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# How to Build Sustainable Productivity Indexes

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# How to Build Sustainable Productivity Indexes

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**Abstract**

Sustainable production requires that firms produce good outputs in ways that minimise the production of bad outputs (e.g., produce electricity in ways that minimise greenhouse gas emissions). Many decision-makers would like statisticians to measure changes in productivity in a way that will reflect well on firms that adopt sustainable production practices. In this paper I describe an approach to building so-called sustainable productivity indexes. This necessarily involves assigning weights to different inputs and outputs. I assign these weights in such a way that the indexes satisfy a set of basic axioms from index theory. I illustrate the properties of different indexes using a toy data set. I discuss ways in which statisticians can assess the sensitivity of index numbers to the choice of weights. Finally, I compute sustainable productivity index numbers for sixteen sectors of the Australian economy.

[1] I acknowledge the helpful comments received from participants in the 7th meeting of the OECD Network on Agricultural TFP and the Environment, held on 5-6 December, 2022. All errors are my own.

## 1. Introduction

According to the OECD (2001, p.11) "productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use ... there is no disagreement on this general notion". Many decision-makers are concerned about the production of bad outputs (e.g., heavy metals, non-biodegradable plastics, greenhouse gases). Consequently, they would like statisticians to measure productivity in such a way that if inputs and good outputs are unchanged, then measures of productivity will increase as bad outputs fall. Most, if not all, of the measures that have been developed to date have serious shortcomings. The measures I propose in this paper overcome these shortcomings.

Pittman (1983) was one of the first to incorporate bad outputs into measures of productivity change. The productivity index he developed has the same form as the multilateral index of Caves et al. (1982*b*) except that bad outputs are valued at shadow prices instead of (nonexistent) market prices. The Pittman index has two shortcomings. First, it does not satisfy a proportionality axiom. This axiom says, for example, that if firm B used twice as much of every input to produce the same outputs as firm A, then the productivity index must say that firm B is half as productive as firm A. The Pittman index will generally say something else. Second, like most multilateral indexes, the Pittman index was designed to make comparisons of productivity across space; it cannot be used to also make comparisons across time (e.g., it cannot tell us whether a firm was more or less productive in period 3 than it was in period 1).

More than a decade later, Shaik (1998) introduced bad outputs into the Malmquist productivity index of Caves et al. (1982*a*). His so-called Malmquist environment-adjusted productivity index also has two shortcomings. First, except in very restrictive special cases (e.g., input and output sets are homothetic and production frontiers exhibit constant returns to scale), Malmquist productivity indexes cannot be written as measures of output volume change divided by measures of input volume change (i.e., they are not, in fact, measures of productivity change). Second, even in special cases where they *are* measures of productivity change, they do not satisfy a transitivity axiom. In a cross-section context, for example, this axiom says that a direct comparison of the productivity of two firms should yield the same index number as an indirect comparison through a third firm. For example, if an index says that firm A is twice as productive as firm B, and that firm B is three times more productive than firm C, then it must also say that firm A is six times more productive than firm C.

More recently, Burgess and Heap (2012) constructed a productivity index by combining a social index, an economic index and an environment index. The social index is a measure of changes in years of adult schooling, expected years of child schooling, and life expectancy. The economic and environment indexes are measures of changes in GDP per capita and greenhouse gas (GHG) emissions per capita. Again, this index has two shortcomings. First, the life expectancy variable is transformed using common logarithms before being standardised to have a mean of one and a standard deviation of one-third, while the two schooling variables are merely standardised. This means the social index (and therefore the productivity index) does not satisfy a commensurability axiom. This axiom say that the value of the index must be invariant to changes in units of measurement (e.g., it says that if years of schooling and life expectancy are measured in months instead of years, then the index numbers must not change). Second, both the economic index and the environment index ignore inputs of capital. This means that the productivity index will measure the benefits of new capital investments (i.e., increases in good outputs and decreases in bad outputs) but will not account for the costs. This incentivises decision-makers to overinvest in capital.

Most recently, Cardenas Rodriquez et al. (2018) use a growth accounting methodology to develop a measure of environmentally-adjusted multifactor productivity growth. This measure is equal to the rate of growth in good outputs minus a weighted sum of rates of growth in bad outputs and inputs. Again, this measure has two shortcomings. First, the weight attached to the inputs is estimated in a regression framework and is generally not equal to one (in their empirical application, it is equal to 0.7)[2]. This means their measure does not satisfy the proportionality axiom. If in a given period a firm used twice as much of every input to produce the same outputs it had produced in the previous period, then we would want all measures of productivity growth to say that productivity had fallen by 50%. However, because it fails the proportionality axiom, the Cardenas Rodriquez et al. (2018) measure will say something else (if the weight attached to inputs is 0.7, then it will say that productivity had fallen by only 1 - $1/2^{0.7} = 38\%$). Second, growth rates are only suitable for making comparisons over time; they cannot be used to make comparisons across space (e.g., they cannot be used to say whether firm A is more or less productive than firm B).

Other attempts to include bad outputs in measures of productivity change are similar to the ones just described, and they have similar shortcomings. In this paper I overcome these shortcomings by defining a sustainable productivity index (SPI) to be any variable

[2] See their equation (8) and the first row of numbers in their Table 2.

of the form:

$$SPI = \frac{GI^{1-\alpha}BI^{-\alpha}}{XI} \tag{1}$$

where $GI$ is an index that measures changes in the volume of good outputs, $BI$ is an index that measures changes in the volume of bad outputs, $XI$ is an input volume index, and $\alpha \in [0,1]$ is a parameter that measures the extent to which we want to account for bad outputs. If $\alpha = 0$, then bad outputs are ignored and the index collapses to a total factor productivity index (TFPI). If $\alpha = 1$, then good outputs are ignored and the index collapses to a measure of changes in input use and the production of bad outputs.

The structure of the paper is as follows. In Section 2 I discuss different types of volume indexes. I focus on indexes that are proper in the sense that they satisfy a set of six axioms from index theory, including proportionality, transitivity and commensurability. In Section 3 I argue that there is no such thing as a 'correct' volume index. Rather, the choice of index is a matter of taste. I then show how to assess the sensitivity of index numbers to the choice of index. In Section 4 I discuss methods for choosing the value of $\alpha$. One method involves estimating shadow output prices and associated average shadow revenue shares. Another method involves regressing the logarithm of a TFP index on a function of good and bad outputs. In Section 5 I discuss a practical method that can be used by statisticians to assess the sensitivity of SPI numbers to the choice of $\alpha$. In Section 6 I demonstrate how national accounts data can be used to implement some of these methods. I compute SPI numbers for the sixteen sectors of the Australian economy and find that accounting for bad outputs generally leads to lower estimates of productivity growth. For example, if I ignore bad outputs, then I find that productivity in the agriculture, forestry and fishing sector increased by 26% between 1995 and 2005. However, if I account for GHG emissions, then I find that productivity in the sector fell by 6.2% in that period.

## 2. Proper Volume Indexes

Computing measures of output and input volume change involves assigning numbers to baskets of outputs and inputs. Measurement theory says that so-called index numbers must be assigned in such a way that the relationships between the numbers reflect the relationships between the baskets (see, for example, Tal, 2016, Section 3). To illustrate this basic principle, consider the baskets of maple syrup and Vegemite and the five associated sets of volume index numbers presented in Table 1. These numbers have been computed as ratios of weighted averages: the Lowe index numbers have been computed as ratios of weighted arithmetic averages using average market prices as weights; the geometric Young (GY) numbers have been computed as ratios of weighted geometric averages using average value shares as weights; the additive data envelopment analysis (ADEA) numbers have been computed as ratios of weighted arithmetic averages using averages of data envelopment analysis (DEA) estimates of shadow prices as weights; the multiplicative data envelopment analysis (MDEA) numbers have been computed as ratios of weighted geometric averages using averages of DEA estimates of shadow value shares as weights; and the benefit-of-the-doubt (BOD) numbers have been computed as ratios of weighted arithmetic averages using weights that vary from one basket to the next. Technical details concerning each of these indexes can be found in O'Donnell (2018, Chs. 3, 6). More important than the way they have been computed is the fact that they are all consistent with measurement theory: in any given column, the number assigned to basket W is twice as large as the number assigned to basket A, reflecting the fact that basket W contains twice as much syrup and Vegemite as basket A; and the number assigned to basket M is the same as the number assigned to basket R, reflecting the fact that both baskets contain the same amounts of the two products.

The Lowe, GY, ADEA, MDEA and BOD indexes are examples of what O'Donnell (2016, 2018) calls 'proper' indexes. They are proper in the sense that they satisfy the six axioms listed in O'Donnell (2018, Ch. 3): weak monotonicity, homogeneity, commensurability, proportionality, time-space reversal, and transitivity.[3] These six axioms only say something about volumes. This means they are weaker than many of the axioms that are described elsewhere in the index literature: those axioms say something about prices as well as volumes and, because they cannot all be satisfied simultaneously, they are often referred to as tests; for more details, see, for example, Eichhorn (1976) and Funke et al. (1979). Importantly, all volume indexes that satisfy the O'Donnell axioms (i.e., all proper volume indexes) yield numbers that are consistent with measurement theory.

Unfortunately, most economists and statistical agencies use volume indexes that are not proper. This means they assign volume index numbers in ways that are not consistent with measurement theory. To illustrate, consider the baskets of syrup and Vegemite and the six associated sets of volume index numbers presented in Table 2. The numbers in this table were computed using indexes that are widely used in economics: the Fisher

[3]O'Donnell (2018, Ch. 3) refers to commensurability as homogeneity type II. O'Donnell (2016) also lists an identity axiom and a circularity axiom. Identity is a special case of proportionality corresponding to a factor of proportionality equal to one. Identity and transitivity together imply circularity. Thus, the six axioms in O'Donnell (2018, Ch. 3) are satisfied if and only if the eight axioms in O'Donnell (2016) are satisfied.

Table 1: Proper Volume Index Numbers

| Basket | Contents | Lowe | GY | ADEA | MDEA | BOD |
|---|---|---|---|---|---|---|
| A |  | 1 | 1 | 1 | 1 | 1 |
| M |  | 2.032 | 1.779 | 2.103 | 1.892 | 2.048 |
| R |  | 2.032 | 1.779 | 2.103 | 1.892 | 2.048 |
| W |  | 2 | 2 | 2 | 2 | 2 |

Source: The Lowe, GY and BOD numbers are from O'Donnell (2018, Tables 1.3 and 3.1).

index is a type of "superlative" index that uses market prices as weights; the Törnqvist index is another type of superlative index that uses value shares as weights; the chained Fisher (CF) and chained Törnqvist (CT) indexes treat the observations in the dataset as a time series and chain together adjacent pairs of Fisher and Törnqvist index numbers; and the Elteto-Koves-Szulc (EKS) and Caves-Christensen-Diewret (CCD) indexes treat the observations as a cross-section and take geometric averages of sets of Fisher and Törnqvist index numbers. Again, technical details concerning these indexes can be found in O'Donnell (2018, Ch. 3). Importantly, none of them are proper indexes and, consequently, none of them yield numbers that are consistent with measurement theory: observe, for example, that the Törnqvist index number assigned to basket M differs from the number assigned to basked R, even though both baskets contain the same amounts of the two products; the CT index number assigned to basket R is 4.914 times larger than the number assigned to basket A, even though basket R contains less than three times as much syrup and Vegemite as basket A; and the EKS index number assigned to basket W is 2.027 times larger than the number assigned to basket A, even though basket W contains exactly twice as much syrup and Vegemite as basket A. This is nonsense. The basic problem is that the Fisher and Törnqvist indexes do not satisfy the transitivity axiom, and the CF, CT, EKS and CCD indexes do not satisfy the proportionality axiom. It would be reasonable to ask why statisticians and economists continue to use these indexes when so many proper alternatives are available. There are several (poor) reasons.

First, the easiest way to construct proper volume indexes is to use constant weights. Fisher (1922, pp. 274, 275) claims that "the only formulae [that satisfy transitivity]

Table 2: Other Volume Index Numbers

| Basket | Contents | Fisher | Törnqvist | CF | CT | EKS | CCD |
|---|---|---|---|---|---|---|---|
| A |  | 1 | 1 | 1 | 1 | 1 | 1 |
| M |  | 1.892 | 1.879 | 2.389 | 4.068 | 1.942 | 2.088 |
| R |  | 1.893 | 1.880 | 2.854 | 4.914 | 1.943 | 2.089 |
| W |  | 2 | 2 | 3.642 | 6.734 | 2.027 | 1.971 |

Source: O'Donnell (2018, Tables 1.3 and 3.2).

are index numbers that have constant weights . . . But, clearly, constant weighting is not theoretically correct". Frisch (1936, p.6) backs him up by claiming that "it is absurd to assume constant [weights]". Today's economists continue to parrot these claims (e.g., Diewert and Fox, 2017, p.279; Färe and Zelenyuk, 2021, p.113). The problems with these claims are that: (a) we do not, in fact, need constant weights in order to satisfy transitivity, as evidenced by the fact that BOD indexes are transitive and use variable weights; (b) there are no economic or statistical theories that say different sets of weights *must* be used for different comparisons; and (c) there is nothing absurd about someone making up their mind about the relative importance of different outputs/inputs and not changing their mind as they look from one output/input bundle to the next.

Second, the US Bureau of Economic analysis (BEA) once measured changes in real GDP by updating the weights in a fixed-weight index every 5 years. However, in 1996 the BEA switched to using a CF index. Landefeld et al. (1995, p.31) defended this switch by saying "[the] use of fixed-weighted measures of real GDP . . . causes an overstatement of growth for periods after the base year and an understatement of growth for periods before the base year . . . [the] BEA's alternative chain-type measure . . . provides unbiased estimates of growth [and] will eliminate the inconvenience and confusion associated with . . . rewriting economic history . . . every 5 years". The problems with these arguments are that: (a) economists only need to change the weights in fixed-weight indexes if they change their minds about the relative importance of different outputs/inputs; (b) proper fixed-weight measures of volume change may overstate or understate growth in values (something they are not designed to measure), but, unlike most variable-weight indexes, they will never overstate or understate growth in volumes, as evidenced by the numbers in Tables 1 and 2; and (c) changing the weights might change the way we view historical input and output bundles, but it does not change those bundles (e.g., today's economists may view nineteenth-century electricity-GHG production plans very differently to the way economists viewed them at the time, but that doesn't change the volumes of electricity produced and GHGs emitted then or at any time before or since).

Third, Caves et al. (1982*a*) have a theorem that is often used to justify the use of (variable-weight) Törnqvist indexes. Their Theorem 2 states that if (i) firms are price takers in output markets, (ii) all firms successfully maximise revenue (i.e., they are always technically and allocatively efficient), (iii) the period-1 and period-2 output distance functions are nondecreasing in outputs, and (iv) the period-1 and period-2 output distance functions are translog functions with identical second-order coefficients, then the Törnqvist index that compares outputs in period 1 with outputs in period 2 is equivalent to a Malmquist index. Economists love this theorem because it means they can effectively compute Malmquist indexes without having to estimate distance functions. The problems are that: (a) there are few, if any, empirical studies that allow for inefficiency and then go on to find that *all* firms are *always* technically and allocatively efficient; (b) except in very restrictive special cases (i.e., homotheticity, etc.), the Malmquist index is not a proper index, so no-one should care that a Törnqvist index might sometimes be equal to a Malmquist index; and (c) if output distance functions are nondecreasing in outputs, then they cannot be translog functions (O'Donnell, 2018, p.100, fn. 17), so the theorem relies on assumptions that simply cannot be true.

Most recently, Rao (2022, pp. 803, 805) argues that "the Lowe index can lead to strange ordering of output quantity vectors ... [and] may lead to counter-intuitive conclusions". To demonstrate, he considers a case where (i) a technically efficient firm uses the same inputs to produce two outputs in two different periods, (ii) it is possible to produce more outputs in period 2 than in period 1 due to technical progress, and (iii) the firm produces more of output 2 and less of output 1 in period 2 than it did in period 1. The carefully-crafted case he considers is one where an increase in productivity due to technical progress is exactly offset by a decrease in productivity due to diseconomies of output substitution (i.e., the loss in productivity due to a change in the output mix). Consequently, the Lowe index plausibly says there has been no output change, and therefore no productivity change. Rao (2022, p. 806) exclaims that "a shift in the frontier should imply productivity change driven by technical change. However, the Lowe index shows no productivity change!" Interestingly, Rao could have used the same argument to criticise indexes that are not proper indexes (e.g., the Laspeyres index, the

Paasche index, and, if there is no price change, the Fisher index). However, the main problem with his argument is that he appears not to understand that the problem of measuring productivity change (where measurement theory is relevant) is quite different to the problem of explaining productivity change (where economic theory and statistical methods are relevant). Relatedly, he fails to account for the fact that that economies of substitution (i.e., changing the input mix and/or output mix) are important drivers productivity change: livestock producers, for example, find that a one-bull-nine-cow input mix is generally more productive than a nine-bull-one-cow input mix; and aircraft component manufacturers generally find that a two-wings-one-tail output mix is better than a one-wing-two-tails output mix.

Finally, some statisticians and economists may not be using proper indexes because (a) they are not fully aware of the practical consequences of using indexes that are not proper, (b) they may face bureaucratic inertia, or (c) they are incorrigible.

## 3. Assessing the Sensitivity of Volume Index Numbers to the Choice of Index

Countless proper volume indexes are available. The differences between them are often due to the choice of weights. Growth accountants often argue that value shares should be used as weights, while profit-maximising business owners often argue that market prices should be used as weights. In the absence of data on value shares or market prices, a production economist might argue that shadow prices should be used as weights, while a government minister might argue that the weights should reflect social or community values (e.g., willingness to pay for ecosystem services). In the absence of any of this information, regression methods can be used to identify weights that minimise the amount of variation in the log-index numbers, and linear programming methods can be used to identify observation-varying weights that make each firm look as productive as possible (i.e., weights that give managers the "benefit of the doubt"). In practice, the choice of weights (and associated proper index) is a matter of taste. Writing in a consumption context, Samuelson and Swamy (1974) put it nicely when they wrote that "we cannot hope for one ideal formula for the index number: if it works for the tastes of Jack Spratt, it won't work for his wife's tastes" (p.568). In a production context, we cannot hope for a correct set of weights or a correct volume index: a set of weights and an index that works for the growth accountant may not work for the business owner, the production economist, or the government minister.

