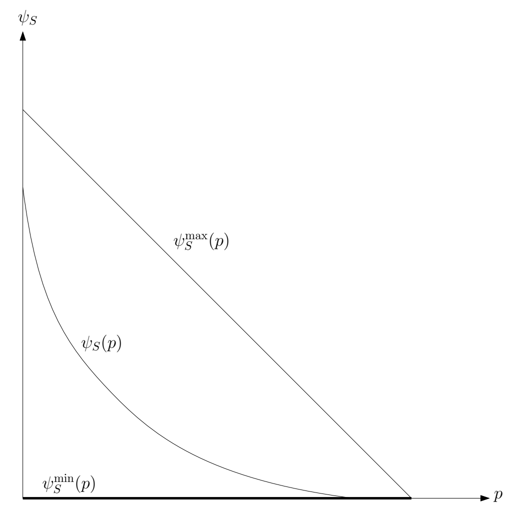


Figure 2: Cumulative multidimensional complaint curve, $\Psi_S(p)$

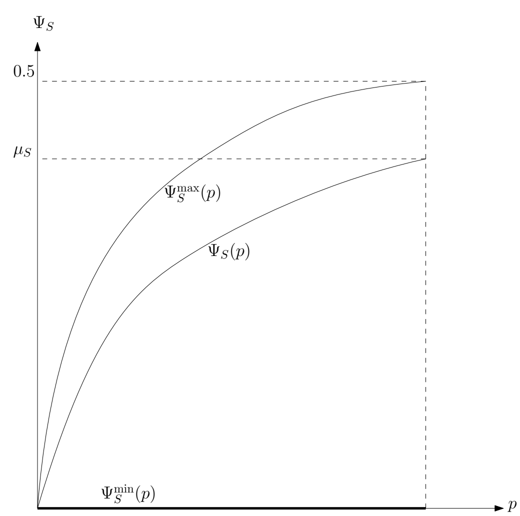


Figure 3: Comparisons of complaint incidence curve, HHIES

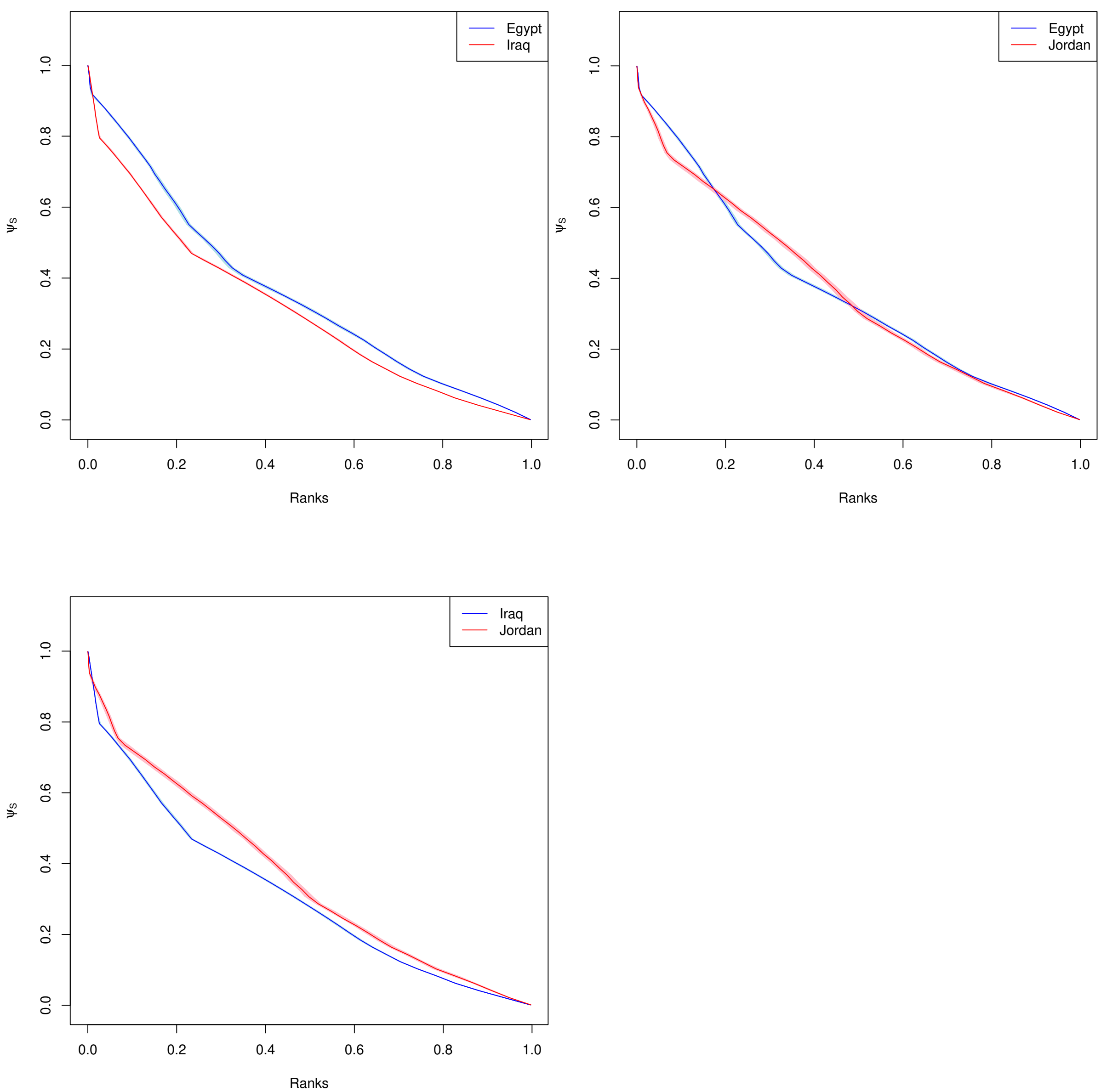


Figure 4: Comparisons of complaint incidence curve, Arab Barometer

