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Estimating the Revenue Efficiency of Public Service Providers in the Presence of Demand Constraints

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Abstract

Evaluating the performance of public service providers is often complicated by the fact that they must choose input levels before demands for their services are known. We consider an even more complicated situation in which service providers have no opportunity to directly influence demands. This means that their predetermined inputs may be more than what is required to meet realised demands. In such cases, conventional measures of revenue efficiency used in the operational research literature will generally mis-classify rational and efficient managers as inefficient. We develop a more appropriate measure of revenue efficiency that accounts for exogenously-determined demands. We explain how data envelopment analysis (DEA) methods can be used to estimate our measure, and also how they can be used to assess the consequences (if any) of providers having to choose input levels before demands are known. The methodology is applied to hospital and health service (HHS) providers in Queensland (Australia). We obtain estimates of revenue efficiency that are quite different from estimates obtained using a conventional approach. Our results also indicate that HHS providers were not disadvantaged by having to choose input levels before demands were known.

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1 Introduction

A public service is any service provided to members of a community by government. Examples include education, electricity, fire, hospital, military, paramedic, police, public transport and waste management services. Evaluating the performance of public service providers is complicated by the fact that demands for services are generally unknown at the time input decisions are made. The managers of public hospitals, for example, must make choices about the types of equipment to purchase and staff to employ without knowing the numbers and types of patients who will need treatment. Models of input choice in the face of demand uncertainty have been developed by De Witte and Geys (2011) and H. N. Nguyen and O'Donnell (2021). Those authors develop measures of how well managers choose their inputs. This paper goes a step further and develops measures of how well managers *use* their inputs.

If inputs have already been chosen, then it is common to assume that managers use them to maximise revenue. Associated measures of how well they do that include various output- and revenue-oriented measures of efficiency. In the operational research (OR) literature on public service efficiency, there have been many studies that take the output-oriented approach; most of them use measures of output-oriented technical efficiency, which is largely due to the unavailability of data on output prices (e.g., Bradley, Johnes, & Millington, 2001; Johnes, 2006; Ancarani, Di Mauro, & Giammanco, 2009; Mitropoulos, Talias, & Mitropoulos, 2015). Unfortunately, conventional measures of efficiency ignore the fact that public service managers are often unable to influence demands for services. The managers of public hospitals, for example, cannot easily influence the number of people who come through the door requiring emergency surgery. This means that their predetermined inputs may be more than what is required to meet realised demands. In such cases, conventional measures of revenue efficiency (RE) will generally mis-classify rational and efficient managers as inefficient.

This paper makes four contributions to the literature. First, we develop a measure of RE that accounts for the fact that public service outputs cannot be stored and cannot exceed exogenously-determined demands (i.e., output levels are constrained). This is not generally done in the OR literature. Second, we develop data envelopment analysis (DEA) estimators that can be used to estimate our measure. Third, we develop DEA estimators that can be used to measure the revenue effects (if any) of managers being required to choose inputs before demands for services are known. Finally, our empirical contribution is to use our methodology to assess the performance of hospital and health service (HHS) managers in

Queensland, Australia.

The structure of the paper is as follows. Section 2 explains how period-and-environment-specific output distance functions can be used to represent the production technologies that exist in a given period. These particular output distance functions are more general (and realistic) than the output distance functions that are found elsewhere in the economics or OR literature because they explicitly recognise the importance of environmental variables (i.e., variables that are physically involved in the production process but never within the control of managers). Section 3 describes the revenue maximisation problems faced by managers in the presence of demand constraints. With a view to our empirical application, we allow for the possibility that output prices depend on the outputs supplied by the firm (i.e., that firms are price-setters in output markets). In Section 4 we look at how well managers solve their revenue maximisation problems. Among other things, we define a measure of RE and explain why it cannot generally be decomposed into measures of technical and allocative efficiency, as is common in the OR literature. In Section 5 we define a revenue-oriented demand uncertainty effect (RDUE). The RDUE is a measure of the revenue effects (if any) of managers being required to choose inputs before demands for services are known. In Section 6 we list the assumptions underpinning standard DEA estimators. We then develop new DEA estimators for estimating our measure of RE and the RDUE. In Section 7 we describe the Queensland HHS data used in the empirical work. Among other things, we describe how observed outputs and measures of "service delivery effectiveness" have been used to construct measures of demands for services. In Section 8 we present the empirical results. Among other things, we find that our estimates of RE are quite different from the estimates obtained using a standard approach that ignores demand constraints. In Section 9 we summarise the paper and discuss some of the shortcomings of our work.

2 Production Technologies

A production technology is a technique, method or system for transforming inputs into outputs (O'Donnell, 2018, p.2). Production technologies can be represented by various types of output and input sets. In this paper, we focus on period-and-environment-specific output sets. A period-and-environment-specific output set is a set of outputs that can be produced using given inputs and the technologies that are available in a given period in a given production environment. For example, the set of outputs that can be produced using the input vector x

and the technologies that are available in period t in a production environment characterised by the vector z is:

$$P^t(x, z) = \{q : x \text{ can produce } q \text{ in period } t \text{ in environment } z\}. \quad (1)$$

This set can be found in O'Donnell (2018, eq. 2.1). If there is no technical or environmental change, then it reduces to the more common output set defined by Shephard (1970, p. 179).

In this paper, we make two assumptions that are common in the OR literature: (i) output sets are bounded, and (ii) outputs are strongly disposable. If these assumptions are true, then $P^t(x, z)$ can be represented by the following period-and-environment-specific output distance function:

$$D_O^t(x, q, z) = \inf\{\rho > 0 : q/\rho \in P^t(x, z)\}. \quad (2)$$

This function gives the reciprocal of the largest factor by which it is possible to scale up q when using x in period t in an environment characterised by z. It is equal to the output distance function defined by O'Donnell (2018, eq. 2.8). If there is no technical or environmental change, then it reduces to the more common output distance function defined by Färe and Primont (1995, eq. 2.1.7). By construction, $D_O^t(x, q, z)$ is nonnegative and linearly homogeneous in outputs. The strong disposability assumption means it is also nondecreasing in outputs. This is one of several assumptions that underpin the DEA linear programs described in Section 5 below.

3 Revenue Maximisation

We assume that characteristics of production environments are known and that inputs have already been chosen by the time output decisions are made. In these cases, it is common in the OR literature to assume that firm managers use their predetermined inputs to maximise revenues (see, e.g., Keh, Chu, & Xu, 2006). In a departure from common practice, this paper accounts for the fact that outputs cannot be stored and cannot exceed exogenously-determined demands. The exact mathematical form of the revenue maximisation problem depends on whether firms are price-setters or price-takers in output markets. We consider both cases. For clarity, let us now introduce firm and time subscripts into the notation so that, for example, x_{it} now represents the input vector of firm i in period t.

First, if firms are price-setters in output markets, then the revenue maximisation problem of manager i in period t is:

$$\max_{q}\{p(q, s_{it})'q : q \leq d_{it},\ D_O^t(x_{it}, q, z_{it}) \leq 1\}, \tag{3}$$

where s_{it} is a vector of nonnegative exogenous variables, d_{it} is a vector of exogenous demands, and $p(q, s_{it})$ is a vector of nonnegative inverse demand functions. There may be more than one output vector that solves this problem. Let $q_{it}^* \equiv q^t(x_{it}, s_{it}, d_{it}, z_{it})$ denote one such vector. Note that if consumer demand is sufficiently weak and/or the demand constraints are sufficiently restrictive, then it is possible that this output vector will lie inside the boundary of the output set (i.e., $D_O^t(x_{it}, q_{it}^*, z_{it}) < 1$). In any event, the associated maximum revenue is $R^t(x_{it}, s_{it}, d_{it}, z_{it}) = p(q_{it}^*, s_{it})'q_{it}^*$. Also note that if we completely ignore the demand constraints, then (3) reduces to:

$$\max_{q}\{p(q, s_{it})'q : D_O^t(x_{it}, q, z_{it}) \leq 1\}. \tag{4}$$

This relatively simple problem can be found in O'Donnell (2018, eq. 4.9). Again, there may be more than one output vector that solves this problem. Let $\ddot{q}_{it} \equiv q^t(x_{it}, s_{it}, z_{it})$ denote one such vector. Again, if consumer demand is sufficiently weak, then it is possible that this output vector will lie inside the boundary of the output set (i.e., $D_O^t(x_{it}, \ddot{q}_{it}, z_{it}) < 1$); see O'Donnell (2016, p.331). In any event, the associated maximum revenue is $R^t(x_{it}, s_{it}, z_{it}) = p(\ddot{q}_{it}, s_{it})'\ddot{q}_{it}$.

