The University of Queensland
AUSTRALIA
CREATE CHANGE

Discussion Paper Series

UQ School of Economics

Contagion Management through Information Disclosure
2021-12-19
Jonas Hedlund, Allan Hernández-Chanto, Carlos Oyarzún

Discussion Paper No. 651

Contagion Management through Information Disclosure*

This version: December, 2021

Jonas Hedlund
University of Texas at Dallas

Allan Hernández-Chanto
University of Queensland

Carlos Oyarzún
University of Queensland

Abstract

We analyze information disclosure as a policy instrument for contagion management in decentralized environments. A benevolent planner (e.g., the government) tests a fraction of the population to learn the infection rate. Individuals meet randomly and exert vigilance effort. Efforts factor in a passage function to probabilistically reduce contagion. We analyze the information disclosure policy that maximizes society's expected welfare. When efforts are strategic substitutes, we provide sufficient conditions and necessary conditions for full disclosure to be optimal. When efforts are strategic complements, pooling intermediate infection rates is optimal whenever individuals' equilibrium effort jumps from no-effort (inaction) to full-effort (frenzy).

JEL: D44, D47, D81, D82

Keywords: Contagion, information design, full-disclosure, obfuscation, vigilance effort, passage function, substitutes, complements.

*Contact information: c.oyarzun@uq.edu.au (Oyarzun, corresponding author), jonas.hedlund@utdallas.edu (Hedlund), a.hernandezchanto@uq.edu.au (Hernandez-Chanto).

We are thankful to Jeffrey Kline, Claudio Mezzetti, and to audiences at University of Costa Rica, Queensland University of Technology, University of Queensland, SAET 2021, and AETW 2021 for insightful questions and comments.

1 Introduction

Some of the greatest economic crises that societies have faced involve some form of contagion. Notable examples include financial contagion, where investors acquire toxic assets that decrease the total value of their portfolios—e.g., the global financial crisis of 2008 (GFC); epidemiological contagion, where an infectious disease is spread in a population, impacting its productivity, growth, and employment—e.g., the Covid-19 pandemic; and monetary contagion, where the debasement of commodity money and the counterfeiting of fiat money reduce the frequency with which goods and services are exchanged—e.g., the debasement crisis of medieval France and England and the current wave of counterfeiting boosted by the digital era.[1]

While contagion problems can be devastating per se, the propagation of their deleterious effects is facilitated by the weak response of economic systems, which is typically hampered by externalities and coordination failure. The reason for this is that individuals fail to internalize the whole effect their efforts have on curbing contagion and are unable to coordinate their responses to increase the effectiveness of their actions. The thesis of this paper is that governments or regulators can alleviate these frictions by managing the disclosure of information they possess in a way that aligns individuals' behavior to what would be the optimal behavior from society's perspective.[2]

We analyze the role of information design to shape individuals' incentives in a decentralized environment susceptible to contagion. In our setup, infected and susceptible individuals match randomly and pairwise. The *infection rate* of the population is initially unknown, as well as the infectious status of each individual. The government (the sender in our setting) has a test available that can reveal the infectious status of any tested individual. There is a continuum of individuals; thus, the government learns the infection rate by testing a positive fraction of the population. Critically, the government may strategically reveal the infection rate to the public or opt to (partially) conceal such information.

[1]According to Quercioli and Smith (2015), about one in 10,000 U.S. dollar notes is counterfeit, with the domestic public losing \$80 million in 2011, more than doubling since 2003. Meanwhile, Rolnick et al. (1996) observe that between 1285 and 1490, France had 123 debasements of silver coins, 112 of more than 5 percent, with the highest being 50 percent.

[2]Information management can be a powerful but fuzzy policy instrument. As a testament of the former, the final report of the Commission that studied the GFC concluded that the "liberal" policies undertaken by credit rating agencies were instrumental in the downfall of financial markets (cf. Financial-Crisis-Inquiry-Commission, 2011). In turn, as an illustration of the latter, it is noteworthy that the WHO advices full transparency regarding the transmission of sanitary information for containing epidemics, but it is silent about the communication policies that governments should follow with respect to their assessment of how widespread contagion is (cf. World-Health-Organization, 2018).

Testing partitions the population in two types: non-tested and tested-negative. Tested-positive individuals are isolated by the government—in line with the policies that have been used to manage contagion in different historical contexts—and play no role in our model. Non-isolated individuals interact in a decentralized environment and make strategic choices of vigilance efforts.

In a matching, a *passage function* maps individuals' vigilance efforts to a probability of contagion, whenever one of the individuals is infected and the other is not. Vigilance reduces the probability of contagion, and, hence, the passage function is decreasing in the effort of both matched individuals. As a consequence, the effort of non-tested individuals—who are uncertain about their infectious status—generates a *positive externality* that benefits both other non-tested individuals who may not be infected with positive probability and tested-negative individuals who are certainly not infected. Moreover, we distinguish two cases with respect to the interaction of efforts: (i) when the marginal benefit of an individual's effort is *decreasing* in the effort of the other individual, i.e, when efforts are *strategic substitutes*; and (ii) when the marginal benefit of an individual's effort is *increasing* in the effort of the other individual, i.e., when efforts are *strategic complements*.

Finally, the government is benevolent and aims to minimize society's expected welfare losses, which include both the cost of vigilance and losses associated to contagion. The only available instrument is the choice of an information disclosure policy that is used to manage individuals' vigilance efforts.

A key departure of our analysis from standard applications of information design is our focus on how the government attempts to correct inefficiencies caused by externalities and coordination failure. Additionally, in our model, testing generates individuals' types—i.e., non-tested and tested-negative—which, in turn, influence individuals' behavior. Finally, individuals can take continuous vigilance efforts based on their testing status, instead of the usual binary-action framework used in models of persuasion. This feature allows us to provide a comprehensive account of the role of strategic complementarity and substitutability in the design of the optimal policy.

Overview of the results Our results reveal that the optimal information disclosure policy can be passive, i.e., fully revealing, or involve obfuscation, driven by three qualitatively different impacts on individuals' vigilance that the government may be aiming for: (i) smoothing the variation of vigilance across different infection rate realizations; (ii) enlarging the set of events for which individuals exert vigilance; and (iii) enlarging the set of events for which individuals' effort is maximal.

Methodologically, we adapt techniques developed in Dworczak and Martini (2019) and Dworczak and Kolotilin (2019), writing the expected payoffs of individuals and the government as functions of the conditional first and second moments of the infection rate. This approach reveals that, if the government's payoff function is convex, then full disclosure is optimal (Lemma 2). Intuitively, convexity implies that, on average, variation in interim beliefs of individuals—and their corresponding efforts—reduce the ex-ante expected loss associated to contagion and vigilance costs.

For strategic substitutes, we provide simple sufficient conditions for convexity of the government's payoff function—and hence, for the optimality of full revelation. As exposure risk increases, when the infection rate gets closer to one half of the population (where the largest exposure risk is attained), the incentives to exert vigilance effort increase, buffering contagion. This results in convexity of the government payoff function, provided that the buffering effects are not dominated by decreasing returns of effort externalities. Proposition 1 provides the precise condition on the magnitude of these effects for convexity to hold.

When convexity is violated, involving convex combinations of beliefs in the support of the distribution of posteriors that arises under full revelation, the optimal information disclosure involves obfuscation (Lemma 3). In particular, Proposition 2 provides an optimal information policy that applies when the government's payoff function is concave over some regions. Here, the government uses obfuscation to smooth non-tested individuals' vigilance effort across different realizations of the infection rate. By concavity, reducing the variation of individuals efforts across different states of the world ex-ante increases the expected payoff of the government.

For strategic complements, the game played by non-isolated individuals is supermodular and exhibits multiple equilibria. Our analysis focuses on the largest equilibria, which involve the highest vigilance effort that can be sustained in equilibrium.[3] When the complementarity of effort is strong, the equilibria that arise are *bang-bang*, jumping from inaction (no effort) to frenzy (full effort), as exposure risk increases. The optimal policy in this scenario requires to pool intermediate infection rates—which is associated to high exposure risk because there are large fractions of both infected and non-infected individuals (Proposition 3).[4] Here, obfuscation allows the government to stretch the set of infection rates within which effort is exerted. This is attained by pooling intermediate infection rates—for which effort has a high private marginal benefit—with more extreme infection rates—for which effort has a low private marginal benefit (not enough to make effort worthy under full disclosure).

[3]The other natural alternative would be considering the smallest equlibria; i.e., those involving the smallest effort. Typically, as in our leading example, those equilibira involve no effort at all. In that scenario, contagion management is better addressed through policies that force behavior, such as lock-downs and curfews (cf. Alvarez et al., 2021).

[4]The government may fully disclose extreme infection rates, either low or high, or can pool such states. Either way individuals will exert no effort.

Finally, when the degree of complementarity is low, the largest equilibrium is not bang-bang, and the analysis is qualitatively similar to the case of strategic substitutes. In Proposition 4, we provide an optimal information disclosure that arises in the presence of kinks in the payoff function of the government. In this case, obfuscation decreases the government expected losses by stretching the set of realizations of infection rates for which effort is maximal.

Related Literature Following the seminal work of Kamenica and Gentzkow (2011) (see Bergemann and Morris, 2019; Kamenica, 2019, for a review), information disclosure has been studied in the analysis of several economic problems. Many of the advances in the literature have been theoretical, yet they have featured diverse applications, including, for instance, disclosure of students' grades by universities to employers (cf. Ostrovsky and Michael, 2010; Boleslavsky and Cotton, 2015), disclosure of medical tests to patients (cf. Schweizer and Szech, 2018), and the disclosure of information by pharmaceutical firms that seek approval by the FDA (cf. Kosterina, 2020). In contrast to ours, however, most of these applications occur in centralized environments in which decision makers act in isolation, and, thus, absent of strategic considerations.

This paper is related to four different strands of literature. First, it provides an application of the recent methodological developments to solve persuasion problems using a duality approach when the sender's payoff can be written as a function of conditional moments of the posterior distribution (e.g., Dworczak and Martini, 2019; Dworczak and Kolotilin, 2019).

Second, our work relates to the analysis of information problems with multiple receivers that interact strategically in the game induced by the sender's information policy. These include Alonso and Câmara (2016), Bardhi and Guo (2018), Galperti and Perego (2018), Taneva (2019), Mathevet et al. (2020), Inostroza and Pavan (2021), and Li et al. (2021), among others. Many of these papers either explicitly model the epistemic conditions of the game or analyze the structure of information propagation in centralized environments. In contrast, we consider the strategic interactions of individuals in a decentralized environment and model explicitly the contagion process through a passage function, as in models that use the SIR framework to study the strategic interaction among individuals' choice of vigilance (cf. Keppo et al., 2021).

From this group of papers, the closest to ours are Inostroza and Pavan (2021) and Li et al. (2021). Inostroza and Pavan (2021) consider a regulator interested in avoiding a bank's default that uses information disclosure to manage investors' beliefs. The regulator performs a stress test to determine the financial soundness of the bank and strategically chooses a policy to disclose such information. The authors model the problem as a *global game* in which the regulator maximizes its objective with respect the most-adverse equilibrium—taking into account the effect that the information policy has on the higher beliefs of individuals. Likewise, Li et al. (2021) considers a game of regime change with continuum of individuals in the realm of a game with strategic complementarities. Unlike these papers, our analysis addresses games with both strategic complements and substitutes.

The third strand of literature analyzes information design in decentralized markets. Here, the closest paper is by Duffie et al. (2017). They study a decentralized market where traders search for dealers to acquire an asset. Dealers post a price and commit to the sell the asset at that price, which is their private information. The authors analyze the impact on welfare of providing a benchmark to traders about the average cost for dealers to provide an asset. Unlike theirs, in our model there is no directed search because individuals match randomly. Hence, information induces individuals to exert more vigilance instead of refining the quality of their search.

Finally, our paper contributes to the literature on economic models of epidemiological contagion boosted by the Covid-19 pandemic. Among these papers, Ely et al. (2020) analyze the benefit of using "rotation" of personnel in an organization as a measurement of contagion mitigation; Heinsalu (2021) studies the incentives of rational agents to "arbitrate" in the time they get infected to maximize their probability of receiving medical treatment without rationing; Whitmeyer (2021) and Gans (2020) analyze the tradeoff in cost and epidemiological control incurred by a government that uses inaccurate tests; and de Vericourt et al. (2021) consider a government that wants to induce individuals to practice social distancing to maximize an objective function that weights (possibly asymmetrically) the health and economic well-being of its population.

Organization The rest of the paper is organized as follows. Section 2 lays down the model. Section 3 describes the vigilance game induced by the government's information policy, including a reformulation of the persuasion problem based on two moments of the posterior distribution. Section 4 determines the equilibria and the optimal disclosure when efforts are strategic substitutes. Section 5 conducts the same analysis when efforts are complements. Section 6 present other applications for which our analysis is relevant. Section 7 concludes.

2 The Environment

We consider a *government* and a continuum of *individuals* with unitary mass. Individuals are susceptible to contract a contagious disease. The government attempts to minimize welfare losses associated to the contagion and the cost of efforts to avoid it.

Initial Infection Nature determines the probability ω with which each individual is infected, i.e., the *infection rate*. This is a random variable drawn from $\Omega \triangleq [0,1]$ according to a full-support and non-atomic distribution F. The infection status of individuals are independent; thus, by the exact law of large numbers of Sun (2006), ω is also the mass of infected individuals.

Testing The government has access to a test that perfectly identifies the infectious status of an individual. Individuals are selected for testing randomly according to an exogenous probability $t \in (0,1)$, which is independent from their infectious status. Thus, the mass of tested individuals in the population also corresponds to t.

The infectious and testing status of each individual is the only source of *interim* heterogeneity. Furthermore, because of the independence between the testing and infectious status of each individual, the mass of positive tests is $t\omega$. Since the government observes this measure, it learns the probability of infection ω.

Vigilance, contagion, and passage function The government enforces the isolation of all individuals whose test result is positive. As a consequence, the mass of non-isolated individuals after testing corresponds to $(1-t)+t(1-\omega)$, i.e., the mass of non-tested individuals plus the mass of tested-negative individuals.

Each individual is randomly and independently matched to another individual in the population. There is *exposure risk* if a *susceptible* individual (i.e., an individual who is not infected) is matched with an infected individual who is not isolated. Matches of two infected individuals or two susceptible individuals cannot result in contagion.

Each individual chooses a *vigilance effort* $e \in [0,1]$, which has a constant marginal cost $\gamma > 0$. The effort exerted by susceptible individuals decreases the probability of getting infected, whereas the effort exerted by infected individuals decreases the probability of infecting susceptible individuals. The probability that a susceptible individual is infected in a meeting with an infected non-isolated individual, provided that their respective efforts are e and $\hat{e}$, is $q(e,\hat{e}) \in [0,1]$. We call q the *passage function*, and assume that it is twice continuously differentiable on $(0,1)^2$ and satisfies decreasing returns, $q_1, -q_{11} < 0$. Furthermore, the effort exerted by infected individuals generates a *positive externality*, $q_2 \leq 0$, which plays an important role in our analysis.

Payoffs Decreasing returns on effort guarantee that all the equilibria in our analysis are symmetric: all non-isolated individuals with the same testing status follow the same strategy, and, as a consequence, exert the same effort. Hence, provided that the effort exerted by non-tested individuals is e^0, a susceptible individual who exerts effort e gets infected with probability $\omega(1-t)q(e,e^0)$. The loss incurred by an infected individual is normalized to 1. Therefore, the payoff for a non-tested individual of exerting effort e is

$$u^0(e,e^0,\omega) = -(1-\omega)\omega(1-t)q(e,e^0) - \gamma e. \tag{1}$$

A non-tested individual has a probability $(1-\omega)$ of being susceptible. In such a case, the individual can get infected only in a matching with another non-tested individual who is infected, which happens with probability $(1-t)\omega$. Conditional on this matching, contagion occurs with probability $q(e,e^0)$. Finally, the last term on the right-hand side of (1) is the individual's private cost of exerting effort e.

Likewise, the payoff for a negative-tested individual of exerting effort e is given by

$$u^1(e,e^0,\omega) = -\omega(1-t)q(e,e^0) - \gamma e. \tag{2}$$

The only difference with the payoff function of non-tested individuals is that, since tested-negative individuals are certaintly not infected, they have a greater marginal benefit of effort.

The government is a benevolent planner. Given a profile of efforts (e^0,e^1), where e^0 and e^1 are the efforts exerted by non-tested and negative-tested individuals, respectively, the government's payoff is

$$v(e^1,e^0,\omega) = (1-t)u^0(e^0,e^0,\omega) + t(1-\omega)u^1(e^1,e^0,\omega), \tag{3}$$

for all $\omega \in \Omega$. The first term in equation (3) corresponds to the mass $(1-t)$ of non-tested individuals, while the second term corresponds to the mass $t(1-\omega)$ of tested-negative individuals.