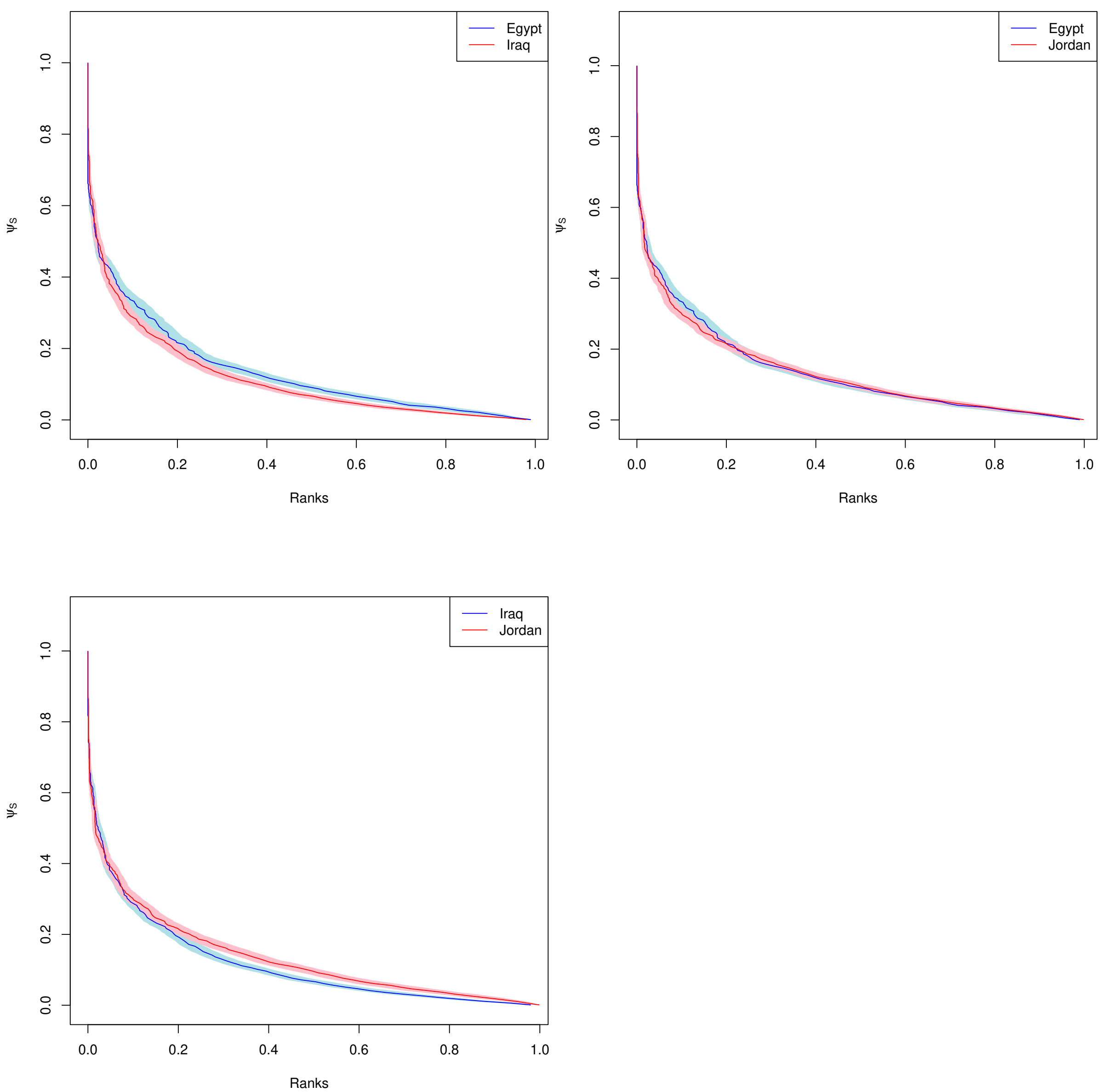
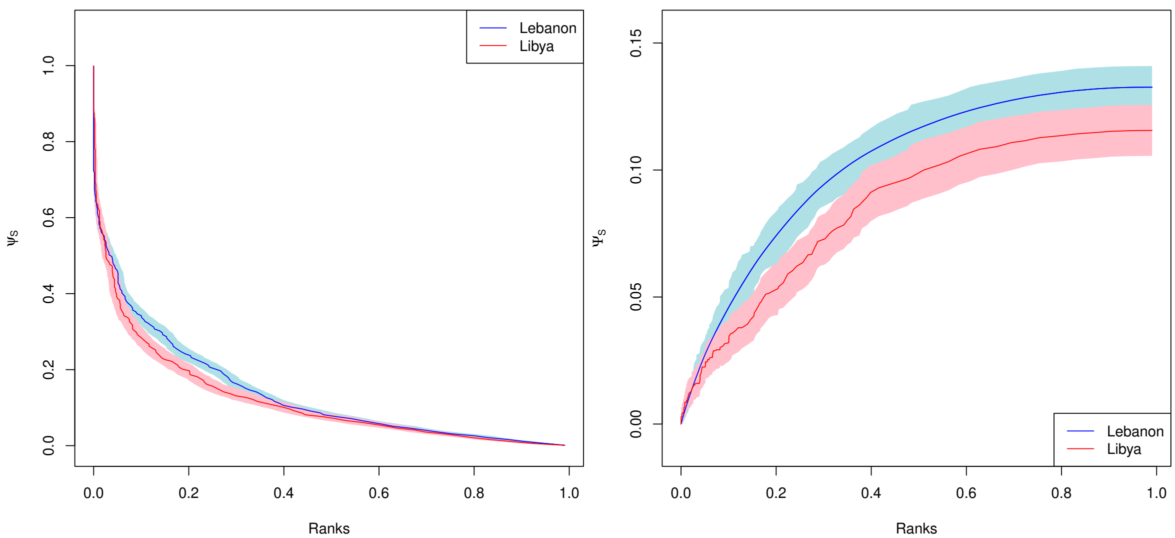


Figure 5: Comparisons of Lebanon and Libya, Arab Barometer



C Tables

Table 1: Dominance tests

Tests	Interpretation
H_0^A not rejected and H_0^B not rejected	$\psi_{S_1}(p) = \psi_{S_2}(p)\ \forall p \in [0,1]$
H_0^A rejected and H_0^B not rejected	$\psi_{S_2}(p) \leqslant \psi_{S_1}(p)\ \forall p \in [0,1]$
H_0^A not rejected and H_0^B rejected	$\psi_{S_1}(p) \leqslant \psi_{S_2}(p)\ \forall p \in [0,1]$
H_0^A rejected and H_0^B rejected	$\psi_{S_1}(p)$ and $\psi_{S_2}(p)$ intersect