Second, if firms are price-takers in output markets, then $\partial p(q, s_{it})/\partial q = 0$ and problem (3) reduces to:

$$\max_{q}\{p_{it}'q : q \leq d_{it},\ D_O^t(x_{it}, q, z_{it}) \leq 1\}, \tag{5}$$

where $p_{it} \equiv p(s_{it})$ is an exogenous output price. Again, there may be more than one output vector that solves this problem. In a slight abuse of notation, let $q_{it}^* \equiv q^t(x_{it}, p_{it}, d_{it}, z_{it})$ denote one such vector. Again, if the demand constraints are sufficiently restrictive, then it is possible that this output vector will lie inside the boundary of the output set (i.e., $D_O^t(x_{it}, q_{it}^*, z_{it}) < 1$). In any event, the associated maximum revenue is $R^t(x_{it}, p_{it}, d_{it}, z_{it}) = p_{it}'q_{it}^*$. Also note that if we completely ignore the demand constraints, then (5) reduces to:

$$\max_{q}\{p_{it}'q : D_O^t(x_{it}, q, z_{it}) \leq 1\}. \tag{6}$$

This relatively simple problem can be found in O'Donnell (2018, eq. 4.12). If there is no technical progress or environmental change, then it reduces to an even simpler problem that can be found in Sickles and Zelenyuk (2019, p.50). Again, there may be more than one output vector that solves (6). In another slight abuse of notation, let $\ddot{q}_{it} \equiv q^t(x_{it}, p_{it}, z_{it})$ denote one such vector. This revenue-maximising vector will always lie on the boundary of the output set (i.e., $D_O^t(x_{it}, \ddot{q}_{it}, z_{it}) = 1$). The associated maximum revenue is $R^t(x_{it}, p_{it}, z_{it}) = p_{it}'\ddot{q}_{it}$.

To illustrate the relationship between problems (5) and (6), Figure 1 depicts the revenue maximisation problem for firm i in period t in the simple case where there are only two outputs and the demand constraint is not binding. In this figure, the (piecewise) frontier passing through point V represents the boundary of the set of outputs that can be produced using the input vector x_{it} in period t in an environment characterised by z_{it}. The dashed lines are iso-revenue lines with slopes of $-p_{1it}/p_{2it}$. Point D represents the demand constraints. Point A represents the observed output vector of the firm. The associated observed revenue is $R_{it} = p_{it}'q_{it}$. Point V represents the output vector that maximises revenue. The associated maximum revenue is $R^t(x_{it}, p_{it}, d_{it}, z_{it})$. Observe that revenue at point A is lower than the maximum revenue, i.e., $R_{it} < R^t(x_{it}, p_{it}, d_{it}, z_{it})$. Also observe that the demand constraint $q \leq d_{it}$ is not binding (i.e., point V is the revenue-maximising point with or without the demand constraint). In this case, the solution to problem (5) is the same as the solution to problem (6).

To further illustrate the relationship between problems (5) and (6) , Figure 2 depicts the revenue maximisation problem in the case where the demand constraint *is* binding. In this figure, the lighter frontier passing through point V is the same frontier that was depicted earlier in Figure 1: it represents the boundary of the set of outputs that can be produced using the input vector x_{it} in period t in an environment characterised by z_{it}. Point D now represents a set of demand constraints that are more restrictive than those depicted in Figure 1. The dark frontier passing through point K is the boundary of the set of outputs that can be produced using the input vector x_{it} in period t in an environment characterised by z_{it} when the demand constraint $q \leq d_{it}$ is binding. Point K represents the output vector that maximises revenue. The associated maximum revenue is $R^t(x_{it}, p_{it}, d_{it}, z_{it})$. Observe that

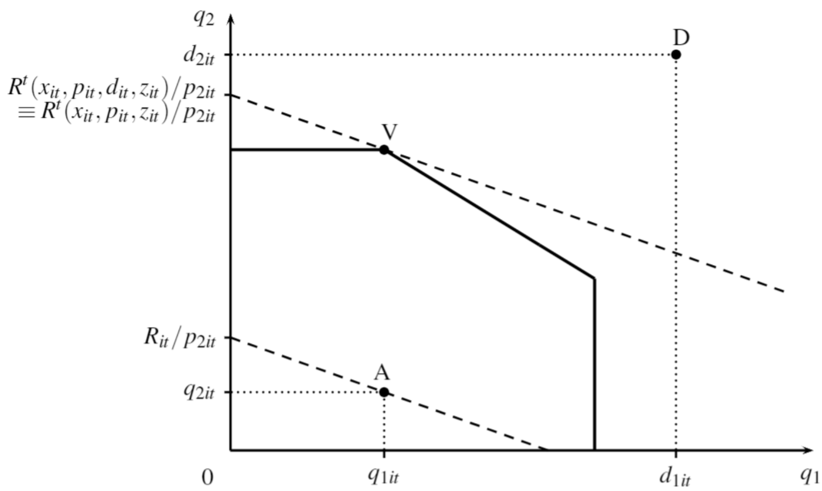

Figure 1: Revenue maximisation problem with non-binding demand constraint

$R^t(x_{it}, p_{it}, d_{it}, z_{it}) < R^t(x_{it}, p_{it}, z_{it})$ (i.e., the binding demand constraint has reduced revenue). In this case, the solution to problem (5) differs from the solution to problem (6) .

4 Measures of Efficiency

Measures of efficiency can be viewed as *ex post* measures of how well managers have solved different optimisation problems (O'Donnell, 2018). This section defines four measures of efficiency associated with the four revenue maximisation problems described in Section 3. It also defines a measure of technical efficiency and a measure of allocative efficiency. All of these measures take values in the closed unit interval.

First, if firms are price-setters in output markets, then the revenue efficiency (RE) of manager i in period t is defined as

$$RE^t(x_{it}, q_{it}, s_{it}, d_{it}, z_{it}) = \frac{R_{it}}{R^t(x_{it}, s_{it}, d_{it}, z_{it})}, \tag{7}$$

where $R_{it} = p(q_{it}, s_{it})'q_{it}$ is the observed revenue of the firm and $R^t(x_{it}, s_{it}, d_{it}, z_{it})$ is the maximum revenue that can be obtained using x_{it} in period t in an environment characterised by z_{it} when the demand market is characterised by s_{it} and outputs cannot exceed d_{it}. This measure of efficiency can be viewed as a measure of how well the manager has solved problem

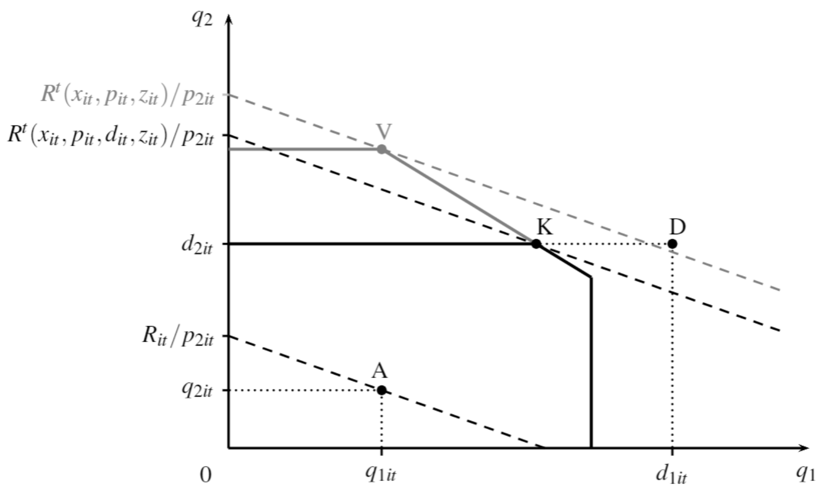

Figure 2: Revenue maximisation problem with binding demand constraint

(3).

Second, if firms are price-setters in output markets and we naively ignore the demand constraints, then what we call the naive revenue efficiency (NRE) of manager i in period t is defined as

$$NRE^t(x_{it}, q_{it}, s_{it}, z_{it}) = \frac{R_{it}}{R^t(x_{it}, s_{it}, z_{it})}, \tag{8}$$

where $R^t(x_{it}, s_{it}, z_{it})$ is the maximum revenue that can be obtained using x_{it} in period t in an environment characterised by z_{it} when the demand market is characterised by s_{it}. This measure of RE can be found in O'Donnell (2018, eq. 5.14). It can be viewed as a measure of how well the manager has solved problem (4).

