Centre for Efficiency and Productivity Analysis

Working Paper Series
No. WP12/2021

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Date: August 2021

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ISSN No. 1932 - 4398

Estimating the Cost Efficiency of Public Service Providers in the Presence of Demand Uncertainty

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August 21, 2021

Abstract

Public service managers generally make input choices in the face of uncertainty about the demand for their services. However, this is generally not taken into account when estimating cost efficiency. The conventional approach to estimating cost efficiency is based on the assumption that managers choose inputs to minimise the cost of producing observed outputs. However, when demand is unknown at the time input decisions are made, many managers will instead choose inputs to minimize the cost of meeting various output targets. This paper explains how data envelopment analysis (DEA) methods can be used to account for demand uncertainty when estimating cost, technical and allocative efficiency. In doing so, it explains how DEA can be used to estimate the effects of demand uncertainty on costs. The methodology is applied to data on hospital and health service providers in the Australian state of Queensland. We obtain estimates of cost, technical and allocative efficiency that are quite different from the estimates obtained using a conventional approach that ignores demand uncertainty. Our empirical results also indicate that demand uncertainty has a significant effect on hospital costs.

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1 Introduction

Most firm managers make production choices in the face of some form of financial, price, technological and/or environmental uncertainty. When it comes to public service provision, another important source of uncertainty is the nature of future demand. The managers of state fire and rescue services, for example, must make equipment and staffing choices in the face of uncertainty about the numbers and types of emergencies that will occur in different jurisdictions; government and community leaders must make choices about the locations and sizes of schools in the face of uncertainty about the sizes of future school-age populations; and hospital managers must make decisions about staff, equipment and supplies in the face of uncertainty about the numbers and types of patients who will arrive at their hospital requiring treatment.

When facing demand uncertainty, most rational managers will choose their inputs to minimise the cost of meeting various output targets (e.g., minimum service levels, predicted maximum demands). These output targets are sometimes set by the managers themselves, and sometimes by governments. Needless to say, when the demand for services is difficult to predict, there can be a significant mismatch between output targets and realised demand. Some managers may find that the inputs they chose to meet a given output target may be insufficient to meet unexpectedly high realised demand: in 2019/20, for example, many Australian fire services found that they did not have enough resources to fight bushfires that were unprecedented on a national scale; in early 2021, many hospitals in India found they did not have enough masks, ventilators and ICU beds to care for unexpectedly large numbers of patients who had contracted COVID-19. At the same time, some managers may find that the inputs they chose to meet a given output target are more than enough to meet unexpectedly low realised demand: in early 2021, for example, unexpectedly low school enrolments forced community leaders in Glendale, Arizona, to consider closing and repurposing five elementary schools. These examples illustrate that demand uncertainty can give rise to both resource shortages and excess capacity in the provision of public services, and both can be costly. The fact that demand uncertainty can affect the cost of providing hospital services has been recognised by Hughes and McGuire (2003) and Lovell, Rodriguez-Alvarez, and Wall (2009); the fact it can affect the cost of providing fire and rescue services has been recognised by Puolokainen (2018).

In this paper, we take the view that rational and competent managers should not be held responsible for events that are outside their control: if they are making good decisions, then the managers of Indian hospitals should not be blamed for COVID-19 deaths resulting from a shortage of ventilators, and community leaders in Glendale should not be held responsible for high per-pupil schooling costs resulting from a combination of fixed infrastructure and unexpected falls in enrolments. Instead, the performance of managers must be assessed in a way that distinguishes the effects of demand uncertainty from the effects of managerial incompetence. This is not usually done when estimating the cost efficiency of public service managers. Instead of estimating how well these managers minimise the cost of meeting output targets, it is common, and inappropriate, to estimate how well they minimise the cost of producing a set of outputs that were unknown at the time their input decisions were made. Examples of this conventional approach can be found in studies of public education services (e.g., Chakraborty, Biswas, and Lewis, 2001; Cherchye, De Witte, Ooghe, and Nicaise, 2010), police services (e.g., Diez-Ticio and Mancebon, 2002; Gorman and Ruggiero, 2008), emergency medical services (e.g., Lambert, Min, and Srinivasan, 2009) and hospital services (e.g., Hollingsworth, 2008; Hussey, Vries, Romley, Wang, Chen, Shekelle, and McGlynn, 2009; Hunt and Link, 2019; Nguyen and Zelenyuk, 2020).