Disclosure In order to incentivize individuals' vigilance effort, the government commits to an *information structure*. Formally, an information structure is a measurable mapping $\pi : \Omega \to \Delta(\mathcal{S})$, where $\mathcal{S}$ denotes a signal space. Given the information structure π and the prior distribution F, each realization of the signal $s \in \mathcal{S}$ induces a posterior distribution μ_s over Ω.[5] Hence, the information structure π ex-ante induces a distribution of posterior beliefs $\tau \in \Delta(\Delta(\Omega))$, given by

$$\tau(\mu) = \int_\Omega \int_{\{s:\mu_s=\mu\}} \pi(s|\omega)dsdF(\omega),$$

for all $\mu \in \Delta(\Omega)$, and satisfying Bayesian plausibility, i.e., $\int_{\Delta(\Omega)} \mu d\tau(\mu) = F$ (cf. Kamenica and Gentzkow, 2011). The set of posteriors satisfying Bayesian Plausibility is denoted by $\mathcal{T}$.

[5]The space $\mathcal{S}$ is equipped with a σ-field. Furthermore, π is a measurable function with respect to the Borel σ-algebra in $[0,1]$.

The posterior beliefs of non-tested individuals are given by

$$\mu_s(\omega) = \frac{\pi(s|\omega)f(\omega)}{\int_0^1 \pi(s|\omega')f(\omega')d\omega'} \tag{4}$$

for all s in the support of the distribution of signals induced by τ.

Meanwhile, the posterior beliefs of individuals whose test is negative are different from those of non-tested individuals, since they also take into account their own test result. Specifically their posterior beliefs are given by

$$\hat{\mu}_s(\omega) = \frac{(1-\omega)\pi(s|\omega)f(\omega)}{\int_0^1 (1-\omega')\pi(s|\omega')f(\omega')d\omega'} \tag{5}$$

for all s in the support of the distribution of signals induced by τ. Beliefs $\hat{\mu}_s$ are more optimistic in the sense that they are first-order stochastically dominated by μ_s—i.e., higher infection rates are more likely under μ_s than under $\hat{\mu}_s$, for all $s \in \mathcal{S}$.[6]

Discussion of the model We aim to provide a tractable approach to the information design problem in decentralized environments exposed to contagion. We are particularly interested in understanding the economic intuition that drives the government's optimal policy. Towards this aim, we have made some assumptions that are worth to discuss in greater detail.

First, as in Keppo et al. (2021)'s analysis of infection, we consider a static model. While our analysis may be constructive towards the specification of a dynamic model, the insights of our results below, regarding to when full disclosure is optimal and when there is a role for obfuscation, seem impervious to dynamic considerations.[7] Second, we consider individuals' payoff functions that are linear in the probability of getting infected and the cost of effort. Quasi-linear utility provides a common and well understood initial approach in several settings (e.g., auction theory and mechanism design more broadly).[8] Third, we follow the search literature in money and labor and assume that matchings are pairwise. This is the minimal matching structure required to model contagion explicitly. The advantage is that individuals' and government's payoff functions

[6] Routine calculations reveal that $\mu_s/\hat{\mu}_s$ is increasing for all $s \in \mathcal{S}$ unless the support of μ_s and $\hat{\mu}_s$ is a singleton.

[7] As Keppo et al. (2021) argue, a static game is adequate for infection analysis because with a continuum, no single individual can impact others' payoffs and the loss from being infected may be regarded as constant over time. This implies that in a dynamic version of our setup, both type of individuals act in a large game in which they have to solve the same strategic problem in each period.

[8] Our model can be reinterpreted as one in which individuals choose "gross efforts" $\xi = (\xi^0, \xi^1)$ and the passage function corresponds to $\tilde{q}(\xi^0, \xi^1)$. In particular, for any effort ξ, and increasing and convex function $c(\xi)$, we can set $e = c(\xi)$. That is, e is the vigilance level of our model induced by the gross effort ξ. Thus, this transformation is innocuous, allowing us to write the passage function directly in terms of vigilance levels, i.e., $q(e^0, e^1)$, and making the presentation simpler.

depend solely on the first two moments of the posterior distribution. Finally, we assume that the testing status of an individual is independent from its infectious status. Assuming correlation does not change the government's ability of learning the true state of the world, but complicates individuals' behavior. In fact, the only difference is that the mass of tested-positive individuals would increase and the mass of tested-negative individuals would decrease. While adding some of these features may help to have a richer environment, the key insights pioneered by our analysis seem robust to such generalizations.

3 The vigilance game

The government's choice of information structure π induces a "downstream" game of incomplete information. In this game, first, Nature chooses ω and a signal s according to $\pi(\cdot|\omega)$. Then, upon observing s, non-tested and negative-tested individuals choose their respective efforts. Finally, payoffs are realized. Figure 1 depicts the timing of the game.

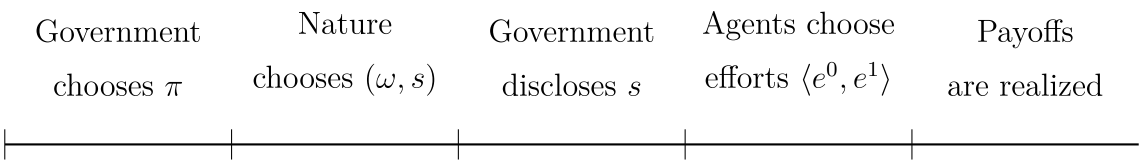


Figure 1: Timing of the vigilance game

Formally, the information structure π defines a *vigilance game* Γ_π played by non-isolated individuals, whose strategies describe the vigilance effort in response to the observed signal s, given their type—i.e., non-tested or tested-negative.

3.1 A two-moment formulation

We can formulate the vigilance game deriving the expected payoffs of non-tested and negative-tested individuals depending on the first two moments of the posterior distributions induced by the prior F and the information structure π. Specifically, for any posterior distribution $\mu \in \Delta(\Omega)$, we define the first and second moments of ω as $x \triangleq \mathbb{E}_\mu[\omega]$ and $y \triangleq \mathbb{E}_\mu[\omega^2]$, respectively. The conditional *exposure risk* of non-tested individuals, $\mathbb{E}_\mu[\omega(1-\omega)]$ is denoted by $z^0 \triangleq x - y$. Likewise, we let X be the closure of the set where (x, y) can take value; i.e., $X \triangleq \{(x, y) \in [0, 1]^2 : x^2 \leq y \leq x\}$.

Meanwhile, the conditional exposure risk of tested-negative individuals is

$$\mathbb{E}_{\hat{\mu}_s}[\omega] = \frac{\int_\omega \omega(1-\omega)\pi(s|\omega)f(\omega)d\omega}{\int_\omega (1-\omega)\pi(s|\omega)f(\omega)d\omega} = \frac{x-y}{1-x} \triangleq z^1, \tag{6}$$

for all $s \in \mathcal{S}$ such that $\int_\omega \pi(s|\omega)f(\omega)d\omega > 0$ and $x < 1$. Because the prior distribution is non-atomic, the right-hand side in (6) is finite almost surely.

Thus, given a choice of effort e^0 by non-tested individuals, the expected payoffs of both type of individuals are given by

$$U^\ell(e^\ell, e^0, x, y) = -z^\ell(1-t)q(e^\ell, e^0) - \gamma e^\ell \tag{7}$$

for all $(x, y) \in X$ and $\ell \in \{0, 1\}$.

Definition 1. *An individual's pure strategy is a tuple of functions $\boldsymbol{\zeta} = \langle \zeta^0, \zeta^1 \rangle$, with*

$$\zeta^\ell : X \to [0,1] \text{ for } \ell \in \{0,1\},$$

where, for all $(x, y) \in X$, $\zeta^0(x, y)$ is the vigilance effort of a non-tested individual and $\zeta^1(x, y)$ is the vigilance effort of a tested-negative individual.

Our assumption on the returns to vigilance being decreasing, namely $q_{11} > 0$, guarantees that, in equilibrium, individuals strategies are pure and symmetric; i.e., every individual follows the same pure strategy $\boldsymbol{\zeta} = \langle \zeta^0, \zeta^1 \rangle$.

Definition 2. *A Bayesian Nash Equilibrium (BNE) is a strategy profile $\boldsymbol{\zeta} = \langle \zeta^0, \zeta^1 \rangle$ such that*

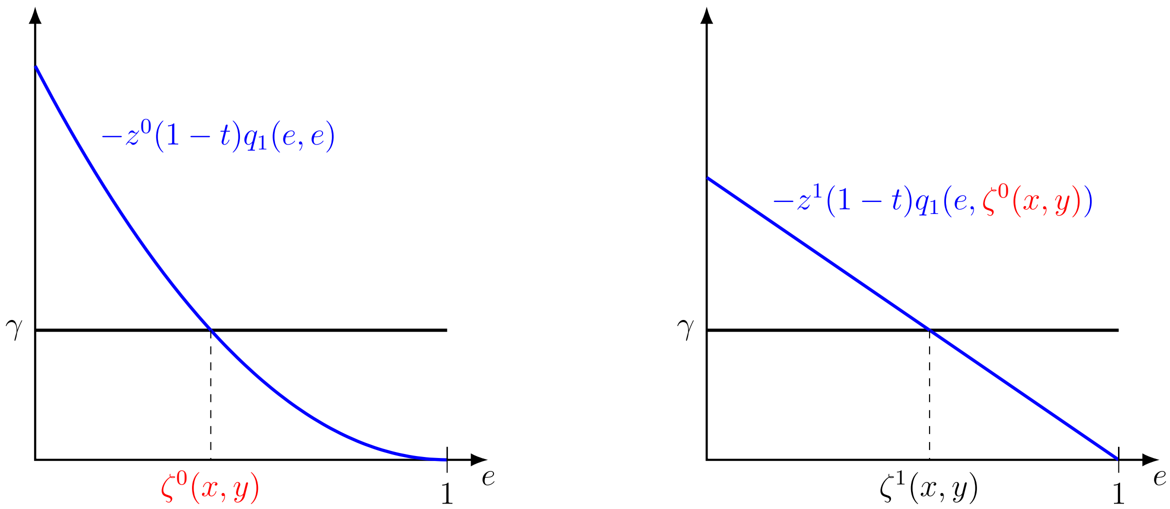
$$\zeta^\ell(x, y) \in \arg\max_{e \in [0,1]} U^\ell(e, \zeta^0(x, y), x, y) \tag{8}$$

for all $(x, y) \in X$ and $\ell \in \{0, 1\}$.

A Bayesian Nash Equilibrium always exists.[9] The fact that the vigilance of non-tested individuals is the one that generates a *positive externality* in the population simplifies the way in which we can obtain the equilibria of the game. In particular, equilibria can be solved sequentially: given a posterior $\mu \in \Delta(\Omega)$, we first find the optimal effort of non-tested individuals $\zeta^0(x, y)$, and then obtain the optimal effort of tested-negative individuals $\zeta^1(x, y)$. Figure 2 depicts how equilibrium efforts are obtained.

Since $z^1 > z^0$ (with probability 1) for any information structure, tested-negative individuals exert more effort than non-tested individuals.

[9] For details see Lemma 6 in the Supplementary Appendix. We also note that a BNE describes the equilibirum behavior of individuals in every vigilance game Γ_π, for any information structure π. I.e., for any two information structures, we assume that the same BNE $\boldsymbol{\zeta}$ describes individual behavior.



(a) Non-tested inidividuals

(b) Tested individuals

Figure 2: Sequential characterization of equilibria for both types of individuals. Panel 2a: equilibrium effort for non-tested individuals for $q(e, e^0) = (1-e)^2(1-e^0)$, $t = 0.05$, $x = 0.2$, $y = 0.04$, and $\gamma = 0.1$. Panel 2b: equilibrium effort of tested-negative individuals, obtained using $\zeta^0(x, y)$ from 2a.

Remark 1. *In any BNE $\boldsymbol{\zeta}$, individuals' efforts satisfy $\zeta^0(x, y) \leq \zeta^1(x, y)$ for all $(x, y) \in X$.*

Proof. From (7), tested-negative individuals' expected marginal benefit from vigilance is strictly greater than non-tested individuals' for all $(x, y) \in X$ such that $x < 1$. (This is also true in the zero-probability event $x = 1$). Therefore, because marginal costs are the same for all individuals, the equilibrium effort of tested-negative individuals is weakly greater than the one for non-tested individuals. ■

3.2 Optimal Disclosure

The government's interim expected payoff is

$$
\begin{aligned}
V(x, y, \boldsymbol{\zeta}) \triangleq & \int_\Omega v(\zeta^1(x, y), \zeta^0(x, y), \omega) d\mu(\omega) \\
= & -(1-t)\left[(x-y)(1-t)q(\zeta^0(x, y), \zeta^0(x, y)) + \gamma\zeta^0(x, y)\right] \\
& - t\left[(x-y)(1-t)q(\zeta^1(x, y), \zeta^0(x, y)) + (1-x)\gamma\zeta^1(x, y)\right], \qquad (9)
\end{aligned}
$$

for all $\mu \in \Delta(\Omega)$. That is, governments interim expected payoff corresponds to the average of each individual's type payoff, weighted by their mass in the population. Furthermore, given a distribution of posteriors τ, its ex-ante expected payoff is $\mathbb{E}_\tau V(x, y, \boldsymbol{\zeta}) = \int_{\Delta(\Omega)} V(x, y, \boldsymbol{\zeta}) d\tau(\mu)$. Hence, given a BNE $\boldsymbol{\zeta}$, the problem of the government consists of choosing a distribution of posteriors

$$\tau \in \arg\max_{\tau' \in \mathcal{T}} \int_{\Delta(\Omega)} V(x, y, \boldsymbol{\zeta}) d\tau'(\mu). \tag{10}$$

Because we reformulate the problem, redefining expected payoffs as a function of two moments of the posterior distribution of ω, we can take advantage of the results in Dworczak and Martini (2019) and Dworczak and Kolotilin (2019) to make our analysis more tractable. In particular, by choosing a distribution of posteriors $\tau \in \mathcal{T}$, the government is also choosing the distribution of conditional moments (x, y) associated to τ. In the sequel, this distribution of conditional moments is denoted by G, and the set of such distributions is denoted by $\mathcal{G}$.

We use an adapted version of Theorem 1b in Dworczak and Martini (2019) to prove the optimality of a distribution of conditional moments G. For clarity in the exposition, we drop the argument $\boldsymbol{\zeta}$ in $V(x, y, \boldsymbol{\zeta})$.

Lemma 1. *Given a BNE $\boldsymbol{\zeta}$, an information disclosure π is optimal if it induces a distribution of conditional moments $G \in \mathcal{G}$ such that there exists a convex function $P : X \to \mathbb{R}$ satisfying that $P \geq V$, and*[10]

$$\int_{\Omega} P(\omega, \omega^2) dF(\omega) = \int_X P(x, y) dG(x, y) = \int_X V(x, y) dG(x, y). \tag{11}$$

Lemma 1 is intuitive. Since P is convex, the first equality in (11) guarantees that the distribution of posteriors G attains the maximum expected value of P among mean preserving contractions. Therefore the second equality in (11) implies that G must also maximizes the expected value of V, because $\mathbb{E}_F[P(\omega, \omega^2)]$ is an upper bound for such expected values. An immediate consequence of this lemma is that the convexity of V implies the optimality of full disclosure.

Lemma 2. *If V is convex, full disclosure is optimal.*

Proof. The result is a direct consequence of Lemma 1 by choosing $P = V$. ■

If V is convex, the government cannot benefit from contracting the distribution of posteriors. The reason is that for any (measurable) set of posteriors $(x, x^2) \in X$, the payoff of the government from the average posterior within that set is less than the corresponding average of payoffs.

4 Strategic Substitutes

In this section, we study the case when efforts are strategic substitutes.

Definition 3. *A passage function q exhibits strategic substitutes if $q_{12} > 0$ over $(0, 1)^2$.*

[10]To avoid notation cluttering, we also omit the dependence of P on $\boldsymbol{\zeta}$.

4.1 Full disclosure

Under strategic substitutability, there is a unique BNE, and it induces a *continuous* payoff function of the government V.[11] However, the BNE may exhibit corner solutions for the maximization problem of individuals for different posteriors, making V to have kinks.[12] Provided that the potential kinks of V caused by corner vigilance efforts $\zeta^0(x,y)$ and $\zeta^1(x,y)$ have no effect on the curvature of V, its convexity can be determined by differentiation. Indeed, when V is convex, this property is impervious to the presence of kinks, except for those corresponding to $\zeta^0(x,y)=1$. The following assumption either rules out those kinks or makes the convexity V resistant to their presence.

Assumption 1 (No-kink condition)**.** *The passage function q satisfies*[13]

$$-\frac{1}{4}(1-t)q_1(1,1) \leq \gamma, \quad \textit{or} \quad \frac{q_2(\cdot,1)}{q_{11}(1,1)+q_{12}(1,1)} = 0. \tag{12}$$

This assumption is used below in our results providing sufficient conditions for the optimality of full disclosure. The inequality in condition (12) guarantees that $\zeta^0(x,y)<1$ over X, except at $(\frac{1}{2},\frac{1}{4})$. If this inequality does not hold, the equality in condition (12) guarantees that the kinks of V caused by the corner's vigilance effort $\zeta^0(x,y)=1$ do not spoil its convexity.