Third, if firms are price-takers in output markets, then the RE of manager i in period t is defined as

$$RE^t(x_{it}, q_{it}, p_{it}, d_{it}, z_{it}) = \frac{R_{it}}{R^t(x_{it}, p_{it}, d_{it}, z_{it})}, \tag{9}$$

where $R_{it} = p_{it}'q_{it}$ is the observed revenue of the firm and $R^t(x_{it}, p_{it}, d_{it}, z_{it})$ is the maximum revenue that can be obtained using x_{it} in period t in an environment characterised by z_{it} when the output price vector is p_{it} and outputs cannot exceed d_{it}. This measure of efficiency can be viewed as a measure of how well the manager has solved problem (5). That problem was depicted earlier in Figures 1 and 2. Observe that the iso-revenue lines passing through the revenue maximising points in those figures (i.e., point V in Figure 1 and point K in Figure 2) have an intercept of $R^t(x_{it}, p_{it}, d_{it}, z_{it})/p_{2it}$, and the iso-revenue line passing through point A has an intercept of R_{it}/p_{2it}. The measure of RE defined by (9) is simply the ratio of these intercepts.

Fourth, if firms are price-takers in output markets and we ignore the demand constraints, then the NRE of manager i in period t is defined as

$$NRE^t(x_{it}, p_{it}, q_{it}, z_{it}) = \frac{R_{it}}{R^t(x_{it}, p_{it}, z_{it})}, \tag{10}$$

where $R^t(x_{it}, p_{it}, z_{it})$ is the maximum revenue that can be obtained when using x_{it} in period t in a production environment characterised by z_{it}. This measure of efficiency can be found in O'Donnell (2018, eq. 5.15). It can be viewed as a measure of how well the manager has solved problem (6). That problem was depicted earlier in Figure 2. Observe that the iso-revenue lines passing through points A and V in that figure have intercepts of R_{it}/p_{2it} and $R^t(x_{it}, p_{it}, z_{it})/p_{2it}$. The measure of RE defined by (10) is the ratio of these intercepts.

Finally, it is common to decompose measures of RE into two components: a measure of output-oriented technical efficiency (OTE) and a measure of output-oriented allocative efficiency (OAE). This decomposition is only meaningful if revenue-maximising points lie on the boundary of the output set; if revenue-maximising points lie inside that boundary, then measures of OAE may lie outside the unit interval and will have no meaningful interpretation. Recall that if consumer demand is sufficiently weak and/or the demand constraints are sufficiently restrictive, then the solutions to problems (3), (4) and (5) may lie inside the boundary of the output set. Consequently, it is generally only meaningful to decompose the NRE measure defined by (10). The OTE component is simply the value of the output distance function:

$$OTE^t(x_{it}, q_{it}, z_{it}) = D^t_O(x_{it}, q_{it}, z_{it}). \tag{11}$$

The OAE component is an output-oriented measure of economies of output substitution (i.e., the benefits associated with changing the output mix) and is defined as:

$$OAE^t(x_{it}, p_{it}, q_{it}, z_{it}) = \frac{R_{it}/D_O^t(x_{it}, q_{it}, z_{it})}{R^t(x_{it}, p_{it}, z_{it})}. \tag{12}$$

This is equivalent to the measure of OAE defined in O'Donnell (2018, eq. 5.17). Equations (10) to (12) imply that OAE can also be computed as

$$OAE^t(x_{it}, p_{it}, q_{it}, z_{it}) = NRE^t(x_{it}, p_{it}, q_{it}, z_{it})/OTE^t(x_{it}, q_{it}, z_{it}).$$

Thus, OAE can be viewed as the component of NRE that remains after accounting for technical inefficiency.

5 The Effect of Demand Uncertainty

This paper assumes managers use predetermined inputs to maximise revenues. This is the second stage of a two-stage optimisation problem. The first stage involves choosing inputs in the face of uncertainty about demands for services, and the second stage involves choosing outputs to maximise revenues. H. N. Nguyen and O'Donnell (2021) considered the first stage. They assumed that public service managers choose inputs to minimise the cost of producing output targets (e.g., minimum service levels, or predicted maximum demands). Importantly, if output targets are less than realised demands, then managers who have successfully solved their first-stage cost minimisation problems will have chosen fewer inputs than are required to meet those demands. On the other hand, if output targets are greater than observed outputs, then managers who have successfully solved their first-stage cost minimisation problems will have chosen inputs that are more than enough to produce those outputs. This section considers the revenue effects (if any) associated with public service managers having been forced to choose inputs before demands for services are known.

Let $\hat{x}_{it}^*$ (resp. x_{it}^*) denote the input vector that minimises the cost of producing the output targets (resp. observed outputs) of firm i in period t. If firms are price-setters in output markets, then the so-called revenue-oriented demand uncertainty effect (RDUE) for firm i in period t is defined as

$$RDUE^t(\hat{x}_{it}^*, x_{it}^*, s_{it}, d_{it}, z_{it}) = \frac{R^t(\hat{x}_{it}^*, s_{it}, d_{it}, z_{it})}{R^t(x_{it}^*, s_{it}, d_{it}, z_{it})}, \tag{13}$$

where $R^t(\hat{x}_{it}^*, s_{it}, d_{it}, z_{it})$ is the maximum revenue that can be obtained using $\hat{x}_{it}^*$ in period t in an environment characterised by z_{it} when the demand market is characterised by s_{it} and outputs cannot exceed d_{it}. On the other hand, if firms are price-takers in output markets, then the RDUE for firm i in period t is defined as

$$RDUE^t(\hat{x}_{it}^*, x_{it}^*, p_{it}, d_{it}, z_{it}) = \frac{R^t(\hat{x}_{it}^*, p_{it}, d_{it}, z_{it})}{R^t(x_{it}^*, p_{it}, d_{it}, z_{it})}, \tag{14}$$

where $R^t(\hat{x}_{it}^*, s_{it}, d_{it}, z_{it})$ is the maximum revenue that can be obtained using $\hat{x}_{it}^*$ in period t in an environment characterised by z_{it} when output prices are given by p_{it} and outputs cannot exceed d_{it}. Unlike the measures of efficiency defined in Section 4, the measures of RDUE defined by (13) and (14) do not always lie in the unit interval. They will generally take values greater than (resp. less than) one whenever output targets are greater than (resp. less than) observed outputs.

6 DEA Models

This section discusses DEA models (or estimators) for estimating the measures of efficiency defined in Section 4 and the uncertainty effects defined in Section 5. The DEA models that are most widely used in the OR literature are implicitly underpinned by the following assumptions: (i) all relevant variables are observed and measured without error; (ii) production frontiers are locally (or piecewise) linear; (iii) inputs, outputs and environmental variables are strongly disposable; and (iv) production possibilities sets are convex (e.g., O'Donnell, 2018, p.219). If these assumptions are true, then measures of efficiency and RDUEs can be estimated by solving various mathematical programs. These programs envelop scatterplots of technically-feasible input-output combinations in such a way that estimated production frontiers do not violate any of assumptions (i) to (v). An unusual feature of our empirical application is that managers are legally required to choose inputs that are capable of producing agreed output targets. With this in mind, this paper follows H. N. Nguyen and O'Donnell (2021) and assumes that the set of technically-feasible input-output combinations includes all combinations of observed inputs and observed outputs as well as all combinations of observed inputs and output targets.

Estimating the measure of RE defined by (7) involves estimating $R^t(x_{it}, s_{it}, d_{it}, z_{it})$. If there are I firms in the dataset, then a DEA estimator that allows for technical progress is the following:

$$\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p(q,s_{it})'q: \; q \leq d_{it}, \; q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}),$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}+\theta_{hr}x_{hr}) \leq x_{it}, \; \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it}, \tag{15}$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1, \; \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\},$$

where $\hat{q}_{hr}$ denotes the vector of output targets for firm h in period r. This nonlinear program (NLP) is quite unlike any DEA model we find in the OR literature. It can be solved using standard NLP packages (e.g., the *NlcOptim* package in *R*). The value of the objective function at the optimum is an estimate of $R^t(x_{it}, s_{it}, d_{it}, z_{it})$. Dividing observed revenue by this estimate yields an estimate of $RE^t(x_{it}, q_{it}, s_{it}, d_{it}, z_{it})$.

Estimating the measures of NRE defined by (8) involves estimating $R^t(x_{it}, s_{it}, z_{it})$. Arguably the easiest way to do this is to simply delete the demand constraint $q \leq d_{it}$ from (15). The NLP then becomes

$$\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p(q,s_{it})'q: \; q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}),$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}+\theta_{hr}x_{hr}) \leq x_{it}, \; \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it}, \tag{16}$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1, \; \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.$$

The value of the objective function at the optimum is an estimate of $R^t(x_{it}, s_{it}, z_{it})$. Dividing observed revenue by this estimate yields an estimate of $NRE^t(x_{it}, q_{it}, s_{it}, z_{it})$.

Estimating the measure of RE defined by (9) involves estimating $R^t(x_{it}, p_{it}, d_{it}, z_{it})$. A

DEA estimator that allows for technical progress is the following:

$$\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p_{it}'q: \ q \leq d_{it}, \ q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}),$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}+\theta_{hr}x_{hr}) \leq x_{it}, \ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it}, \tag{17}$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1, \ \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.$$

This is now a linear program (LP) that can be solved using standard LP packages (e.g., the *lpSolve* package in *R*). The value of the objective function at the optimum is an estimate of $R^t(x_{it}, p_{it}, d_{it}, z_{it})$. Dividing observed revenue by this estimate yields an estimate of $RE^t(x_{it}, q_{it}, p_{it}, d_{it}, z_{it})$.