Witte and Geys (2011) were among the first to acknowledge and take into account the effects of demand factors when measuring the performance of public service providers. These authors estimated the input-oriented technical efficiency of library service managers in Flanders. They assumed that managers seek to minimise the inputs required to produce measures of "service potential" (e.g., available facilities and opening hours). They argued that the conversion of service potential into final outputs is affected by public demand, something that is not within the control of managers. More recently, Puolokainen (2018) accounted for demand uncertainty when measuring the performance of rescue service managers in Estonia, Finland, and Sweden. He assumed that managers choose inputs to minimise the cost of meeting "minimum service levels" (i.e., minimum numbers of services to be provided in given jurisdictions in given periods). Again, he argued that differences between minimum service levels and observed service levels are partly due to demand factors that are outside the control of managers.

This paper contributes to this literature by measuring the performance of hospital and health service (HHS) managers in the Australian state of Queensland. Instead of assuming that HHS managers choose inputs to minimise the cost of producing observed outputs, we assume they choose inputs to minimise the cost of producing output targets that are specified in service level agreements (SLAs) negotiated annually between HHS managers and the relevant government department, namely Queensland Health. Queensland Health is viewed as a system manager who purchases local healthcare services from HHSs in different locations across Queensland.

The methodology developed in this paper can be used to estimate the cost efficiency of any public service providers facing demand uncertainty. Our application to healthcare service providers in Queensland is partly motivated by growing uncertainty about the future incidence and treatment of certain illnesses (e.g., variants of COVID-19, drug-resistant tuberculosis). It is also partly motivated by recent rapid expansions in public healthcare spending in many high-income countries: in the period from 2000 to 2016, public healthcare spending in high-income countries[1] increased from an average of 4.5% of GDP (and 11.6% of total public spending) to an average of 6.1% of GDP (and 14.9% of total public spending) (Xu, Soucat, Kutzin, et al., 2018); in the same period, public healthcare spending in Australia increased from 5.2% of GDP (and 15.2% of total public spending) to 6.3% of GDP (and 17.4% of total public spending).[2] In Queensland, spending on health is now the largest component of the state budget; in 2017-18, it accounted for 36% of state government spending (Queensland Health, 2018). Not surprisingly, managers of healthcare services are coming under increased pressure to minimise costs. Measures of how well they do that must account for the fact that they must choose their inputs before the demand for many of their services is known.

The structure of the paper is as follows. In Section 2 we explain how production technologies can be represented using period-and-environment-specific variable-input sets. We then make enough regularity assumptions to ensure that these sets can be represented using variable-input distance functions. In Section 3 we describe the cost minimisation problems faced by firm managers in the presence of demand uncertainty; we assume that firms are price takers in input markets, and that managers choose variable inputs in order to minimise the cost of meeting various output targets. In Section 4 we describe measures of how well managers solve their cost minimisation problems. In Section 5 we explain how to measure of the effect of demand uncertainty on costs. In Section 6 we list the assumptions underpinning DEA estimation methods. We then presents the DEA estimators (or models) that we use to estimate minimum costs. In Section 7 we describe the data used in the empirical work. The dataset comprises observations on 16 firms over 5 years. In Section 8 we present the empirical results.

[1] Based on World Bank income classification in 2016.

[2] Source: Global Health Expenditure Database, World Health Organization.

Among other things, we find that our estimates of cost, technical and allocative efficiency are quite different from the estimates obtained using a conventional approach that ignores demand uncertainty. In Section 9 we summarise the paper and offer some concluding remarks.

2 Production Technologies

We follow O'Donnell (2018) and divide the possibly millions of variables that are physically involved in the production process into those that are at some point chosen or controlled by managers and those that are never within their control. The variables that are never controlled by managers are referred to as environmental variables. The variables that are chosen or controlled by managers are divided into predetermined inputs (i.e., goods going into the production process that have been chosen in a previous period), variable inputs (i.e., goods going into the production process that are chosen in the current period) and outputs (i.e., goods and bads coming out of the production process). We also follow O'Donnell (2018) and view production technologies as techniques or methods for transforming inputs into outputs.