In this context, decision-makers may want to know how sensitive volume index numbers are to the choice of index and weights. The easiest way forward is to compute many sets of index numbers and see how they vary. In practice, this can be done in several ways. If, for example, market prices (or shadow prices) are available, then we can compute at least as many sets of index numbers as there are observations in the dataset. To illustrate, I used artificial (or 'toy') volume and market price data on five firms in five periods (i.e., 25 observations) to compute 50 sets of volume index numbers. The data were the good-output data reported in Appendix A. The indexes were 25 additive indexes and 25 multiplicative indexes. Additive indexes are ratios of weighted arithmetic means, while multiplicative indexes are ratios of weighted geometric means; for more details, see O'Donnell (2018, Ch. 3). Descriptive statistics for selected index numbers are reported in Table 3. The interpretation of these statistics is straightforward: the statistics in row M, for example, reveal that the 50 additive and multiplicative index numbers that compare the good outputs of firm 3 in period 3 with the good outputs of firm 1 in period 1 have a mean of 1.905 and a standard deviation of 0.363. Some index numbers are evidently more sensitive than others to the choice of index and weights. The degree of sensitivity largely depends on whether the output mix of the comparison firm/period is close to the output mix of firm 1 in period 1 (the reference firm/period). Some of the standard deviations in Table 3 are zero because the output mixes are identical. Histograms and density plots can be used to convey similar information. Figure 1, for example, presents a histogram and density plot for the index number that compares the outputs of firm 3 in period 3 with the outputs of firm 1 in period 1. For comparison purposes, this figure also reports the Lowe and GY index numbers (2.032 and 1.779 respectively). Note from Appendix A that the outputs of firm 3 in period 3 are the same as the outputs of firm 2 in period 4. Thus, Figure 1 is also the histogram and density plot for the index number that compares the outputs of firm 2 in period 4 with the outputs of firm 1 in period 1.

Many proper volume indexes are constructed using weights that have been estimated using some type of statistical procedure. ADEA indexes, for example, are constructed using non-parametric estimates of shadow prices, BOD indexes are constructed using non-parametric estimates of observation-varying weights, and it is possible to construct multiplicative indexes using weights that have been estimated econometrically. Standard errors and other measures of reliability associated with these estimates can be used to measure the reliability of the associated index numbers. To illustrate, I used the good-output data in the toy dataset to compute a set of BOD volume index numbers. I then used the `dea.boot` function in the software of Bogetoft and Otto (2015) to generate 10,000 sets of index numbers. Table 4 reports descriptive statistics for selected index numbers. Figure 2 presents a histogram and density plot for the index number that compares the outputs of firm 3 in period 3 with the outputs of firm 1 in period 1. For

Table 3: Descriptive Statistics for 50 Sets of Additive and Multiplicative Index Numbers

| | | | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Row | Firm | Period | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| A | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| B | 2 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| C | 3 | 1 | 2.37 | 0 | 2.37 | 2.37 | 2.37 | 2.37 | 2.37 |
| D | 4 | 1 | 2.11 | 0 | 2.11 | 2.11 | 2.11 | 2.11 | 2.11 |
| E | 5 | 1 | 2.665 | 0.329 | 1.870 | 2.483 | 2.674 | 2.901 | 3.175 |
| : | : | : | : | : | : | : | : | : | : |
| M | 3 | 3 | 1.905 | 0.363 | 1.066 | 1.674 | 1.952 | 2.137 | 2.507 |
| : | : | : | : | : | : | : | : | : | : |
| V | 1 | 5 | 2.778 | 0.767 | 1.137 | 2.247 | 2.941 | 3.320 | 4.139 |
| W | 2 | 5 | 2 | 0 | 2 | 2 | 2 | 2 | 2 |
| X | 3 | 5 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Y | 4 | 5 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Z | 5 | 5 | 2.665 | 0.329 | 1.870 | 2.483 | 2.674 | 2.901 | 3.175 |

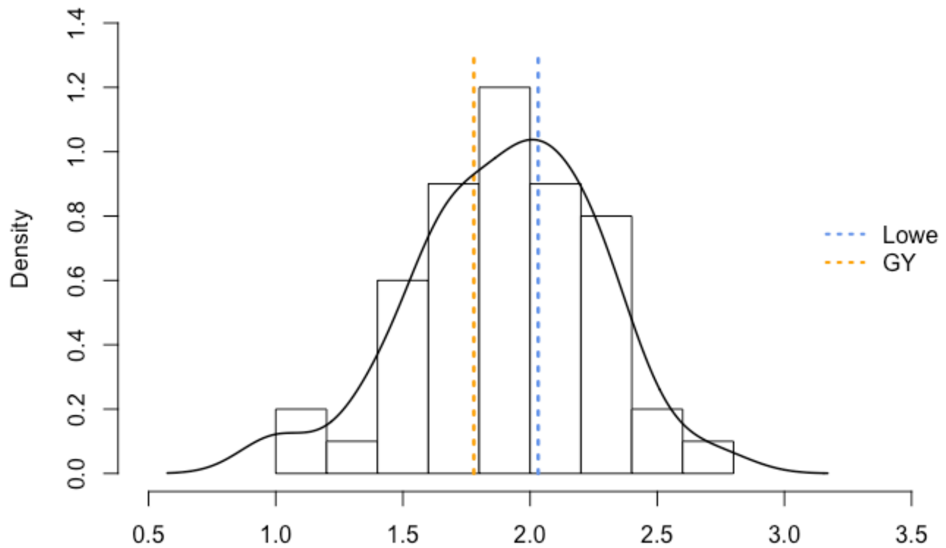



Figure 1: Distribution of 50 Additive & Multiplicative Index Numbers That Compare the Outputs of Firm 3 in Period 3 With the Outputs of Firm 1 in Period 1

comparison purposes, this figure again presents the values of the Lowe and GY indexes. Note that the density in Figure 1 is reasonably symmetric, while the density in Figure 2 is right-skewed. However, the modes of the two densities are similar.

Table 4: Descriptive Statistics for 10,000 Sets of BOD Index Numbers

| | | | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Row | Firm | Period | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| A | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| B | 2 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| C | 3 | 1 | 2.37 | 0 | 2.37 | 2.37 | 2.37 | 2.37 | 2.37 |
| D | 4 | 1 | 2.11 | 0 | 2.11 | 2.11 | 2.11 | 2.11 | 2.11 |
| E | 5 | 1 | 2.846 | 0.186 | 2.554 | 2.735 | 2.810 | 2.926 | 3.328 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| M | 3 | 3 | 2.199 | 0.201 | 1.897 | 2.059 | 2.162 | 2.299 | 2.711 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| V | 1 | 5 | 3.664 | 0.402 | 3.032 | 3.382 | 3.600 | 3.899 | 4.638 |
| W | 2 | 5 | 2 | 0 | 2 | 2 | 2 | 2 | 2 |
| X | 3 | 5 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Y | 4 | 5 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| Z | 5 | 5 | 2.846 | 0.186 | 2.554 | 2.735 | 2.810 | 2.926 | 3.328 |

## 4. Choosing the Value of $\alpha$

Any volume indexes can be used to build the SPI defined by equation 1. If the volume indexes satisfy the six axioms listed in O'Donnell (2018, Ch. 3) (i.e., if they are proper indexes), then the SPI will satisfy five axioms: weak monotonicity, commensurability, proportionality, time-space reversal, and transitivity.[4] Formal definitions of these axioms are provided in Appendix B. The weak monotonicity axiom says, among other things, that productivity doesn't fall when a firm uses fewer inputs to produce more good outputs and/or fewer bad outputs. The commensurability axiom says that the value of the index is invariant to changes in units of measurement. The proportionality axiom tells us what happens to productivity if all outputs and inputs are scaled up or down; whether productivity rises or falls depends on the value of $\alpha$. The time-space-reversal axiom says that the SPI number that compares firm $i$ in period $t$ with firm $k$ in period $s$ is

[4]It will also satisfy an homogeneity axiom that is analogous to the one listed in O'Donnell (2018, Ch. 3). However, the homogeneity axiom is implied by the proportionality and transitivity axioms.

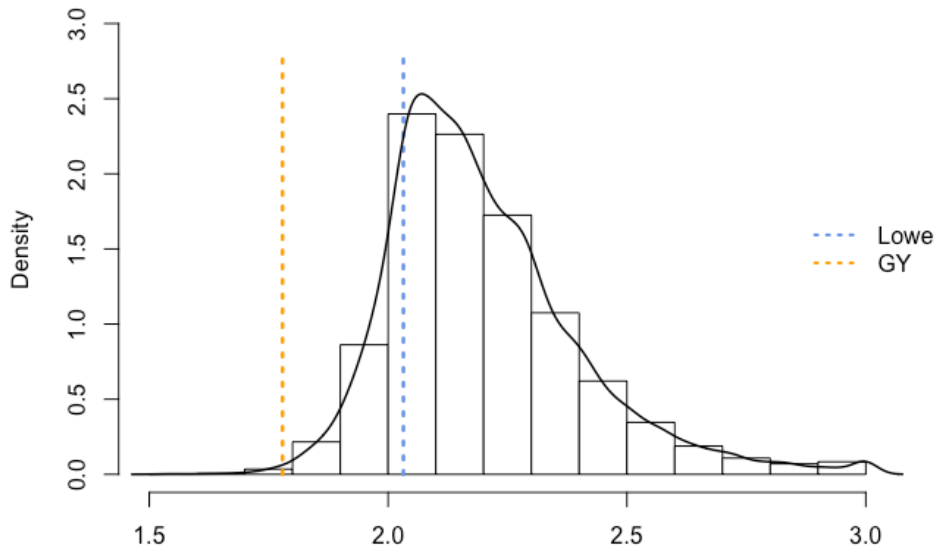



Figure 2: Distribution of 10,000 BOD Index Numbers That Compare the Outputs of Firm 3 in Period 3 With the Outputs of Firm 1 in Period 1.

the reciprocal of the number that compares firm $k$ in period $s$ with firm $i$ in period $t$ (i.e., when the reference and comparison observations are flipped). Finally, the transitivity axiom says that a direct comparison of the productivity of two firms/periods must yield the same index number as an indirect comparison through a third firm/period.

In practice, we need to choose the value of $\alpha$. Again, there is no correct value – the choice is a matter of taste. If a decision-maker does not prescribe the value of $\alpha$, then there are at least two ways we could proceed. First, if bad outputs are freely disposable, then we could estimate the shadow prices of good and bad outputs, and then set $\alpha$ to the average estimated shadow value share of bad outputs. The problem with this approach is that bad outputs are generally not freely disposable. Second, whether or not bad outputs are freely disposable, we could set $\alpha$ to the value that minimises the amount of variation in the log-SPI numbers. This involves rewriting equation (1) in the form of the following regression model:

$$\ln GI - \ln XI = \alpha[\ln GI + \ln BI] + e \tag{2}$$

where $e = \ln SPI$ is an unobserved variable that in any other context would be interpreted as statistical noise. The dependent variable in this model is the logarithm of a proper TFP index. The explanatory variable is the logarithm of a measure of change in total output. Estimates of $\alpha$ can be obtained using least squares methods. However, in many empirical applications the associated confidence intervals will extend beyond the unit interval. The most satisfactory way of solving this problem is to estimate $\alpha$ in a

Bayesian framework using a prior that assigns zero probability to any value of $\alpha$ that lies outside the unit interval (or, for that matter, any subinterval of the unit interval that reflects the views of decision-makers).

To illustrate, I considered the toy dataset and an SPI comprising a GY good-output volume index, an MDEA bad-output volume index, and a GY input volume index. I then used Bayesian methods to estimate the value of $\alpha$ in equation (2). I assumed that the unobserved "noise" variables in the model were independent normal random variables with mean zero and constant variance, and I used a prior that assigned zero probability to values of $\alpha$ outside the unit interval. I generated an MCMC chain of length 5,000 using the sampling package of Plummer (2019). The mean of the 5,000 MCMC draws was 0.336. Selected SPI numbers computed using $\alpha = 0.336$ are reported in Table 5. For convenience, this table also reproduces the volume data from Appendix A – this makes it easier to confirm that the SPI numbers are sensible. Observe, for example, that the SPI number in row B is $1/0.56 = 1.786$, reflecting the fact that firm 2 used $1 - 0.56 = 44\%$ fewer inputs than firm 1 to produce the same outputs. Also observe that the SPI number in row W is $2/0.919 = 2.176$, reflecting the fact that, in period 5, firm 2 used $1 - 0.919 = 8.1\%$ less input to produce twice as much good output and half as much bad output as firm 1 in period 1. As a final example, observe that the SPI number in row Z is half as large as the number in row E, reflecting the fact that, in period 5, firm 5 used twice as much input to produce exactly the same outputs it had produced in period 1.

## 5. Assessing the Sensitivity of SPI Numbers to the Choice of $\alpha$.

Decision-makers may want to know how sensitive SPI numbers are to the choice of $\alpha$. Again, the easiest way forward is to choose many values for $\alpha$ and see how the associated sets of index numbers vary. These many values could be chosen in at least three ways. First, since the choice of $\alpha$ is a matter of taste (or politics), one possibility is to simply ask a small number of decision-makers what values they would choose. Second, if we used the average estimated shadow value share of bad outputs as an estimate of $\alpha$, we could use each estimated share that was used in the calculation of the average. Finally, if we used Bayesian methods to estimate $\alpha$, and if we estimated the posterior pdf using MCMC sampling, then we could use each of the MCMC draws.

To illustrate, I used the toy dataset and the 5,000 MCMC draws described in the previous section to estimate the posterior pdfs of SPI numbers that compare the productivity of each firm in each period with the productivity of firm 1 in period 1. Characteristics of selected pdfs are reported in Table 6. Observe that some of the estimated pdfs are

Table 5: SPI Numbers Computed Using a Bayesian Estimate of $\alpha$

| Row | Firm | Period | $g_1$ | $g_2$ | $b_1$ | $b_2$ | $x_1$ | $x_2$ | SPI |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| B | 2 | 1 | 1 | 1 | 1 | 1 | 0.56 | 0.56 | 1.786 |
| C | 3 | 1 | 2.37 | 2.37 | 1 | 1 | 1 | 1 | 1.770 |
| D | 4 | 1 | 2.11 | 2.11 | 0.4 | 0.4 | 1.05 | 0.7 | 2.693 |
| E | 5 | 1 | 1.81 | 3.62 | 0.3 | 0.4 | 1.05 | 0.7 | 3.299 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| M | 3 | 3 | 1 | 3 | 1.4 | 1.6 | 1.354 | 1 | 1.133 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| V | 1 | 5 | 1 | 5.166 | 1.8 | 1.5 | 1 | 1 | 1.490 |
| W | 2 | 5 | 2 | 2 | 0.5 | 0.5 | 0.919 | 0.919 | 2.176 |
| X | 3 | 5 | 1 | 1 | 3 | 2.7 | 1.464 | 0.215 | 1.446 |
| Y | 4 | 5 | 1 | 1 | 1 | 1 | 0.74 | 0.74 | 1.351 |
| Z | 5 | 5 | 1.81 | 3.62 | 0.3 | 0.4 | 2.1 | 1.4 | 1.649 |

degenerate. For example, and for reasons given earlier, the estimated pdf of the SPI number that compares firm 2 in period 5 with firm 1 in period 1 has support only at $2/0.919 = 2.176$. More information can sometimes be obtained from posterior density plots. For example, Figure 3 plots the estimated posterior density for the SPI number that compares firm 3 in period 3 with firm 1 in period 1. This figure and the results reported in row M of Table 6 indicate that there is a better than 75% chance that firm 3 in period 3 was more productive than firm 1 in period 1 (the area under the density to the right of 1.0 is slightly greater than 0.75).