Our next result provides a simple condition for the convexity of V, which yields that the optimal disclosure policy is full revelation.

Proposition 1. *Suppose Assumption 1 holds. Then, full disclosure is optimal if*

$$\begin{aligned}(1-t)z^0q_{11}(\zeta^0,\zeta^0)(\zeta_1^0)^2 + (1-t)\left(-z^0q_{22}(\zeta^0,\zeta^0))(\zeta_1^0)^2 - q_2(\zeta^0,\zeta^0)(2\zeta_1^0+z^0\zeta_{11}^0)\right)\\ +t\left(-z^0q_{22}(\zeta^1,\zeta^0)(\zeta_1^0)^2 - q_2(\zeta^1,\zeta^0)(2\zeta_1^0+z^0\zeta_{11}^0)\right) \geq 0,\end{aligned} \tag{13}$$

for all (x,y) in the interior of $\{(x,y)\in X : \zeta^0(x,y)>0\}$.[14]

[11] As illustrated in Figure 2, since equilibria are symmetric and $q_{11}+q_{12}>0$ on $(0,1)^2$, the marginal benefit from vigilance for non-tested individuals, $-z^0(1-t)q_1(e,e)$, is decreasing in the effort e, for each $(x,y)\in X$. Hence, the marginal cost γ equalizes the marginal benefit for at most one effort level. Similarly, there is a unique optimal level of effort for tested-negative individuals for each posterior (x,y), given the effort level of non-tested individuals.

[12] From Remark 1, V may be a piece-wise function, conformed by five different pieces in which (i) no individual type make effort; (ii) negative-tested individuals make positive—but less than full—effort and non-tested individuals make no effort; (iii) negative-tested individuals make full effort and non-tested make no effort; (iv) both types make positive—but less than full—effort; (v) negative-tested individuals make full effort and non-tested make positive—but less than full—effort; and (vi) both types make full effort.

[13] Up to this point, we have not needed to refer to the derivatives of q in the boundary of $(0,1)^2$. Here and in the sequel, whenever we refer to these derivatives at a boundary point, they are defined as the corresponding limit, which we assume to exist whenever we use them.

[14] We have ommitted the dependence of ζ^0 and ζ^1 and their derivatives on (x,y) to avoid notation cluttering.

The first term of the sum in (13) is always positive. It reflects how the own-effect of non-tested individuals' effort affects the curvature of V in response to variations of (x, y). The second and third terms reflect how the externality of non-tested individuals' effort affects the curvature of V through the probability of contagion of non-tested individuals and negative-tested individuals, respectively. In the discussion of our next result we will say more about what determines the sign of these terms and the corresponding economic interpretation.

In the absence of externalities (i.e., if $q_2 = q_{22} = 0$), individuals' and government's preferences would be aligned. As a consequence, the choices of vigilance effort by both type of individuals would be the same as if they were chosen by the government. Thus, if vigilance efforts were chosen to maximize the government's expected payoff, then V would be convex. Intuitively, full disclosure would be optimal in that scenario. More broadly, when the effect of externalities is tenuous, i.e., when the terms including q_2 and q_{22} on the right-hand side of (13) are relatively small, full revelation is the optimal information disclosure.

Example 1. *Consider $q(e, e^0) = (\alpha(1-e)^\rho + (1-\alpha)(1-e^0)^\rho)^{\frac{v}{\rho}}$ with $v > \rho \geq 1$. If $\alpha v \geq 1$, then for α close enough to 1 or large enough v, V is convex and thus, full disclosure is optimal.*[15]

The following corollary reinforces the intuition of Proposition 1. Although the presence of externalities is a necessary condition to break the convexity of the function V, it is not sufficient to conclude that the government must obfuscate information in the optimal policy. In fact, if the effect of externalities on the curvature of V make the function more convex than in their absence, then full disclosure would be optimal. This allows us to provide a simple sufficient condition for full disclosure that only requires the sum of the terms of the right-hand side of (13) that contain q_2 or q_{22} to be positive. The sign of this sum is determined by the difference between two magnitudes. The first one is given by the semi-elasticity of the *rate of change of the marginal effect from non-tested individuals' equilibrium effort* in the probability of passing the infection. The second one is the semi-elasticity of the marginal externality effect of non-tested individuals effort on the probability of passing the infection. In particular, let $\tilde{q}(e) \triangleq q_1(e, e)$ for all $e \in [0, 1]$; that is, $\tilde{q}(e)$ is the marginal effect from non-tested individuals' equilibrium effort in the probability of passing the infection. Also, let $\xi^{\tilde{q}_1}$ and $\xi^{q_2(e,\cdot)}$ be the semi-elasticities of $\tilde{q}_1$ and $q_2(e, \cdot)$:

$$\xi^{\tilde{q}_1}(e') \triangleq -\frac{d \ln \tilde{q}_1(e')}{de'} \quad \text{and} \quad \xi^{q_2(e,\cdot)}(e') \triangleq -\frac{d \ln q_2(e, e')}{de'},$$

for all $e, e' \in [0, 1]^2$.

[15] Detailed computations for this example are provided in the Supplementary Appendix.

Corollary 1. *Suppose Assumption 1 holds. If*

$$\xi^{\tilde{q}_1}(\zeta^0(x,y)) \geq \xi^{q_2(e,\cdot)}(\zeta^0(x,y)) \tag{14}$$

for $e = \zeta^0(x,y), \zeta^1(x,y)$ *in the interior of* $\{(x,y) \in X : \zeta^0(x,y) > 0\}$, *then full disclosure is optimal.*

We provide two interpretations for this corollary. First, if condition (14) holds for $e = \zeta^0(x,y)$ and $e = \zeta^1(x,y)$, then tested and non-tested individuals, respectively, solve a maximization problem that is convex in (x,y). Thus, by Danskin's Theorem, the value functions of both tested and non-tested individuals in (8) are convex functions of (x,y). The government's payoff function, as a convex combination of these two value functions, is then also a convex function of (x,y).

Second, in economics terms, the presence of externalities could lead to a non-convexity in V when $q_{22} > 0$, that is, whenever the effect of the externality in the probability of contagion exhibits decreasing returns. Nonetheless, this non-convexity is fully offset if the effort of non-tested individuals is convex enough as a function of exposure risk. The magnitude of the former effect is determined by $\xi^{q_2(e,\cdot)}$, whereas the latter is driven by $\xi^{\tilde{q}_1}$. Moreover, the larger $\xi^{\tilde{q}_1}$, the more convex $-q_1(\cdot,\cdot)$ is, increasing the marginal effort of non-tested individuals as a response to increments in exposure (see the left panel of Figure 2). The inequality in (14) requires the second effect to dominate.

Example 2. *Consider* $q(e, e_0) = (1-e)^\alpha (1-e_0)^\beta$ *with* $\alpha > 1$ *and* $\beta > 0$. *Direct computations reveal that* $\alpha + \beta \geq 1$ *implies (14). In addition, Assumption 1 holds. Thus, for this passage function, full revelation is the optimal information disclosure.*

4.2 Obfuscation

The following result provides a necessary condition for the optimality of full disclosure.

Lemma 3. *Assume strategic substitubility. Full disclosure is optimal only if, for* $k \in \mathbb{N}$ *such that* $k \geq 2$,

$$V\left(\sum_{i=1}^{k} \lambda_i (x_i, x_i^2)\right) \leq \sum_{i=1}^{k} \lambda_i V(x_i, x_i^2) \tag{15}$$

for all $(\lambda_i, x_i)_{i=1}^k \in [0,1]^{2k}$ *such that* $\sum_{i=1}^k \lambda_i = 1$.

If condition (15) does not hold, them the government is better-off by pooling intervals around the points (x_i, x_i^2) for $i = 1, \ldots, k$.

Example 3. *We revisit Example 1. Consider* $q(e, e^0) = (\alpha(1-e)^\rho + (1-\alpha)(1-e^0)^\rho)^{\frac{v}{\rho}}$ *with* $v = 2, \rho = 1$. *If* $\alpha v < 1$, *then* V *is not convex; indeed, as illustrated in Figure 3, for* $\alpha = 0.15$, $\gamma = 0.05$ *and* $t = 0.05$, *condition (15) does not hold.*[16] *Hence, full revelation is not optimal.*

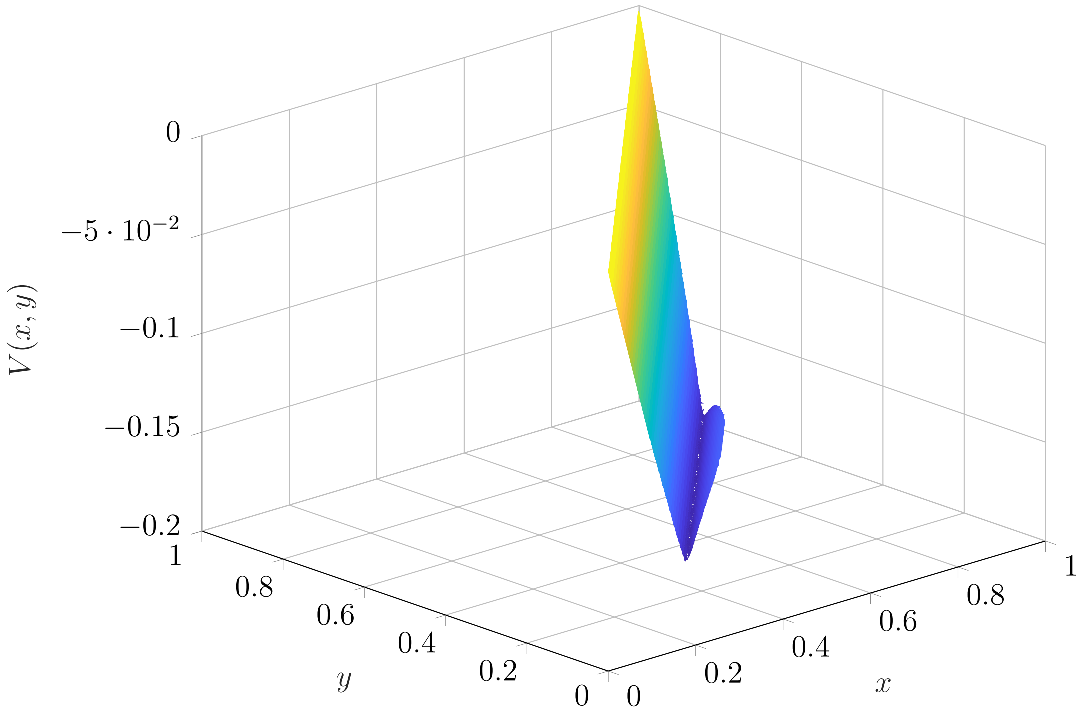


Figure 3: Government's payoff function under a CES passage function $q(e, e^0) = (\alpha(1-e)^\rho + (1-\alpha)(1-e^0)^\rho)^{\frac{v}{\rho}}$ with parameters $v = 2$, $\rho = 1$, $\alpha = 0.15$, $\gamma = 0.05$, and $t = 0.05$.

In particular, if V is concave over some regions of X, then the optimal information disclosure will involve some obfuscation. Our next result provides an optimal information disclosure for the case in which V is concave within the whole set of conditional posteriors that induce effort from non-tested individuals. Let $\tilde{x}(a, b) \triangleq \mathbb{E}_F[\omega|\omega \in [a, b]]$ and $\tilde{y}(a, b) \triangleq \mathbb{E}_F[\omega^2|\omega \in [a, b]]$ for all $0 \leq a \leq b \leq 1$. We will often omit the dependence of $\tilde{x}$ and $\tilde{y}$ on a and b. Also let $\underline{\omega}$ and $\overline{\omega}$ be the smallest and largest roots of $-\omega(1-\omega)(1-t)q_1(0, 0) - \gamma = 0$, respectively. In other words, under full disclosure, non-tested individuals exert a positive effort for all infection rates in $(\underline{\omega}, \overline{\omega})$. The result is based on the solution to the problem

$$\max_{0 \leq a \leq b \leq 1} \int_0^a V(\omega, \omega^2) dF(\omega) + (F(b) - F(a)) V(\tilde{x}(a, b), \tilde{y}(a, b)) + \int_b^1 V(\omega, \omega^2) dF(\omega). \tag{16}$$

[16] For instance, for $x_1 = 0.45$, $x_2 = 0.6$, and $\lambda = 0.5$, $V(\lambda(x_1, x_1^2) + (1-\lambda)(x_2, x_2^2)) - (\lambda V(x_1, x_1^2) + (1-\lambda) V(x_2, x_2^2)) = 1.99 \times 10^{-5}$.

This problem identifies an "intermediate" interval where infection rates are pooled, whereas outside of the interval, infection rates are disclosed. The solution of this problem provides an "educated guess" to design the optimal information disclosure.

Proposition 2. *Suppose that V is concave on $\{(x,y) \in X : \zeta^0(x,y) > 0\}$ and*

$$f(a)\left(V(\tilde{x},\tilde{y}) - V(a,a^2)\right) = (F(b) - F(a))\left(V_1(\tilde{x},\tilde{y})\tilde{x}_1 + V_2(\tilde{x},\tilde{y})\tilde{y}_1\right) \tag{17}$$

$$f(b)\left(V(\tilde{x},\tilde{y}) - V(b,b^2)\right) = -\left(F(b) - F(a)\right)\left(V_1(\tilde{x},\tilde{y})\tilde{x}_2 + V_2(\tilde{x},\tilde{y})\tilde{y}_2\right), \tag{18}$$

for some $a \in (0,\underline{\omega})$ and $b \in (\overline{\omega},1)$. Then, full revelation in $[0,1]\setminus(a,b)$ and pooling over (a,b) is an optimal information disclosure.

The F.O.C.'s of problem (16), which correspond to (17)-(18), have a natural interpretation. The left-hand sides correspond to the marginal increase in the expected payoff of the government from increasing the size of the pooling interval, whereas the the right-hand sides correspond to the marginal cost from stretching the pooling interval–for the optimal interval $[a,b]$, those magnitudes must coincide both at the low end and the high end. In the proof of Proposition 2, we verify the global optimality of this policy using Lemma 1.

Example 4. *We revisit Example 3. For $\alpha = 0.15$, $\gamma = 0.05$ and $t = 0.05$, we have that V is concave on $\{(x,y) \in X : \zeta^0(x,y) > 0\}$.*[17] *Furthermore, $\underline{\omega} = 0.23$ and $\overline{\omega} = 0.77$, and $(a,b) = (0.22, 0.78)$ solves (17)-(18). Thus, fully revealing the realization of ω over $[0,0.22]\cup[0.78,1]$ and pooling all other realizations is an optimal information disclosure.*

The kind of obfuscation described in Proposition 2 and Example 4 renders non-tested individuals' effort constant, at $\zeta^0(\tilde{x},\tilde{y})$, when it is positive. The concavity of V implies that the government prefers the outcome associated to this level of effort to the average of outcomes resulting when individuals are fully informed. Intuitively, this case is the flip-side of the one that occurs in Proposition 1 and Corollary 1. Here, the buffering effect on contagion of the higher vigilance effort of non-tested individuals, triggered by higher exposure risk, is dominated by the counteracting effect of the decreasing returns of externalities. Thus, contagion losses are convex as a function of (x,y), and hence, V is concave (over a subset of X).

This motivation for obfuscation is qualitatively different from the one arising under strategic complements, to be discussed in the next section. In that case, by pooling states the government simply stretches the range of infection rates for which individuals exert effort.[18] Finally,

[17] More generally, V is concave over $\left\{(x,y) \in X : \zeta^0(x,y) > 0\right\}$ as long as this set is contained in $\left\{(x,y) \in X : \zeta^1(x,y) = 1\right\}$ and $2(1-t)\alpha^2 - 2(1-\alpha)^2 < 0$.

[18] Incidentally, in the obfuscation described here for strategic substitutes, the interval for which non-tested individuals exert effort is larger than under full revelation as well.

a different type of obfuscation arises when V does not satisfy Assumption 1. In this case, V features kinks over all (x, y) such that $-z^0(1-t)q_1(1,1) = \gamma$; that is, over the set of points that separate the region of conditional moments where non-tested individuals make full effort. These kinks–and the obfuscation associated to them–can also arise in the case of strategic complements when complementarity between efforts is relatively low. We consider such a case in Section 5.2.

5 Strategic complements

In this section, we study the case when the vigilance game corresponds to a game of strategic complements.

Definition 4. *A passage function q exhibits strategic complements if $q_{12} < 0$ over $(0,1)^2$.*

From the theory of games with strategic complementarities (e.g., Vives, 1990, 2005), when q exhibits strategic complements, we can guarantee the existence of a largest, $\overline{\boldsymbol{\zeta}} = \langle \overline{\zeta}^0, \overline{\zeta}^1 \rangle$, and a smallest, $\underline{\boldsymbol{\zeta}} = \langle \underline{\zeta}^0, \underline{\zeta}^1 \rangle$, BNE (for details, see Section A.2 in the Supplementary Appendix).[19] Our analysis focuses on the largest BNE equilibria.[20]

Let $\gamma_I \triangleq -\frac{1}{4}(1-t)q_1(1,1)$. If $\gamma \geq \gamma_I$, under full disclosure, the cost of effort is strictly greater than benefit with probability 1 and for any choice of the other individuals. Thus, there is no obfuscation that yields a strictly positive probability that the conditional expected value of the benefit of effort is weakly greater than the cost. Therefore, any information structure results in a BNE involving $\zeta^0(x,y) = \zeta^1(x,y) = 0$ with probability 1. To rule out trivial cases, in the sequel we assume $\gamma < \gamma_I$.