Production technologies can be represented by various input sets, output sets and production possibilities sets. In this paper we focus on period-and-environment-specific variable-input sets. A period-and-environment-specific variable-input set is a set of variable inputs that can be used with given predetermined (or fixed) inputs and the technologies that are available in a given period to produce given outputs in a given production environment. For example, the set of variable inputs that can be used with the fixed input vector x^f and the technologies that are available in period t to produce the output vector q in a production environment characterised by the vector z is the following:

$$L^t(x^f, q, z) = \{x^v : x^v \text{ and } x^f \text{ can produce } q \text{ in period } t \text{ in environment } z\}. \tag{1}$$

If there are no fixed inputs (i.e., if all inputs are variable), then the v superscript and all references to x^f can be deleted. In that case, the input set defined by (1) is equal to the period-and-environment-specific input set defined by O'Donnell (2018, p.59).

In this paper we assume that inputs are strongly disposable and input sets are convex. These so-called regularity assumptions are common in the efficiency literature and are more than sufficient to ensure that $L^t(x^f, q, z)$ can be represented by the following period-and-environment-specific variable-input distance function:

$$D_I^t(x^v, x^f, q, z) = \sup\{\theta > 0 : x^v/\theta \in L^t(x^f, q, z)\}. \tag{2}$$

This function gives the reciprocal of the smallest fraction of x^v that can be used with x^f to produce q in period t in an environment characterised by z. If there are no fixed inputs, then it is equal to the period-and-environment-specific input distance function defined by O'Donnell (2018, p.67). By construction, $D_I^t(x^v, x^f, q, z)$ is nonnegative and linearly homogeneous in variable inputs. The strong disposability assumption means it is also nondecreasing in variable inputs. These properties will be used to motivate the DEA linear programs described in Section 5 below.

3 Cost Minimisation

We follow common practice and assume that firms are price takers in input markets, and that firm managers choose variable inputs to minimise costs. However, in a significant departure from common practice, we assume that outputs are uncertain at the time these variable input decisions are made. To handle this uncertainty, we assume that firm managers choose variable inputs to minimise the cost of meeting known output targets. For clarity, it is convenient to introduce firm and time subscripts into the notation so that, for example, x_{it}^f now represents the fixed input vector of firm i in period t. Mathematically, the variable cost minimisation problem of manager i in period t is the following:

$$\min_{x^v}\{w_{it}^{v\prime} x^v : D_I^t(x^v, x_{it}^f, \hat{q}_{it}, z_{it}) \geq 1\} \tag{3}$$

where $w_{it}^v = (w_{1it}^v, \ldots, w_{Mit}^v)'$ is a vector of variable input prices, $\hat{q}_{it}$ is a vector of output targets to be met, and z_{it} is a vector of environmental variables. There may be more than one variable input vector that solves this problem. Let $\hat{x}_{it}^{v*} \equiv \hat{x}^{vt}(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ denote one such vector. The associated minimum variable cost is $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) = w_{it}^{v\prime}\hat{x}_{it}^{v*}$.

Figure 1 illustrates this variable cost minimisation problem in a simple case where there are only two variable inputs. In this figure, the (piecewise) frontier passing through point S represents the boundary of the set of variable inputs that can be used with the fixed input vector x_{it}^f to produce the target vector $\hat{q}_{it}$ in period t in an environment characterised by z_{it}. The dashed lines are iso-cost lines with slopes of $-w_{1it}^v/w_{2it}^v$. Point A represents the variable input