Table 2: Variables in the Harmonized Household Income and Expenditure Surveys (HHIES)

Total expenditures	*Education*	*Disability status*
Continuous variable	None	Disabled
	Primary/Lower secondary	No disability
	Secondary	
	Post secondary or equivalent	
	University	
	Postgraduate	

Table 3: Dominance tests, HHIES

Theorem 2	p-value
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.7960
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.7692
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
Theorem 3	**p-value**
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.0000

Table 4: Inequality of opportunity orderings, HHIES

	Egypt	Iraq	Jordan
Egypt	-		ND
Iraq	Υ^{***}	-	Υ^{***}
Jordan			-

$**$: Dominance at the 0.05 level for the indicated set of indices
$***$: Dominance at the 0.01 level for the indicated set of indices
ND: No dominance

Note: Table 5 is displayed on the next page.

Table 5: Variables in the Arab Barometer

Income	*Education*	*Feeling that own personal/family's safety and security are ensured of not*	Free to make decision for myself	Feel depressed	Feel stressed
Lebanon 9 intervals	No formal education	Not at all ensured	I strongly disagree	Most of the time	Most of the time
	Elementary	Not ensured	I disagree	Often	Often
Others 12 intervals	Preparatory/basic	Ensured	I agree	Sometimes	Sometimes
	Secondary	Fully ensured	I strongly agree	Never	Never
	Mid-level diploma/ professional or technical				
	BA				
	MA and above				

Table 6: Dominance tests (Theorem 2), HHIES

Beginning of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8930
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.2207
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.3645
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5484
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.2609
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.1037
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.2341
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9465
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.9967
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.2575
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.9197
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5117
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.3746
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.2676
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.3077
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0201
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8361
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9365
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.5084
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8763
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3512
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.7826
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.5452
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.8662
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0201
Table 6 continues on the next page	

Continuation of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.8495
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.1472
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9967
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.8896
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.6187
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.8462
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0167
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.0970
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.7726
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.7124
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5017
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.8027
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.1204
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.4749
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1906
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0502
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9766
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.7157
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.0669
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0134
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9666
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.6388
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.2207
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.1572
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7324
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9933
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.2876
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.6823
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0602
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.6823
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0435
Table 6 continues on the next page	

Continuation of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3980
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9933
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.8261
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9565
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.9632
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0067
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0134
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.5753
End of Table 6	

Note: Table 7 is displayed on the next page.

Table 7: Dominance tests, HHIES

	p-value
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0468
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.9030
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0635
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.6856
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.1416
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.7191
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Tunisia}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.1137
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1806
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.7492
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.1304
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0368
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.0970
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8328
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.2776
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.8629
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.5552
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0508
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.2074
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.8395
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0167
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0368
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8328
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1137
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0067
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7425
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.8094
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.4047
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.4682
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7458
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	1.000
$H_0 : \Psi_{S_{\text{Tunisia}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.1739
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.6656
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.3880
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8127
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0468
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3579
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9331

Table 8: Inequality of opportunity orderings, Arab Barometer

	Algeria	Egypt	Iraq	Jordan	Lebanon	Libya	Morocco	Palestine	Sudan	Tunisia
Algeria	-	Υ^{***}		Υ^{***}	Υ^{***}	ND	Υ^{***}	ND	Υ^{***}	ND
Egypt		-		ND	ND		ND		ND	
Iraq	Υ_R^{**}	Υ^{***}	-	Υ^{***}	Υ^{**}	ND	Υ^{***}	ND	Υ^{***}	Υ^{**}
Jordan				-	Υ_R^{**}		Υ_R^{**}		ND	Υ^{***}
Lebanon					-		ND		ND	ND
Libya		Υ^{**}		Υ^{***}	Υ_R^{***}	-	Υ^{***}	ND	Υ_R^{**}	Υ^{**}
Morocco							-		ND	
Palestine		Υ^{***}		Υ^{***}	Υ^{**}		Υ^{***}	-	Υ^{***}	Υ^{***}
Sudan									-	
Tunisia		Υ^{***}					Υ^{***}		Υ^{**}	-

$**$: Dominance at the 0.05 level for the indicated set of indices

$***$: Dominance at the 0.01 level for the indicated set of indices

ND: No dominance