5.1 Bang-bang equilibria

In spite of the multiplicity of equilibria, we can show that under mild conditions of the passage function, in the smallest equilibrium of the game, no individual exerts effort, whereas in the largest equilibrium, both types exert full effort if the exposure risk is sufficiently high.

[19] Furthermore, in this case, the payoff function of non-tested individuals is supermodular in (e, e^0) and z^0, and the payoff function for tested-negative individuals is supermodular in (e, e^0) and z^1. From Topkis' Monotonicity Theorem, it follows that in the smallest and largest equilibria, ζ^0 and ζ^1, are increasing in z^0 and z^1, respectively.

[20] While other equilibria could be handled as well, their analysis does not seem to add many insights beyond what we provide here. For instance, in many cases the smallest equilibrium involves no effort, in which case any information disclosure yields the same outcome.

Assumption 2 (Bang-bang)**.** *The passing function satisfies*

$$q_{11}(e,e) + q_{12}(e,e) < 0 \tag{19}$$

for all $e \in (0,1)$*,*

$$-(1-t)q_1(0,0) < \gamma, \tag{20}$$

and $-q_1(1,1) < \infty$*.*

The first part of this assumption implies that non-tested individuals' marginal benefit from effort is increasing in the equilibrium effort, as illustrated in Figure 4a. The second part of the assumption, namely, inequality (20), requires that, regardless of the level of exposure risk, when non-tested individuals do not make effort, negative-tested follow suit. Throughout the rest of this section, we assume that Assumption 2 holds; we will drop this assumption in Section 5.2.

Since $-q_1(1,1)$ is finite, there exists a positive level of exposure risk at which effort is not worthy, even if all other individuals are making full effort. Now, since we assume that $\gamma < \gamma_I$, a non-tested individual's marginal benefit from effort is greater than the marginal cost when the exposure risk is maximum, provided that the other non-tested individuals are making full effort. Thus, by continuity, there exists an exposure risk $\hat{z} \in (0, \frac{1}{4})$ that equalizes the marginal benefit and the marginal cost of effort when non-tested individuals make full effort. In particular, $\hat{z}$ is the root of $-z^0(1-t)q_1(1,1) = \gamma$ with respect to z^0. The following lemma follows immediately.

Lemma 4. *The smallest equilibrium is* $\underline{\boldsymbol{\zeta}}(x,y) = (0,0)$ *and the largest equilibrium is given by*

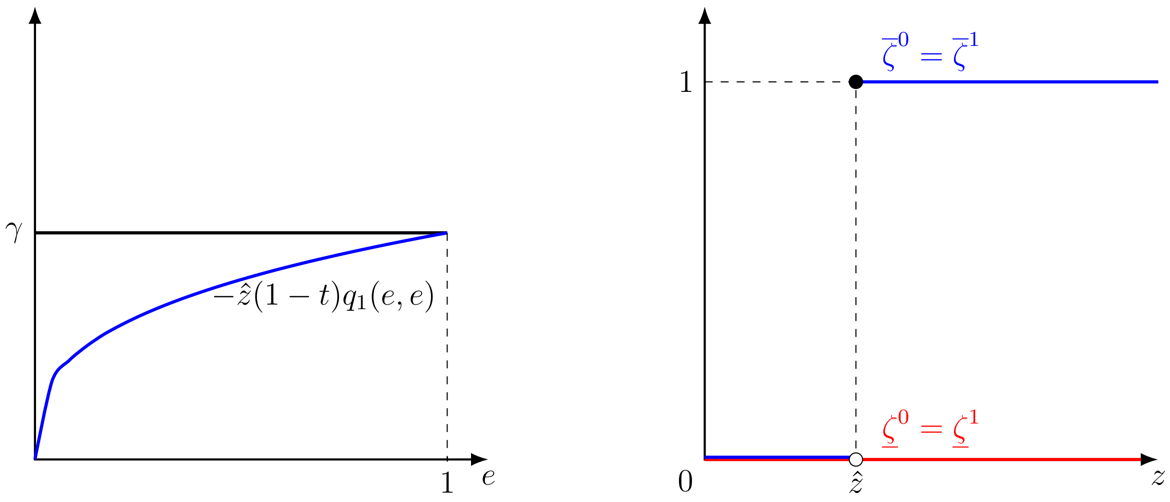
$$\overline{\boldsymbol{\zeta}}(x,y) = \begin{cases} (0,0) & \text{if} \quad z^0 < \hat{z} \\ (1,1) & \text{if} \quad z^0 \geq \hat{z}. \end{cases}$$

The largest BNE is bang-bang: there is a cut-off for non-tested individuals' exposure risk at which behavior jumps from inaction to full effort. Figure 4b shows the equilibrium effort in both the smallest and largest equilibria for Example 5 (provided below). In the sequel we assume that individuals adopt the largest BNE. Then, the payoff function of the government is given by

$$V(x,y) = \begin{cases} -(1-t)z^0 q(0,0) & \text{if } z^0 < \hat{z} \\ -(1-t)z^0 q(1,1) - (1-tx)\gamma & \text{if } z^0 \geq \hat{z}. \end{cases}$$

Under full disclosure individuals make a positive vigilance effort for all $\omega \in (\underline{\omega}_I, \overline{\omega}_I)$, where $\underline{\omega}_I$ and $\overline{\omega}_I$ are the roots of

$$-(1-t)\omega(1-\omega)q_1(1,1) = \gamma \tag{21}$$



(a) (b)

Figure 4: Representation of the Bang-bang equilibrium. Panel 4a shows the increasing marginal benefit of equilibrium effort. Here, $q(e, e^0) = 1 - (e^0 e)^{1/\eta}$, $\eta = 1.5$, $t = 0.05$, $\hat{z} = \frac{\gamma\eta}{1-t}$, and $\gamma = 0.1$. Panel 4b depicts the smallest and largest equilibria, in red and blue respectively, as a function of exposure risk.

with respect to ω. On the other hand, the government prefers that individuals exert full vigilance effort rather than no effort at all, for all $\omega \in (\underline{\omega}_G, \overline{\omega}_G)$, where $\underline{\omega}_G$ and $\overline{\omega}_G$ are the roots of

$$(1-t)\omega(1-\omega)(q(0,0) - q(1,1)) = \gamma(1 - t\omega). \tag{22}$$

This expression merely equalizes the social benefit of exerting full effort for both type of individuals to its cost. The next lemma shows that the government always prefers to induce full effort, rather than no effort at all, for a larger set of infection rates than the one corresponding to full effort under full revelation. Obfuscation allows the government to increase the alignment between its and individuals' preferences.

Lemma 5. *Under strategic complements, $[\underline{\omega}_I, \bar{\omega}_I] \subsetneq [\underline{\omega}_G, \bar{\omega}_G]$ and the government always benefits from obfuscation.*

The main result of this section characterizes the situations where the government may use obfuscation in order to align, either partially or completely (depending on effort's marginal cost), individuals effort to its preferences. The result is based on the solution of the problem

$$\begin{aligned}
\max_{\{0 \le a \le b \le 1\}} -(1-t)q(0,0)\int_0^a \omega(1-\omega)f(\omega)d\omega - \int_a^b \left((1-t)\omega(1-\omega)q(1,1) + \gamma\left(1 - t\omega\right)\right)f(\omega)d\omega \\
-(1-t)q(0,0)\int_b^1 \omega(1-\omega)f(\omega)d\omega \\
\text{s.t.} \quad -(1-t)q_1(1,1)\int_a^b \omega(1-\omega)f(\omega)d\omega \ge \gamma \int_a^b f(\omega)d\omega. \quad (23)
\end{aligned}$$

Let $\gamma_0 \triangleq q_1(1,1)(1-t)\mathbb{E}_F[\omega(1-\omega)]$; i.e., under full obfuscation, individuals may exert effort in equilibrium if and only if $\gamma < \gamma_0$.

Proposition 3. *Suppose that q exhibits strategic complements. The following information structure is optimal: the government discloses ω if $\omega \in [0, \underline{\omega}^*) \cup (\overline{\omega}^*, 1]$, and reveals only that $\omega \in [\underline{\omega}^*, \overline{\omega}^*]$ if $\omega \in [\underline{\omega}^*, \overline{\omega}^*]$, for some $\underline{\omega}^* \in [\underline{\omega}_G, \underline{\omega}_I)$ and $\overline{\omega}^* \in (\overline{\omega}_I, \overline{\omega}_G]$. Furthermore, there exist $\underline{\gamma} \leq \overline{\gamma} \in (\gamma_0, \gamma_I)$ such that*

i) for all $\gamma \in \left(0, \underline{\gamma}\right]$, $(\underline{\omega}^*, \overline{\omega}^*) = (\underline{\omega}_G, \overline{\omega}_G)$,

ii) for all $\gamma \in (\overline{\gamma}, \gamma_I)$, $\underline{\omega}^*$ and $\overline{\omega}^*$ are the solution to the following system of equations in a and b:

$$\frac{(1-t)a(1-a)(q(0,0)-q(1,1)) - \gamma(1-ta)}{-(1-t)a(1-a)q_1(1,1) - \gamma} = \frac{(1-t)b(1-b)(q(0,0)-q(1,1)) - \gamma(1-tb)}{-(1-t)b(1-b)q_1(1,1) - \gamma}, \tag{24}$$

$$-(1-t)q_1(1,1)\int_a^b \omega(1-\omega)f(\omega)d\omega = \gamma \int_a^b f(\omega)d\omega \tag{25}$$

and

iii) if $\gamma \in \left(\underline{\gamma}, \overline{\gamma}\right)$, then either $(\underline{\omega}^*, \overline{\omega}^*) = (\underline{\omega}_G, \overline{\omega}_G)$ or $(\underline{\omega}^*, \overline{\omega}^*)$ is the solution to (24)-(25).

In part i) of Proposition 3, the cost of effort is relatively low, allowing the government to induce individuals to exert effort whenever it prefers so. Conversely, in part ii), the constraint on the conditional expectation of the private marginal benefit from effort is binding. Here, the first-order conditions of the constrained maximization problem yield equalities (24)-(25). Condition (25) is the constraint on the conditional expectation of the private marginal benefit from effort, whenever the government discloses that $\omega \in [\underline{\omega}^*, \overline{\omega}^*]$. Condition (24), on the other hand, is a standard "bang-for-buck" condition. That is, the government's marginal gain from stretching the vigilance interval $[a, b]$ on the left end, divided by its effect on the non-tested individuals' vigilance constraint, must coincide with its counterpart on the right end.

Example 5. *Consider the passage function*

$$q(e, e^0) = 1 - (ee^0)^{1/\eta},$$

where $\eta \in (1, 2)$. We have that $q_1(0,0) = 0$, $q_1(1,1) = -\frac{1}{\eta}$, and

$$q_{11}(e,e) + q_{12}(e,e) = \frac{1}{\eta}\left(1 - \frac{2}{\eta}\right)e^{2(1/\eta - 1)} < 0$$

for all $e \in (0,1)$. Therefore, Assumption 2 holds and $\hat{z} = \frac{\eta\gamma}{1-t}$.

The smallest BNE is $\langle \underline{\zeta}^0(x,y), \underline{\zeta}^1(x,y)\rangle = (0,0)$ *for all* $(x,y) \in X$. *The largest BNE is shaped by the value of the parameters* η, t, *and* γ. *If* $z^0(1-t) \geq \gamma\eta$, *then* $\langle \overline{\zeta}^0(x,y), \overline{\zeta}^1(x,y)\rangle = (1,1)$; *otherwise,* $\langle \overline{\zeta}^0(x,y), \overline{\zeta}^1(x,y)\rangle = (0,0)$.[21]

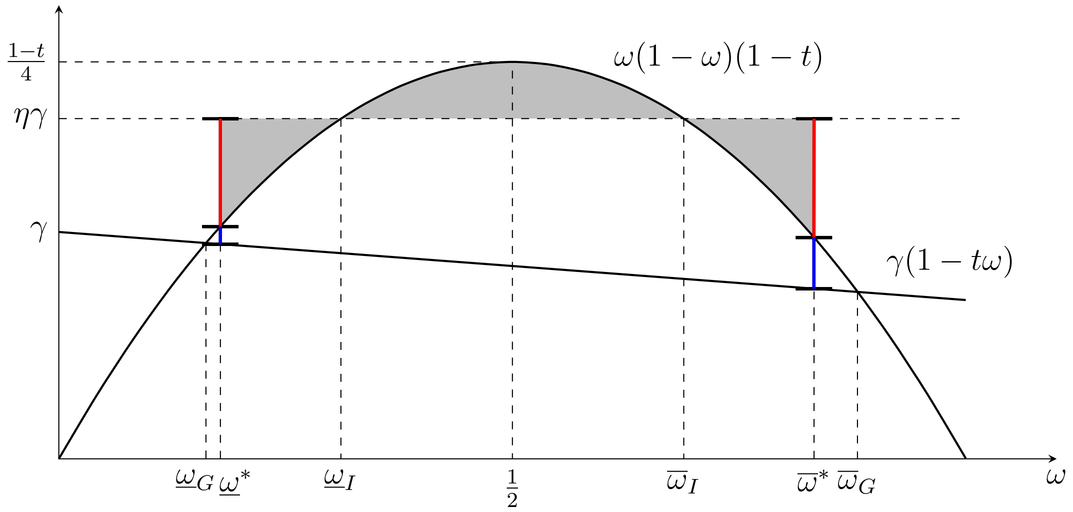


Figure 5: Government's information disclosure under the bang-bang equilibrium of Example 5 with $\gamma = 0.1$, $t = 0.3$, and $\eta = 1.5$.

Plugging the strategies of the largest BNE, the payoff function of the government is

$$V(x,y) = \begin{cases} -(1-t)z^0 & \text{if } (1-t)z^0 < \gamma\eta \\ -(1-tx)\gamma & \text{if } (1-t)z^0 \geq \gamma\eta. \end{cases} \tag{26}$$

Figure 5 illustrates the analysis of Proposition 3. Individuals prefer to exert effort for all $\omega \in [\underline{\omega}_I, \overline{\omega}_I]$, *where the parabola* $(1-t)\omega(1-\omega)$ *is above the marginal cost* γ *(re-scaled by* η*). Meanwhile, the government prefers to induce effort for all* $\omega \in [\underline{\omega}_G, \overline{\omega}_G]$, *where the parabola is above the line* $\gamma(1-t\omega)$. *However, for the parameter values in the example, restriction (23) is binding, and, thus, the government can only induce effort in the interval* $[\underline{\omega}^*, \overline{\omega}^*] \subset [\underline{\omega}_G, \overline{\omega}_G]$, *where* $\underline{\omega}^*$ *and* $\overline{\omega}^*$ *are the solution to the system of equations (24)-(25).*

Condition (24) can be represented by the equality of two analogous ratios evaluated at $\underline{\omega}^*$ *and* $\overline{\omega}^*$, *respectively. Each ratio is obtained by dividing two segments: (i) the first one given by the distance between the line* $\eta\gamma$ *and the parabola* $(1-t)\omega(1-\omega)$; *and (ii) the second one given by*

[21] There is a third BNE in which

$$\zeta^0(x,y) = \left[\frac{\gamma\eta}{z^0(1-t)}\right]^{\frac{\eta}{2-\eta}} \quad \text{and} \quad \zeta^1(x,y) = \left[\frac{z^1(1-t)}{\gamma\eta}\right]^{\frac{\eta-1}{\eta}} \left[\frac{z^0(1-t)}{\gamma\eta}\right]^{\frac{(2-\eta)(1-\eta)}{\eta}},$$

if $z^0(1-t) \geq \gamma\eta$, and $(\zeta^0(x,y), \zeta^1(x,y)) = (0,0)$, otherwise. Other equilibria can be constructed by using the strategies of this equilibrium, the smallest, or the largest equilibrium at each $(x,y) \in X$.

the distance between the parabola and the straight line $\gamma(1 - t\omega)$. (Both segments are highlighted in red and blue, respectively, in Figure 5.)

Furthermore, when F is a uniform distribution, condition (25) is equivalent to having the shaded area of the "dome" equal to the sum of of the shaded areas on the sides. The area of the dome represents the "incentive budget" of the government to pool infection rates realizations that induce effort. Hence, the optimal solution is attained when the government "spends" its budget enlarging the effort interval beyond $[\underline{\omega}_I, \overline{\omega}_I]$ while equalizing the marginal benefit at both extremes of the interval.

5.2 Low complementarity

When the degree of complementarity is relatively low, the analysis is similar to the one in the case of substitutes.

Assumption 3 (Low complementarity)**.** *The passing function satisfies*

$$q_{11}(e, e) + q_{12}(e, e) > 0 \tag{27}$$

for all $e \in (0, 1)$.