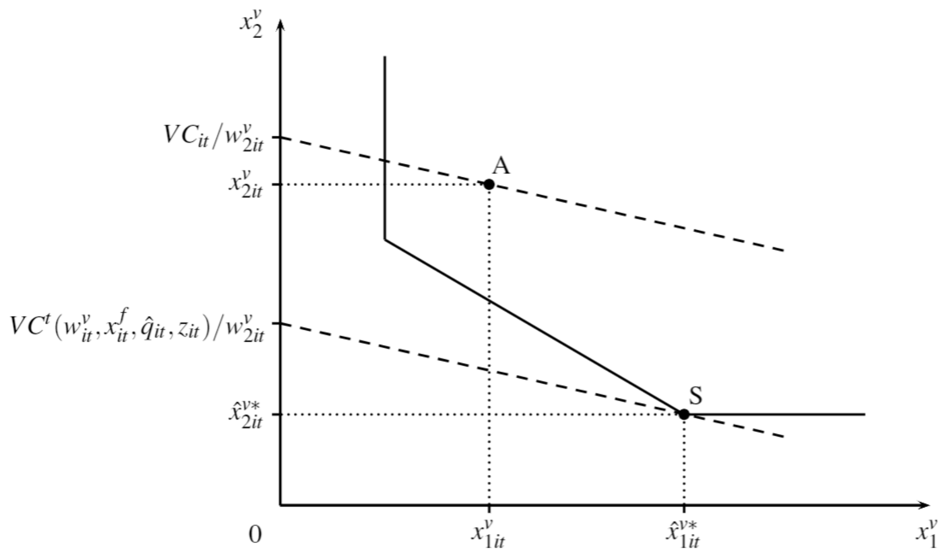

Figure 1: Variable cost minimisation in the presence of demand uncertainty

vector chosen by manager i in period t, with the associated variable cost given by $VC_{it} = w_{it}^{v\prime} x_{it}^v$. Point S represents the variable input vector that minimises the cost of producing $\hat{q}_{it}$, with the associated minimum variable cost given by $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$. Observe that variable cost at point A is greater than minimum variable cost, i.e., $VC_{it} > VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$.

To get a better understanding of the implications of demand uncertainty, it is useful to consider the variable cost minimisation problem under the common but naive assumption that outputs are known at the time input decisions are made. Let q_{it} denote the observed output vector of firm i in period t. If q_{it} is known at the time manager i chooses his/her variable inputs, then his/her period-t variable cost minimisation problem is the following:

$$\min_{x^v}\{w_{it}^{v\prime} x^v : D_I^t(x^v, x_{it}^f, q_{it}, z_{it}) \geq 1\}. \tag{4}$$

If there are no fixed inputs, then this problem is equivalent to the cost minimisation problem specified in O'Donnell (2018, eq. 4.17). If there are no fixed inputs and there is no technical or environmental change, then it is equivalent to the cost minimisation problem discussed in Sickles and Zelenyuk (2019, Sect. 2.1). Again, there may be more than one variable input vector that solves problem (4). Let $x_{it}^{v*} \equiv x^{vt}(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ denote one such vector. The associated minimum variable cost is $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it}) = w_{it}^{v\prime} x_{it}^{v*}$. We can generally expect

x_{it}^{v*} and $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ to differ from $\hat{x}_{it}^{v*}$ and $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ whenever q_{it} differs from $\hat{q}_{it}$.

Figure 2 compares the cost minimisation problems (3) and (4) in a case where there are only two variable inputs and q_{it} is less than $\hat{q}_{it}$. In this figure, the frontier passing through point S is the same frontier that was depicted earlier in Figure 1: it represents the boundary of the set of variable inputs that can be used with x_{it}^f to produce the target output vector $\hat{q}_{it}$. The lighter frontier passing through point X is the boundary of the set of variable inputs that can be used with x_{it}^f to produce the smaller observed vector q_{it}. Point X represents the variable input vector that minimises the cost of producing q_{it}, with the associated minimum variable cost given by $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$. Observe that $\hat{x}_{it}^{v*} > x_{it}^{v*}$ and $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) > VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ (i.e., demand uncertainty has led to higher input use and higher variable costs).

Finally, some managers may face situations where their variable input mix is predetermined (or fixed). If the manager of firm i in period t can only choose variable input vectors that are scalar multiples of x_{it}^v, then his/her variable cost minimisation problem is the following:

$$\min_{x^v} \{w_{it}^{v\prime} x^v : x^v \propto x_{it}^v,\ D_I^t(x^v, x_{it}^f, \hat{q}_{it}, z_{it}) \geq 1\}. \tag{5}$$