## 6. Productivity Change in the Market Sectors of the Australian Economy

The Australian economy comprises sixteen market sectors that use different types of capital, labour and other inputs to produce a wide range of products and services. Importantly, the types of inputs that are used in any one sector are generally quite different from the types of inputs that are used in any other sector: the tractors and other types of capital used in the agriculture sector, for example, are quite different from the rock drills and other types of capital used in the mining sector, and the types of labour used in the transportation sector are quite different from the types of labour used in the arts and recreation services sector. For this reason, it is not meaningful to compare levels of productivity across sectors (e.g., it is not meaningful to compare tonnes of coal per miner with the number of concertos per orchestra). Consequently, I measured productivity

Table 6: Characteristics of Estimated Posterior Densities of SPI Numbers

| | | | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Row | Firm | Period | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| A | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| B | 2 | 1 | 1.786 | 0 | 1.786 | 1.786 | 1.786 | 1.786 | 1.786 |
| C | 3 | 1 | 1.770 | 0.200 | 1.389 | 1.635 | 1.755 | 1.899 | 2.184 |
| D | 4 | 1 | 2.693 | 0.060 | 2.581 | 2.652 | 2.694 | 2.732 | 2.820 |
| E | 5 | 1 | 3.299 | 0.064 | 3.178 | 3.255 | 3.299 | 3.340 | 3.434 |
| : | : | : | : | : | : | : | : | : | : |
| M | 3 | 3 | 1.133 | 0.142 | 0.865 | 1.036 | 1.121 | 1.224 | 1.429 |
| : | : | : | : | : | : | : | : | : | : |
| V | 1 | 5 | 1.490 | 0.271 | 1.003 | 1.302 | 1.461 | 1.658 | 2.074 |
| W | 2 | 5 | 2.176 | 0 | 2.176 | 2.176 | 2.176 | 2.176 | 2.176 |
| X | 3 | 5 | 1.446 | 0.201 | 1.071 | 1.308 | 1.428 | 1.574 | 1.868 |
| Y | 4 | 5 | 1.351 | 0 | 1.351 | 1.351 | 1.351 | 1.351 | 1.351 |
| Z | 5 | 5 | 1.649 | 0.032 | 1.589 | 1.627 | 1.650 | 1.670 | 1.717 |

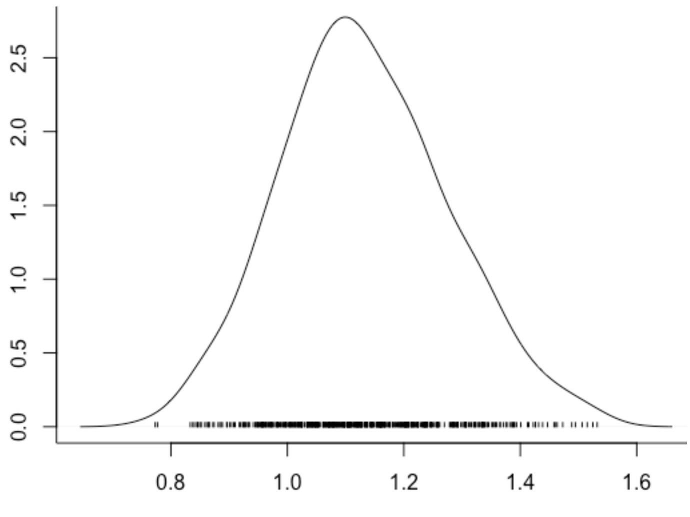



Figure 3: Estimated Pdf of the SPI Number That Compares Firm 3 in Period 3 With Firm 1 in Period 1.

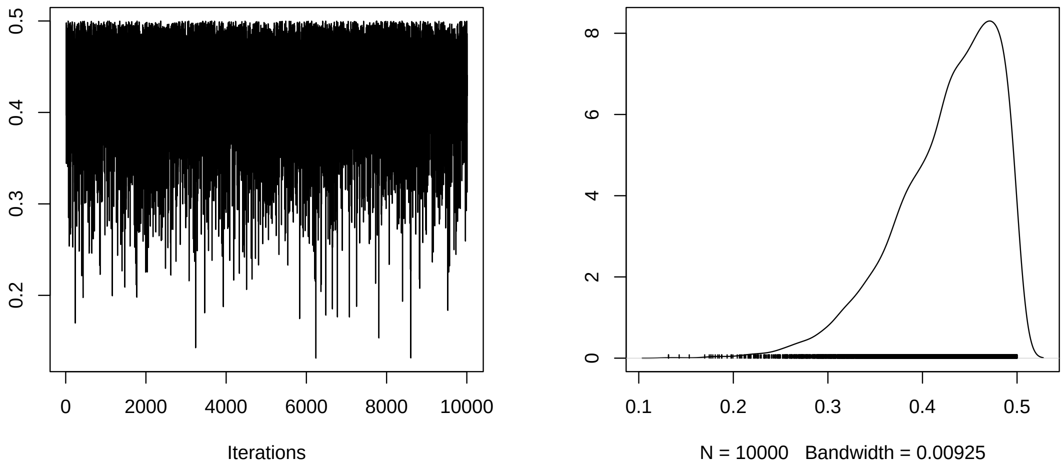
change in each sector separately.

The dataset contained twenty-five observations on the volume of one good output (market goods and services), the volume of one bad output (direct GHG emissions) and the volumes and cost shares of three inputs (capital, labour and other inputs) for each market sector over the financial years from 1995 to 2019. The good output and input data were compiled from national accounts data published by the Australian Bureau of Statistics (ABS). The bad output data were compiled from data published by the Australian Department of Industry, Science, Energy and Resources (ADISER).

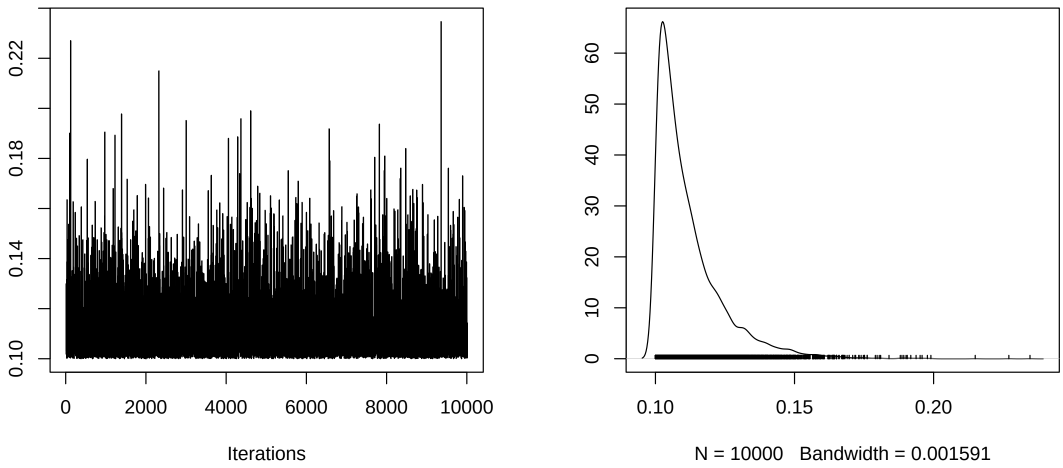
The first step in constructing an SPI involves choosing the good output, bad output and input volume indexes. There was only one good output and one bad output in each sector, so all good and bad output volume indexes yield the same numbers. Thus, I only needed to choose the input volume index. I chose the GY input volume index over other indexes because the Australian government appears to have a preference for using cost shares as measures of input value, as evidenced by the fact that most Australian government agencies currently compute measures of input volume change using Törnqvist, CT or CCD indexes. All of those indexes use cost shares as weights. However, they are not proper indexes. The GY index is a proper index that uses average cost shares as weights.

The second step in constructing an SPI involves choosing the value of $\alpha$. I chose to set $\alpha$ in each sector to the value that minimises the amount of variation in the log-SPI numbers. I again estimated equation (2) in a Bayesian framework under the assumption that the unobserved noise variables were independent normal random variables with mean zero and constant variance. However, my knowledge of Australian politics led me to believe that any value of $\alpha$ less than 0.1 would be unacceptable to many members of the Australian community (and parliament), and any value greater than 0.5 would "scare the horses". I therefore used a prior that assigned zero probability to values of $\alpha$ outside the interval from 0.1 to 0.5. For each sector, I generated an MCMC chain of length 10,000 using the sampling package of Plummer (2019). Figure 4 summarises the results for (a) the agriculture, forestry and fishing sector and (b) the mining sector. Figures that summarise the results for other sectors are presented in Appendix C. The left-hand panels in these figures plot the evolution of the MCMC chains; they indicate that the chains are stationary. The right-hand panels are the estimated posterior pdfs for $\alpha$. In the case of the agriculture, forestry and fishing sector (resp. mining sector), the mean and standard deviation of the estimated posterior pdf were 0.426 and 0.055 (resp. 0.112 and 0.012).

For each sector, I used the 10,000 MCMC draws to estimate the posterior pdfs of




(a) Agriculture, Forestry and Fishing: Mean = 0.426; St. Dev. = 0.055.




(b) Mining: Mean = 0.112; St. Dev. = 0.012.

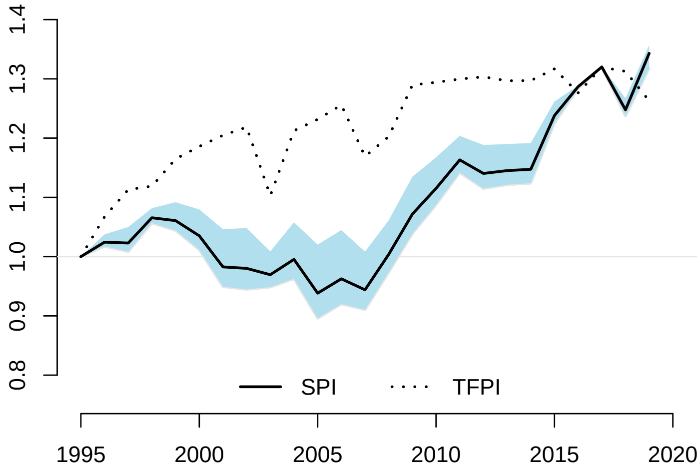
Figure 4: MCMC Chains and Estimated Posterior Pdfs.

SPI numbers that compare productivity in each year with productivity in 1995. Characteristics of estimated posterior pdfs (i.e., means, standard deviations and percentiles) for each sector are reported in tabular form in Appendix D. Figure 5 summarises the results for (a) the agriculture, forestry and fishing sector and (b) the mining sector. Figures that summarise the results for other sectors are presented in Appendix E. The solid lines in these figures are the means of the posterior pdfs. The shaded areas are 95% highest posterior density intervals (the Bayesian equivalents of confidence intervals). For comparison purposes, Figure 5 also presents TFPI numbers (i.e., the numbers obtained when $\alpha$ is set to zero and GHG emissions are ignored). The TFPI numbers in panel (a) of Figure 5 indicate that productivity in the agriculture, forestry and fishing sector increased by 26% between 1995 and 2005. However, the SPI numbers indicate that productivity in the sector fell by 6.2% in that period. The TFPI numbers in panel (b) indicate productivity in the mining sector fell by 15% between 1995 and 2019. However, the SPI numbers indicate that productivity in the sector fell by 28.7%.

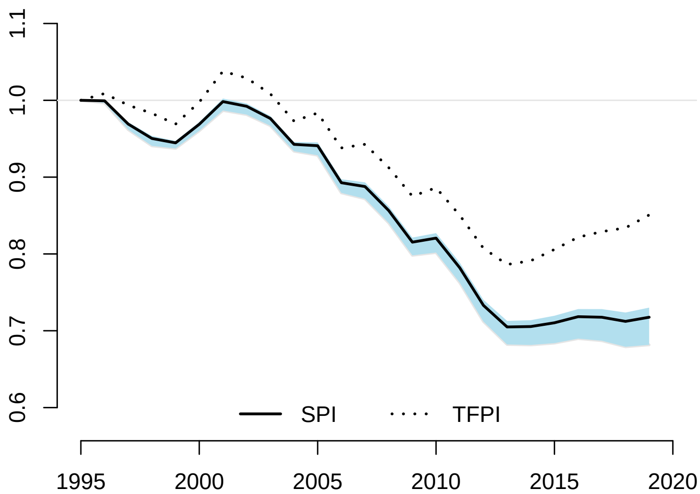
## 7. Conclusion

My aim in this paper was to build productivity indexes in such a way that if volumes of inputs and good outputs remained unchanged, then productivity numbers would increase as volumes of bad outputs fell. The first step involved measuring changes in volumes of inputs, good outputs and bad outputs. Many volume indexes are available, and the choice of index is a matter of taste. Unfortunately, most economists and statistical agencies have a taste for indexes that produce nonsensical numbers. When choosing a volume index, the overriding consideration should be whether the index produces numbers that are consistent with measurement theory (i.e., whether the patterns in the numbers mirror the patterns in the volumes). Only proper indexes can do this. For no good reason, the volume indexes that are most widely chosen by economists and statistical agencies are not proper indexes, and they do not yield numbers that are consistent with measurement theory. As tasty as economists and statisticians may find them, they are bad for economics and should be taken off the menu.

The second step in building a so-called sustainable productivity index (SPI) involved combining the good output and bad output volume indexes into a total output index. In turn, this involved choosing the value of a parameter, $\alpha \in [0, 1]$, that measured the extent to which the total output index accounted for bad outputs. In practice, the choice of $\alpha$ is again a matter of taste (or politics). If a decision-maker does not prescribe a value for $\alpha$, then one possibility is to set $\alpha$ to the value that minimises the amount of variation in the log-SPI numbers. This can be done using regression methods. In this paper, I suggested




(a) Agriculture, Forestry and Fishing




(b) Mining

Figure 5: Sustainable Productivity Index Numbers.

that $\alpha$ be estimated in a Bayesian framework using a prior that assigns zero probability to any value of $\alpha$ that lies outside the unit interval, or, for that matter, any subinterval of the unit interval that reflects the views of decision-makers. One of the advantages of estimating $\alpha$ in this way is that it is possible to compute measures of reliability for SPI numbers.

To illustrate the methodology, I used data from the Australian system of national accounts and the Australian national inventory of greenhouse gas (GHG) emissions to compute SPI numbers for sixteen sectors of the Australian economy. I found that SPI numbers were generally lower than total factor productivity index (TFPI) numbers (i.e., measures of productivity change that ignore bad outputs). For example, the SPI numbers indicate that productivity in the mining sector fell by 28.7% between 1995 and 2019, whereas the TFPI numbers indicate that productivity fell by only 15%.

Measuring changes in productivity is one thing. Explaining those changes is an entirely different matter. O'Donnell (2022) used stochastic production frontier models to explain variations in the TFPI numbers reported in this paper. The same models can be used to explain the SPI numbers. If this were done, then the estimated rates of technical progress, environmental change and technical efficiency change would be exactly the same as those reported in O'Donnell (2022). The only driver of productivity change that would differ would be the measure of scale and mix efficiency change (i.e., the measure of economies and diseconomies of scale and substitution). Interestingly, some authors define productivity change in way that deliberately ignores this component. For example, rather than define productivity change to be a measure of output volume change divided by input volume change, Diewert (1992, p.228) writes that "a change in productivity is taken to be a shift in the production function" (i.e., a measure of technical change), while Färe et al. (1994, p.72) "define productivity growth as the product of [technical] efficiency change and technical change". This suggests that the first step in building a productivity index should, in fact, be defining what is meant by the terms 'productivity' and 'productivity change'

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## Appendix A
## Toy Data†

| | | | Volumes | | | | | | Prices | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Goods | | Bads | | Inputs | | Goods | | Inputs | |
| Row | Firm | Period | $g_1$ | $g_2$ | $b_1$ | $b_2$ | $x_1$ | $x_2$ | $p_1$ | $p_2$ | $w_1$ | $w_2$ |
| A | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.57 | 0.41 | 0.28 | 1.91 |
| B | 2 | 1 | 1 | 1 | 1 | 1 | 0.56 | 0.56 | 0.26 | 0.25 | 0.22 | 0.58 |
| C | 3 | 1 | 2.37 | 2.37 | 1 | 1 | 1 | 1 | 0.57 | 0.41 | 0.28 | 1.91 |
| D | 4 | 1 | 2.11 | 2.11 | 0.4 | 0.4 | 1.05 | 0.7 | 0.58 | 0.53 | 0.16 | 0.41 |
| E | 5 | 1 | 1.81 | 3.62 | 0.3 | 0.4 | 1.05 | 0.7 | 0.26 | 0.26 | 0.07 | 1.02 |
| F | 1 | 2 | 1 | 1 | 1.6 | 2.8 | 0.996 | 0.316 | 0.59 | 0.76 | 0.24 | 0.29 |
| G | 2 | 2 | 1.777 | 3.503 | 1.7 | 1.6 | 1.472 | 0.546 | 0.63 | 0.65 | 0.16 | 0.16 |
| H | 3 | 2 | 0.96 | 0.94 | 1.1 | 1.3 | 0.017 | 0.346 | 0.34 | 0.31 | 0.17 | 0.70 |
| I | 4 | 2 | 5.82 | 0.001 | 0.9 | 0.7 | 4.545 | 0.01 | 0.46 | 0.58 | 0.27 | 0.39 |
| J | 5 | 2 | 6.685 | 0.001 | 0.9 | 1.3 | 4.45 | 0.001 | 0.61 | 1.43 | 0.29 | 0.79 |
| K | 1 | 3 | 1.381 | 4.732 | 1.1 | 1.8 | 1 | 1 | 0.57 | 0.41 | 0.28 | 1.91 |
| L | 2 | 3 | 0.566 | 4.818 | 2.8 | 2.5 | 1 | 1 | 0.49 | 0.65 | 0.21 | 0.56 |
| M | 3 | 3 | 1 | 3 | 1.4 | 1.6 | 1.354 | 1 | 0.51 | 0.46 | 0.16 | 0.74 |
| N | 4 | 3 | 0.7 | 0.7 | 0.9 | 2 | 0.33 | 0.16 | 0.52 | 0.23 | 0.24 | 2.30 |
| O | 5 | 3 | 2 | 2 | 1.1 | 2 | 1 | 1 | 0.37 | 0.17 | 0.24 | 0.15 |
| P | 1 | 4 | 1 | 1 | 2.4 | 4.6 | 0.657 | 0.479 | 0.41 | 0.76 | 0.26 | 0.61 |
| R | 2 | 4 | 1 | 3 | 2.6 | 2.1 | 1 | 1 | 0.53 | 0.48 | 0.16 | 0.22 |
| S | 3 | 4 | 1 | 1 | 1.4 | 1.9 | 1.933 | 0.283 | 0.53 | 0.37 | 0.19 | 0.62 |
| T | 4 | 4 | 1.925 | 3.722 | 2 | 1.3 | 1 | 1 | 0.91 | 0.53 | 0.17 | 0.26 |
| U | 5 | 4 | 1 | 1 | 1.2 | 1.5 | 1 | 0.31 | 0.31 | 1.03 | 0.27 | 0.91 |
| V | 1 | 5 | 1 | 5.166 | 1.8 | 1.5 | 1 | 1 | 0.47 | 0.08 | 0.29 | 0.78 |
| W | 2 | 5 | 2 | 2 | 0.5 | 0.5 | 0.919 | 0.919 | 0.57 | 0.27 | 0.39 | 0.81 |
| X | 3 | 5 | 1 | 1 | 3 | 2.7 | 1.464 | 0.215 | 0.31 | 0.51 | 0.21 | 0.31 |
| Y | 4 | 5 | 1 | 1 | 1 | 1 | 0.74 | 0.74 | 0.31 | 0.67 | 0.23 | 0.69 |
| Z | 5 | 5 | 1.81 | 3.62 | 0.3 | 0.4 | 2.1 | 1.4 | 0.42 | 0.69 | 0.31 | 0.22 |

† The good-output and input volume and price data are from O'Donnell (2018, Tables 1.1, 1.4 and 1.5).