In particular, when Assumption 3 holds, the largest equilibrium efforts and the associated payoff function of the government, $V(\cdot, \cdot; \overline{\boldsymbol{\zeta}})$ are continuous over X. Figure 6 illustrates the equilibrium effort of non-tested individuals for the passage function of Example 5, but with $\eta = 4$, which satisfies Assumption 3.

We end this section with a result that provides an optimal obfuscation policy for this example. This result describes the optimal obfuscation in presence of kinks; i.e., when Assumption 1 does not hold. The result is based on the solution of the problem

$$\max_{\{0 \le a \le b \le 1\}} \int_0^a V(\omega, \omega^2) f(\omega) d\omega + (F(b) - F(a)) V(\tilde{x}(a, b), \tilde{y}(a, b)) + \int_b^1 V(\omega, \omega^2) f(\omega) d\omega$$
$$\text{s.t.} \quad -(1 - t) q_1(1, 1) \int_a^b \omega(1 - \omega) f(\omega) d\omega \ge \gamma \int_a^b f(\omega) d\omega. \tag{28}$$

Proposition 4. *Suppose Assumption 3 and equation (13) hold but Assumption 1 does not. Furthermore, assume*

$$\frac{V(\tilde{x}(a, b), \tilde{y}(a, b)) - V(a, a^2) - (F(b) - F(a)) (V_1(\tilde{x}, \tilde{y}) \tilde{x}_1 + V_2(\tilde{x}, \tilde{y}) \tilde{y}_1)}{V(\tilde{x}(a, b), \tilde{y}(a, b)) - V(b, b^2) + (F(b) - F(a)) (V_1(\tilde{x}, \tilde{y}) \tilde{x}_2 + V_2(\tilde{x}, \tilde{y}) \tilde{y}_2)} = \frac{f(a) ((1 - t) q_1(1, 1) a(1 - a) + \gamma)}{f(b) ((1 - t) q_1(1, 1) b(1 - b) + \gamma)}$$

and the restriction of problem (28) holds with equality, for some $a \in [0, \underline{\omega}_I]$ and $b \in [\overline{\omega}_I, 1]$. Then, full revelation in $[0, 1] \setminus (a, b)$ and pooling over $[a, b]$ is an optimal information disclosure.

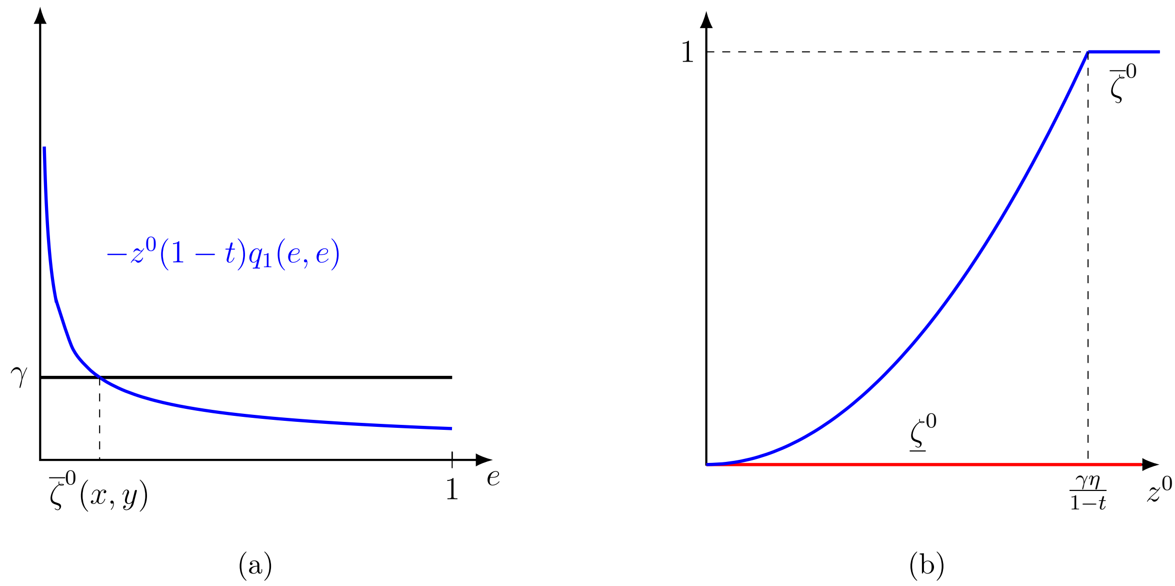


Figure 6: Equilibrium for $q(e,e^0) = 1-(ee^0)^{1/\eta}$, $t = 0.05$, $\eta = 4$, and $\gamma = 0.05$. Panel 6a depicts the equilibrium marginal benefit of effort for non-tested when $x - y = 0.16$. Panel 6b depicts the smallest and largest equilibria effort of non-tested individuals, in red and blue respectively, as a function of the exposure risk. For $z^0 < \frac{\gamma\eta}{1-t}$, $\overline{\zeta}^0$ is given by the expression for ζ^0 in footnote 21.

A pair (a, b) satisfying the hypotheses of Proposition 4 solves problem (28), and therefore, provides an optimal information disclosure among those that obfuscate only within an interval of moderate infection rates. The argument showing that such an obfuscation is optimal among all information disclosures is similar to that in the proof of Propositions 2 and 3, and it is omitted.

Example 6. *Consider the passage function* $q(e,e^0) = 1-(ee^0)^{1/\eta}$*, with* $t = 0.05$*,* $\eta = 4$*, and* $\gamma = 0.05$*. In this example, under full disclosure, non-tested individuals exert maximum effort for all* $\omega \in [\underline{\omega}_I, \overline{\omega}_I] = [0.30, 0.70]$*. The solution to problem (28) is* $(a, b) = (0.16, 0.84)$*. Thus, the optimal disclosure is pooling all infection rates in* $[0.16, 0.84]$ *and disclosing them in* $[0, 0.16) \cup (0.84, 1]$*. By pooling all infection rates in* $[0.16, 0.84]$*, the government enlarges the set of infection rates for which non-tested individuals exert maximum effort. Meanwhile, the effort exerted for the rest of the infection rates remains the same as under full disclosure.*

6 Extensions and applications

We discuss how our theoretical framework and results are useful to analyze similar applications in epidemiological, monetary, and financial contagion environments.

6.1 Epidemiological contagion: Vaccination

We adapt our baseline model for the case when the government has access to a stronger policy instrument, such as a vaccination, to manage contagion. In the vaccination setup, unlike in the testing model, sterilized individuals' losses are zero, because vaccinated individuals are assumed to receive full immunization (cf. Brito et al., 1991). Therefore, the only contagion event is when two non-vaccinated individuals meet and one of them is infected whereas the other is not.

Let $\nu \in [0, 1]$ be the fraction of vaccinated individuals. Following closely the same notation as before, the expected payoff function of a non-vaccinated individual exerting effort e when other non-vaccinated individuals exert effort e^0, and given conditional moments (x, y), is

$$U(e, e^0, x, y, \nu) = -z^0(1-\nu)q(e, e^0) - \gamma e. \tag{29}$$

An unvaccinated individual's strategy now depends on the vaccination rate, in addition to the conditional moments. That is, $\zeta : X \times [0, 1] \to [0, 1]$. In equilibrium,

$$\zeta(x, y, \nu) \in \arg\max_{e\in[0,1]} U(e, \zeta(x, y, \nu), x, y, \nu).$$

The assumption of full immunization makes the government's problem of choosing the optimal policy more tractable. In particular, here our analysis allows the government not only to select the optimal disclosure policy, but also the vaccination rate. As such, we now include vaccination costs in the payoff function of the government. Given an unvaccinated individuals' strategy ζ, the government's expected payoff is

$$\begin{aligned} V(x, y, \nu) &= -(1-\nu)\left[z^0(1-\nu)q(\zeta(x, y, \nu), \zeta(x, y, \nu)) + \gamma\zeta(x, y, \nu)\right] - C(\nu) \\ &= (1-\nu)U(e, e^0, x, y, \nu) - C(\nu), \end{aligned} \tag{30}$$

where the cost function $C : [0, 1] \to \mathbb{R}$ is increasing and convex.

By the Principle of Optimality, the government's problem is

$$\max_{\nu\in[0,1]} \left\{ \int_X V(x, y, \nu) dG^*_\nu \right\} \tag{31}$$

where

$$G^*_\nu \in \arg\max_{G\in\mathcal{G}} \int_X V(x, y, \nu) dG.$$

for $\nu \in [0, 1]$.

Substitutes In this setup, we obtain a simplified version of the sufficient condition for full revelation that arises when the passage function exhibits strategic substitutes.

Proposition 5. *Suppose that q exhibits strategic substitutes and satisfies Assumption 1. Then, for any $\nu \in [0,1]$, the optimal policy of the government is full disclosure if*

$$(q_{11}(\zeta,\zeta) - q_{22}(\zeta,\zeta))(q_{11}(\zeta,\zeta) + q_{22}(\zeta,\zeta)) + (q_{111}(\zeta,\zeta) + 2q_{112}(\zeta,\zeta) + q_{122}(\zeta,\zeta))q_2(\zeta,\zeta) \geq 0. \tag{32}$$

The interpretation of (32) is analogous to that of (13), but only with the terms corresponding to non-tested individuals.[22]

Under full-disclosure, provided that the government's payoff function is concave in ν and the solution of (31) is interior, the optimal level of vaccination ν^* is the root of[23]

$$\int_\Omega \left\{ z^0(1-\nu^*)\left[2q(\zeta(\nu^*),\zeta(\nu^*)) - (1-\nu^*)q_2(\zeta(\nu^*),\zeta(\nu^*))\zeta_3(\nu^*)\right] + \gamma\zeta(\nu^*) \right\} dF(\omega) = C_1(\nu^*). \tag{33}$$

In this setup, externalities still play an important role to determine the optimal vaccination rate. Higher vaccination rates decrease equilibrium effort because they reduce its private marginal benefit. Thus, externalities make the optimal vaccination rates to be smaller. However, in this scenario, there is no interaction between the optimal vaccination rate and the optimal disclosure policy, because the latter is full disclosure.

Example 7. *If $q(e,e^0) = (1-e)^\alpha(1-e^0)^\beta$ with $\alpha > 1$ and $\beta > 0$ the optimal policy of the government is full disclosure. Furthermore, if $1 < \alpha + \beta < 2$, then $V(x,y,\cdot)$ is concave for all $(x,y) \in X$, and, hence, the optimal level of vaccination is given by the root of (33).*

Complements If a suitably adjusted version of Assumption 2 is satisfied, the largest equilibrium under endogeneous vaccination is bang-bang. Furthermore, the optimal information disclosure policy is analogous to the one in Proposition 3 (replacing t by ν). The only difference is that the numerator in (24) includes the term γ, instead of $\gamma(1-\nu a)$. The reason is that the losses associated to vaccinated individuals, other than the vaccination costs $C(\nu)$, are zero, as they cannot get infected or exert vigilance. This modification renders a tractable direct comparative statics exercise that permits to inspect how the optimal level of obfuscation varies with the vaccination rate.

Remark 2. *If q exhibits strategic complements and Assumption 2 is satisfied, then, for any vaccination rates $\nu' > \nu$, the obfuscation bounds satisfy $\underline{\omega}^*(\nu) \leq \underline{\omega}^*(\nu')$ and $\overline{\omega}^*(\nu) \geq \overline{\omega}^*(\nu')$.*[24]

[22] Condition (32) also looks slightly different from (13) because in (32) we have already substituted the expressions for the derivatives of effort with respect to the conditional expected moments (x, y).

[23] In order to avoid notation cluttering, we have omitted the dependence on (x, y) of ζ and its derivatives.

[24] Here condition (20) is not needed as vaccinated individuals play no role in the vaccination setup.

Remark 2, which follows immediately from the modified version of equations (24)-(25), states that it is optimal for the government to obfuscate less when the level of vaccination increases. Intuitively, as the mass of vaccinated individuals increases, both the private and social benefit of unvaccinated individuals' effort decreases; therefore, the optimal interval of infection rates for which the government induces effort shrinks.

6.2 Monetary contagion: Counterfeiting money

Money counterfeiting is a widespread criminal activity that generates millions in losses every year.[25] When counterfeiting increases, individuals' trust in notes gets eroded, compromising the fundamental principle that permits to sustain a positive value of fiat money. In extreme cases, counterfeiting can cause the same problems to trade as those caused by hyperinflation.

To avoid the losses of counterfeiting, both the police and good citizens can engage in vigilant activities to decrease the probability with which a fake note is accepted in a transaction. When the police discovers a fake note, it seizes it and puts it out of circulation. In turn, when a good citizen discovers a fake note, she rejects it upfront. Most of the literature focuses on the "seizuring game" played by the police and counterfeiters, avoiding to analyze the role of the transmission of fake notes among good citizens, who may do this unknowingly. A remarkable exception is Quercioli and Smith (2015), who formalize a "vigilance game" played by good citizens, but still restrict the role of the police to a verifier that aims to confiscate fake notes.

Our framework can be used to analyze information disclosure in decentralized environments susceptible to counterfeiting. We assume that there is a continuum of counterfeiters with measure ω and a continuum of good citizens with measure $1 - \omega$, where $\omega \in [0, 1]$ is a random variable drawn according to a full-support and non-atomic distribution F. Each bad individual can produce exactly one fake note. Matching is random and pairwise, and no agent can tell whether a transactor is a counterfeiter or a good agent. Each period a good citizen handling a note can be matched exogenously with a random transactor.

Good citizens can invest in vigilance effort e that permits to verify if a note is fake in a transaction. In turn, counterfeiters can invest in the quality c to obfuscate the verification of fake notes. The passage function of fake note is then given by $q(e, c)$.

The police inspects individuals randomly, using a technology (e.g. cyber-security protocols) that perfectly identifies the type of each individual, i.e., whether they are good citizens or coun-

[25] According to the data published by Quercioli and Smith (2015), which belongs to the Secret Service, in the period 1995-2004, $5,594,062$ fake notes were passed and $8,541,972$ were seized in the United States. These amounts reflect the magnitude of the problem, since they represent only a lower bound.

terfeiters. When the police discovers a counterfeiter, it puts her in jail. Furthermore, the police inspects an exogenous mass $t \in (0, 1)$ of individuals and learns the mass of counterfeiters in the population. Therefore, if the police is benevolent, and aims to minimize the social losses associated to counterfeiting, it can use an optimal information disclosure about ω to persuade good agents to exert effort e and discourage counterfeiters effort; much in the same spirit of our benchmark model.

6.3 Financial contagion: Toxic assets

Our model is also suitable to analyze information design problems in economies under financial contagion. The financial crisis of 2008 provides an example. The crisis was triggered by the crash of the subprime mortgage market and the subsequent bankruptcy of Lehman Brothers (see Financial-Crisis-Inquiry-Commission (2011)). The sharp declines in currency, commodity, and equity values rapidly transmitted to Europe, Asia, and Latin America.[26]

Because the crisis started when investors traded mortgage-backed securities comprised by many "toxic" home loans that were not repayable, we can model it by using our baseline model. In particular, we can assume that there is a unit mass of buyers and sellers that meet randomly in a decentralized market. Each seller holds an asset. The fraction of "toxic" assets in the population ω is private information drawn from $[0, 1]$ according to a distribution F.[27]

Sellers can invest in the sophistication s used to construct their synthetic mortgage backed securities. At the same time, buyers can invest in their vigilance efforts e through the quality controls conducted by their back offices (e.g., stress testing, value at risk, etc). The passage function corresponds to $q(e, s)$. Both types of agents maximize profits.

A regulator can inspect a mass $t > 0$ of assets using stress testing. If a seller is tested and the regulator finds that the asset is toxic, the seller loses the face value of its asset, which is fixed at 1. Hence, if the regulator is interested in maximizing the social welfare, it has to choose the optimal information design to disuade sellers to obscure the quality of their assets and persuade buyers to increase their vigilance. We can then apply our theoretical framework to analyze how the optimal information disclosure depends on the shape of the passage function q.

[26]The fundamental channels of contagion have been extensively studied in the literature (cf. Schmukler et al., 2003), and, more recently, there have been attempts to study the informational channels using global games in experimental settings (Trevino, 2020). However, to our knowledge, a proper information design analysis in decentralized financial contagion problems has not been formally laid out.

[27]Our framework uses a global game to model the contagion problem in the spirit of Dasgupta (2004).

7 Concluding remarks

Our paper provides a framework to study optimal information design in descentralized economies that are susceptible to contagion. In our setup, (i) the government takes an action to learn the state of the world, which affects the amount of information individuals have; (ii) the vigilance effort exerted by one type of individuals (the non-tested) generate a positive externality to the rest of the population; and (iii) the vigilance efforts of matched individuals can exhibit complementarities or substitutabilities. We provide conditions under which the optimal information policy is full disclosure or involves obfuscation. We also determine the optimal information structures that arise under obfuscation for broad sets of conditions.