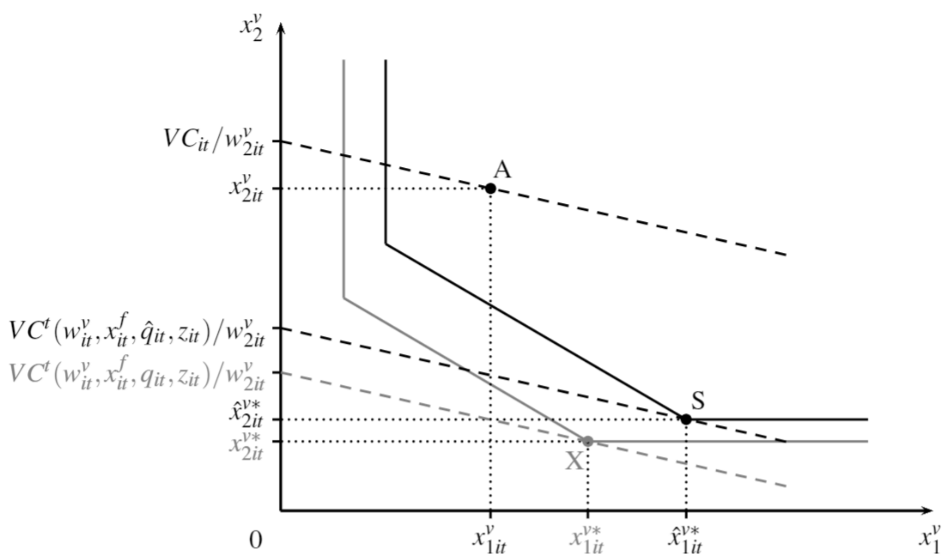

Figure 2: Variable cost minimisation in the presence and absence of demand uncertainty

The input vector that solves this problem is $\bar{x}_{it}^{v} \equiv \bar{x}^{vt}(x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it}) = x_{it}^{v}/D_{I}^{t}(x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it})$. This solution is unique and does not depend on input prices. The associated minimum variable cost is $VC^{t}(w_{it}^{v}, x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it}) = w_{it}^{v\prime}\bar{x}_{it}^{v} = VC_{it}/D_{I}^{t}(x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it})$. Figure 3 illustrates this problem in a simple case where there are only two variable inputs. Again, in this figure, the frontier passing through point S is the same frontier that was depicted earlier in Figure 1. Point B represents the scalar multiple of x_{it}^{v} that minimises the cost of producing $\hat{q}_{it}$. The associated minimum variable cost is $VC^{t}(w_{it}^{v}, x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it})$. Observe that $VC^{t}(w_{it}^{v}, x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it}) > VC^{t}(w_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it})$ (i.e., restricting the input mix has led to higher variable costs).

4 Measures of Efficiency

Measures of efficiency can be viewed as *ex post* measures of how well managers solve different optimisation problems (O'Donnell, 2018). This section defines four measures of efficiency associated with the three cost minimisation problems described in Section 3. All four measures take values in the closed unit interval.

The first measure of efficiency recognises that outputs are unknown at the time variable input decisions are made. In this case, the variable cost efficiency (VCE) of manager i in

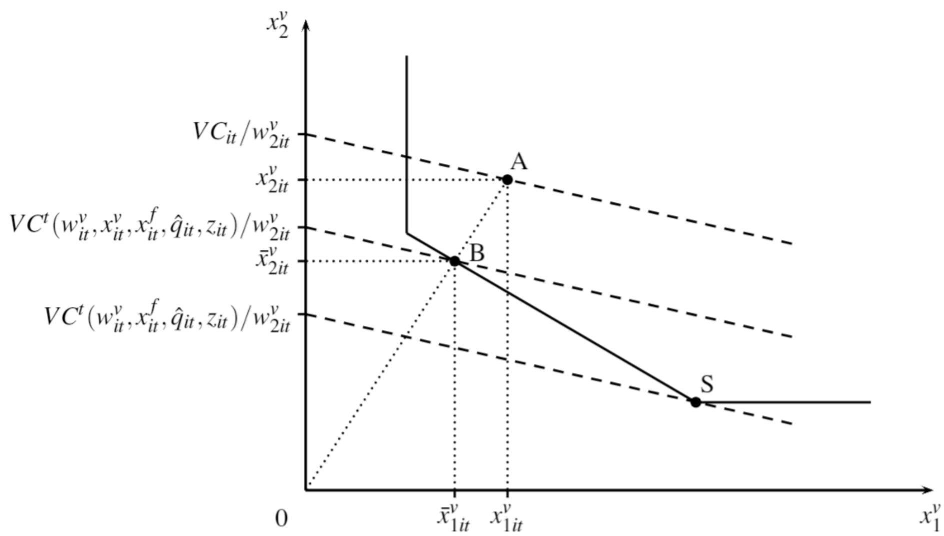

Figure 3: Variable cost minimisation in the presence of demand uncertainty when the input mix is fixed