## Appendix B
## Sustainable Productivity Indexes

Let $g_{it} \in \mathbb{R}^K_+$, $b_{it} \in \mathbb{R}^J_+$ and $x_{it} \in \mathbb{R}^N_+$ denote the volumes of of good outputs, bad outputs and inputs of firm $i$ in period $t$. An SPI that compares the productivity of firm $i$ in period $t$ with the productivity of firm $k$ in period $s$ is defined as any variable of the form

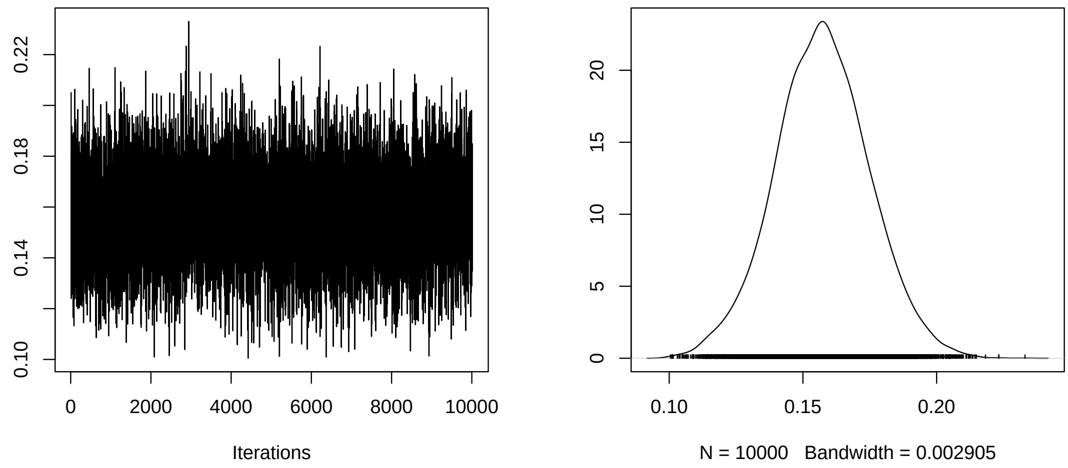
$$SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}) \equiv \frac{GI(g_{ks}, g_{it})^{1-\alpha} BI(b_{ks}, b_{it})^{-\alpha}}{XI(x_{ks}, x_{it})} \tag{3}$$

where $GI(.)$ is a proper good-output index, $BI(.)$ is a proper bad-output index, $XI(.)$ is a proper input index, and $\alpha \in [0,1]$ measures the extent to which we want to account for bad outputs. For more details on the construction and properties of proper volume indexes, see (O'Donnell, 2018, Ch. 3). If all inputs and outputs are positive,[5] then all SPIs of the form given by equation (3) satisfy the following axioms:

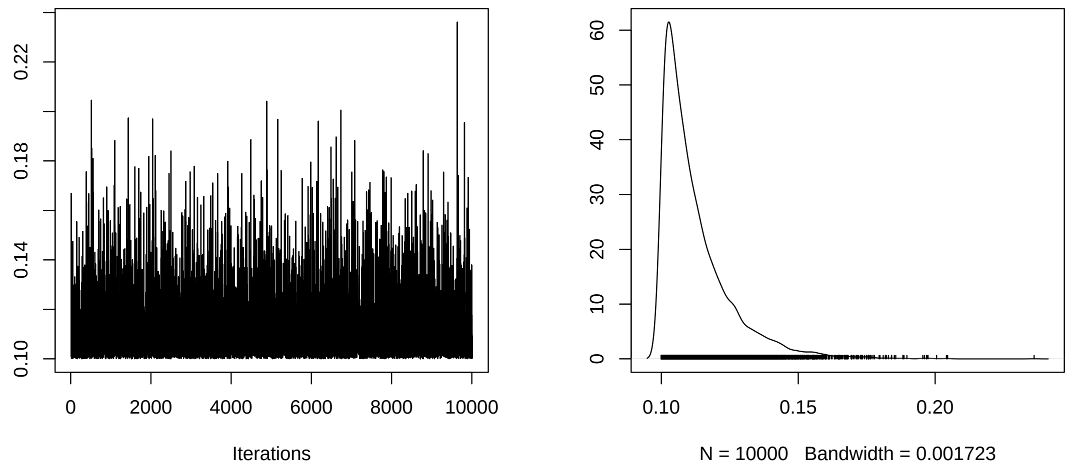
- SI1 Weak monotonicity: $g_{rl} \geq g_{it}$, $b_{rl} \leq b_{it}$ and $x_{rl} \leq x_{it} \Rightarrow SPI(x_{ks}, g_{ks}, b_{ks}, x_{rl}, g_{rl}, b_{rl}) \geq SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it})$;
- SI2 Commensurability: $SPI(\delta x_{ks}, \lambda g_{ks}, \theta b_{ks}, \delta x_{it}, \lambda g_{it}, \theta b_{it}) = SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it})$ for $(\lambda, \delta, \theta)' > 0$;
- SI3 Proportionality: $SPI(x_{ks}, g_{ks}, b_{ks}, \delta x_{ks}, \lambda g_{ks}, \theta b_{ks}) = (\lambda^{1-\alpha}\theta^{-\alpha}/\delta)$ for $\lambda > 0$, $\delta > 0$ and $\theta > 0$;
- SI4 Time-space reversal: $SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}) = 1/SPI(x_{it}, g_{it}, b_{it}, x_{ks}, g_{ks}, b_{ks})$; and
- SI5 Transitivity: $SPI(x_{ks}, g_{ks}, b_{ks}, x_{it}, g_{it}, b_{it}) = SPI(x_{ks}, g_{ks}, b_{ks}, x_{rl}, g_{rl}, b_{rl}) \times SPI(x_{rl}, g_{rl}, b_{rl}, x_{it}, g_{it}, b_{it})$.

[5]If some outputs or inputs are zero, then some SPIs may be either zero or mathematically undefined and may therefore not satisfy some axioms.

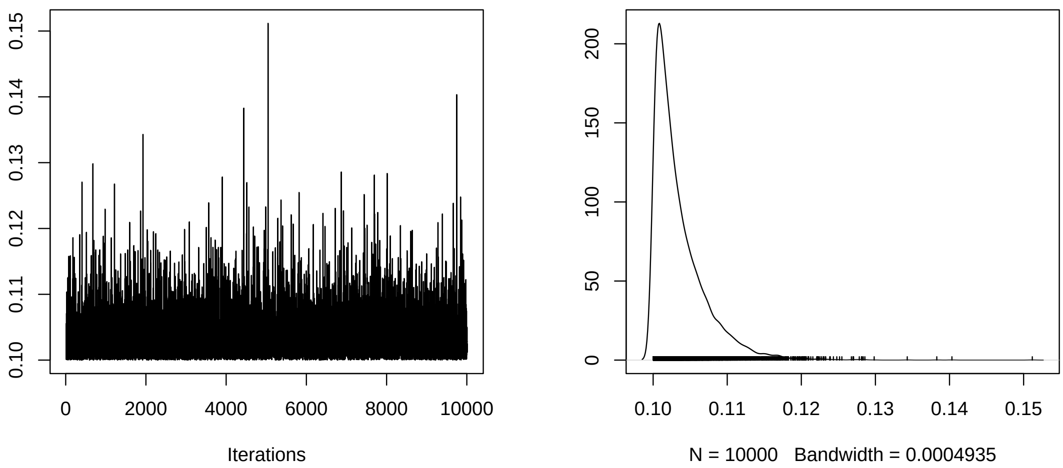
**Appendix C**
**MCMC Chains and Estimated Posterior Pdfs**



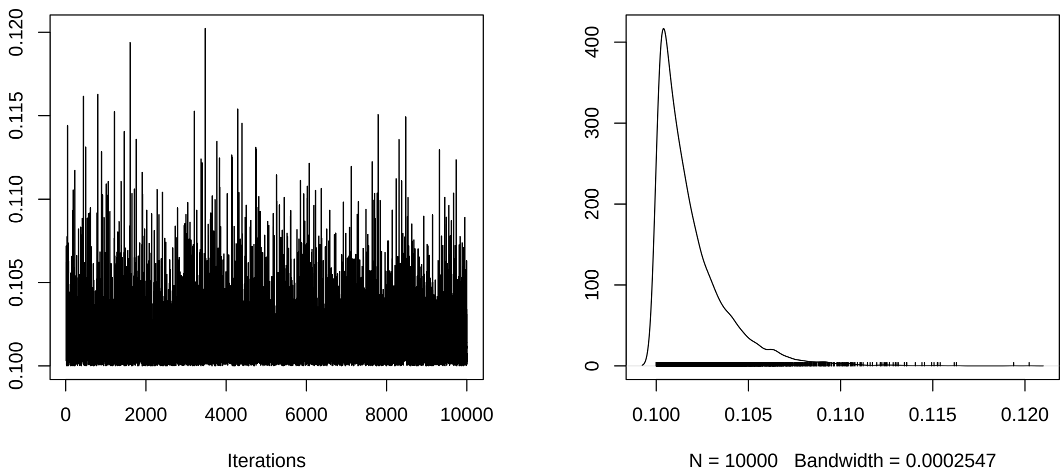

(c) Manufacturing: Mean = 0.157; St. Dev. = 0.017.



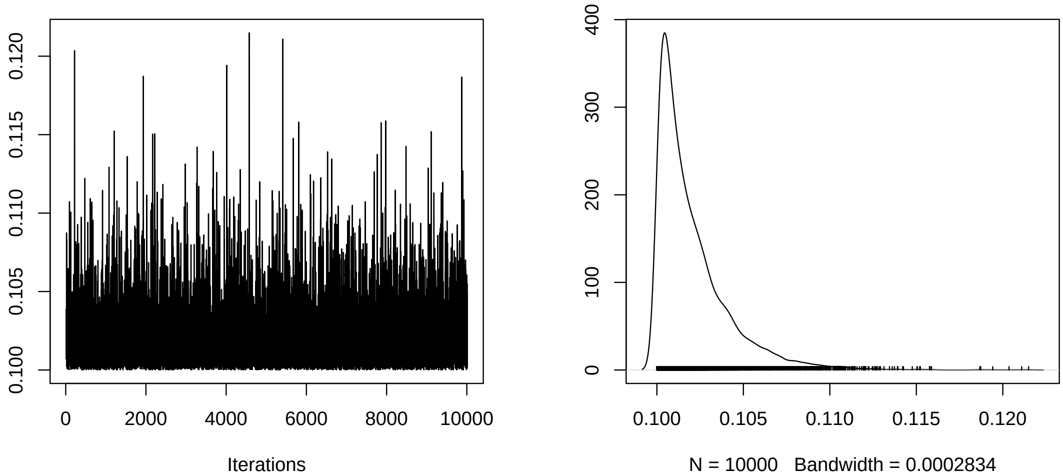

(d) Electricity, Gas, Water and Waste Services: Mean = 0.113; St. Dev. = 0.013.



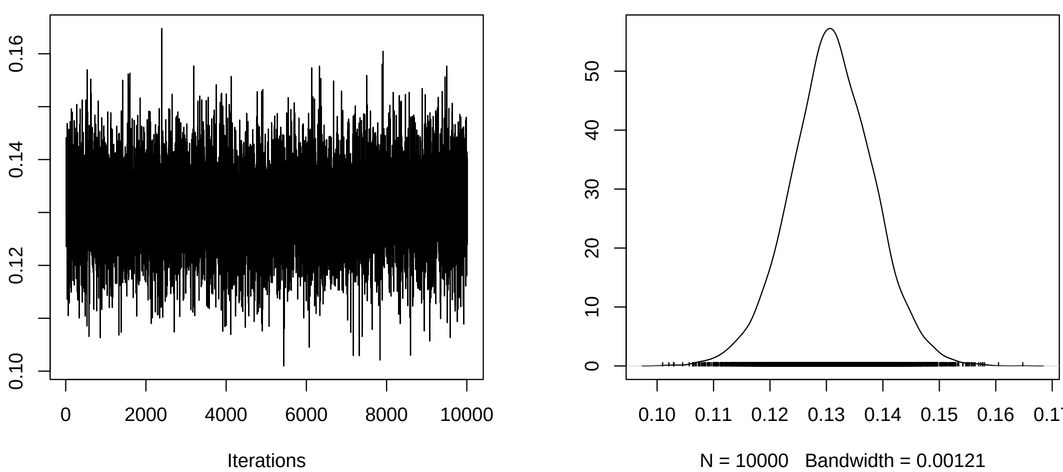

(e) Construction: Mean = 0.103; St. Dev. = 0.004.



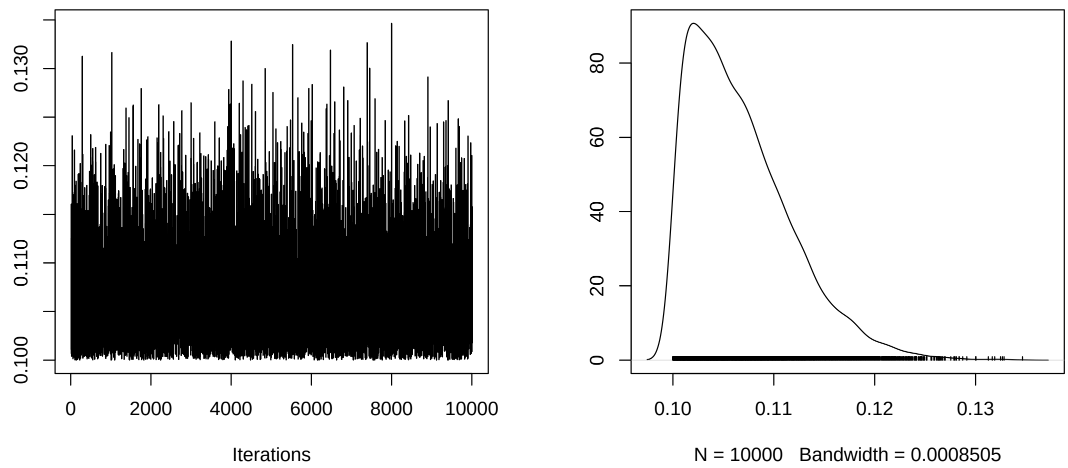

(f) Wholesale Trade: Mean = 0.102; St. Dev. = 0.002.



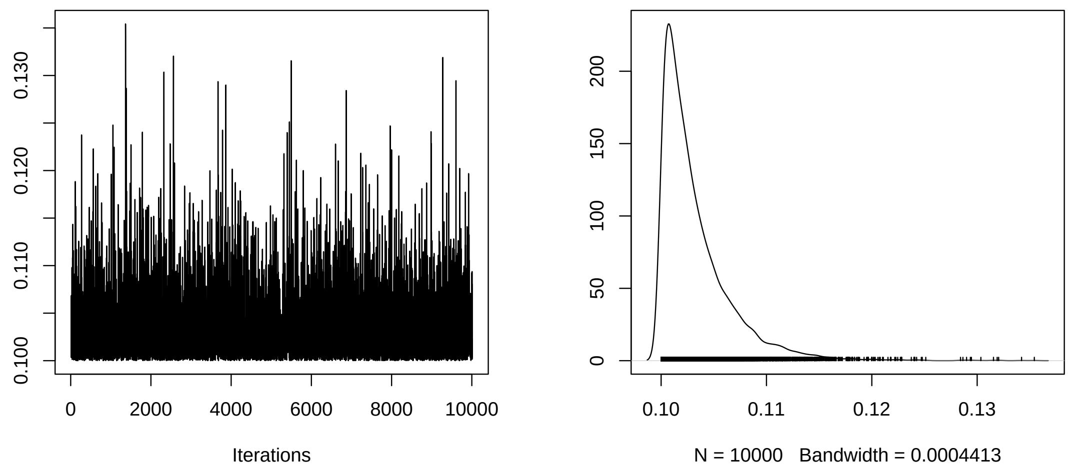

(g) Retail Trade: Mean = 0.102; St. Dev. = 0.002.



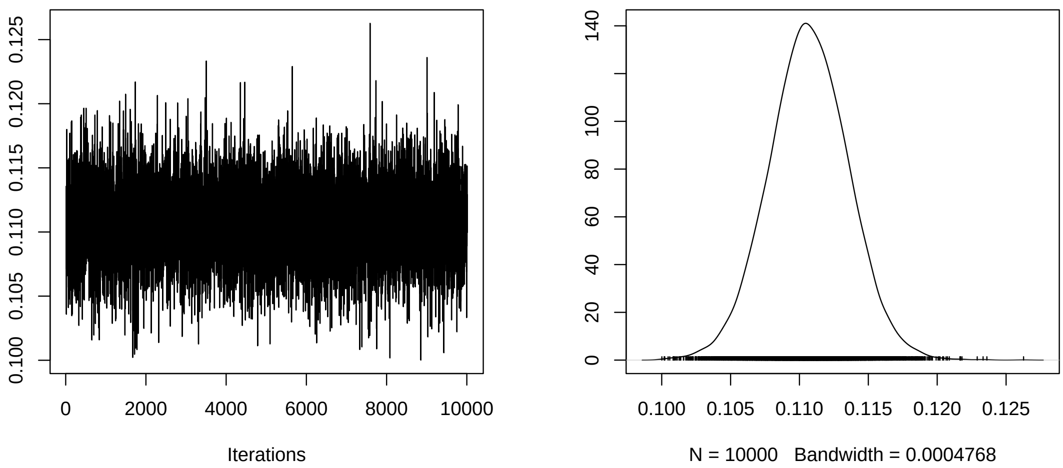

(h) Accommodation and Food Services: Mean = 0.131; St. Dev. = 0.007.