Estimating the measure of NRE defined by (10) involves estimating $R^t(x_{it}, p_{it}, z_{it})$. Again, we can estimate this value by simply deleting the demand constraint $q \leq d_{it}$ from (17). The LP then becomes:

$$\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p_{it}'q: \ q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}),$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}+\theta_{hr}x_{hr}) \leq x_{it}, \ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it}, \tag{18}$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1, \ \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.$$

This is an LP that can be solved using standard DEA packages (e.g., the *Benchmarking* package in *R*). The value of the objective function at the optimum is an estimate of $R^t(x_{it}, p_{it}, z_{it})$. Dividing observed revenue by this estimate yields an estimate of $NRE^t(x_{it}, p_{it}, q_{it}, z_{it})$.

It is worth noting at this point that the naive estimates of RE obtained by solving (16) and (18) are still not as naive as estimates that are normally found in the OR literature. In that literature, estimates of revenue efficiency are normally computed using data on observed inputs and outputs only (see, e.g., Sahoo, Mehdiloozad, & Tone, 2014, eq. 17). An estimate of $R^t(x_{it}, s_{it}, z_{it})$, for example, would normally be obtained by solving the following NLP:

$$\begin{aligned}\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p(q, s_{it})'q :\ & q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}q_{hr},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}x_{hr} \leq x_{it}, \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}z_{hr} \leq z_{it},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr} = 1,\ \lambda_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{19}$$

Similarly, an estimate of $R^t(x_{it}, p_{it}, z_{it})$ would normally be obtained by solving an LP that can be found in O'Donnell (2018, eq. 6.11):

$$\begin{aligned}\max_{q,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{p_{it}'q :\ & q \leq \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}q_{hr},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}x_{hr} \leq x_{it}, \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}z_{hr} \leq z_{it},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr} = 1,\ \lambda_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{20}$$

Dividing observed revenue by these alternative estimates of maximum revenue yield estimates of what we call "super-naive" revenue efficiency (SNRE).

Estimating the measures of OTE and OAE defined by (11) and (12) involves estimating $D_O^t(x_{it}, q_{it}, z_{it})$. A DEA estimator that can be used for this purpose is the following:

$$\begin{aligned}\max_{\mu,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{\mu :\ & \mu q_{it} \leq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}), \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}+\theta_{hr}x_{hr}) \leq x_{it},\ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it}, \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1,\ \lambda_{hr}, \theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{21}$$

The value of μ at the optimum is an estimate of the reciprocal of $OTE^t(x_{it}, q_{it}, z_{it})$. Dividing the estimate of $NRE^t(x_{it}, p_{it}, q_{it}, z_{it})$ by the estimate of $OTE^t(x_{it}, q_{it}, z_{it})$ yields an estimate of $OAE^t(x_{it}, p_{it}, q_{it}, z_{it})$

Estimating the measures of RDUE defined by (13) and (14) involves estimating $\hat{x}_{it}^*$ and x_{it}^*. In this paper, we follow H. N. Nguyen and O'Donnell (2021) and partition each input vector into a vector of predetermined (or fixed) inputs and a vector of variable inputs. To be specific, we let $\hat{x}_{it}^* = (x_{it}^f, \hat{x}_{it}^{v*})$ and $x_{it}^* = (x_{it}^f, x_{it}^{v*})$ where x_{it}^f is a vector of fixed inputs and $\hat{x}_{it}^{v*}$ and x_{it}^{v*} are vectors of variable inputs that solve the variable cost minimisation problems defined in H. N. Nguyen and O'Donnell (2021, eq. 3 and eq. 4). Estimates of these cost-minimising variable input vectors can be obtained by solving LPs (12) and (14) in H. N. Nguyen and O'Donnell (2021). Estimates of $R^t(\hat{x}_{it}^*, s_{it}, d_{it}, z_{it})$, $R^t(x_{it}^*, s_{it}, d_{it}, z_{it})$, $R^t(\hat{x}_{it}^*, p_{it}, d_{it}, z_{it})$ and $R^t(x_{it}^*, p_{it}, d_{it}, z_{it})$ can be obtained by replacing x_{it} in (15) and (17) with $\hat{x}_{it}^*$ and x_{it}^* as appropriate. Estimates of RDUE can then be obtained using (13) and (14).

Finally, the DEA LPs and NLPs described above allow production frontiers to exhibit variable returns to scale (VRS). In the efficiency literature, it is common to assume that production frontiers exhibit constant returns to scale (CRS). To impose this restriction, the right-hand sides of the constraints involving the environmental variables must be replaced with "ρz_{it}", and all instances of "$= 1$" must be replaced with "$\leq \rho$". For a rationale, see O'Donnell (2018, Section 6.2.3).

7 Data

The data comprises observations on $I = 16$ Queensland hospital and health services (HHSs) for the $T = 5$ financial years from 2012/13 to 2016/17. We have quantity data on one fixed input (x^f = beds and bed-alternatives), seven variable inputs (x_1^v = medical officers, x_2^v = nurses, x_3^v = other personal care staff, x_4^v = diagnostic and professional staff, x_5^v = administrative staff, x_6^v = domestic and other staff, x_7^v = non-labour inputs) and six outputs (q_1 = acute inpatient services, q_2 = outpatient services, q_3 = sub-acute care services, q_4 = emergency department services, q_5 = mental health services, q_6 = other interventions and procedures). These data have been used by, and are described in detail in, H. N. Nguyen and O'Donnell (2021). Importantly, the output quantities are measured in Queensland Weighted Activity Units (QWAUs); we assume that these measures account for variations in service quality insofar as they are designed to reflect service complexity and resource intensity. We also assume that the input measures account for variations in input quality to the extent that inputs of different quality have been treated as different inputs (e.g., that the nurses input has been constructed using an index procedure that treats registered nurses and enrolled nurses as different types of labour input).

This paper supplements the quantity data described above with observations on output prices and service demands. If the number of QWAUs is below an agreed threshold, then the price received by an HHS for each QWAU is a price that is common to all HHSs and is referred to as the Queensland Efficient Price (QEP). Data on QEPs was sourced from Service Level Agreements (SLAs) negotiated annually between HHS managers and the relevant government department, namely Queensland Health. Data on service demands were derived from Service Delivery Statements (SDSs) reported annually by Queensland Health: the demands for the first four outputs were computed by dividing the reported number of services delivered by the HHS by the corresponding percentage of services performed within recommended timeframes (i.e., services that were not performed with recommended time frames were considered as excess demand); the demand for the fifth output (mental health services) was computed by dividing the reported number of services by the rate of community follow-up within 1-7 days following discharge from an acute mental health inpatient unit; and the demand for the last output (other interventions and procedures) was constructed by dividing the reported number of services by the percentage of the Queensland population who engaged in levels of physical activity for health benefit. Unfortunately, the effectiveness measures reported in the SDSs were not an exact match for the acute inpatient and outpatient service categories. In this paper, the percentage of elective surgery patients treated on time was used as a measure of the percentage of acute inpatients treated on time, and the percentage of specialist outpatients treated on time was used as a measure of the percentage of all outpatients treated on time. Descriptive statistics for all variables are reported in Table 1. These statistics reveal that observed outputs generally fall short of the corresponding demands.

8 Results

This section reports DEA estimates of the measures of efficiency defined in Section 4 and the demand uncertainty effects defined in Section 5. The specific LPs and NLPs used to generate these results differed from the LPs and NLPs discussed in Section 6 in two ways: first, we had no observations on characteristics of the production environment, so all constraints involving environmental variables were removed; and, second, the data cover a period of only five years, so we felt it was not necessary to allow for technical progress. To formulate the objective functions in problems (15), (16) and (19), we needed to specify the vector of inverse demand functions. The inverse demand functions in this empirical application are unusual in two respects. First, even though there are six outputs, there is in fact only one inverse demand function; the reason for this is that all outputs are measured in QWAUs, and the price of one