period t is defined as

$$VCE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) = \frac{VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}{VC_{it}} \tag{6}$$

where VC_{it} is the observed variable cost and $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ is the minimum cost of producing $\hat{q}_{it}$ in period t when using x_{it}^f in a production environment characterised by z_{it}. This measure of efficiency can be viewed as a measure of how well the manager has solved problem (3). That problem was depicted earlier in Figure 1. Observe that the isocost lines passing through points S and A in that figure have intercepts of $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})/w_{2it}^v$ and VC_{it}/w_{2it}^v. The VCE of manager i in period t is given by the ratio of these intercepts.

The second measure of efficiency also recognises that outputs are unknown at the time variable input decisions are made. However, it also assumes that input mixes are fixed. In this case, what we call the variable-input-oriented technical efficiency (VITE) of manager i in period t is defined as

$$VITE^t(x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) = \frac{VC^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}{VC_{it}} \tag{7}$$

where $VC^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ is the minimum cost of producing $\hat{q}_{it}$ in period t when using x_{it}^f and a scalar multiple of x_{it}^v in a production environment characterised by z_{it}. Equivalently, $VITE^t(x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) = 1/D_I^t(x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$. If there are no fixed inputs (i.e., if all inputs are variable) and $\hat{q}_{it} = q_{it}$, then this measure of efficiency is equivalent to the measure of input-oriented technical efficiency defined in O'Donnell (2018, eq. 5.8). As it stands, it can be viewed as a measure of how well the manager has solved problem (5). That problem was depicted earlier in Figure 3. Observe that the isocost lines passing through points B and A in that figure have intercepts of $VC^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})/w_{2it}^v$ and VC_{it}/w_{2it}^v. The VITE of manager i in period t is given by the ratio of these intercepts.

It is common to break measures of cost efficiency into separate measures of technical and allocative efficiency. If outputs are unknown at the time variable input decisions are made, then the technical efficiency component is given by (7). The associated measure of variable-input-oriented allocative efficiency (VIAE) is defined as

$$VIAE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) = \frac{VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}{VC^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}. \tag{8}$$

This can be viewed as a cost-oriented measure of how well the manager has captured economies of input substitution (i.e., the cost savings obtained by substituting some inputs for others). Notice from Figure 3 that the isocost lines passing through points S and B have intercepts of $VC^t(w^v_{it}, x^f_{it}, \hat{q}_{it}, z_{it})/w^v_{2it}$ and $VC^t(w^v_{it}, x^v_{it}, x^f_{it}, \hat{q}_{it}, z_{it})/w^v_{2it}$. The VIAE of manager i in period t is given by the ratio of these intercepts. Also notice that equations (6), (7) and (8) together imply that

$$VIAE^t(w^v_{it}, x_{it}, \hat{q}_{it}, z_{it}) = \frac{VCE^t(w^v_{it}, x^v_{it}, x^f_{it}, \hat{q}_{it}, z_{it})}{VITE^t(x^v_{it}, x^f_{it}, \hat{q}_{it}, z_{it})}. \tag{9}$$

Thus, VIAE can also be viewed as the component of VCE that remains after accounting for VITE. If there are no fixed inputs and $\hat{q}_{it} = q_{it}$, then it is equivalent to the measure of input-oriented allocative efficiency defined in O'Donnell (2018, eq. 5.23).

The final measure of efficiency makes the common but naive assumption that outputs are known at the time variable input decisions are made – it ignores demand uncertainty. In this case, what we call the naive variable cost efficiency (NVCE) of manager i in period t is defined as

$$NVCE^t(w^v_{it}, x^v_{it}, x^f_{it}, q_{it}, z_{it}) = \frac{VC^t(w^v_{it}, x^f_{it}, q_{it}, z_{it})}{VC_{it}} \tag{10}$$

where $VC^t(w^v_{it}, x^f_{it}, q_{it}, z_{it})$ is the minimum cost of producing q_{it} in period t when using x^f_{it} in a production environment characterised by z_{it}. If there are no fixed inputs (i.e., if all inputs are variable), then this measure of efficiency is equivalent to the measure of cost efficiency defined in O'Donnell (2018, eq. 5.20). As it stands, it can be viewed as a measure of how well the manager has solved problem (4). That problem was depicted earlier in Figure 2. Observe that the isocost lines passing through points X and A in that figure have intercepts of $VC^t(w^v_{it}, x^f_{it}, q_{it}, z_{it})/w^v_{2it}$ and VC_{it}/w^v_{2it}. The NVCE of manager i in period t is given by the ratio of those two intercepts.