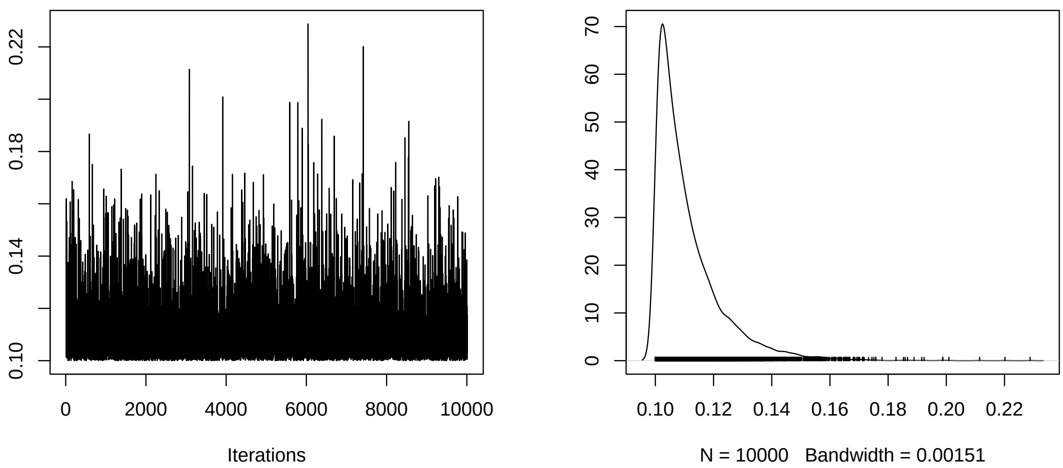

(i) Transport, Postal and Warehousing: Mean = 0.107; St. Dev. = 0.005.



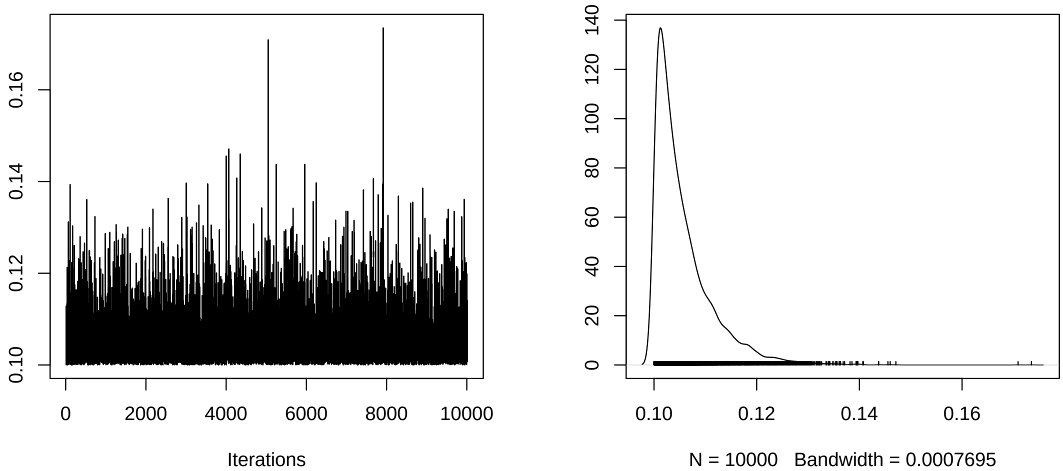

(j) Information, Media and Telecommunications: Mean = 0.103; St. Dev. = 0.003.



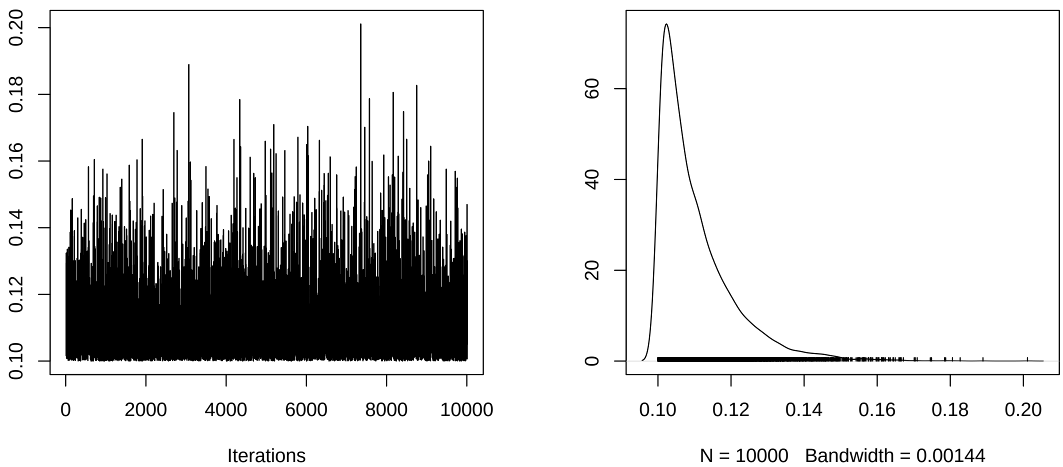

(k) Financial and insurance Services: Mean = 0.111; St. Dev. = 0.003.



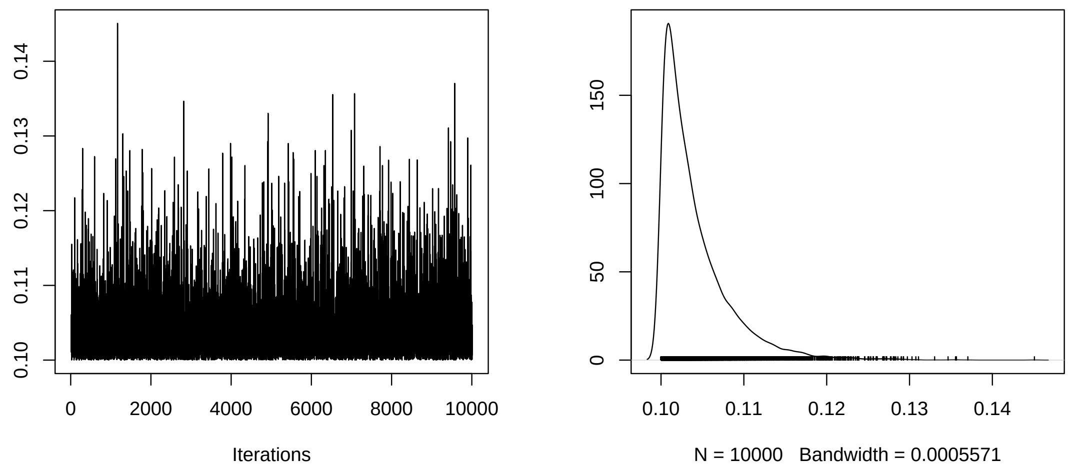

(l) Rental, Hiring and Real Estate Services: Mean = 0.112; St. Dev. = 0.012.



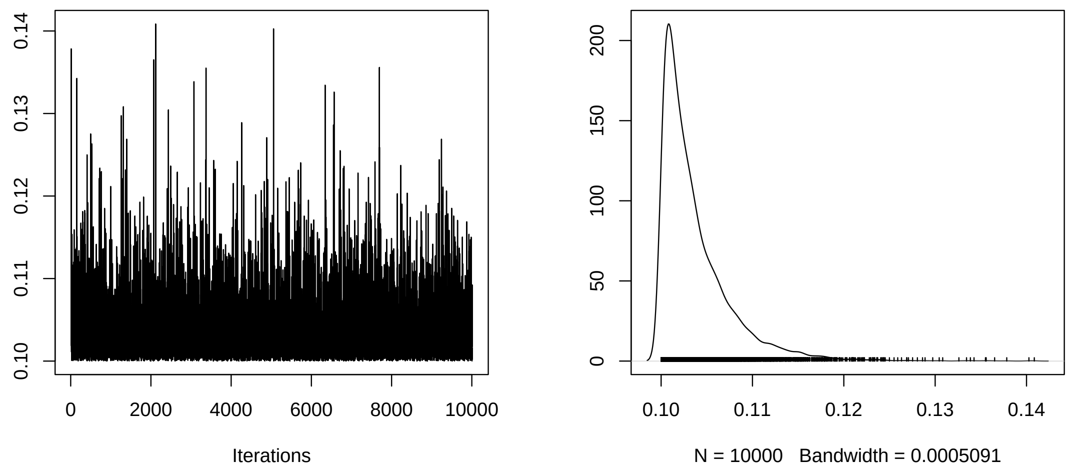

(m) Professional, Scientific and Technical Services: Mean = 0.106; St. Dev. = 0.006.




(n) Administrative and Support Services: Mean = 0.11; St. Dev. = 0.01.




(r) Arts and Recreation Services: Mean = 0.104; St. Dev. = 0.004.




(s) Other Services: Mean = 0.104; St. Dev. = 0.004.

## Appendix D
## Characteristics of Estimated Posterior Densities of SPI Numbers

(a) Agriculture, Forestry and Fishing

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.024 | 0.005 | 1.017 | 1.020 | 1.023 | 1.027 | 1.037 |
| 1997 | 1.023 | 0.011 | 1.009 | 1.014 | 1.021 | 1.029 | 1.050 |
| 1998 | 1.066 | 0.007 | 1.057 | 1.060 | 1.064 | 1.069 | 1.082 |
| 1999 | 1.061 | 0.013 | 1.044 | 1.051 | 1.058 | 1.068 | 1.092 |
| 2000 | 1.035 | 0.018 | 1.012 | 1.021 | 1.032 | 1.045 | 1.079 |
| 2001 | 0.982 | 0.026 | 0.949 | 0.962 | 0.977 | 0.997 | 1.046 |
| 2002 | 0.980 | 0.028 | 0.945 | 0.958 | 0.974 | 0.995 | 1.047 |
| 2003 | 0.969 | 0.016 | 0.949 | 0.957 | 0.966 | 0.978 | 1.008 |
| 2004 | 0.995 | 0.026 | 0.963 | 0.975 | 0.990 | 1.009 | 1.057 |
| 2005 | 0.938 | 0.033 | 0.896 | 0.912 | 0.931 | 0.956 | 1.019 |
| 2006 | 0.962 | 0.033 | 0.920 | 0.936 | 0.956 | 0.980 | 1.044 |
| 2007 | 0.944 | 0.026 | 0.910 | 0.923 | 0.939 | 0.958 | 1.008 |
| 2008 | 1.004 | 0.024 | 0.974 | 0.985 | 0.999 | 1.017 | 1.061 |
| 2009 | 1.072 | 0.026 | 1.039 | 1.052 | 1.067 | 1.086 | 1.134 |
| 2010 | 1.115 | 0.021 | 1.088 | 1.098 | 1.111 | 1.127 | 1.167 |
| 2011 | 1.163 | 0.017 | 1.142 | 1.150 | 1.160 | 1.172 | 1.203 |
| 2012 | 1.140 | 0.020 | 1.115 | 1.125 | 1.136 | 1.151 | 1.188 |
| 2013 | 1.145 | 0.018 | 1.121 | 1.131 | 1.141 | 1.155 | 1.190 |
| 2014 | 1.147 | 0.018 | 1.124 | 1.133 | 1.144 | 1.157 | 1.191 |
| 2015 | 1.238 | 0.010 | 1.225 | 1.230 | 1.236 | 1.243 | 1.262 |
| 2016 | 1.287 | 0.001 | 1.284 | 1.286 | 1.287 | 1.288 | 1.289 |
| 2017 | 1.320 | 0.000 | 1.320 | 1.320 | 1.320 | 1.320 | 1.320 |
| 2018 | 1.248 | 0.008 | 1.237 | 1.241 | 1.246 | 1.252 | 1.267 |
| 2019 | 1.343 | 0.011 | 1.318 | 1.337 | 1.345 | 1.351 | 1.357 |

(b) Mining

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.999 | 0.001 | 0.997 | 0.999 | 1.000 | 1.000 | 1.000 |
| 1997 | 0.969 | 0.003 | 0.962 | 0.968 | 0.970 | 0.971 | 0.972 |
| 1998 | 0.950 | 0.004 | 0.941 | 0.949 | 0.951 | 0.953 | 0.954 |
| 1999 | 0.945 | 0.003 | 0.937 | 0.944 | 0.945 | 0.946 | 0.947 |
| 2000 | 0.969 | 0.003 | 0.960 | 0.968 | 0.970 | 0.971 | 0.972 |
| 2001 | 0.998 | 0.004 | 0.987 | 0.997 | 1.000 | 1.001 | 1.002 |
| 2002 | 0.992 | 0.004 | 0.981 | 0.991 | 0.993 | 0.995 | 0.996 |
| 2003 | 0.976 | 0.003 | 0.967 | 0.975 | 0.977 | 0.979 | 0.980 |
| 2004 | 0.943 | 0.003 | 0.934 | 0.941 | 0.944 | 0.945 | 0.946 |
| 2005 | 0.941 | 0.005 | 0.928 | 0.939 | 0.942 | 0.944 | 0.945 |
| 2006 | 0.893 | 0.005 | 0.880 | 0.891 | 0.894 | 0.896 | 0.897 |
| 2007 | 0.888 | 0.006 | 0.872 | 0.886 | 0.890 | 0.892 | 0.893 |
| 2008 | 0.856 | 0.006 | 0.840 | 0.854 | 0.858 | 0.861 | 0.862 |
| 2009 | 0.815 | 0.006 | 0.798 | 0.813 | 0.817 | 0.820 | 0.821 |
| 2010 | 0.821 | 0.007 | 0.802 | 0.818 | 0.823 | 0.825 | 0.827 |
| 2011 | 0.782 | 0.007 | 0.763 | 0.779 | 0.784 | 0.787 | 0.789 |
| 2012 | 0.733 | 0.008 | 0.712 | 0.730 | 0.735 | 0.738 | 0.740 |
| 2013 | 0.705 | 0.008 | 0.682 | 0.702 | 0.708 | 0.711 | 0.713 |
| 2014 | 0.705 | 0.009 | 0.682 | 0.702 | 0.708 | 0.712 | 0.714 |
| 2015 | 0.710 | 0.010 | 0.684 | 0.707 | 0.713 | 0.717 | 0.720 |
| 2016 | 0.718 | 0.011 | 0.690 | 0.714 | 0.722 | 0.726 | 0.728 |
| 2017 | 0.718 | 0.011 | 0.687 | 0.713 | 0.721 | 0.725 | 0.728 |
| 2018 | 0.712 | 0.012 | 0.679 | 0.708 | 0.716 | 0.721 | 0.724 |
| 2019 | 0.717 | 0.013 | 0.682 | 0.713 | 0.722 | 0.727 | 0.730 |

(c) Manufacturing

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.010 | 0.000 | 1.009 | 1.010 | 1.010 | 1.010 | 1.010 |
| 1997 | 1.000 | 0.001 | 0.999 | 1.000 | 1.000 | 1.001 | 1.002 |
| 1998 | 1.001 | 0.001 | 0.999 | 1.001 | 1.001 | 1.002 | 1.003 |
| 1999 | 1.002 | 0.002 | 0.998 | 1.000 | 1.002 | 1.003 | 1.005 |
| 2000 | 1.005 | 0.002 | 1.001 | 1.004 | 1.005 | 1.007 | 1.009 |
| 2001 | 1.018 | 0.002 | 1.014 | 1.017 | 1.018 | 1.019 | 1.022 |
| 2002 | 1.023 | 0.003 | 1.018 | 1.021 | 1.023 | 1.025 | 1.028 |
| 2003 | 1.012 | 0.004 | 1.003 | 1.009 | 1.012 | 1.015 | 1.021 |
| 2004 | 1.010 | 0.005 | 1.000 | 1.007 | 1.010 | 1.014 | 1.021 |
| 2005 | 0.999 | 0.005 | 0.989 | 0.996 | 0.999 | 1.003 | 1.010 |
| 2006 | 0.993 | 0.005 | 0.983 | 0.989 | 0.993 | 0.996 | 1.003 |
| 2007 | 0.984 | 0.006 | 0.971 | 0.979 | 0.984 | 0.988 | 0.997 |
| 2008 | 0.982 | 0.007 | 0.968 | 0.978 | 0.982 | 0.987 | 0.997 |
| 2009 | 0.989 | 0.005 | 0.979 | 0.986 | 0.989 | 0.992 | 0.999 |
| 2010 | 0.990 | 0.005 | 0.979 | 0.986 | 0.990 | 0.993 | 1.000 |
| 2011 | 0.992 | 0.005 | 0.981 | 0.988 | 0.992 | 0.995 | 1.002 |
| 2012 | 1.002 | 0.005 | 0.992 | 0.998 | 1.002 | 1.005 | 1.011 |
| 2013 | 1.003 | 0.004 | 0.995 | 1.001 | 1.003 | 1.006 | 1.011 |
| 2014 | 1.012 | 0.003 | 1.005 | 1.010 | 1.012 | 1.014 | 1.019 |
| 2015 | 1.024 | 0.002 | 1.020 | 1.023 | 1.024 | 1.025 | 1.028 |
| 2016 | 1.029 | 0.001 | 1.026 | 1.028 | 1.029 | 1.030 | 1.031 |
| 2017 | 1.029 | 0.001 | 1.027 | 1.028 | 1.029 | 1.029 | 1.031 |
| 2018 | 1.028 | 0.001 | 1.026 | 1.027 | 1.028 | 1.029 | 1.031 |
| 2019 | 1.026 | 0.001 | 1.024 | 1.026 | 1.026 | 1.027 | 1.029 |