For the case of substitutes, the presence of externalities is a necessary condition to break the convexity of the government's payoff function, and, as a consequence, the optimality of full disclosure. Nonetheless, externalities are not sufficient for obfuscation. In spite of the preference misalignment between a benevolent government and descentralized decision making, there is no role for obfuscation when the government's payoff is a convex function of the conditional first two moments of the underlying random variable of the model—i.e., the initial infection rate.

We also provide a sufficient condition for the government's payoff function to be convex in terms of the semi-elasticicities of: (i) the curvature of the private benefit obtained from non-tested individuals's equilibrium effort; and (ii) the externality in the passage function. When the former dominates the latter, an increase in exposure risk causes increments in effort to produce a greater effect than the one produced by decrements in the marginal benefit of the externality. Such a dominance guarantees convexity.

On the other hand, if the semi-elasticity of the externalities dominates, then the payoff function of the government can be concave over a set of posteriors. Pooling that set within a larger interval of initial contagion rates allows the government to reduce the variations in effort. This yields a greater ex-ante expected payoff for the government due to the concavity of the payoff function. The higher effort allowed by obfuscation in pooled states with relatively low infection rates more than compensates the lower effort in pooled states with relatively high infection rates, due to the fast decay in the effect of the externality.

Our analysis of the case of complements focuses on bang-bang equilibria, in which, as the initial infection rate varies, individuals' efforts jump from no-effort at all to full-effort. The optimal policy under the bang-bang equilibrium involves pooling of intermediate initial infection rates, for which exposure risk is sufficiently high, stretching the set of infection rates for which individuals make effort.[28]

We outline alternate applications for which the insights of our model may be useful, including the analysis of counterfeiting money and financial contagion. Within the context of an epidemic, we also consider the case when the government has access to a vaccine, which is a stronger instrument to contain a contagion. Vaccines offer full immunization, and, thus, the decisions of non-vaccinated individuals are the only ones that matter in this setup. The resulting simplified environment is more tractable, allowing for the analysis of the optimal level of vaccination for specific functional forms of the passage function.

Finally, there are interesting directions in which our analysis can be extended. First, further analysis could consider inaccurate tests. In such a setup, the equilibrium of the vigilance game cannot be obtained sequentially, as tested individuals can also be infected. For the case of substitutes, a natural conjecture is that the presence of cross-externalities reinforces the use of obfuscation as the optimal policy, since concavity of the government's payoff is likely to arise more often. Another natural extension is considering a dynamic version of our model, in which both individuals and the government live infinitely. The further insights that might be captured in the analysis of these extensions would contribute to a more complete understanding of optimal information design for contagion problems.

References

Alonso, R. and Câmara, O. (2016). Persuading voters. *American Economic Review*, 106(11):3590–3605.

Alvarez, F., Argente, D., and Lippi, F. (2021). A simple planning problem for covid-19 lock-down, testing, and tracing. *American Economic Review: Insights*, 3(3):367–82.

Bardhi, A. and Guo, Y. (2018). Modes of persuasion toward unanimous consent. *Theoretical Economics*, 13(3):1111–1149.

Bergemann, D. and Morris, S. (2019). Information design: A unified perspective. *Journal of Economic Literature*, 57:44–95.

Boleslavsky, R. and Cotton, C. (2015). Grading standards and education quality. *American Economic Journal: Microeconomics*, 7(2):248–279.

[28] Exposure risk is not monotone in the infection rate. Because contagion involves matching of an infected and a susceptible individual, it is maximised when the initial infection rate is 0.5.

Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge University press.

Brito, D. L., Sheshinski, E., and Intriligator, M. D. (1991). Externalities and compulsory vaccinations. *Journal of Public Economics*, 45(1):69–90.

Dasgupta, A. (2004). Financial contagion through capital connections: A model of the origin and spread of bank panics. *Journal of the European Economic Association*, 2(6):1049–1084.

de Vericourt, F., Gurkan, H., and Wang, S. (2021). Informing the public about a pandemic. *Management Science*.

Duffie, D., Dworczak, P., and Zhu, H. (2017). Benchmarks in search markets. *The Journal of Finance*, 72(5):1983–2044.

Dworczak, P. and Kolotilin, A. (2019). The persuasion duality. *arXiv preprint arXiv:1910.11392*.

Dworczak, P. and Martini, G. (2019). The simple economics of optimal persuasion. *Journal of Political Economy*, 127(5):1993–2048.

Ely, J., Galeotti, A., and Steiner, J. (2020). Rotation as contagion mitigation.

Financial-Crisis-Inquiry-Commission (2011). *The financial crisis inquiry report: The final report of the National Commission on the causes of the financial and economic crisis in the United States*. Cosimo, Inc.

Galperti, S. and Perego, J. (2018). A dual perspective on information design.

Gans, J. S. (2020). Test sensitivity for infection versus infectiousness of sars-cov-2. Technical report, National Bureau of Economic Research.

Heinsalu, S. (2021). Infection arbitrage.

Inostroza, N. and Pavan, A. (2021). Persuasion in global games with applications to stress testing.

Kamenica, E. (2019). Bayesian persuasion and information design. *Annual Review of Economics*, 11:249–272.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.

Keppo, J., Kudlyack, M., Quercioli, E., Smith, L., and Wilson, A. (2021). The behavioral sir model, with applications to the swine flu and covid-19 pandemics.

Kosterina, S. (2020). Persuasion with unknown beliefs.

Li, F., Song, Y., and Zhao, M. (2021). Global manipulation by local obfuscation.

Mathevet, L., Perego, J., and Taneva, I. (2020). On information design in games. *Journal of Political Economy*, 128(4):1370–1404.

Müller, A. and Stoyan, D. (2002). *Comparison methods for stochastic models and risks*, volume 389. Wiley.

Ostrovsky, M. and Michael, S. (2010). Information disclosure and unraveling in matching markets. *American Economic Journal: Microeconomics*, 57:34–63.

Quercioli, E. and Smith, L. (2015). The economics of counterfeiting. *Econometrica*, 83(3):1211–1236.

Rolnick, A. J., Velde, F. R., and Weber, W. E. (1996). The debasement puzzle: an essay on medieval monetary history. *The Journal of Economic History*, 56(4):789–808.

Schmukler, S. L., Zoido, P., and Halac, M. (2003). Financial globalization, crises, and contagion. *Globalization World Bank Policy Research Report*, pages 1–30.

Schweizer, N. and Szech, N. (2018). Optimal revelation of life-changing information. *Management Science*, 64(11):5250–5262.

Sun, Y. (2006). The exact law of large numbers via fubini extension and characterization of insurable risks. *Journal of Economic Theory*, 126(1):31–69.

Taneva, I. (2019). Information design. *American Economic Journal: Microeconomics*, 11(4):151–85.

Trevino, I. (2020). Informational channels of financial contagion. *Econometrica*, 88(1):297–335.

Vives, X. (1990). Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, 19(3):305–321.

Vives, X. (2005). Complementarities and games: New developments. *Journal of Economic Literature*, 43(2):437–479.

Whitmeyer, M. (2021). An imperfect test for a virus can be worse than no test at all. *Health Economics*, 30(6):1347–1360.

World-Health-Organization (2018). *Managing epidemics: key facts about major deadly diseases.* Published by the World Health Organization.

Appendices

A Proofs

A.1 Proof of Lemma 1

For any distribution of conditional moments $G' \in \mathcal{G}$,

$$\begin{aligned}
\int_X V(x,y) - P(x,y)dG(x,y) &\geq \int_X V(x,y) - P(x,y)dG'(x,y) \\
\int_X V(x,y)dG(x,y) - \int_X V(x,y)dG'(x,y) &\geq \int_X P(x,y)dG(x,y) - \int_X P(x,y)dG'(x,y) \\
&= \int_\Omega P(\omega,\omega^2)dF(\omega) - \int_X P(x,y)dG'(x,y) \\
&\geq \int_\Omega P(\omega,\omega^2)dF(\omega) - \int_\Omega P(\omega,\omega^2)dF(\omega).
\end{aligned}$$

The first inequality follows from the second equality in (11) and $P \geq V$, which, respectively, guarantee that the left hand side is zero and the righthand side is negative. The equality follows from the first equality in (11). The last inequality follows from the fact that $-P$ is concave and that $(x,y) \leq_{cx} (\omega,\omega^2)$ for all $\tau' \in \mathcal{T}$ (for details see part (a) of Theorem 3.4.2 in Müller and Stoyan (2002)).

A.2 Proof of Proposition 1

Given a BNE $\boldsymbol{\zeta}$, let $\mathrm{U}^0(x,y) \triangleq U^0(\zeta^0(x,y),\zeta^0(x,y),x,y)$ for all $(x,y) \in X$. Also, for each $e \in [0,1]$, define the function $\mathrm{V}^e : X \to \mathbb{R}$, by

$$\mathrm{V}^e(x,y) = (1-t)\mathrm{U}^0(x,y) + t\left(-(1-t)(x-y)q(e,\zeta^0(x,y)) - \gamma(1-x)\cdot e\right)$$

for all $(x,y) \in X$.

Claim 1. *Suppose the hypotheses of Proposition 1 hold. Then,* V^e *is convex for all* $e \in [0,1]$.

Proof. First, define

$$X^0 \triangleq \{(x,y) \in X : \zeta^0(x,y) = 0\}$$
$$X^{>0} \triangleq \{(x,y) \in X : \zeta^0(x,y) > 0\}$$
$$X^1 \triangleq \{(x,y) \in X : \zeta^0(x,y) = 1\}.$$

Empty X^1. Consider first the case in which the inequality in (12) holds. Thus, X^1 at most contains only one point, $(\frac{1}{2}, \frac{1}{4})$.

First, V^e is affine, and therefore convex, on X^0. Second, V^e_{11} and V^e_{22} both equal the left hand side of (13) times $(1-t)$, which is assumed positive, and $\mathrm{V}^e_{12} = \mathrm{V}^e_{21} = -\mathrm{V}^e_{11}$. Therefore, $|H^{\mathrm{V}^e}| = 0$ and V^e is convex on $X^{>0}$.

Thus, if $X^0 = \varnothing$ or if $X^{>0} = \varnothing$, then V^e is convex. The same conclusion follows immediately if X^0 or $X^{>0}$ have an empty interior.

From here on, suppose that both X^0 and $X^{>0}$ have a non-empty interior. Let D be the set of pairs of points $w = ((x,y),(x',y')) \in X^2$ such that $(x,y) = (x',y')$. Also, define $l(s;w) \triangleq s(x,y) + (1-s)(x',y')$, with $s \in [0,1]$, be the points on the line through arbitrary points (x,y) and (x',y') for $w = ((x,y),(x',y')) \in X^2 \setminus D$. Define, as well, $f(s;w) \triangleq \mathrm{V}^e(l(s;w))$ for all $s \in [0,1]$ and $w \in X^2 \setminus D$. Then, V^e is convex if and only if $f(\cdot;w)$ is convex for all $w \in X^2 \setminus D$.

Define

$$S^0(w) \triangleq \{s \in [0,1] : \zeta^0(l(s;w)) = 0\}$$
$$S^{>0}(w) \triangleq \{s \in [0,1] : \zeta^0(l(s;w)) > 0\},$$

for all $w \in X^2 \setminus D$.

Pick an arbitrary $w \in X^2 \setminus D$ and assume, without loss, that $0 \in S^0(w)$ and that $1 \in S^{>0}(w)$. This implies $x - y > x' - y'$. Let $\hat{s}$ be the value of s such that $l(\hat{s};w) \in S^0(w)$, for $s \le \hat{s}$, and $l(s;w) \in S^{>0}(w)$, for $s > \hat{s}$. The convexity of V^e over X^0 and $X^{>0}$ guarantees that $f(\cdot;w)$ is convex on $[0,\hat{s}]$ and $[\hat{s},1]$, respectively. Thus, V^e is convex if $\lim_{s\uparrow\hat{s}} f_1(s;w) \le \lim_{s\downarrow\hat{s}} f_1(s;w)$ for all $w \in X^2 \setminus D$ such that $0 \in S^0(w)$ and $1 \in S^{>0}(w)$. For all such w, we have

$$\begin{aligned}\lim_{s\uparrow\hat{s}} f_1(s;w) =& \lim_{s\uparrow\hat{s}} \mathrm{V}^e_1(l(s;w))(x-x') + \lim_{s\uparrow\hat{s}} \mathrm{V}^e_2(l(s;w))(y-y') \\ =& -(1-t)^2 q(0,0)(x-x'-(y-y')) - t(1-t)q(e,0)(x-x'-(y-y')) + t\gamma(x-x')e\end{aligned}$$

and

$$\lim_{s\downarrow\hat{s}} f_1(s,w) = -(1-t)^2\left[q(0,0)+(\hat{s}(x-y)+(1-\hat{s})(x'-y'))q_2(0,0)\zeta_1^0(l(\hat{s};w))\right](x-x'-(y-y'))$$
$$-t(1-t)\left[q(1,0)+(\hat{s}(x-y)+(1-\hat{s})(x'-y'))q_2(e,0)\zeta_1^0(l(\hat{s};w))\right](x-x'-(y-y'))$$
$$+t\gamma(x-x')e.$$

Therefore,

$$\lim_{s\downarrow\hat{s}} f_1(s;w)-\lim_{s\uparrow\hat{s}} f_1(s;w) = -(1-t)^2\left[(\hat{s}(x-y)+(1-\hat{s})(x'-y'))q_2(0,0)\zeta_1^0(l(\hat{s};w))\right](x-x'-(y-y'))$$
$$-t(1-t)\left[(\hat{s}(x-y)+(1-\hat{s})(x'-y'))q_2(e,0)\zeta_1^0(l(\hat{s};w))\right](x-x'-(y-y')),$$

which is positive because $x-x'>y-y'$.

Non-empty X^1. Now consider the case in which the inequality in (12) does not hold. Thus, the interior of X^1 is non-empty. The function V^e is affine, and therefore convex, over X^1.

Analogously to what we did above, define

$$S^1(w) \triangleq \{s\in[0,1]:\zeta^0(l(s;w))=1\}$$

for all $w\in X^2\setminus D$.

Without loss, consider $w\in X^2\setminus D$ such that $1\in S^1(w)$ and $0\in[0,1]\setminus S^1(w)$; that is $\zeta^0(x',y')<1$ and $\zeta^0(x,y)=1$, respectively. Let $\widetilde{s}$ be the value of s such that $l(s;w)\in S^1(w)$ for $s\geq\widetilde{s}$ and $l(s;w)\in[0,1]\setminus S^1(w)$ for $s<\widetilde{s}$. Thus, V^e is convex if $\lim_{s\uparrow\widetilde{s}} f_1(s;w)\leq\lim_{s\downarrow\widetilde{s}} f_1(s;w)$, for all $w\in X^2\setminus D$ such that $0\in[0,1]\setminus S^1(w)$ and $1\in S^1(w)$. For all such w, we have

$$\lim_{s\downarrow\widetilde{s}} f_1(s;w) = \lim_{s\downarrow\widetilde{s}} \mathrm{V}_1^e(x-x')+\lim_{s\downarrow\widetilde{s}} \mathrm{V}_2^e(y-y')$$
$$= -(1-t)^2q(1,1)(x-x'-(y-y'))-t(1-t)q(e,1)(x-x'-(y-y'))+t\gamma(x-x')e$$

and

$$\lim_{s\uparrow\widetilde{s}} f_1(s;w) = -(1-t)^2\left[q(1,1)+(\widetilde{s}(x-y)+(1-\widetilde{s})(x'-y'))q_2(1,1)\zeta_1^0(l(\widetilde{s};w))\right](x-x'-(y-y'))$$
$$-t(1-t)\left[q(e,1)+(\widetilde{s}(x-y)+(1-\widetilde{s})(x'-y'))q_2(e,1)\zeta_1^0(l(\widetilde{s};w))\right](x-x'-(y-y')),$$
$$+t\gamma(x-x')e.$$

Therefore,

$$\begin{aligned}\lim_{s\downarrow\tilde{s}} f_1(s;w) - \lim_{s\uparrow\tilde{s}} f_1(s;w) =& (1-t)^2(\tilde{s}(x-y)+(1-\tilde{s})(x'-y'))q_2(1,1)\zeta_1^0(l(\tilde{s};w))(x-x'-(y-y')) \\ &+ t(1-t)(\tilde{s}(x-y)+(1-\tilde{s})(x'-y'))q_2(e,1)\zeta_1^0(l(\tilde{s};w))(x-x'-(y-y')) \\ =&(1-t)(\tilde{s}(x-y)+(1-\tilde{s})(x'-y'))((1-t)q_2(1,1)+tq_2(e,1))\zeta_1^0(l(\tilde{s};w))\times \\ &(x-x'-(y-y')) \\ \leq& 0,\end{aligned}$$

where the last inequality follows from $x-y>x'-y'$. Thus, the convexity of V^e is implied by $q_2(\cdot,1)\zeta_1^0(l(\tilde{s};w))=0$; that is,[29]

$$\frac{q_2(\cdot,1)}{q_{11}(1,1)+q_{12}(1,1)}=0.$$

■

To conclude the argument, notice that

$$V(x,y)=\max_{e\in[0,1]}\mathrm{V}^e(x,y),$$

for all $(x,y)\in X$. Thus, V is the maximum (with respect to e) of functions that are convex in the parameters (x,y). Then, Danskin's theorem (see, e.g., ch. 3.2.3 in Boyd and Vandenberghe (2004)) gives that V is convex.