5 The Effect of Demand Uncertainty

If demand is uncertain at the time variable input choices are made, then managers will solve the cost minimisation problem given by (3). The input vector that solves that problem is $\hat{x}^{v*}_{it}$, and the associated minimum cost is $VC^t(w^v_{it}, x^f_{it}, \hat{q}_{it}, z_{it}) = w^{v\prime}_{it}\hat{x}^{v*}_{it}$. On the other hand, if there is no demand uncertainty (i.e., if outputs are known at the time variable input choices are made), then managers will solve the cost minimisation problem given by (4). The input vector that solves that problem is x_{it}^{v*}, and the associated minimum cost is $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it}) = w_{it}^{v\prime} x_{it}^{v*}$. In this paper, we attribute any difference between $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ and $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ to demand uncertainty. Mathematically, the cost-oriented demand uncertainty effect (CDUE) on manager i in period t is defined as

$$CDUE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, q_{it}, z_{it}) = \frac{VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}{VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})} = \frac{VCE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})}{NVCE^t(w_{it}^v, x_{it}^v, x_{it}^f, q_{it}, z_{it})} \quad (11)$$

where $VCE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$ and $NVCE^t(w_{it}^v, x_{it}^v, x_{it}^f, q_{it}, z_{it})$ are the measures of efficiency defined by (6) and (10). Unlike the measures of efficiency defined in Section 4, the CDUE defined by (11) can take a value greater than one. Indeed, it will generally take a value greater than (resp. less than) one whenever the target output vector $\hat{q}_{it}$ is greater than (resp. less than) the observed output vector q_{it}. Relatedly, if outputs are unknown at the time variable input decisions are made, then the (inappropriate) use of the naive measure of variable cost efficiency, $NVCE^t(w_{it}^v, x_{it}^v, x_{it}^f, q_{it}, z_{it})$, will malign (resp. flatter) the manager whenever the CDUE defined by (11) takes a value greater than (resp. less than) one. To illustrate, reconsider the cost minimisation problems depicted earlier in Figure 2. This figure depicts a case where there are only two inputs and where $\hat{q}_{it} > q_{it}$. Observe that $\hat{x}_{it}^{v*} > x_{it}^{v*}$ and $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) > VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$. Consequently, $VCE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it}) > NVCE^t(w_{it}^v, x_{it}^v, x_{it}^f, q_{it}, z_{it})$ and $CDUE^t(w_{it}^v, x_{it}^v, x_{it}^f, \hat{q}_{it}, q_{it}, z_{it}) > 1$. In this example, the inappropriate use of the naive measure of variable cost efficiency would unfairly malign the manager.

6 DEA Models

Data envelopment analysis (DEA) is a non-parametric estimation approach that is widely-used to estimate production frontiers and associated measures of efficiency. The approach is underpinned by the following assumptions: (i) production possibilities sets can be represented by distance, revenue, cost and/or profit functions; (ii) all relevant variables are observed and measured without error; (iii) production frontiers are locally (or piecewise) linear; (iv) inputs, outputs and environmental variables are strongly disposable; and (v) production possibilities sets are convex (O'Donnell, 2018, p.219). Basic DEA estimators (or models) envelop scatterplots of technically-feasible input-output combinations in such a way that estimated production frontiers are consistent with these assumptions. The set of technically-feasible input-output combinations obviously includes all pairs of observed inputs and outputs, i.e., the pairs (x_{it}, q_{it}) for all values of i and t. The empirical application in this paper is a little unusual in that firm managers sign legally-binding contracts (service level agreements) that require them to choose inputs that are capable of producing agreed output targets. We assume that managers meet these legal requirements, and for this reason we assume the set of technically-feasible input-output combinations also includes all pairs of observed inputs and output targets, i.e., the pairs $(x_{it}, \hat{q}_{it})$ for all values of i and t. In this section we describe how this expanded set of observations can be used to estimate the measures of VCE, VITE and VIAE defined in Section 4 and the CDUE defined in Section 5.