Table 1: Descriptive Statistics

	Variable	Mean	St. Dev.	Min	Max
Input Quantities					
x^f	Beds & alternatives	684.55	605.70	64	2354
x_1^v	Medical officers	494.28	491.55	10.87	1836.24
x_2^v	Nurses	1475.83	1334.36	81.65	4961.12
x_3^v	Other personal care staff	84.06	96.77	0.01	395.61
x_4^v	Diagnostic & professionals	359.79	395.55	0.00	1457.78
x_5^v	Administrative staff	491.07	451.92	19.37	1859.83
x_6^v	Domestic & other staff	442.93	347.81	28.96	1621.26
x_7^v	Non-labour variable inputs	1869.87	1862.03	63.48	9496.35
Outputs Quantities					
q_1	Acute inpatients	57943.35	58702.99	1775	267791
q_2	Outpatients	14209.53	14810.45	453	72277
q_3	Sub-acute	5830.06	6002.38	177	24956
q_4	Emergency department	13579.61	10420.77	1037	39962
q_5	Mental health	8945.13	10874.08	30	55788
q_6	Interventions & prevention	6938.66	8284.98	0	32356
Output prices		4612.34	73.68	4558	4755.66
Demands for Service					
d_1	Acute inpatients	63167.14	64747.30	1975.52	279562.90
d_2	Outpatients	25575.15	29248.33	991.46	124913.01
d_3	Sub-acute	6395.76	6735.34	184.42	26246.34
d_4	Emergency department	19307.23	15960.87	1047.47	60530.30
d_5	Mental health	14045.21	17085.77	46.22	77667.23
d_6	Interventions & prevention	11784.02	14258.10	0.00	61513.31

QWAU of acute inpatient services, for example, is the same as the price of one QWAU of any other service. Second, the inverse demand function is a step function that depends on both the total of the observed outputs and the total of the output targets specified in the SLAs; this is because HHSs generally receive no funding for any outputs in excess of the total of the agreed output targets (see, e.g., Queensland Health, 2019). Mathematically, the price paid to

HHS i in period t for one QWAU is:

$$p(q_{it}, s_{it}) = \begin{cases} QEP_t & \text{if } \sum_n q_{nit} < \sum_n \hat{q}_{nit}, \\ QEP_t \times \sum_n \hat{q}_{nit} / \sum_n q_{nit} & \text{otherwise,} \end{cases} \tag{22}$$

where $s_{it} = (QEP_t, \hat{q}_{it})$ and QEP_t is the Queensland Efficient Price in period t.

Selected estimates of efficiency are reported in Table 2; results for all HHSs in all periods are reported in the Appendix. The estimates of RE, NRE and SNRE reported in columns A, B and C were obtained by solving problems (15), (16) and (19); the estimates of RE, NRE and SNRE reported in columns D, E and H were obtained by solving problems (17), (18) and (20); and the estimates of of OTE and OAE reported in columns F and G were obtained by solving problem (21).

Table 2: Selected Estimates of Efficiency

		Price setters			Price takers				
Period	HHS	(A) RE	(B) NRE	(C) SNRE	(D) RE	(E) NRE	(F) OTE	(G) OAE	(H) SNRE
1	6	0.9945	0.9945	0.9945	0.8227	0.7543	0.9436	0.7994	0.7543
1	10	0.9912	0.9456	1.0000	0.9912	0.9456	1.0000	0.9456	1.0000
2	11	0.9546	0.9546	0.9546	0.6858	0.4996	0.5725	0.8727	0.4996
3	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	13	1.0000	1.0000	1.0000	0.9956	0.9618	1.0000	0.9618	0.9618
4	10	0.9564	0.8772	0.8847	0.9564	0.8772	1.0000	0.8772	0.8847
5	1	1.0000	1.0000	1.0000	0.9163	0.8042	1.0000	0.8042	0.8042
5	16	1.0000	1.0000	1.0000	0.9931	0.9072	1.0000	0.9072	0.9072
min	1	0.8576	0.8576	0.8800	0.6858	0.4996	0.5725	0.7661	0.4996
mean		0.9928	0.9912	0.9938	0.9512	0.8906	0.9818	0.9064	0.8942
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The results reported in Table 2 (and the Appendix) seem plausible. The estimates reported in column A were obtained under our most realistic set of assumptions and so are our preferred estimates of RE. The fact that they are on average very close to one indicates that most HHS managers have done a good job of maximising revenue. The fact that most of the OTE estimates reported in column F are also close to one indicates that most HHSs operated close to the production frontier.

Several other things about Table 2 (and the Appendix) are noteworthy. First, the RE and OTE estimates reported in columns A and F indicate that there were several HHSs that operated on the production frontier but did not maximise revenue (i.e., OTE = 1 but RE < 1). This indicates that they did not choose a revenue-maximising output mix (i.e., they were allocatively inefficient). Recall that all outputs are measured in QWAUs, and the price of a QWAU of one type of service is the same as the price of a QWAU of any other type of service. This means that producing one less QWAU of one type of service and one more QWAU of another type of service will not change revenue. Our finding that some HHSs operated on the frontier but did not maximise revenue indicates that marginal rates of transformation (MRTs) (i.e., the rates that one QWAU of one type of service can be substituted for one QWAU of another type of service) differ from one. In turn, this indicates that QWAUs may not reflect service complexity and resource intensity as well as intended.

Second, the RE estimates obtained under the assumption that HHSs are price setters in output markets (i.e., the estimates reported in column A) are always greater than or equal to the estimates obtained under the (false) assumption that HHSs are price takers in output markets (i.e., the estimates reported in column D). This is because estimates of maximum revenue obtained by solving problem (15) are always less than or equal to estimates obtained by solving problem (17). In turn, this is due to the fact that the inverse demand function in problem (15) places a cap on maximum revenue (recall that HHSs are not paid for QWAUs in excess of targets written into SLAs).

Third, the RE estimates reported in columns A and D are always greater than or equal to the NRE estimates reported in columns B and E. This is because estimates of maximum revenue obtained by solving problems (15) and (17) are always less than or equal to estimates obtained by solving problems (16) and (18). In turn, this is because problems (15) and (17) contain demand constraints that place a cap on maximum revenue. HHS 10 is an example of an HHS that faced binding demand constraints in periods 1 and 4 (i.e., the demand for its services was less than the number of services it could provide using its predetermined inputs). This explains why, for this HHS in these periods, (a) the estimates of RE are greater than the estimates of NRE, and (b) the RE and NRE estimates obtained under the assumption that HHSs are price setters are equal to the estimates obtained under the assumption that HHSs are price takers (i.e., the inverse demand function did not play a role).

Finally, the NRE estimates reported in columns B and E are always less than or equal to the SNRE estimates reported in columns C and H. This is because the frontiers that are used to compute the NRE estimates envelop the frontiers that are used to compute the SNRE estimates. In turn, this is because twice as many data points are used to solve problems (16) and (18) as are used to solve problems (19) and (20).

Table 2 provides a somewhat incomplete picture of our different estimates of revenue efficiency. A slightly more complete picture is provided by the box-and-whisker plots in Figure 3. The three plots on the left-hand side summarise estimates of RE, NRE and SNRE that have been obtained under the assumption that HHSs are price setters in output markets; all three plots are qualitatively similar and indicate that most HHSs were fully revenue efficient. The three plots on the right-hand side summarise estimates of RE, NRE and SNRE that have been obtained under the assumption that HHSs are price takers in output markets; these three plots are also qualitatively similar but indicate that most HHSs were inefficient, albiet less so over time. These results tell us that (a) accounting for demand constraints and/or making use of observations on output targets (in addition to observed outputs) has relatively little impact on estimates of revenue efficiency, and (b) accounting for price setting behaviour (i.e., the fact that the prices HHSs receive for their outputs depends on the total number of those outputs) makes a huge difference. Conventional approaches to estimating revenue efficiency make no allowance for price-setting behaviour and will almost certainly mis-classify competent managers as being revenue inefficient.

The plot in the top left-hand panel of Figure 3 reveals that our preferred RE estimates (i.e., those obtained under the assumption that HHSs are price setters in output markets, and that they face demand constraints) are almost all equal to one. The lowest estimates are those for HHSs 3 and 13 in period 1. These two HHSs are located in rural and remote regions of Queensland and are the two smallest HHSs in the state (sorted by their total QWAUs). To better understand this finding, Figure 4 classifies our preferred RE estimates by size and location. It indicates that location, rather than size, is the characteristic that matters. This finding is consistent with the findings of other studies (not necessarily in Queensland) that have used output-oriented models to measure HHS performance but have ignored demand constraints (e.g., Paul, 2002; B. H. Nguyen & Zelenyuk, 2021; Andrews, 2020).