(d) Electricity, Gas, Water and Waste Services

| Year | Mean | SD | Percentiles 2.5% | 25% | 50% | 75% | 97.5% |
|---|---|---|---|---|---|---|---|
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.006 | 0.000 | 1.005 | 1.006 | 1.006 | 1.006 | 1.006 |
| 1997 | 1.019 | 0.001 | 1.017 | 1.019 | 1.019 | 1.020 | 1.020 |
| 1998 | 1.016 | 0.002 | 1.011 | 1.016 | 1.017 | 1.018 | 1.018 |
| 1999 | 1.004 | 0.003 | 0.997 | 1.003 | 1.005 | 1.006 | 1.007 |
| 2000 | 0.998 | 0.003 | 0.990 | 0.997 | 0.999 | 1.001 | 1.001 |
| 2001 | 0.991 | 0.004 | 0.980 | 0.989 | 0.992 | 0.994 | 0.995 |
| 2002 | 0.980 | 0.004 | 0.969 | 0.979 | 0.981 | 0.983 | 0.984 |
| 2003 | 0.974 | 0.004 | 0.964 | 0.973 | 0.975 | 0.977 | 0.978 |
| 2004 | 0.961 | 0.005 | 0.949 | 0.960 | 0.963 | 0.964 | 0.965 |
| 2005 | 0.942 | 0.005 | 0.930 | 0.940 | 0.943 | 0.945 | 0.946 |
| 2006 | 0.929 | 0.005 | 0.916 | 0.927 | 0.930 | 0.932 | 0.934 |
| 2007 | 0.919 | 0.005 | 0.905 | 0.918 | 0.921 | 0.923 | 0.924 |
| 2008 | 0.897 | 0.005 | 0.882 | 0.895 | 0.898 | 0.900 | 0.902 |
| 2009 | 0.897 | 0.006 | 0.881 | 0.895 | 0.899 | 0.901 | 0.903 |
| 2010 | 0.879 | 0.006 | 0.864 | 0.877 | 0.881 | 0.883 | 0.885 |
| 2011 | 0.877 | 0.006 | 0.863 | 0.875 | 0.879 | 0.881 | 0.882 |
| 2012 | 0.864 | 0.005 | 0.850 | 0.862 | 0.866 | 0.868 | 0.869 |
| 2013 | 0.862 | 0.005 | 0.849 | 0.860 | 0.863 | 0.865 | 0.866 |
| 2014 | 0.850 | 0.004 | 0.839 | 0.849 | 0.852 | 0.853 | 0.854 |
| 2015 | 0.847 | 0.005 | 0.834 | 0.845 | 0.849 | 0.850 | 0.852 |
| 2016 | 0.853 | 0.006 | 0.838 | 0.851 | 0.855 | 0.857 | 0.859 |
| 2017 | 0.852 | 0.006 | 0.838 | 0.850 | 0.854 | 0.856 | 0.858 |
| 2018 | 0.849 | 0.006 | 0.834 | 0.847 | 0.850 | 0.853 | 0.854 |
| 2019 | 0.837 | 0.005 | 0.823 | 0.835 | 0.839 | 0.841 | 0.842 |

(e) Construction

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.998 | 0.000 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 |
| 1997 | 0.989 | 0.000 | 0.988 | 0.989 | 0.989 | 0.989 | 0.989 |
| 1998 | 1.001 | 0.000 | 1.000 | 1.001 | 1.001 | 1.002 | 1.002 |
| 1999 | 0.996 | 0.001 | 0.994 | 0.996 | 0.997 | 0.997 | 0.997 |
| 2000 | 0.992 | 0.001 | 0.990 | 0.992 | 0.993 | 0.993 | 0.993 |
| 2001 | 0.969 | 0.000 | 0.968 | 0.969 | 0.969 | 0.969 | 0.969 |
| 2002 | 1.002 | 0.001 | 1.001 | 1.002 | 1.002 | 1.003 | 1.003 |
| 2003 | 0.989 | 0.001 | 0.986 | 0.989 | 0.990 | 0.990 | 0.990 |
| 2004 | 0.977 | 0.002 | 0.972 | 0.976 | 0.977 | 0.978 | 0.978 |
| 2005 | 0.970 | 0.002 | 0.964 | 0.969 | 0.970 | 0.971 | 0.972 |
| 2006 | 0.967 | 0.002 | 0.961 | 0.966 | 0.967 | 0.968 | 0.969 |
| 2007 | 0.945 | 0.002 | 0.939 | 0.944 | 0.946 | 0.947 | 0.948 |
| 2008 | 0.934 | 0.003 | 0.926 | 0.933 | 0.934 | 0.936 | 0.936 |
| 2009 | 0.938 | 0.003 | 0.931 | 0.937 | 0.939 | 0.940 | 0.940 |
| 2010 | 0.933 | 0.003 | 0.926 | 0.932 | 0.934 | 0.935 | 0.936 |
| 2011 | 0.923 | 0.003 | 0.915 | 0.922 | 0.923 | 0.925 | 0.925 |
| 2012 | 0.941 | 0.003 | 0.933 | 0.940 | 0.942 | 0.944 | 0.944 |
| 2013 | 0.942 | 0.003 | 0.933 | 0.941 | 0.943 | 0.944 | 0.945 |
| 2014 | 0.949 | 0.003 | 0.940 | 0.948 | 0.950 | 0.952 | 0.953 |
| 2015 | 0.927 | 0.003 | 0.918 | 0.926 | 0.928 | 0.930 | 0.931 |
| 2016 | 0.910 | 0.004 | 0.900 | 0.909 | 0.911 | 0.913 | 0.913 |
| 2017 | 0.904 | 0.003 | 0.895 | 0.903 | 0.905 | 0.906 | 0.907 |
| 2018 | 0.900 | 0.004 | 0.890 | 0.898 | 0.901 | 0.902 | 0.903 |
| 2019 | 0.887 | 0.004 | 0.877 | 0.885 | 0.888 | 0.889 | 0.890 |

(f) Wholesale Trade

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.001 | 1.001 |
| 1997 | 1.011 | 0.000 | 1.010 | 1.011 | 1.011 | 1.011 | 1.011 |
| 1998 | 1.005 | 0.001 | 1.004 | 1.005 | 1.006 | 1.006 | 1.006 |
| 1999 | 0.972 | 0.001 | 0.969 | 0.971 | 0.972 | 0.972 | 0.973 |
| 2000 | 0.969 | 0.001 | 0.966 | 0.968 | 0.969 | 0.969 | 0.970 |
| 2001 | 0.983 | 0.001 | 0.980 | 0.982 | 0.983 | 0.983 | 0.984 |
| 2002 | 0.987 | 0.001 | 0.984 | 0.986 | 0.987 | 0.988 | 0.988 |
| 2003 | 0.991 | 0.002 | 0.987 | 0.990 | 0.991 | 0.992 | 0.992 |
| 2004 | 1.004 | 0.002 | 1.000 | 1.003 | 1.005 | 1.005 | 1.006 |
| 2005 | 0.982 | 0.002 | 0.977 | 0.981 | 0.983 | 0.984 | 0.984 |
| 2006 | 0.980 | 0.002 | 0.975 | 0.979 | 0.981 | 0.982 | 0.982 |
| 2007 | 0.954 | 0.002 | 0.948 | 0.953 | 0.955 | 0.955 | 0.956 |
| 2008 | 0.935 | 0.002 | 0.929 | 0.934 | 0.935 | 0.936 | 0.937 |
| 2009 | 0.929 | 0.002 | 0.923 | 0.928 | 0.930 | 0.931 | 0.932 |
| 2010 | 0.941 | 0.002 | 0.935 | 0.940 | 0.942 | 0.943 | 0.943 |
| 2011 | 0.914 | 0.003 | 0.907 | 0.913 | 0.915 | 0.916 | 0.917 |
| 2012 | 0.929 | 0.003 | 0.922 | 0.928 | 0.930 | 0.931 | 0.932 |
| 2013 | 0.936 | 0.003 | 0.929 | 0.935 | 0.937 | 0.938 | 0.939 |
| 2014 | 0.931 | 0.003 | 0.923 | 0.930 | 0.932 | 0.933 | 0.933 |
| 2015 | 0.932 | 0.003 | 0.924 | 0.931 | 0.932 | 0.934 | 0.934 |
| 2016 | 0.954 | 0.003 | 0.946 | 0.952 | 0.954 | 0.956 | 0.956 |
| 2017 | 0.954 | 0.003 | 0.946 | 0.953 | 0.955 | 0.957 | 0.957 |
| 2018 | 0.959 | 0.003 | 0.951 | 0.958 | 0.960 | 0.962 | 0.962 |
| 2019 | 0.946 | 0.003 | 0.938 | 0.945 | 0.947 | 0.948 | 0.949 |

(g) Retail Trade

| Year | Mean | SD | Percentiles 2.5% | 25% | 50% | 75% | 97.5% |
|---|---|---|---|---|---|---|---|
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.001 | 0.000 | 1.000 | 1.000 | 1.001 | 1.001 | 1.001 |
| 1997 | 0.982 | 0.000 | 0.981 | 0.982 | 0.982 | 0.982 | 0.982 |
| 1998 | 0.979 | 0.000 | 0.978 | 0.979 | 0.979 | 0.980 | 0.980 |
| 1999 | 0.969 | 0.001 | 0.967 | 0.969 | 0.969 | 0.970 | 0.970 |
| 2000 | 0.962 | 0.001 | 0.960 | 0.962 | 0.962 | 0.962 | 0.963 |
| 2001 | 0.974 | 0.001 | 0.971 | 0.973 | 0.974 | 0.974 | 0.975 |
| 2002 | 0.987 | 0.001 | 0.984 | 0.986 | 0.987 | 0.988 | 0.988 |
| 2003 | 0.990 | 0.001 | 0.986 | 0.989 | 0.990 | 0.990 | 0.991 |
| 2004 | 0.995 | 0.001 | 0.991 | 0.994 | 0.995 | 0.996 | 0.996 |
| 2005 | 0.971 | 0.002 | 0.966 | 0.970 | 0.972 | 0.972 | 0.973 |
| 2006 | 0.965 | 0.002 | 0.961 | 0.965 | 0.966 | 0.966 | 0.967 |
| 2007 | 0.966 | 0.002 | 0.961 | 0.965 | 0.967 | 0.967 | 0.968 |
| 2008 | 0.968 | 0.002 | 0.963 | 0.968 | 0.969 | 0.970 | 0.970 |
| 2009 | 0.934 | 0.002 | 0.927 | 0.933 | 0.934 | 0.935 | 0.936 |
| 2010 | 0.932 | 0.002 | 0.926 | 0.932 | 0.933 | 0.934 | 0.935 |
| 2011 | 0.919 | 0.003 | 0.912 | 0.918 | 0.920 | 0.921 | 0.921 |
| 2012 | 0.936 | 0.003 | 0.929 | 0.935 | 0.937 | 0.938 | 0.938 |
| 2013 | 0.935 | 0.003 | 0.927 | 0.934 | 0.936 | 0.937 | 0.938 |
| 2014 | 0.945 | 0.003 | 0.937 | 0.944 | 0.946 | 0.947 | 0.947 |
| 2015 | 0.945 | 0.003 | 0.937 | 0.944 | 0.946 | 0.947 | 0.948 |
| 2016 | 0.949 | 0.003 | 0.941 | 0.948 | 0.950 | 0.951 | 0.952 |
| 2017 | 0.944 | 0.003 | 0.935 | 0.943 | 0.945 | 0.946 | 0.947 |
| 2018 | 0.947 | 0.003 | 0.939 | 0.946 | 0.948 | 0.950 | 0.950 |
| 2019 | 0.933 | 0.003 | 0.924 | 0.932 | 0.934 | 0.935 | 0.936 |

(h) Accommodation and Food Services

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.992 | 0.001 | 0.991 | 0.992 | 0.992 | 0.992 | 0.993 |
| 1997 | 0.998 | 0.001 | 0.996 | 0.997 | 0.998 | 0.998 | 0.999 |
| 1998 | 0.992 | 0.001 | 0.990 | 0.991 | 0.992 | 0.993 | 0.995 |
| 1999 | 0.997 | 0.003 | 0.992 | 0.996 | 0.997 | 0.999 | 1.002 |
| 2000 | 0.978 | 0.003 | 0.972 | 0.976 | 0.978 | 0.981 | 0.985 |
| 2001 | 0.992 | 0.003 | 0.987 | 0.990 | 0.992 | 0.994 | 0.997 |
| 2002 | 1.010 | 0.003 | 1.005 | 1.009 | 1.010 | 1.012 | 1.015 |
| 2003 | 1.012 | 0.003 | 1.007 | 1.011 | 1.013 | 1.014 | 1.018 |
| 2004 | 1.011 | 0.003 | 1.005 | 1.009 | 1.012 | 1.014 | 1.018 |
| 2005 | 1.014 | 0.004 | 1.007 | 1.012 | 1.014 | 1.017 | 1.021 |
| 2006 | 1.029 | 0.004 | 1.022 | 1.027 | 1.029 | 1.031 | 1.036 |
| 2007 | 1.039 | 0.004 | 1.032 | 1.037 | 1.039 | 1.042 | 1.047 |
| 2008 | 1.031 | 0.004 | 1.024 | 1.028 | 1.031 | 1.033 | 1.038 |
| 2009 | 1.014 | 0.004 | 1.006 | 1.011 | 1.014 | 1.016 | 1.021 |
| 2010 | 0.996 | 0.004 | 0.989 | 0.993 | 0.996 | 0.998 | 1.002 |
| 2011 | 1.003 | 0.004 | 0.995 | 1.000 | 1.003 | 1.006 | 1.011 |
| 2012 | 1.007 | 0.004 | 0.999 | 1.004 | 1.007 | 1.009 | 1.014 |
| 2013 | 0.989 | 0.004 | 0.981 | 0.986 | 0.989 | 0.992 | 0.997 |
| 2014 | 0.985 | 0.005 | 0.975 | 0.982 | 0.985 | 0.988 | 0.994 |
| 2015 | 0.989 | 0.005 | 0.979 | 0.986 | 0.989 | 0.993 | 0.999 |
| 2016 | 0.993 | 0.005 | 0.982 | 0.989 | 0.993 | 0.997 | 1.004 |
| 2017 | 0.990 | 0.005 | 0.979 | 0.986 | 0.990 | 0.994 | 1.001 |
| 2018 | 0.982 | 0.006 | 0.971 | 0.978 | 0.982 | 0.985 | 0.993 |
| 2019 | 0.986 | 0.005 | 0.975 | 0.982 | 0.986 | 0.989 | 0.997 |

(i) Transport, Postal and Warehousing

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.994 | 0.001 | 0.993 | 0.994 | 0.994 | 0.995 | 0.995 |
| 1997 | 1.026 | 0.001 | 1.023 | 1.025 | 1.026 | 1.027 | 1.027 |
| 1998 | 1.044 | 0.001 | 1.042 | 1.044 | 1.044 | 1.045 | 1.045 |
| 1999 | 1.045 | 0.001 | 1.043 | 1.045 | 1.046 | 1.046 | 1.047 |
| 2000 | 1.031 | 0.001 | 1.028 | 1.030 | 1.032 | 1.032 | 1.033 |
| 2001 | 1.034 | 0.002 | 1.030 | 1.033 | 1.035 | 1.036 | 1.037 |
| 2002 | 1.051 | 0.002 | 1.046 | 1.050 | 1.051 | 1.052 | 1.053 |
| 2003 | 1.058 | 0.002 | 1.053 | 1.057 | 1.059 | 1.060 | 1.061 |
| 2004 | 1.045 | 0.003 | 1.038 | 1.044 | 1.046 | 1.047 | 1.049 |
| 2005 | 1.030 | 0.003 | 1.022 | 1.028 | 1.031 | 1.033 | 1.034 |
| 2006 | 1.017 | 0.004 | 1.008 | 1.015 | 1.018 | 1.020 | 1.022 |
| 2007 | 1.019 | 0.004 | 1.009 | 1.017 | 1.020 | 1.022 | 1.025 |
| 2008 | 1.015 | 0.005 | 1.003 | 1.012 | 1.016 | 1.018 | 1.021 |
| 2009 | 1.008 | 0.005 | 0.997 | 1.006 | 1.009 | 1.012 | 1.014 |
| 2010 | 0.998 | 0.005 | 0.987 | 0.996 | 0.999 | 1.002 | 1.004 |
| 2011 | 0.989 | 0.005 | 0.976 | 0.986 | 0.990 | 0.993 | 0.995 |
| 2012 | 0.993 | 0.006 | 0.980 | 0.990 | 0.994 | 0.997 | 1.000 |
| 2013 | 0.992 | 0.006 | 0.978 | 0.989 | 0.994 | 0.997 | 1.000 |
| 2014 | 0.984 | 0.006 | 0.970 | 0.980 | 0.985 | 0.988 | 0.991 |
| 2015 | 0.980 | 0.006 | 0.965 | 0.976 | 0.981 | 0.984 | 0.987 |
| 2016 | 0.975 | 0.006 | 0.960 | 0.971 | 0.976 | 0.979 | 0.982 |
| 2017 | 0.975 | 0.006 | 0.960 | 0.971 | 0.976 | 0.979 | 0.982 |
| 2018 | 0.956 | 0.006 | 0.940 | 0.952 | 0.957 | 0.961 | 0.964 |
| 2019 | 0.945 | 0.007 | 0.929 | 0.941 | 0.946 | 0.950 | 0.953 |