A.3 Proof of Corollary 1

Straightforward calculations reveal that for $e=\zeta^0(x,y),\zeta^1(x,y)$ in the interior of $\{(x,y)\in X:\zeta^0(x,y)>0\}$, we have

$$\begin{aligned}&\zeta^0(-z^0q_{22}(e,\zeta^0)(\zeta_1^0)^2-z^0q_2(e,\zeta^0)(2\zeta_1^0+\zeta_{11}^0)) \\ =\ & -\zeta^0\left[-\frac{q_{111}(\zeta^0,\zeta^0)+2q_{121}(\zeta^0,\zeta^0)+q_{122}(\zeta^0,\zeta^0)}{q_{11}(\zeta^0,\zeta^0)+q_{12}(\zeta^0,\zeta^0)}+\frac{q_{22}(e,\zeta^0)}{q_2(e,\zeta^0)}\right]\frac{q_1(\zeta^0,\zeta^0)^2q_2(e,\zeta^0)}{(q_{11}(\zeta^0,\zeta^0)+q_{12}(\zeta^0,\zeta^0))^2} \\ =\ & -\left[\xi^{\tilde{q}_1}(\zeta^0(x,y))-\xi^{q_2(e,\cdot)}(\zeta^0(x,y))\right]\frac{q_1(\zeta^0,\zeta^0)^2q_2(e,\zeta^0)}{(q_{11}(\zeta^0,\zeta^0)+q_{12}(\zeta^0,\zeta^0))^2}.\end{aligned}$$

Therefore, if $\xi^{\tilde{q}_1}(\zeta^0(x,y))\geq\xi^{q_2(e,\cdot)}(\zeta^0(x,y))$ for $e=\zeta^0(x,y),\zeta^1(x,y)$, we obtain

$$-z^0q_{22}(e,\zeta^0)(\zeta_1^0)^2-z^0q_2(e,\zeta^0)(2\zeta_1^0+\zeta_{11}^0)\geq 0.$$

Thus, (13) holds and full disclosure is optimal.

[29] Recall that $\zeta^0(l(\tilde{s};w))=1$ implies $q_1(\zeta^0(l(\tilde{s};w)),\zeta^0(l(\tilde{s};w)))>0$.

A.4 Proof of Lemma 3

We provide the argument for $k = 2$; the extension to $k > 2$ is direct. Suppose

$$V(\lambda_1(x_1, x_1^2) + \lambda_2(x_2, x_2^2)) > \lambda_1 V(x_1, x_1^2) + \lambda_2 V(x_2, x_2^2)$$

for some $\lambda_1 \in (0, 1)$, $\lambda_2 = 1 - \lambda_1$ and $0 \leq x_1 < x_2 \leq 1$.

Since strategic substitubility guarantees that V is continuous, there exists $\varepsilon > 0$ such that

$$V(\lambda_1(x_1+\varepsilon_1, (x_1+\varepsilon_1)^2)+\lambda_2(x_2-\varepsilon_2, (x_2-\varepsilon_2)^2)) > \lambda_1 V(x_1+\varepsilon_1, (x_1+\varepsilon_1)^2)+\lambda_2 V(x_2-\varepsilon_2, (x_2-\varepsilon_2)^2)$$

for all $\varepsilon_1, \varepsilon_2 \in (0, \varepsilon)$.

If the government pools $(x_1, x_1+\varepsilon_1)$ and $(x_2-\varepsilon_2, x_2)$, such that the ratio of the pooled masses is λ_1/λ_2, then the expected payoff of the contracted distribution is greater than the expected payoff under full disclosure.

A.5 Proof of Proposition 2

Let $h : X \to \mathbb{R}$ be the projection over X of the hyperplane containing $(a, a^2, V(a, a^2))$, $(b, b^2, V(b, b^2))$, and $(\tilde{x}, \tilde{y}, V(\tilde{x}, \tilde{y}))$. Let $P \triangleq \max\{V, h\}$.

By the argument in the proof of Proposition 1, V is convex over X^0. Furthermore, the points $(a, a^2, V(a, a^2))$, $(b, b^2, V(b, b^2))$, and $(\tilde{x}, \tilde{y}, V(\tilde{x}, \tilde{y}))$ are in the graph of h; thus,

$$\begin{aligned} \alpha + \beta a + \delta a^2 &= V(a, a^2) \\ \alpha + \beta b + \delta b^2 &= V(b, b^2) \\ \alpha + \beta \tilde{x} + \delta \tilde{y} &= V(\tilde{x}, \tilde{y}), \end{aligned}$$

for some real numbers α, β, and δ. Subtracting the first two equations from the last we obtain, respectively,

$$\begin{aligned} \beta(\tilde{x} - a) + \delta(\tilde{y} - a^2) &= V(\tilde{x}, \tilde{y}) - V(a, a^2) \\ \beta(\tilde{x} - b) + \delta(\tilde{y} - b^2) &= V(\tilde{x}, \tilde{y}) - V(b, b^2). \end{aligned}$$

Then, from the hypothesis of the result,

$$\begin{aligned} \beta(\tilde{x} - a) + \delta(\tilde{y} - a^2) &= \frac{(F(b) - F(a))\,(V_1(\tilde{x}, \tilde{y})\tilde{x}_1 + V_2(\tilde{x}, \tilde{y})\tilde{y}_1)}{f(a)} \\ \beta(\tilde{x} - b) + \delta(\tilde{y} - b^2) &= -\frac{(F(b) - F(a))\,(V_1(\tilde{x}, \tilde{y})\tilde{x}_2 + V_2(\tilde{x}, \tilde{y})\tilde{y}_2)}{f(b)}. \end{aligned}$$

Using the facts that

$$\frac{(F(b)-F(a))\tilde{x}_1}{f(a)} = \tilde{x} - a \quad \text{and} \quad \frac{(F(b)-F(a))\tilde{y}_1}{f(a)} = \tilde{y} - a^2,$$

$$\frac{-(F(b)-F(a))\tilde{x}_2}{f(b)} = \tilde{x} - b \quad \text{and} \quad \frac{-(F(b)-F(a))\tilde{y}_2}{f(b)} = \tilde{y} - b^2,$$

we equivalently have

$$\begin{aligned}
\beta(\tilde{x}-a)+\delta(\tilde{y}-a^2) &= V_1(\tilde{x},\tilde{y})(\tilde{x}-a)+V_2(\tilde{x},\tilde{y})(\tilde{y}-a^2)\\
\beta(\tilde{x}-b)+\delta(\tilde{y}-b^2) &= V_1(\tilde{x},\tilde{y})(\tilde{x}-b)+V_2(\tilde{x},\tilde{y})(\tilde{y}-b^2).
\end{aligned}$$

Thus, $\beta = V_1(\tilde{x},\tilde{y})$ and $\delta = V_2(\tilde{x},\tilde{y})$. Since h is tangent to V at $(\tilde{x},\tilde{y})$ and V is concave over $X^{>0}$, we have that $h > V$ on $X^{>0}$, and thus, $P = h$ on $X^{>0}$.

Let $\widetilde{V} : X \to \mathbb{R}$ be the payoff of the government if non-tested individuals are constrained to exert no effort. That is, $\widetilde{V} \triangleq V$ over X^0, and over $X^{>0}$, $\widetilde{V}$ is given by the right hand side of (9), replacing, mutatis mutandis, $\zeta^0(x,y)$ by 0, both in (9) itself and in the definition of $\zeta^1(x,y)$ in (8).

We use the argument in the proof of Proposition 1, to show that $\widetilde{V}$ is convex. In particular, for each $e \in [0,1]$, define the function $\widetilde{\mathrm{V}}^e : X \to \mathbb{R}$, by

$$\widetilde{\mathrm{V}}^e(x,y) = (1-t)U^0(0,0,x,y) + t\left(-(1-t)(x-y)q(e,0) - \gamma(1-x)\cdot e\right)$$

for all $(x,y) \in X$. Since $\widetilde{\mathrm{V}}^e$ is affine, it is convex for all $e \in [0,1]$. And since $\widetilde{\mathrm{V}} = \max_{e\in[0,1]} \widetilde{\mathrm{V}}^e$, we have that $\widetilde{\mathrm{V}}$ is convex, by Danskin's Therorem.

Furthermore, since the government's payoff is increasing in the effort of non-tested individuals over $[0,\zeta^0(x,y)]$, for all $(x,y) \in \{(x,y) \in X : \zeta^0(x,y) > 0\}$, we have that $\widetilde{V} \leq V$.[30] Thus, $P = \max\left\{\widetilde{V}, h\right\}$. Since $\widetilde{V}$ and h are convex, P is convex as well.

Additionally, the first equality in (11) holds here as well. Let G be the distribution of posteriors associated to the information disclosure described in the Proposition. Then,

$$\begin{aligned}
\int_X P(x,y)dG(x,y) =& \alpha + \int_0^a \beta\omega + \delta\omega^2 dF(\omega) + \left(\beta\tilde{x}(a,b)+\delta\tilde{y}(a,b)\right)(F(b)-F(a)) + \int_b^1 \beta\omega + \delta\omega^2 dF(\omega)\\
=& \int_\Omega \alpha + \beta\omega + \delta\omega^2 dF(\omega)\\
=& \int_\Omega P(\omega,\omega^2)dF(\omega).
\end{aligned}$$

Finally, the support of the distribution of posteriors is the set of Dirac delta distributions defined at each of the elements of

$$\{(x,x^2) \in X : x \in [0,a] \cup [b,1]\} \cup \{(\tilde{x},\tilde{y})\} \tag{A.1}$$

[30] The marginal effect of non-tested individuals' effort on the government payoff is bounded below by $-(1-t)^2 z^0(q_1(\cdot,\cdot)+q_2(\cdot,\cdot)) - (1-t)\gamma$, which is positive over $[0,\zeta^0(x,y)]$ for all $(x,y) \in \left\{(x,y) \in X : \zeta^0(x,y) > 0\right\}$.

and $P = V$ over this set; hence, the second equality in (11) holds. Therefore, Lemma 1 implies the result.

A.6 Proof of Lemma 5 and Proposition 3

We first prove Lemma 5. We start showing that $[\underline{\omega}_I, \bar{\omega}_I] \subsetneq [\underline{\omega}_G, \bar{\omega}_G]$. This follows from the following observation.

Claim 2. *We have that* $q(1,1) - q(0,0) < q_1(1,1)$.

Proof. The mean value theorem implies that there is $c \in (0,1)$ such that

$$q_1(c,1) = q(1,1) - q(0,1).$$

As $q_{11} > 0$, we have that $q_1(c,1) < q_1(1,1)$. Therefore, we can write

$$q(1,1) - q(0,0) < q(1,1) - q(0,1) < q_1(1,1),$$

where the first inequality follows because q is decreasing in its second argument. ■

The strict inclusion $[\underline{\omega}_I, \bar{\omega}_I] \subsetneq [\underline{\omega}_G, \bar{\omega}_G]$ follows because we can rewrite the equations that define $(\underline{\omega}_I, \bar{\omega}_I)$ and $(\underline{\omega}_G, \bar{\omega}_G)$ as

$$(1-t)\omega(1-\omega) = \frac{\gamma}{-q_1(1,1)}$$
$$(1-t)\omega(1-\omega) = \frac{\gamma(1-t\omega)}{q(0,0) - q(1,1)},$$

respectively, and the right hand side of the first equality is larger than that of the second one for all $\omega \in [0,1]$ whenever $q(1,1) - q(0,0) < q_1(1,1)$, as shown in Claim 2.

Now we prove that the government always benefits from obfuscation. Notice that

$$-\omega(1-\omega)(1-t)q_1(1,1) > (<)\gamma$$

for all $\omega \in (\underline{\omega}_I, \overline{\omega}_I)$ ($\omega \in [0, \underline{\omega}_I) \cup (\overline{\omega}_I, 1]$). Thus, there exists $\varepsilon \in (0, \min\{\underline{\omega}_I - \underline{\omega}_G, \overline{\omega}_G - \overline{\omega}_I\})$ such that

$$-\mathbb{E}_F\left[\omega(1-\omega)|\omega \in [\underline{\omega}_I - \varepsilon, \overline{\omega}_I + \varepsilon]\right](1-t)q_1(1,1) > \gamma.$$

Consider the information structure that reveals ω when $\omega \in [0, \underline{\omega}_I - \varepsilon) \cup (\overline{\omega}_I + \varepsilon, 1]$ and reveals only that $\omega \in [\underline{\omega}_I - \varepsilon, \overline{\omega}_I + \varepsilon]$, otherwise. The government prefers this information structure to full disclosure. This completes the proof of Lemma 5.

Now we prove Proposition 3. Consider arbitrary $\gamma \in (0, \gamma_I)$. We postulate that the optimal information structure involves choosing an interval $[\underline{\omega}^*, \overline{\omega}^*] \subseteq [\underline{\omega}_G, \overline{\omega}_G]$ with the disclosure policy described in Proposition 3. We find the optimal information structure within this class and subsequently we show that such an information structure is optimal globally.

Full alignment of behavior to the Government preferences First we observe that if

$$-(1-t)\mathbb{E}_F[\omega(1-\omega)|\omega\in[\underline{\omega}_G,\overline{\omega}_G]]q_1(1,1)\geq\gamma, \tag{A.2}$$

then by setting $(\underline{\omega}^*,\overline{\omega}^*)=(\underline{\omega}_G,\overline{\omega}_G)$ the Government attains its preferred amount of effort in a bang-bang BNE: $e^0=e^1=1$ if $\omega\in[\underline{\omega}_G,\overline{\omega}_G]$ and $e^0=e^1=0$ if $\omega\in[0,\underline{\omega}_G)\cup(\overline{\omega}_G,1]$. So the government cannot make any better. In particular, for all $\gamma\in(0,\gamma_0]$, (A.2) holds with strict inequality. Thus, by continuity, there exists $\underline{\gamma}>\gamma_0$ such that for all $\gamma\leq\underline{\gamma}$, the government can induce its preferred amount of effort, among those that can be attained in a BNE, by disclosing ω when $\omega\in[0,\underline{\omega}_G)\cup(\overline{\omega}_G,1]$ and pooling infection rates $\omega\in[\underline{\omega}_G,\overline{\omega}_G]$. This proves part i) of the proposition.

Optimal interval obfuscation Alternatively, if (A.2) does not hold, then the government faces the problem (23) and the restriction is binding. Thus, the solution of the problem satisfies (24)-(25). Now we use Lemma 1 to show that this information disclosure is optimal.

Optimality of interval obfuscation First, V is discontinuous at all (x,y) such that $x-y=\hat{z}$ (where $\hat{z}$ is the exposure risk at which individuals effort jumps from inaction to full effort). Let h_λ be a family of hyperplanes, parameterized by $\lambda\in\mathbb{R}$, such that $V(x,y)$ coincides with $h_\lambda(x,y)$ at all such discontinuity points:

$$h_\lambda(x,y)=-(1-t)(x-y)q(1,1)-\gamma(1-tx)\gamma+\lambda\left(x-y-\hat{z}\right)$$

for all $(x,y)\in X$.

Then, let $\underline{\omega}^*$ and $\overline{\omega}^*$ be the the solution of (24)-(25). Notice that $\underline{\omega}^*<\underline{\omega}_I$ and $\overline{\omega}^*>\overline{\omega}_I$. We choose $\lambda=\lambda^*$ such that $V(x,y)=h_{\lambda^*}(x,y)$ at $(x,y)=(\underline{\omega}^*,\underline{\omega}^{*2})$. That is,

$$-(1-t)\underline{\omega}^*(1-\underline{\omega}^*)q(0,0)=-(1-t)\underline{\omega}^*(1-\underline{\omega})q(1,1)-\gamma(1-t\underline{\omega}^*)+\lambda^*\left(\underline{\omega}^*(1-\underline{\omega}^*)-\hat{z}\right) \tag{A.3}$$

$$\lambda^*=\frac{-(1-t)\underline{\omega}^*(1-\underline{\omega}^*)(q(0,0)-q(1,1))+\gamma(1-t\underline{\omega}^*)}{\underline{\omega}^*(1-\underline{\omega}^*)-\hat{z}}. \tag{A.4}$$

Since $(1-t)\omega(1-\omega)(q(0,0)-q(1,1))-\gamma(1-t\omega)$ is a concave function of ω, with roots $\underline{\omega}_G$ and $\overline{\omega}_G$, and $\underline{\omega}_G<\underline{\omega}^*<\overline{\omega}_G$, we have that $\lambda^*>0$.

From (24), (A.3) holds, mutatis mutandis, replacing $\underline{\omega}^*$ with $\overline{\omega}^*$. Thus, $h_{\lambda^*}(x,y)=V(x,y)$ at $(x,y)=(\overline{\omega}^*,\overline{\omega}^{*2})$.