Elsewhere in the literature, high efficiency scores of the type presented in the top left-hand panel of Figure 3 are sometimes attributed to overfitting. Overfitting tends to occur in situations where there is little theory to guide variable selection, and where the number of observations in the sample is low relative to the number of variables in the model. We do not

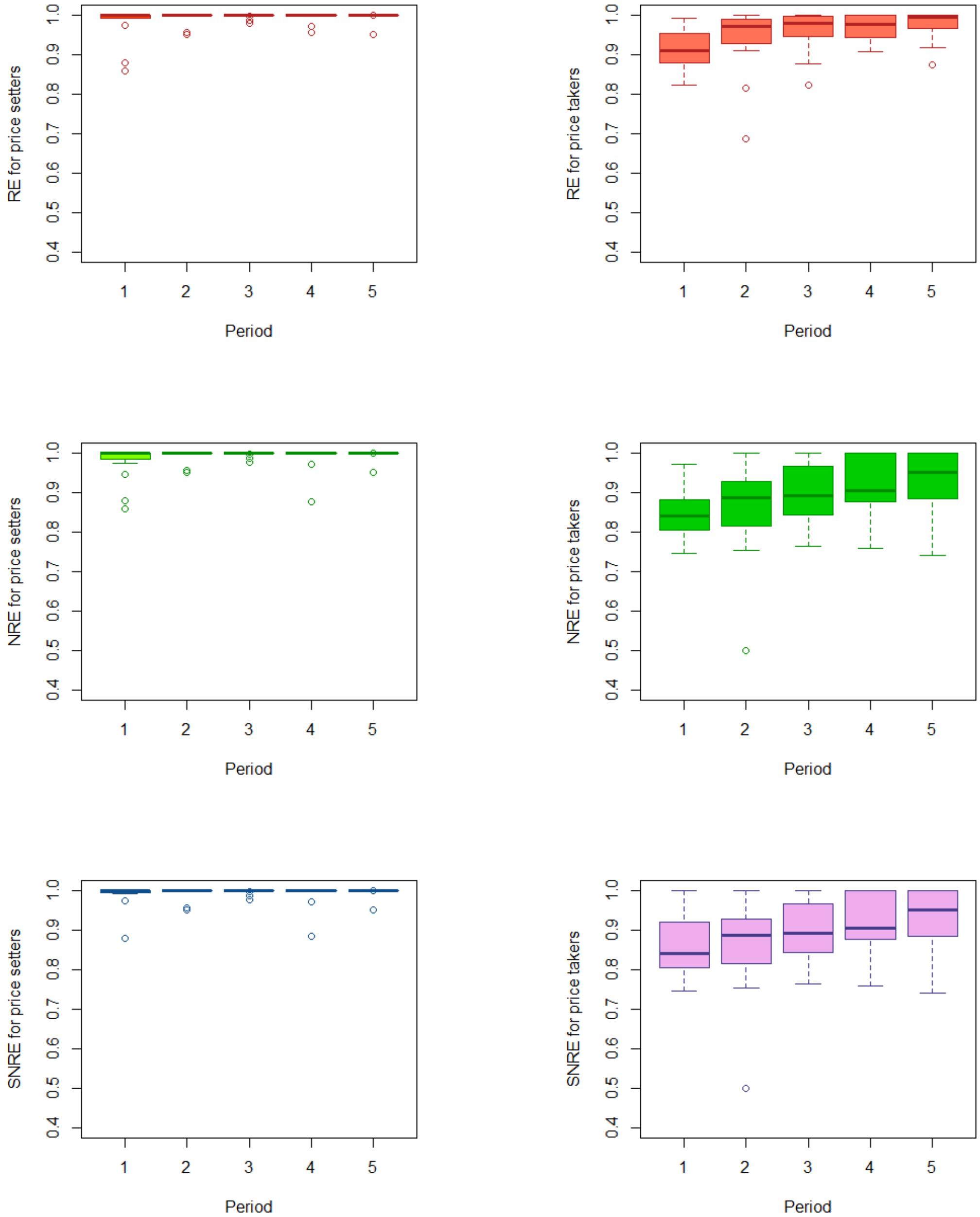

Figure 3: Estimates of Revenue Efficiency

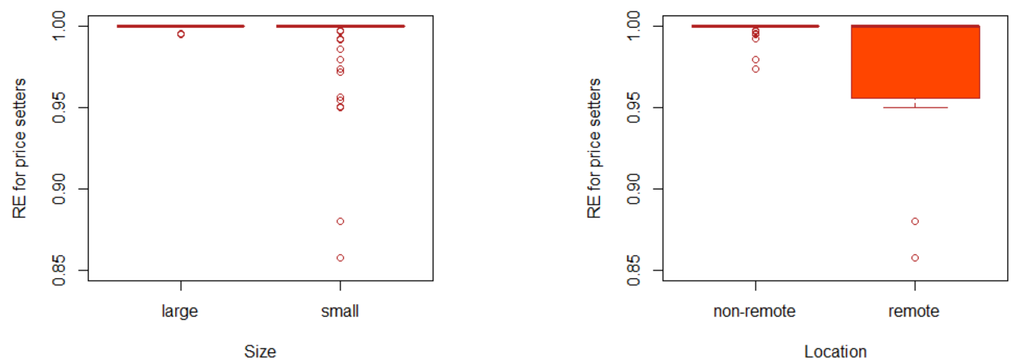

Figure 4: Estimates of Revenue Efficiency of Queensland HHSs by Size and Location

believe overfitting is an issue in our case, partly because we have relied heavily on economic theory to guide variable selection, and also because the ratio of the number of observations to the number of variables is much higher than what is normally recommended in the DEA literature: Golany and Roll (1989) suggest that the number of observations used for DEA estimation should be at least twice the total number of inputs and outputs, Banker, Charnes, Cooper, Swarts, and Thomas (1989), suggest it should be at least three times the total number of inputs and outputs, while Dyson et al. (2001) recommend it be at least twice the product of the number of inputs and number of outputs. In our study, the number of observations (160) is more than eleven times greater than the total number of inputs and outputs (14), and more than three times greater than the product of the number of inputs and number of outputs (48).

Finally, selected estimates of maximum revenue and associated RDUEs are reported in Table 3 (again results for all HHSs in all periods are reported in the Appendix). A more complete picture is provided by the box-and-whisker plots in Figure 5. The estimates reported in the three left-hand columns of Table 3 and in the left-hand panel of Figure 5 are our preferred estimates; they indicate that the "second-round" impacts of demand uncertainty on maximum revenue are very small. On the other hand, the estimates reported in the three right-hand columns of Table 3 and in the left-hand panel of Figure 5 indicate that the second-round impacts can be large. These results suggest that the funding cap built into the inverse demand function is enough to blunt the effect that different first-stage input choices may have on maximum revenue.

Table 3: Selected Estimates of Revenue-Oriented Demand Uncertainty Effects

		Price setters			Price takers		
Period	HHS	$R(\hat{x}^*_{it},\dots)$	$R(x^*_{it},\dots)$	RDUE	$R(\hat{x}^*_{it},\dots)$	$R(x^*_{it},\dots)$	RDUE
1	6	623.92	623.92	1.0000	715.25	678.26	1.0545
1	10	65.65	68.66	0.9563	65.65	68.66	0.9563
2	11	48.12	46.78	1.0286	48.36	46.78	1.0338
3	8	1613.36	1613.36	1.0000	1637.35	1637.35	1.0000
3	13	34.66	34.66	1.0000	39.76	39.76	1.0000
4	10	73.08	72.67	1.0056	73.08	72.67	1.0056
5	1	594.38	594.38	1.0000	645.49	731.39	0.8826
5	16	398.49	398.49	1.0000	409.80	451.04	0.9086
min	1	18.51	18.51	0.9563	19.14	19.14	0.7527
mean		474.50	474.52	0.9999	510.06	517.28	0.9823
max	16	1808.25	1808.25	1.0286	2109.61	2109.61	1.0789

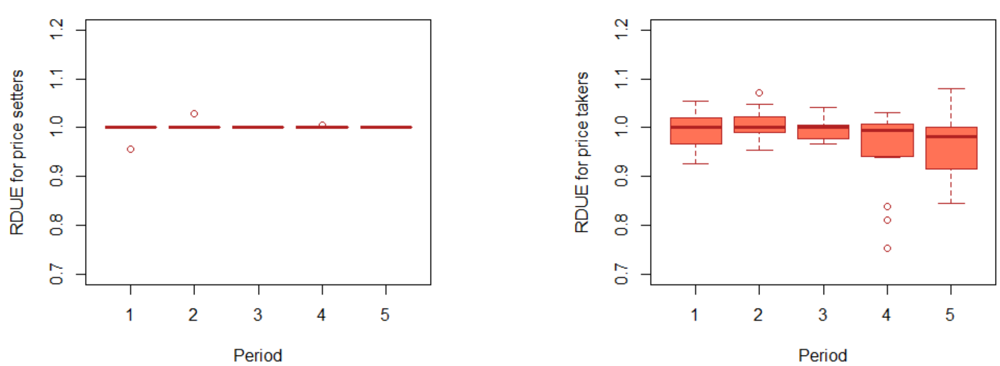

Figure 5: Estimates of Revenue-Oriented Demand Uncertainty Effects

9 Conclusion

Most public service managers make input and output decisions in the face of demand uncertainty. We take the view that the performance of these managers must be assessed in a way that distinguishes the effects of uncertainty (if any) from the effects of managerial incompetence. To do this, we break the decision making process into two separate stages: the first stage is a resource planning stage in which managers make input choices before demand is revealed; the second stage is a production stage in which the chosen inputs are used to meet realised demand. H. N. Nguyen and O'Donnell (2021) focused on the first stage. This paper focused on the second stage. In a departure from common practice in the OR literature, and with a view to our empirical application, we assumed that firms are price setters in output markets. We then assumed that managers choose outputs in order to maximise the revenue that can be obtained using their predetermined inputs. In a further departure from common practice in OR, we assumed that service outputs cannot be stored and that managers are unable to influence service demands. This led us to include a demand constraint in what is already an unusual revenue maximisation problem. We defined measures of revenue efficiency (i.e., measures of how well managers have solved their revenue maximisation problems) and explained why they cannot generally be decomposed into measures of output-oriented technical and allocative efficiency. We also developed a measure of the revenue effects (if any) of managers being required to choose inputs before demands for services are known. We explained how data envelopment analysis (DEA) methods could be used to estimate all of these measures.