(j) Information, Media and Telecommunications

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.958 | 0.000 | 0.958 | 0.958 | 0.959 | 0.959 | 0.959 |
| 1997 | 0.951 | 0.001 | 0.950 | 0.951 | 0.952 | 0.952 | 0.952 |
| 1998 | 0.975 | 0.001 | 0.972 | 0.974 | 0.975 | 0.975 | 0.976 |
| 1999 | 0.975 | 0.001 | 0.971 | 0.974 | 0.975 | 0.976 | 0.976 |
| 2000 | 0.955 | 0.002 | 0.950 | 0.954 | 0.955 | 0.956 | 0.956 |
| 2001 | 0.941 | 0.002 | 0.936 | 0.940 | 0.941 | 0.942 | 0.942 |
| 2002 | 0.940 | 0.002 | 0.934 | 0.939 | 0.941 | 0.941 | 0.942 |
| 2003 | 0.951 | 0.002 | 0.945 | 0.950 | 0.952 | 0.953 | 0.953 |
| 2004 | 0.939 | 0.003 | 0.930 | 0.937 | 0.940 | 0.941 | 0.941 |
| 2005 | 0.903 | 0.004 | 0.893 | 0.902 | 0.904 | 0.905 | 0.906 |
| 2006 | 0.884 | 0.004 | 0.874 | 0.883 | 0.886 | 0.887 | 0.888 |
| 2007 | 0.886 | 0.004 | 0.875 | 0.885 | 0.888 | 0.889 | 0.890 |
| 2008 | 0.888 | 0.004 | 0.876 | 0.886 | 0.889 | 0.891 | 0.892 |
| 2009 | 0.883 | 0.004 | 0.870 | 0.881 | 0.884 | 0.886 | 0.887 |
| 2010 | 0.879 | 0.005 | 0.866 | 0.877 | 0.880 | 0.882 | 0.883 |
| 2011 | 0.867 | 0.005 | 0.853 | 0.865 | 0.868 | 0.870 | 0.871 |
| 2012 | 0.852 | 0.005 | 0.839 | 0.850 | 0.854 | 0.856 | 0.857 |
| 2013 | 0.851 | 0.005 | 0.837 | 0.849 | 0.853 | 0.855 | 0.856 |
| 2014 | 0.858 | 0.005 | 0.844 | 0.857 | 0.860 | 0.862 | 0.863 |
| 2015 | 0.873 | 0.006 | 0.858 | 0.871 | 0.875 | 0.877 | 0.879 |
| 2016 | 0.874 | 0.006 | 0.858 | 0.872 | 0.876 | 0.878 | 0.880 |
| 2017 | 0.865 | 0.006 | 0.848 | 0.863 | 0.867 | 0.869 | 0.871 |
| 2018 | 0.874 | 0.006 | 0.856 | 0.871 | 0.876 | 0.878 | 0.880 |
| 2019 | 0.881 | 0.006 | 0.864 | 0.878 | 0.883 | 0.885 | 0.887 |

(k) Financial and Insurance Services

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.997 | 0.000 | 0.996 | 0.997 | 0.997 | 0.997 | 0.997 |
| 1997 | 1.027 | 0.001 | 1.026 | 1.027 | 1.027 | 1.028 | 1.029 |
| 1998 | 1.031 | 0.001 | 1.028 | 1.030 | 1.031 | 1.031 | 1.033 |
| 1999 | 1.019 | 0.002 | 1.016 | 1.018 | 1.019 | 1.020 | 1.023 |
| 2000 | 1.029 | 0.002 | 1.025 | 1.028 | 1.029 | 1.031 | 1.033 |
| 2001 | 1.004 | 0.002 | 1.000 | 1.003 | 1.004 | 1.006 | 1.008 |
| 2002 | 1.010 | 0.002 | 1.006 | 1.009 | 1.010 | 1.012 | 1.015 |
| 2003 | 1.000 | 0.003 | 0.995 | 0.998 | 1.000 | 1.002 | 1.005 |
| 2004 | 1.005 | 0.003 | 0.999 | 1.003 | 1.005 | 1.007 | 1.010 |
| 2005 | 0.985 | 0.003 | 0.979 | 0.983 | 0.985 | 0.987 | 0.992 |
| 2006 | 0.978 | 0.003 | 0.972 | 0.976 | 0.978 | 0.981 | 0.985 |
| 2007 | 0.993 | 0.004 | 0.985 | 0.990 | 0.993 | 0.995 | 1.000 |
| 2008 | 0.991 | 0.004 | 0.982 | 0.988 | 0.991 | 0.993 | 0.999 |
| 2009 | 0.981 | 0.004 | 0.972 | 0.978 | 0.981 | 0.984 | 0.990 |
| 2010 | 0.992 | 0.004 | 0.983 | 0.989 | 0.992 | 0.995 | 1.001 |
| 2011 | 0.979 | 0.005 | 0.970 | 0.976 | 0.979 | 0.983 | 0.989 |
| 2012 | 0.995 | 0.005 | 0.985 | 0.992 | 0.995 | 0.998 | 1.005 |
| 2013 | 1.025 | 0.005 | 1.015 | 1.021 | 1.025 | 1.028 | 1.034 |
| 2014 | 1.012 | 0.005 | 1.002 | 1.009 | 1.012 | 1.016 | 1.023 |
| 2015 | 1.012 | 0.005 | 1.002 | 1.009 | 1.012 | 1.016 | 1.023 |
| 2016 | 1.015 | 0.005 | 1.004 | 1.011 | 1.015 | 1.019 | 1.026 |
| 2017 | 1.008 | 0.006 | 0.997 | 1.004 | 1.008 | 1.012 | 1.019 |
| 2018 | 0.995 | 0.006 | 0.984 | 0.992 | 0.995 | 0.999 | 1.006 |
| 2019 | 0.986 | 0.006 | 0.975 | 0.983 | 0.986 | 0.990 | 0.998 |

(l) Rental, Hiring and Real Estate Services

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 1.008 | 0.002 | 1.004 | 1.007 | 1.008 | 1.009 | 1.009 |
| 1997 | 0.973 | 0.003 | 0.966 | 0.972 | 0.974 | 0.975 | 0.976 |
| 1998 | 0.955 | 0.004 | 0.944 | 0.954 | 0.957 | 0.958 | 0.959 |
| 1999 | 0.924 | 0.005 | 0.911 | 0.922 | 0.926 | 0.927 | 0.929 |
| 2000 | 0.904 | 0.006 | 0.888 | 0.902 | 0.906 | 0.908 | 0.909 |
| 2001 | 0.870 | 0.006 | 0.855 | 0.868 | 0.872 | 0.874 | 0.875 |
| 2002 | 0.867 | 0.007 | 0.849 | 0.865 | 0.869 | 0.872 | 0.873 |
| 2003 | 0.838 | 0.009 | 0.815 | 0.835 | 0.841 | 0.845 | 0.847 |
| 2004 | 0.803 | 0.009 | 0.781 | 0.800 | 0.806 | 0.809 | 0.811 |
| 2005 | 0.765 | 0.009 | 0.741 | 0.762 | 0.768 | 0.771 | 0.774 |
| 2006 | 0.754 | 0.009 | 0.729 | 0.750 | 0.757 | 0.760 | 0.762 |
| 2007 | 0.711 | 0.009 | 0.688 | 0.708 | 0.714 | 0.717 | 0.719 |
| 2008 | 0.677 | 0.009 | 0.653 | 0.674 | 0.680 | 0.683 | 0.685 |
| 2009 | 0.684 | 0.009 | 0.660 | 0.681 | 0.687 | 0.691 | 0.693 |
| 2010 | 0.681 | 0.009 | 0.657 | 0.678 | 0.684 | 0.688 | 0.690 |
| 2011 | 0.658 | 0.010 | 0.632 | 0.655 | 0.661 | 0.665 | 0.668 |
| 2012 | 0.656 | 0.011 | 0.627 | 0.652 | 0.659 | 0.664 | 0.666 |
| 2013 | 0.670 | 0.011 | 0.641 | 0.666 | 0.674 | 0.678 | 0.681 |
| 2014 | 0.673 | 0.013 | 0.639 | 0.668 | 0.677 | 0.682 | 0.685 |
| 2015 | 0.663 | 0.013 | 0.629 | 0.658 | 0.667 | 0.672 | 0.675 |
| 2016 | 0.671 | 0.014 | 0.635 | 0.666 | 0.676 | 0.681 | 0.684 |
| 2017 | 0.670 | 0.015 | 0.632 | 0.665 | 0.675 | 0.680 | 0.684 |
| 2018 | 0.659 | 0.015 | 0.621 | 0.653 | 0.663 | 0.669 | 0.672 |
| 2019 | 0.658 | 0.015 | 0.619 | 0.652 | 0.662 | 0.668 | 0.672 |

(m) Professional, Scientific and Technical Services

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.964 | 0.000 | 0.963 | 0.963 | 0.964 | 0.964 | 0.964 |
| 1997 | 0.940 | 0.001 | 0.937 | 0.940 | 0.940 | 0.941 | 0.941 |
| 1998 | 0.950 | 0.002 | 0.946 | 0.949 | 0.950 | 0.951 | 0.951 |
| 1999 | 0.923 | 0.003 | 0.915 | 0.922 | 0.924 | 0.925 | 0.925 |
| 2000 | 0.903 | 0.003 | 0.895 | 0.902 | 0.904 | 0.905 | 0.906 |
| 2001 | 0.895 | 0.003 | 0.887 | 0.894 | 0.896 | 0.898 | 0.898 |
| 2002 | 0.889 | 0.003 | 0.880 | 0.888 | 0.890 | 0.892 | 0.893 |
| 2003 | 0.881 | 0.004 | 0.872 | 0.880 | 0.882 | 0.883 | 0.884 |
| 2004 | 0.875 | 0.004 | 0.864 | 0.873 | 0.876 | 0.877 | 0.878 |
| 2005 | 0.848 | 0.004 | 0.836 | 0.846 | 0.849 | 0.851 | 0.852 |
| 2006 | 0.837 | 0.005 | 0.825 | 0.835 | 0.839 | 0.840 | 0.842 |
| 2007 | 0.823 | 0.005 | 0.810 | 0.821 | 0.825 | 0.827 | 0.828 |
| 2008 | 0.814 | 0.005 | 0.800 | 0.812 | 0.815 | 0.817 | 0.818 |
| 2009 | 0.810 | 0.005 | 0.796 | 0.808 | 0.812 | 0.814 | 0.815 |
| 2010 | 0.836 | 0.006 | 0.820 | 0.834 | 0.838 | 0.840 | 0.841 |
| 2011 | 0.850 | 0.006 | 0.833 | 0.848 | 0.852 | 0.855 | 0.856 |
| 2012 | 0.852 | 0.007 | 0.834 | 0.850 | 0.855 | 0.857 | 0.859 |
| 2013 | 0.848 | 0.007 | 0.829 | 0.845 | 0.850 | 0.853 | 0.855 |
| 2014 | 0.843 | 0.007 | 0.824 | 0.840 | 0.845 | 0.848 | 0.850 |
| 2015 | 0.849 | 0.008 | 0.829 | 0.846 | 0.852 | 0.855 | 0.857 |
| 2016 | 0.830 | 0.008 | 0.809 | 0.827 | 0.832 | 0.835 | 0.837 |
| 2017 | 0.840 | 0.008 | 0.819 | 0.837 | 0.843 | 0.846 | 0.848 |
| 2018 | 0.830 | 0.009 | 0.808 | 0.827 | 0.833 | 0.836 | 0.839 |
| 2019 | 0.816 | 0.009 | 0.793 | 0.812 | 0.818 | 0.822 | 0.824 |

(n) Administrative and Support Services

| Year | Mean | SD | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | | | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.972 | 0.001 | 0.970 | 0.972 | 0.972 | 0.972 | 0.972 |
| 1997 | 1.094 | 0.013 | 1.082 | 1.085 | 1.090 | 1.098 | 1.128 |
| 1998 | 1.473 | 0.057 | 1.421 | 1.434 | 1.456 | 1.492 | 1.626 |
| 1999 | 1.171 | 0.023 | 1.150 | 1.156 | 1.164 | 1.179 | 1.231 |
| 2000 | 0.866 | 0.004 | 0.855 | 0.864 | 0.867 | 0.868 | 0.869 |
| 2001 | 0.855 | 0.003 | 0.848 | 0.854 | 0.856 | 0.857 | 0.858 |
| 2002 | 0.836 | 0.004 | 0.826 | 0.835 | 0.837 | 0.839 | 0.840 |
| 2003 | 0.786 | 0.007 | 0.768 | 0.784 | 0.788 | 0.791 | 0.793 |
| 2004 | 0.754 | 0.007 | 0.736 | 0.751 | 0.756 | 0.758 | 0.760 |
| 2005 | 0.780 | 0.004 | 0.769 | 0.778 | 0.781 | 0.783 | 0.784 |
| 2006 | 0.757 | 0.008 | 0.737 | 0.754 | 0.759 | 0.762 | 0.764 |
| 2007 | 0.693 | 0.011 | 0.663 | 0.689 | 0.697 | 0.701 | 0.704 |
| 2008 | 0.674 | 0.013 | 0.639 | 0.669 | 0.678 | 0.683 | 0.687 |
| 2009 | 0.696 | 0.012 | 0.664 | 0.692 | 0.700 | 0.705 | 0.708 |
| 2010 | 0.681 | 0.011 | 0.653 | 0.677 | 0.685 | 0.689 | 0.692 |
| 2011 | 0.659 | 0.012 | 0.626 | 0.654 | 0.663 | 0.668 | 0.671 |
| 2012 | 0.677 | 0.008 | 0.654 | 0.674 | 0.679 | 0.683 | 0.685 |
| 2013 | 0.686 | 0.007 | 0.668 | 0.683 | 0.688 | 0.691 | 0.692 |
| 2014 | 0.692 | 0.008 | 0.672 | 0.689 | 0.694 | 0.698 | 0.699 |
| 2015 | 0.662 | 0.011 | 0.634 | 0.658 | 0.665 | 0.670 | 0.673 |
| 2016 | 0.687 | 0.009 | 0.662 | 0.683 | 0.689 | 0.693 | 0.696 |
| 2017 | 0.699 | 0.009 | 0.674 | 0.695 | 0.702 | 0.705 | 0.708 |
| 2018 | 0.792 | 0.002 | 0.788 | 0.792 | 0.793 | 0.794 | 0.794 |
| 2019 | 0.749 | 0.008 | 0.728 | 0.746 | 0.751 | 0.754 | 0.756 |

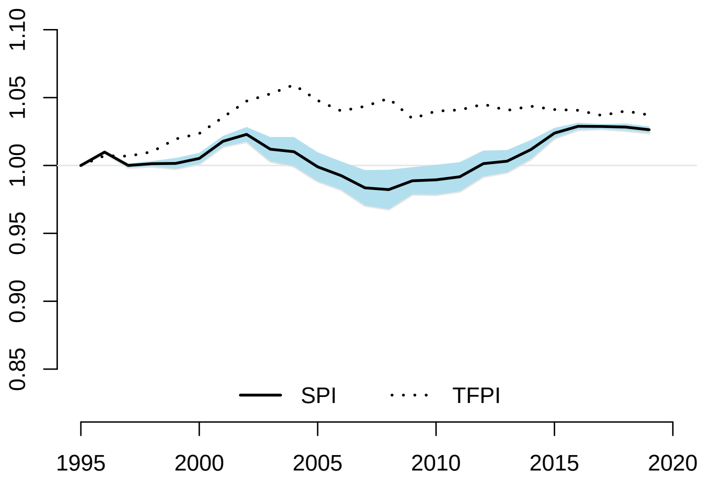
(r) Arts and Recreation Services

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.968 | 0.000 | 0.968 | 0.968 | 0.968 | 0.968 | 0.968 |
| 1997 | 0.974 | 0.000 | 0.974 | 0.974 | 0.974 | 0.974 | 0.974 |
| 1998 | 0.912 | 0.002 | 0.908 | 0.912 | 0.913 | 0.913 | 0.914 |
| 1999 | 0.880 | 0.003 | 0.872 | 0.879 | 0.880 | 0.882 | 0.882 |
| 2000 | 0.819 | 0.005 | 0.805 | 0.817 | 0.820 | 0.822 | 0.824 |
| 2001 | 0.760 | 0.008 | 0.739 | 0.757 | 0.762 | 0.765 | 0.767 |
| 2002 | 0.749 | 0.008 | 0.728 | 0.746 | 0.751 | 0.754 | 0.756 |
| 2003 | 0.729 | 0.009 | 0.706 | 0.726 | 0.731 | 0.735 | 0.737 |
| 2004 | 0.719 | 0.010 | 0.694 | 0.716 | 0.722 | 0.726 | 0.729 |
| 2005 | 0.740 | 0.010 | 0.714 | 0.736 | 0.743 | 0.747 | 0.749 |
| 2006 | 0.774 | 0.008 | 0.752 | 0.771 | 0.777 | 0.780 | 0.782 |
| 2007 | 0.798 | 0.008 | 0.777 | 0.795 | 0.801 | 0.804 | 0.806 |
| 2008 | 0.779 | 0.009 | 0.756 | 0.775 | 0.781 | 0.785 | 0.787 |
| 2009 | 0.805 | 0.009 | 0.782 | 0.802 | 0.808 | 0.811 | 0.813 |
| 2010 | 0.776 | 0.009 | 0.752 | 0.773 | 0.779 | 0.783 | 0.785 |
| 2011 | 0.745 | 0.010 | 0.719 | 0.742 | 0.748 | 0.752 | 0.754 |
| 2012 | 0.711 | 0.011 | 0.682 | 0.707 | 0.715 | 0.719 | 0.722 |
| 2013 | 0.700 | 0.011 | 0.671 | 0.696 | 0.703 | 0.708 | 0.710 |
| 2014 | 0.707 | 0.011 | 0.678 | 0.703 | 0.710 | 0.715 | 0.717 |
| 2015 | 0.706 | 0.011 | 0.677 | 0.702 | 0.710 | 0.714 | 0.717 |
| 2016 | 0.687 | 0.012 | 0.656 | 0.683 | 0.691 | 0.695 | 0.698 |
| 2017 | 0.698 | 0.012 | 0.667 | 0.694 | 0.702 | 0.706 | 0.709 |
| 2018 | 0.694 | 0.012 | 0.663 | 0.690 | 0.698 | 0.702 | 0.705 |
| 2019 | 0.710 | 0.011 | 0.680 | 0.706 | 0.713 | 0.718 | 0.721 |