Let

$$P(x,y)=\max\{V(x,y),h_{\lambda^*}(x,y)\}$$

for all $(x, y) \in X$. Thus, P is convex and satisfies $P \geq V$. The first equality in (11) follows because

$$\begin{aligned}\int_X P(x,y)dG(x,y) &= \int_0^{\underline{\omega}^*} P(\omega,\omega^2)dF + P(\mathbb{E}_F[\omega|\omega \in [\underline{\omega}^*,\overline{\omega}^*]], \mathbb{E}_F[\omega^2|\omega \in [\underline{\omega}^*,\overline{\omega}^*]]) \\ &\quad \times (F(\overline{\omega}^*) - F(\underline{\omega}^*)) + \int_{\overline{\omega}^*}^1 P(\omega,\omega^2)dF \\ &= \int_\Omega P(\omega,\omega^2)dF.\end{aligned}$$

The last equality follows because $P(x, y)$ coincides with $h_\lambda(x, y)$ in the convex hull of $\{(x, x^2) \in X : x \in (\underline{\omega}^*, \overline{\omega}^*)\}$ and, thus, P is affine in that region.

Finally, the support of the distribution of posterior beliefs associated to the information structure described in the proposition is the set of Dirac delta distributions defined at each of the elements of

$$\{(x,x^2) \in X : x \in [0,\underline{\omega}^*] \cup [\underline{\omega}^*,1]\} \cup \{(\mathbb{E}_F[\omega|\omega \in [\underline{\omega}^*,\overline{\omega}^*]], \mathbb{E}_F[\omega^2|\omega \in [\underline{\omega}^*,\overline{\omega}^*]])\} \tag{A.5}$$

and $P = V$ over this set; hence, the second equality in (11) holds.

Partial alignment of behavior to the Government preferences Since

$$-(1-t)\mathbb{E}_F[\omega(1-\omega)|\omega \in [\underline{\omega}_G, \overline{\omega}_G]]q_1(1,1) < \gamma_I,$$

and by continuity, there exists $\overline{\gamma} < \gamma_I$ such that for all $\gamma > \overline{\gamma}$, in any BNE, the government cannot induce its preferred amount of effort in every state, and solution of the government problem is as described in the proposition, with a and b solving (24)-(25). This proves part ii) of the proposition.

Finally, for $\gamma \in (\underline{\gamma}, \overline{\gamma})$, the government solves problem (23) and the solution of the problem is given by (24)-(25) when the restriction of the problem is binding, or $a = \underline{\omega}$ and $b = \overline{\omega}$, otherwise. This proves part iii) of the proposition.

For Online Publication: Supplementary Appendix

Contagion Management through Information Disclosure

by Jonas Hedlund, Allan Hernández-Chanto, and Carlos Oyarzún.

This supplement comprises an appendix that includes proofs of some technical propositions and lemmas in the main paper, and detailed versions of some of the examples.

A Omitted technical results

A.1 Existence of equilibrium

Lemma 6. *There exists a Bayesian Nash Equilibrium* $\boldsymbol{\zeta} = \langle \zeta^0, \zeta^1 \rangle$.

Proof. From the Theorem of the Maximum,

$$\varphi(e^0, x, y) \triangleq \arg\max_{e\in[0,1]} U^0(e, e^0, x, y)$$

defines a continuous function $\varphi(\cdot, x, y) : [0, 1] \to [0, 1]$ for each $(x, y) \in X$. Therefore, Brouwer's fixed point theorem guarantees that $\varphi(\cdot, x, y)$ has a fixed point at each $(x, y) \in X$. That is, there exists $\zeta^0(x, y)$ such that $\zeta^0(x, y) = \varphi(\zeta^0(x, y), x, y)$.

The existence of the equilibrium effort $\zeta^1(x, y)$ follows directly from Weierstrass' Extreme Value Theorem. ■

A.2 Existence of largest and smallest equilibria in games with strategic complementarities

Let $\varphi(e^0, z^0)$ be the best response of a non-tested individual when all other non-tested individuals exert effort $e^0 \in [0, 1]$ and the exposure risk is $z^0 \in [0, \frac{1}{4}]$. That is,

$$\varphi(e^0, z^0) \triangleq \arg\max_{e\in[0,1]} \widetilde{U}^0(e, e^0, z^0),$$

where $\widetilde{U}^0(e, e^0, z^0) \triangleq U^0(e, e^0, x, y)$ and $z^0 = x - y$ for all $(x, y) \in X$.

Lemma 7. *If q exhibits strategic complements, $\varphi(\cdot, z^0)$ has a largest and a smallest fixed point at each $z^0 \in [0, \frac{1}{4}]$.*

Proof. The function $\varphi(\cdot, z^0) : [0,1] \to [0,1]$ is increasing by an application of the Topkis' Monotonicity Theorem, given the supermodularity assumption, i.e., given that $q_{12} < 0$.

Define $\overline{E}(z^0) \triangleq \{e^0 \in [0,1] : \varphi(e^0, z^0) \geq e^0\}$ and let $e^*(z^0)$ be its supremum, for all $z^0 \in [0, \frac{1}{4}]$. Since $\varphi(0, z^0) \geq 0$, we have that $\overline{E}(z^0)$ is non-empty for all $z^0 \in [0, \frac{1}{4}]$. For any $e^0 \in \overline{E}(z^0)$,

$$e^0 \leq \varphi(e^0, z^0) \leq \varphi(e^*(z^0), z^0),$$

since $e^0 \leq e^*(z^0)$ and $\varphi(\cdot, z^0)$ is increasing. Furthermore, $\varphi(e^*(z^0), z^0) \geq e^*(z^0)$ because $e^*(z^0) \in \overline{E}(z^0)$ since $\overline{E}(z^0)$ is a compact subset of $\mathbb{R}$ and $\varphi(\cdot, z^0)$ is continuous. Hence, since $\varphi(\cdot, z^0)$ is increasing, $\varphi(\varphi(e^*(z^0), z^0), z^0) \geq \varphi(e^*(z^0), z^0)$, and thus, $\varphi(e^*(z^0), z^0) \in \overline{E}(z^0)$. But, then, because $e^*(z^0)$ is the supremum of $\overline{E}(z^0)$, $\varphi(e^*(z^0), z^0) \leq e^*(z^0)$. Therefore, $\varphi(e^*(z^0), z^0) = e^*(z^0)$. Because $\overline{E}(z^0)$ contains all the fixed points of $\varphi(\cdot, z^0)$, $e^*(z^0)$ is its largest fixed point.

The analogous argument proves that

$$e_*(z^0) \triangleq \inf\{e^0 \in [0,1] : \varphi(e^0, z^0) \leq e^0\}$$

is the smallest fixed point of $\varphi(\cdot, z^0)$ for each $z^0 \in [0, \frac{1}{4}]$. ■

The strategies of non-tested individuals in the largest and smallest BNE correspond, respectively, to the largest and smallest fixed points of $\varphi(\cdot, z^0)$, which we have shown to exist in Lemma 7, for each $(x, y) \in X$. The corresponding equilibrium strategies for negative-tested individuals are given by the solution to the decision problem in (8) for $\ell = 1$.

A.3 Optimal vaccination policy

A.3.1 Proof of Proposition 5

In the vaccination setup, the government's expected payoff depends on (x, y) only through $z^0 = x - y$, and, thus, abusing notation, we write its payoff function as $V(z^0; \zeta, \nu)$. Likewise, the equilibrium strategy of non-vaccinated individuals will be denoted by $\zeta(z^0, \nu)$. In what follows, we omit the dependence of functions on ζ and ν for simplicity in the exposition.

First, notice that

$$z^0 \zeta_1(z^0) = -\frac{q_1}{q_{11} + q_{12}}$$

and

$$\begin{aligned}
\frac{d}{dz} z^0 \zeta_1(z^0) &= \zeta_1(z^0) + z\zeta_{11}(z^0) \\
&= -\frac{(q_{11} + q_{12})\zeta_1(z^0)(q_{11} + q_{12}) - (q_{111} + q_{112} + q_{121} + q_{122})\zeta_1(z^0) q_1}{(q_{11} + q_{12})^2} \\
&= -\zeta_1(z^0) + \frac{(q_{111} + 2q_{112} + q_{122})\zeta_1(z^0) q_1}{(q_{11} + q_{12})^2}.
\end{aligned}$$

The third line collects similar terms and applies the Young theorem on q_1 to conclude that $q_{112} = q_{121}$.

Now, computing the second derivative of government's payoff function we have

$$\begin{aligned}
\frac{V_{11}(z^0)}{(1-\nu)^2} &= -(q_1+q_2)\zeta_1(z^0) - q_2\zeta_1(z^0) - z^0(q_{12}+q_{22})\zeta_1(z^0)^2 - z^0 q_2\zeta_{11}(z^0) \\
&= z^0(q_{11}+q_{12})\zeta_1(z^0)^2 - z^0(q_{12}+q_{22})\zeta_1(z^0)^2 - q_2(2\zeta_1(z^0) + z^0\zeta_{11}(z^0)) \\
&= z^0(q_{11}-q_{22})\zeta_1(z^0)^2 - \frac{q_{111}+2q_{112}+q_{122}}{(q_{11}+q_{12})^2} q_1 q_2 \zeta_1(z^0) \\
&= \frac{(q_{11}-q_{22})(q_{11}+q_{12}) + (q_{111}+2q_{112}+q_{122})q_2}{z^0(q_{11}+q_{12})^3} q_1^2.
\end{aligned}$$

The first line is straightforward differentiation of V_1. The second line plugs in the formula for $\zeta_1(z^0)$ in the term $-q_1\zeta_1(z^0)$ and reorganizes. In turn, the third line reorganizes and plugs in the formula for $2\zeta_1(z^0) + \zeta_{11}(z^0)$ derived above. Finally, the fourth line puts everything in terms of the primitives. Therefore, the convexity of V, and, thus, the optimality of full disclosure, is obtained if the hypothesis of the proposition are satisfied.

A.3.2 Proof of Example 7

Direct computations reveal that $-q_{22}(q_{11}+q_{12}) + (q_{111}+2q_{112}+q_{122})q_2 \geq 0$, which implies $V_{11}(z^0) \geq 0$ (see proof of Proposition 5), is equivalent to $\alpha + \beta \geq 1$.

A.4 Detailed examples

A.4.1 CES passage function

Consider $q(e, e_0) = (\alpha(1-e)^\rho + (1-\alpha)(1-e_0)^\rho)^{\frac{v}{\rho}}$ with $v > \rho \geq 1$. The convexity of the government's payoff function is satisfied if and only if for all e and for all e^0 in the image of $\zeta^0(x, y)$ we have

$$\Psi(e, e^0) \triangleq \alpha(v-\rho-1)(1-e)^\rho + (\alpha-1)(1-e^0)^\rho \geq 0.$$

This condition never holds for all e and for all e^0 in the image of $\zeta^0(x, y)$, since $\Psi(1, e^0) = (\alpha-1)(1-e^0)^\rho$, which is negative whenever $e^0 < 1$. Therefore, we instead examine $\Psi(e, e^0)$ around the equilibrium values for e and e^0.[31]

First, notice that in equilibrium we have $\zeta^1 \geq \zeta^0$, by Remark 1. Thus, if $v - \rho \leq 1$, then $\Psi(e, e^0) < 0$ for all relevant values (e, e^0). Likewise, if $v - \rho > 1$, then $\Psi(e, e^0) < 0$ for all relevant

[31] The results we get differentiating the value function imply that there is no loss in looking at $\Psi(e, e^0)$ around the equilibrium values.

values of (e, e^0) whenever $\alpha(v-\rho) < 1$. This is obtained because maximizing $\Psi(e, e^0)$, given $e \geq e^0$, requires $e = e^0$ and

$$\Psi(e, e) = (\alpha(v-\rho) - 1)(1-e).$$

Hence, if $\alpha(v-\rho) < 1$, the value function of tested-negative individuals, denoted by $\mathrm{U}^1(x, y)$, is not convex.[32] We cannot conclude the same about the value function of the non-tested individuals, $\mathrm{U}^0(x, y)$, because the Danskin's condition is not tight in their case. However, routine calculations reveal that $\mathrm{U}^0_{11}(x, y) < 0$ if $v\alpha < 1$, which is a stronger condition than $\alpha(v-\rho) < 1$.

In light of the above, we can conclude that if $v\alpha < 1$, then neither the value function of non-tested individuals, $\mathrm{U}^0(x, y)$, nor the value function of the tested individuals, $\mathrm{U}^1(x, y)$, are convex functions. Therefore, the government's payoff function is not a convex function. Optimal disclosure should then involve obfuscation.[33]

We now turn to find sufficient conditions for the convexity of the value function of the tested individuals $\mathrm{U}^1(x, y)$. A neat sufficient condition is hard to obtain and we resign to obtaining some limit results, illustrating that convexity is possible.

The objective function of the tested is

$$U^1(e, e^0, x, y) = \begin{cases} -z^1 q(e, e^0) - \gamma e & \text{if} \quad x \neq 1 \\ -x q(e, e^0) - \gamma e & \text{if} \quad x = 1, \end{cases}$$

for all $(x, y) \in X$ and $e^0 \in [0, 1]$.

The corresponding F.O.C is given by

$$\begin{cases} (x-y)\alpha v \left(\alpha(1-\hat{e})^\rho + (1-\alpha)(1-e^0)^\rho\right)^{\frac{v-\rho}{\rho}} (1-\hat{e})^{\rho-1} = (1-x)\gamma & \text{if} \quad y \neq x^2 \\ x\alpha v \left(\alpha(1-\hat{e})^\rho + (1-\alpha)(1-e^0)^\rho\right)^{\frac{v-\rho}{\rho}} (1-\hat{e})^{\rho-1} = \gamma & \text{if} \quad y = x^2 \end{cases}$$

If follows that $\hat{e} < 1$ for all $(x, y) \in X$.

Let $e^* = \max_{(x,y)\in X} \hat{e}$ be the maximal equilibrium effort of tested individuals across all values of (x, y), and let (x^*, y^*) be its maximizer. Because e^0 is a function only of $z^0 = x - y$, it follows that $\hat{e} > 0$ strictly increases in the value of x along the line $\{(x, y) \in X : z^0 = x - y = k,\ k > 0\}$. Thus, at the point $(x, y) \in X$ that maximizes the value of x along such a line, we must have $y = x^2$. This means that we can focus on the portion of the first order condition that has $y = x^2$. It is then clear that $(x^*, y^*) = (1, 1)$, since at this point we have $e^0 = 0$ and we are looking at strategic substitutes.

[32]This follows from the results derived for the Hessian of $\mathrm{U}^1(x, y)$, which illustrate that the Danskin sufficient condition is tight for tested individuals.

[33]Having a small α means that the externality is large as others' behavior has a large effect, whereas v is harder to interpret.

The maximal equilibrium e^* is defined by

$$\alpha v\left(\alpha(1-e^*)^\rho + (1-\alpha)\right)^{\frac{v-\rho}{\rho}}(1-e^*)^{\rho-1} = \gamma.$$

Hence, e^* is increasing in α and $\lim_{\alpha\to 1} e^* = 1 - \left(\frac{\gamma}{v}\right)^{\frac{1}{v-1}}$.

Furthermore, if $v - \rho > 1$, we obtain

$$\begin{aligned}\Psi(e, e^0) &= \alpha(v-\rho-1)(1-e)^\rho - (1-\alpha)(1-e^0)^\rho \\ &\geq \alpha(v-\rho-1)\left(\frac{\gamma}{v}\right)^{\frac{\rho}{v-1}} - (1-\alpha).\end{aligned}$$

Therefore, if

$$\alpha\left[1 + (v-\rho-1)\left(\frac{\gamma}{v}\right)^{\frac{\rho}{v-1}}\right] \geq 1,$$

then $\Psi(e, e^0) \geq 0$ for all equilibrium values of (e, e^0).

One can conduct a similar analysis for v. The crucial observation here is that e^* does not approach 1 as v approaches infinity. Suppose (v^n, e^n) is a sequence with $v^n = n$ and $e^n \to 1$. Then, we get that

$$\lim_{n\to\infty} \alpha v^n\left(\alpha(1-e^n)^\rho + (1-\alpha)\right)^{\frac{v^n-\rho}{\rho}}(1-e^n)^{\rho-1} = \lim_{n\to\infty} \alpha v^n (1-\alpha)^{\frac{v^n-\rho}{\rho}}(1-e^n)^{\rho-1} = 0.$$

This result follows because the derivative of $v(1-\alpha)^{(v-\rho)/\rho}$ with respect to v becomes negative in the limit. As a result, e^* is bounded away from 1 as v approaches infinity. We can conclude that if v is sufficiently large, then $\Psi(e, e^0)$ is positive for all equilibrium values of (e, e^0).

Putting the different pieces together, we can conclude that, whenever there is some (x, y) such that both tested and non-tested exert effort, then (i) if $\alpha v < 1$, then neither value function is convex and there will be obfuscation, and (ii) if $\alpha v \geq 1$, and α or v are sufficiently large then both value functions are convex, and there will be full disclosure.

Finally, there is a third case. If negative-tested individuals are the only ones to ever exert effort, then full disclosure will be the optimal policy. This happens whenever $\gamma \geq 4\alpha v(1-t)$.