Our methodology can be used to assess the performance of any of public service managers who face demand uncertainty and/or demand constraints. In our application to Queensland hospital and health services (HHSs), we found that most HHSs (a) did not face binding demand constraints, (b) operated reasonably close to the production frontier (i.e., levels of output-oriented technical efficiency were generally high), (c) were able to maximise the revenue they could obtain from their predetermined inputs (i.e., levels of revenue efficiency were generally high), and (d) were not particularly affected by having to choose inputs before demands for service were known (i.e., revenue-oriented demand uncertainty effects were generally small). We also found that HHSs located in regional and remote areas were likely to be less revenue-efficient than their metropolitan counterparts. These findings are consistent with the findings of previous hospital studies that have ignored demand constraints.

Our work has two main shortcomings. First, our revenue maximisation model does not explain why many HHSs produced outputs in excess of the output targets specified in their agreements with Queensland Health; hospitals receive no funding for these excess outputs. This implies that HHS output decisions are also driven by non-financial considerations. Community expectations and system-wide healthcare pressures can certainly make it difficult for individual HHSs to turn patients away from their doors. Political considerations may also play a part if historical levels of service delivery can be used as leverage in negotiations for higher future funding. Second, the DEA models we used to generate our results are underpinned by a number of restrictive assumptions (e.g., that all relevant variables are observed and measured without error, and that distance functions are locally linear). To relax these assumptions we must allow for statistical noise. Stochastic frontier analysis (SFA) estimators allow for noise. We are currently using SFA methods to replicate, and therefore check the robustness of, our results.

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Appendices

Table A1: Selected Estimates of Efficiency

		Price setters			Price takers				
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Period	HHS	RE	NRE	SNRE	RE	NRE	OTE	OAE	SNRE
1	1	1.0000	1.0000	1.0000	0.9754	0.8056	1.0000	0.8056	0.8056
1	2	0.9923	0.9923	0.9923	0.9126	0.8007	1.0000	0.8007	0.8007
1	3	N/A[1]	0.8576	0.9818	0.8819	0.8576	1.0000	0.8576	1.0000
1	4	1.0000	1.0000	1.0000	0.8896	0.8281	0.9325	0.8880	0.8281
1	5	1.0000	1.0000	1.0000	0.9075	0.8766	1.0000	0.8766	0.8766
1	6	0.9945	0.9945	0.9945	0.8227	0.7543	0.9436	0.7994	0.7543
1	7	0.9733	0.9733	0.9733	0.9320	0.8437	0.9766	0.8639	0.8437
1	8	1.0000	1.0000	1.0000	0.8575	0.8518	1.0000	0.8518	0.8518
1	9	1.0000	1.0000	1.0000	0.9131	0.8838	1.0000	0.8838	0.8838
1	10	0.9912	0.9456	0.9638	0.9912	0.9456	1.0000	0.9456	1.0000
1	11	1.0000	1.0000	1.0000	0.9683	0.9415	1.0000	0.9415	0.9546
1	12	0.9950	0.9950	0.9950	0.8819	0.8272	0.9223	0.8969	0.8272
1	13	0.8800	0.8800	0.8800	0.8747	0.8049	1.0000	0.8049	0.8049
1	14	1.0000	1.0000	1.0000	0.9384	0.8372	1.0000	0.8372	0.8372
1	15	1.0000	1.0000	1.0000	0.9711	0.9711	1.0000	0.9711	0.9711
1	16	1.0000	1.0000	1.0000	0.8634	0.7468	0.9402	0.7943	0.7468
2	1	1.0000	1.0000	1.0000	0.9628	0.8351	1.0000	0.8351	0.8351
2	2	1.0000	1.0000	1.0000	0.9905	0.8680	1.0000	0.8680	0.8680
2	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	4	1.0000	1.0000	1.0000	0.9819	0.8883	1.0000	0.8883	0.8883
2	5	1.0000	1.0000	1.0000	0.9393	0.7661	1.0000	0.7661	0.7661
2	6	1.0000	1.0000	1.0000	0.9444	0.8829	1.0000	0.8829	0.8829
2	7	1.0000	1.0000	1.0000	0.9935	0.9082	1.0000	0.9082	0.9101
2	8	1.0000	1.0000	1.0000	0.9101	0.9101	0.9965	0.9133	0.9101
2	9	1.0000	1.0000	1.0000	0.9135	0.8971	1.0000	0.8971	0.8971
2	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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[1]The NLP did not converge (i.e., did not find the maximum revenue).

Table A1 (continued).

		Price setters			Price takers				
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Period	HHS	RE	NRE	SNRE	RE	NRE	OTE	OAE	SNRE
2	11	0.9546	0.9546	0.9546	0.6858	0.4996	0.5725	0.8727	0.4996
2	12	1.0000	1.0000	1.0000	0.9888	0.9444	1.0000	0.9444	0.9444
2	13	0.9506	0.9506	0.9506	0.8150	0.7526	0.9344	0.8054	0.7526
2	14	1.0000	1.0000	1.0000	0.9884	0.8854	1.0000	0.8854	0.8854
2	15	1.0000	1.0000	1.0000	0.9765	0.9765	1.0000	0.9765	0.9765
2	16	1.0000	1.0000	1.0000	0.9414	0.7971	0.9822	0.8115	0.7971
3	1	1.0000	1.0000	1.0000	0.9766	0.8566	1.0000	0.8566	0.8566
3	2	0.9792	0.9772	0.9772	0.9792	0.8384	0.9998	0.8386	0.8384
3	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	4	0.9972	0.9972	0.9972	0.8222	0.7628	0.8372	0.9111	0.7628
3	5	1.0000	1.0000	1.0000	0.9427	0.8617	1.0000	0.8617	0.8617
3	6	1.0000	1.0000	1.0000	0.9537	0.8916	0.9799	0.9099	0.8916
3	7	1.0000	1.0000	1.0000	0.9945	0.9327	1.0000	0.9327	0.9342
3	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	9	1.0000	1.0000	1.0000	0.9644	0.8908	1.0000	0.8908	0.8908
3	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	11	0.9859	0.9859	0.9859	0.8956	0.8235	0.9350	0.8807	0.8313
3	12	1.0000	1.0000	1.0000	0.9971	0.9718	1.0000	0.9718	0.9718
3	13	1.0000	1.0000	1.0000	0.9956	0.9618	1.0000	0.9618	0.9618
3	14	1.0000	1.0000	1.0000	0.9899	0.9130	1.0000	0.9130	0.9130
3	15	0.9964	0.9964	0.9964	0.8772	0.8497	0.9145	0.9291	0.8497
3	16	1.0000	1.0000	1.0000	0.9500	0.7800	0.9567	0.8153	0.7800
4	1	1.0000	1.0000	1.0000	0.9059	0.7825	0.9758	0.8019	0.7825
4	2	1.0000	1.0000	1.0000	0.9936	0.8541	1.0000	0.8541	0.8559
4	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	6	1.0000	1.0000	1.0000	0.9434	0.8770	0.9634	0.9103	0.8770
4	7	1.0000	1.0000	1.0000	0.9950	0.9272	1.0000	0.9272	0.9288
4	8	1.0000	1.0000	1.0000	0.9535	0.8885	1.0000	0.8885	0.8885
4	9	1.0000	1.0000	1.0000	0.9399	0.8772	1.0000	0.8772	0.8772
4	10	0.9564	0.8772	0.8772	0.9564	0.8772	1.0000	0.8772	0.8847
4	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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Table A1 (continued).