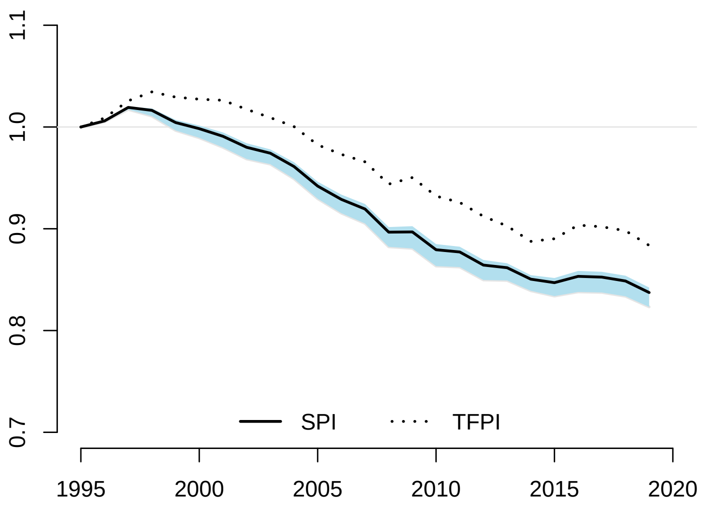
(s) Other Services

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| Year | Mean | SD | 2.5% | 25% | 50% | 75% | 97.5% |
| 1995 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1996 | 0.984 | 0.000 | 0.983 | 0.984 | 0.984 | 0.984 | 0.984 |
| 1997 | 0.996 | 0.001 | 0.994 | 0.995 | 0.996 | 0.996 | 0.996 |
| 1998 | 0.977 | 0.001 | 0.975 | 0.977 | 0.977 | 0.978 | 0.978 |
| 1999 | 0.985 | 0.001 | 0.981 | 0.984 | 0.985 | 0.985 | 0.986 |
| 2000 | 0.978 | 0.001 | 0.974 | 0.977 | 0.978 | 0.978 | 0.979 |
| 2001 | 1.018 | 0.001 | 1.015 | 1.018 | 1.019 | 1.019 | 1.020 |
| 2002 | 1.001 | 0.001 | 0.997 | 1.000 | 1.001 | 1.002 | 1.002 |
| 2003 | 1.014 | 0.001 | 1.010 | 1.013 | 1.014 | 1.015 | 1.015 |
| 2004 | 1.024 | 0.002 | 1.020 | 1.023 | 1.024 | 1.025 | 1.025 |
| 2005 | 1.003 | 0.002 | 0.997 | 1.002 | 1.003 | 1.004 | 1.004 |
| 2006 | 0.985 | 0.002 | 0.980 | 0.985 | 0.986 | 0.987 | 0.987 |
| 2007 | 0.986 | 0.002 | 0.980 | 0.985 | 0.987 | 0.988 | 0.988 |
| 2008 | 0.954 | 0.002 | 0.948 | 0.953 | 0.955 | 0.956 | 0.956 |
| 2009 | 0.960 | 0.002 | 0.953 | 0.959 | 0.960 | 0.961 | 0.962 |
| 2010 | 0.962 | 0.003 | 0.955 | 0.961 | 0.963 | 0.964 | 0.964 |
| 2011 | 0.953 | 0.003 | 0.945 | 0.952 | 0.954 | 0.955 | 0.955 |
| 2012 | 0.967 | 0.003 | 0.958 | 0.966 | 0.968 | 0.969 | 0.970 |
| 2013 | 0.949 | 0.003 | 0.941 | 0.948 | 0.950 | 0.951 | 0.951 |
| 2014 | 0.928 | 0.003 | 0.920 | 0.927 | 0.929 | 0.930 | 0.931 |
| 2015 | 0.924 | 0.003 | 0.915 | 0.923 | 0.925 | 0.926 | 0.927 |
| 2016 | 0.928 | 0.003 | 0.919 | 0.926 | 0.929 | 0.930 | 0.931 |
| 2017 | 0.915 | 0.003 | 0.906 | 0.914 | 0.916 | 0.917 | 0.918 |
| 2018 | 0.900 | 0.003 | 0.891 | 0.899 | 0.901 | 0.902 | 0.903 |
| 2019 | 0.902 | 0.003 | 0.892 | 0.901 | 0.903 | 0.904 | 0.905 |

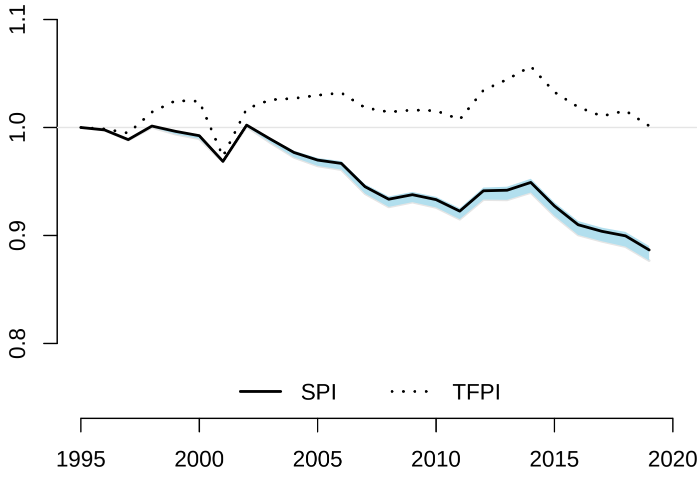
## Appendix E
## Sustainable Productivity Index Numbers



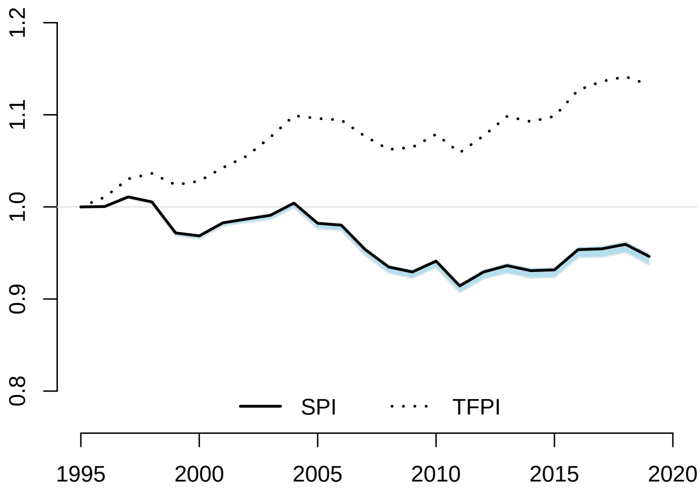

(c) Manufacturing



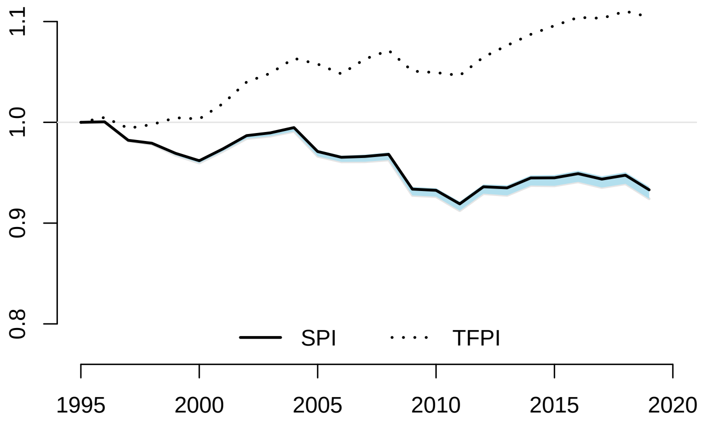

(d) Electricity, Gas, Water and Waste Services



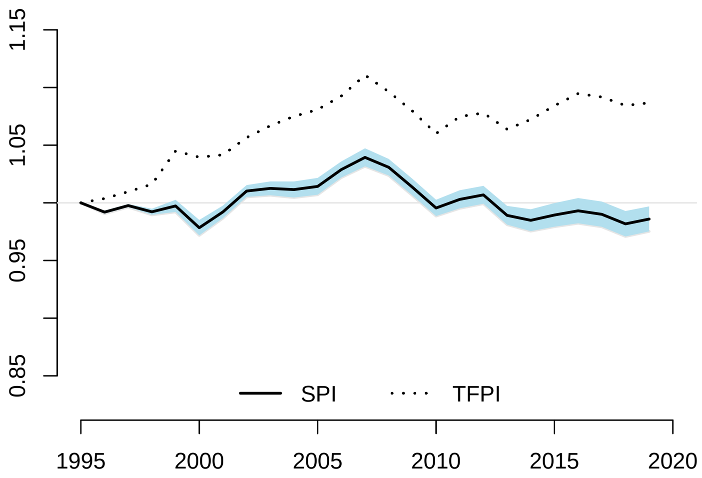

(e) Construction



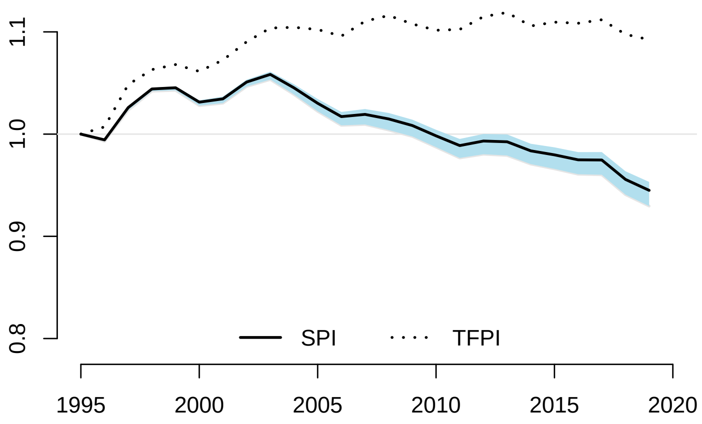

(f) Wholesale Trade



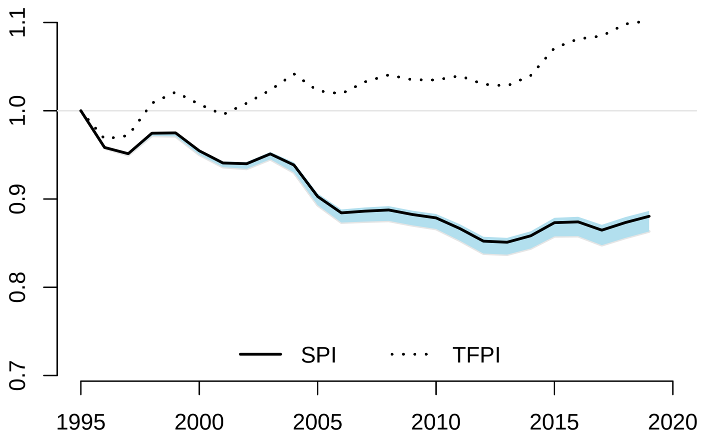

(g) Retail Trade



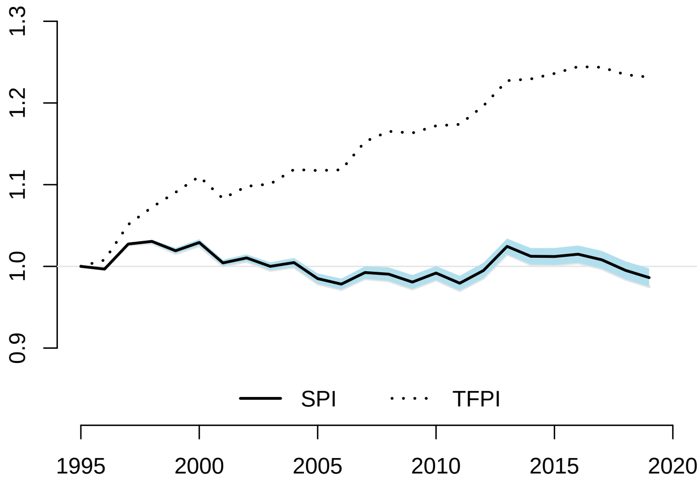

(h) Accommodation and Food Services



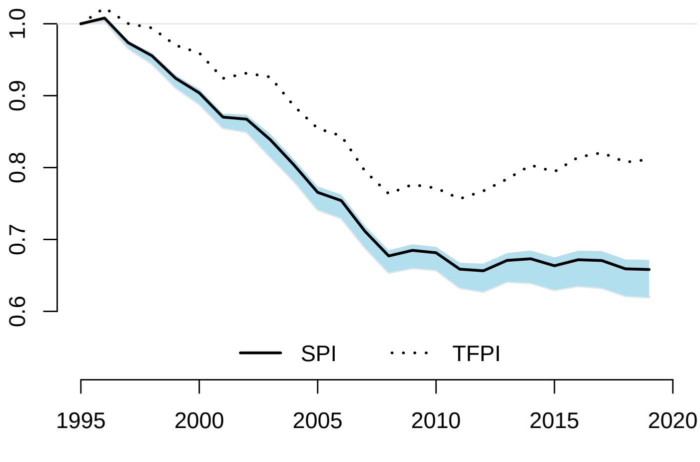

(i) Transport, Postal and Warehousing



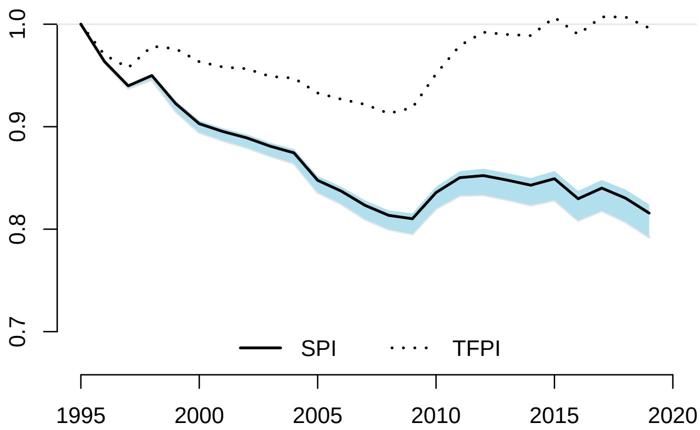

(j) Information, Media and Telecommunications



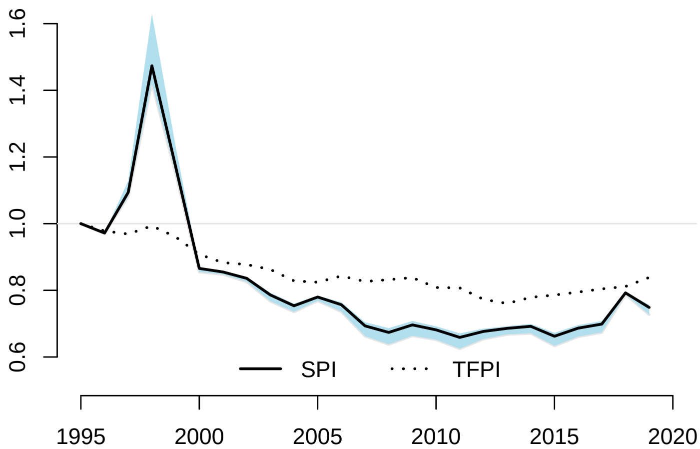

(k) Financial and Insurance Services



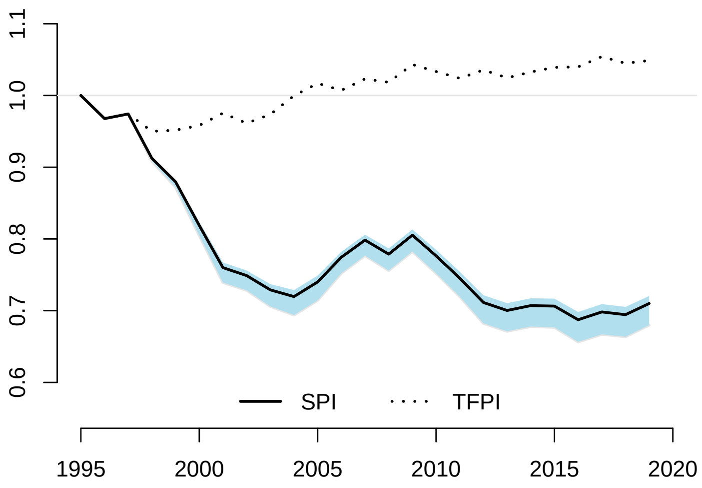

(l) Rental, Hiring and Real Estate Services



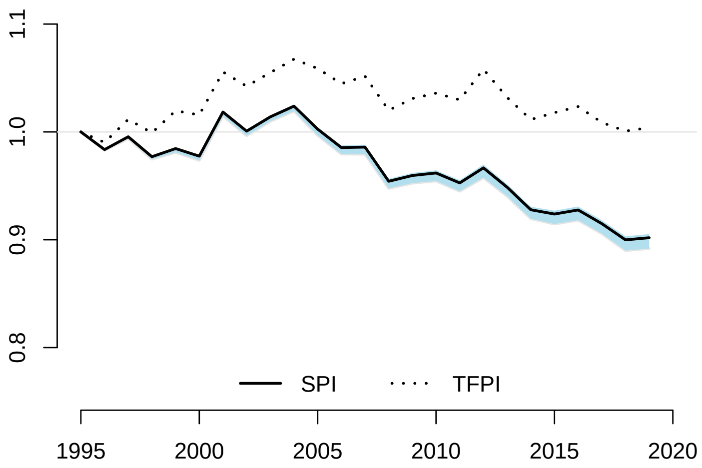

(m) Professional, Scientific and Technical Services




(n) Administrative and Support Services




(r) Arts and Recreation Services




(s) Other Services