		Price setters			Price takers				
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Period	HHS	RE	NRE	SNRE	RE	NRE	OTE	OAE	SNRE
4	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	13	0.9715	0.9715	0.9715	0.9606	0.9211	1.0000	0.9211	0.9211
4	14	1.0000	1.0000	1.0000	0.9406	0.8881	1.0000	0.8881	0.8881
4	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	16	1.0000	1.0000	1.0000	0.9431	0.7583	0.9208	0.8235	0.7583
5	1	1.0000	1.0000	1.0000	0.9163	0.8042	1.0000	0.8042	0.8042
5	2	1.0000	1.0000	1.0000	0.9451	0.8432	0.9585	0.8797	0.8432
5	3	0.9497	0.9497	0.9497	0.9828	0.9497	1.0000	0.9497	1.0000
5	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	5	1.0000	1.0000	1.0000	0.9737	0.8595	1.0000	0.8595	0.8595
5	6	1.0000	1.0000	1.0000	0.9630	0.9502	1.0000	0.9502	0.9502
5	7	1.0000	1.0000	1.0000	0.9953	0.9440	1.0000	0.9440	0.9448
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	9	1.0000	1.0000	1.0000	0.9980	0.9980	1.0000	0.9980	0.9980
5	10	0.9999	0.9998	0.9998	0.9999	0.9998	1.0000	0.9998	1.0000
5	11	0.9991	0.9991	0.9991	0.9985	0.9282	1.0000	0.9282	0.9282
5	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	14	1.0000	1.0000	1.0000	0.9691	0.9518	1.0000	0.9518	0.9518
5	15	1.0000	1.0000	1.0000	0.8743	0.7397	0.9010	0.8210	0.7397
5	16	1.0000	1.0000	1.0000	0.9931	0.9072	1.0000	0.9072	0.9072
min	1	0.8576	0.8576	0.8800	0.6858	0.4996	0.5725	0.7661	0.4996
mean		0.9928	0.9912	0.9938	0.9512	0.8906	0.9818	0.9064	0.8942
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A2: Selected Estimates of Revenue-Oriented Demand Uncertainty Effects

		Price setters			Price takers		
Period	HHS	$R(\hat{x}_{it}^{*},\dots)$	$R(x_{it}^{*},\dots)$	RDUE	$R(\hat{x}_{it}^{*},\dots)$	$R(x_{it}^{*},\dots)$	RDUE
1	1	474.95	474.95	1.0000	520.93	504.52	1.0325
1	2	276.91	276.91	1.0000	287.25	292.02	0.9837
1	3	22.17	22.17	1.0000	21.56	23.29	0.9255
1	4	166.59	166.59	1.0000	169.62	177.24	0.9571
1	5	382.49	382.49	1.0000	441.16	460.17	0.9587
1	6	623.92	623.92	1.0000	715.25	678.26	1.0545
1	7	205.20	205.20	1.0000	210.36	215.47	0.9763
1	8	1491.21	1491.21	1.0000	1751.28	1751.28	1.0000
1	9	1241.61	1241.61	1.0000	1385.49	1385.49	1.0000
1	10	65.65	68.66	0.9563	65.65	68.66	0.9563
1	11	46.14	46.14	1.0000	48.07	48.69	0.9872
1	12	457.76	457.76	1.0000	495.89	482.92	1.0269
1	13	39.89	39.89	1.0000	40.14	40.14	1.0000
1	14	531.84	531.84	1.0000	572.12	569.29	1.0050
1	15	318.81	318.81	1.0000	340.89	332.83	1.0242
1	16	313.69	313.69	1.0000	346.99	341.61	1.0158
2	1	512.54	512.54	1.0000	533.07	533.07	1.0000
2	2	298.81	298.81	1.0000	302.34	302.34	1.0000
2	3	19.07	19.07	1.0000	19.14	19.14	1.0000
2	4	172.42	172.42	1.0000	177.94	183.37	0.9704
2	5	405.70	405.70	1.0000	460.78	459.25	1.0033
2	6	717.59	717.59	1.0000	761.02	748.82	1.0163
2	7	223.26	223.26	1.0000	223.46	226.87	0.9850
2	8	1553.25	1553.25	1.0000	1717.04	1668.33	1.0292
2	9	1306.99	1306.99	1.0000	1451.48	1451.48	1.0000
2	10	82.23	82.23	1.0000	84.98	88.42	0.9612
2	11	48.12	46.78	1.0286	48.36	46.78	1.0338
2	12	506.88	506.88	1.0000	525.52	525.52	1.0000
2	13	34.48	34.48	1.0000	36.84	38.62	0.9540
2	14	555.28	555.28	1.0000	613.88	572.55	1.0722
2	15	351.37	351.37	1.0000	381.77	364.10	1.0485
2	16	348.18	348.18	1.0000	371.13	372.02	0.9976
3	1	528.32	528.32	1.0000	547.60	554.06	0.9883
3	2	304.72	304.72	1.0000	310.16	305.18	1.0163
3	3	18.79	18.79	1.0000	22.69	22.69	1.0000

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Table A2 (continued).

		Price setters			Price takers		
Period	HHS	$R(\hat{x}^*_{it}, \dots)$	$R(x^*_{it}, \dots)$	RDUE	$R(\hat{x}^*_{it}, \dots)$	$R(x^*_{it}, \dots)$	RDUE
3	4	267.65	267.65	1.0000	280.94	289.57	0.9702
3	5	472.66	472.66	1.0000	521.87	517.54	1.0084
3	6	778.33	778.33	1.0000	838.10	857.38	0.9775
3	7	234.75	234.75	1.0000	236.92	236.92	1.0000
3	8	1613.36	1613.36	1.0000	1637.35	1637.35	1.0000
3	9	1404.20	1404.20	1.0000	1470.32	1470.32	1.0000
3	10	80.69	80.69	1.0000	82.97	85.91	0.9658
3	11	45.53	45.53	1.0000	47.50	47.07	1.0091
3	12	523.14	523.14	1.0000	530.61	544.31	0.9748
3	13	34.66	34.66	1.0000	39.76	39.76	1.0000
3	14	587.27	587.27	1.0000	615.16	629.26	0.9776
3	15	333.79	333.79	1.0000	354.81	340.34	1.0425
3	16	343.56	343.56	1.0000	359.08	366.84	0.9788
4	1	539.07	539.07	1.0000	564.57	601.62	0.9384
4	2	313.12	313.12	1.0000	320.80	325.64	0.9851
4	3	18.51	18.51	1.0000	20.56	20.79	0.9890
4	4	338.62	338.62	1.0000	356.52	347.48	1.0260
4	5	475.99	475.99	1.0000	513.99	633.14	0.8118
4	6	877.51	877.51	1.0000	941.81	921.43	1.0221
4	7	234.39	234.39	1.0000	237.29	250.88	0.9459
4	8	1655.51	1655.51	1.0000	1820.09	1820.09	1.0000
4	9	1465.62	1465.62	1.0000	1571.10	1571.10	1.0000
4	10	73.08	72.67	1.0056	73.08	72.67	1.0056
4	11	44.19	44.19	1.0000	52.69	62.86	0.8383
4	12	565.79	565.79	1.0000	574.12	568.53	1.0098
4	13	36.81	36.81	1.0000	40.03	38.86	1.0299
4	14	614.65	614.65	1.0000	660.27	660.33	0.9999
4	15	352.53	352.53	1.0000	365.87	486.10	0.7527
4	16	359.38	359.38	1.0000	369.94	383.97	0.9634
5	1	594.38	594.38	1.0000	645.49	731.39	0.8826
5	2	342.55	342.55	1.0000	351.75	367.90	0.9561
5	3	22.62	22.62	1.0000	21.86	21.86	1.0000
5	4	370.53	370.53	1.0000	390.52	390.52	1.0000
5	5	432.80	432.80	1.0000	463.21	543.45	0.8524

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Table A2 (continued).

		Price setters			Price takers		
Period	HHS	$R(\hat{x}^*_{it}, \dots)$	$R(x^*_{it}, \dots)$	RDUE	$R(\hat{x}^*_{it}, \dots)$	$R(x^*_{it}, \dots)$	RDUE
5	6	942.49	942.49	1.0000	982.44	1064.34	0.9231
5	7	249.47	249.47	1.0000	255.05	301.51	0.8459
5	8	1808.25	1808.25	1.0000	2109.61	2109.61	1.0000
5	9	1618.55	1618.55	1.0000	1755.77	1765.57	0.9944
5	10	86.00	86.00	1.0000	86.00	86.00	1.0000
5	11	48.56	48.56	1.0000	48.59	48.59	1.0000
5	12	661.07	661.07	1.0000	675.26	664.68	1.0159
5	13	43.43	43.43	1.0000	46.88	43.45	1.0789
5	14	650.33	650.33	1.0000	689.75	723.84	0.9529
5	15	359.52	359.52	1.0000	373.30	385.78	0.9676
5	16	398.49	398.49	1.0000	409.80	451.04	0.9086
min	1	18.51	18.51	0.9563	19.14	19.14	0.7527
mean		474.50	474.52	0.9999	510.06	517.28	0.9823
max	16	1808.25	1808.25	1.0286	2109.61	2109.61	1.0789