Estimating the measure of VCE defined by (6) and the CDUE defined by (11) involves estimating $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$. If there are I firms in the dataset and assumptions (i) to (v) are true, then a DEA LP that allows for technical progress is the following:

$$\begin{aligned}\min_{x^v,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}} \{w_{it}^{v\prime} x^v : \ & \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr} q_{hr} + \theta_{hr}\hat{q}_{hr}) \geq \hat{q}_{it},\ x^v \geq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr} x_{hr}^v + \theta_{hr} x_{hr}^v), \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr} x_{hr}^f + \theta_{hr} x_{hr}^f) \leq x_{it}^f,\ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr} z_{hr} + \theta_{hr} z_{hr}) \leq z_{it}, \\ & \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr} + \theta_{hr}) = 1,\ \lambda_{hr}, \theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{12}$$

This is a linear program (LP) that can be solved using standard DEA software packages. The value of the objective function at the optimum is an estimate of $VC^t(w_{it}^v, x_{it}^f, \hat{q}_{it}, z_{it})$. Dividing this estimate by observed cost yields an estimate of VCE. Dividing it by an estimate of $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ yields an estimate of the CDUE; the estimation of $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$ is discussed below.

Estimating the measures of VITE and VIAE defined by (7) and (8) involves estimating the reciprocal of the variable-input distance function. A DEA LP that will do this is

$$\begin{aligned}\min_{\mu,\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}}\{\mu : &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}) \geq \hat{q}_{it},\ \mu x_{it}^{v} \geq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}^{v}+\theta_{hr}x_{hr}^{v}),\\ &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}^{f}+\theta_{hr}x_{hr}^{f}) \leq x_{it}^{f},\ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it},\\ &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1,\ \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{13}$$

Again, this is an LP that can be solved using standard DEA software packages. The value of μ at the optimum is an estimate of $VITE^{t}(x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it}) = 1/D_{I}^{t}(x_{it}^{v}, x_{it}^{f}, \hat{q}_{it}, z_{it})$. Dividing an estimate of VCE by this estimate of VITE yields an estimate of VIAE.

Estimating the measure of NVCE defined by (10) and the CDUE defined by (11) involves estimating $VC^{t}(w_{it}^{v}, x_{it}^{f}, q_{it}, z_{it})$. Arguably the easiest way to do this is to simply replace $\hat{q}_{it}$ in LP (12) with q_{it}. The DEA LP then becomes

$$\begin{aligned}\min_{x^{v},\lambda_{11},\ldots,\lambda_{It},\theta_{11},\ldots,\theta_{It}}\{w_{it}^{v\prime}x^{v} : &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}q_{hr}+\theta_{hr}\hat{q}_{hr}) \geq q_{it},\ x^{v} \geq \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}^{v}+\theta_{hr}x_{hr}^{v}),\\ &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}x_{hr}^{f}+\theta_{hr}x_{hr}^{f}) \leq x_{it}^{f},\ \sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}z_{hr}+\theta_{hr}z_{hr}) \leq z_{it},\\ &\sum_{h=1}^{I}\sum_{r=1}^{t}(\lambda_{hr}+\theta_{hr}) = 1,\ \lambda_{hr},\theta_{hr} \geq 0 \text{ for all } h \text{ and } r\}.\end{aligned} \tag{14}$$

The value of the objective function at the optimum is an estimate of $VC^{t}(w_{it}^{v}, x_{it}^{f}, q_{it}, z_{it})$. Dividing this estimate of minimum variable cost by observed cost yields a naive estimate of variable cost efficiency. Importantly, naive estimates of variable cost efficiency obtained in this way are still not as naive as estimates that are normally found in the efficiency literature. In that literature, estimates of variable cost efficiency are normally computed using data on observed inputs and outputs only; they are obtained by solving DEA LPs of the following form:

$$\min_{x^v,\lambda_{11},\ldots,\lambda_{It}} \{w_{it}^{v}{}'x^v : \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}q_{hr} \geq q_{it},\ x^v \geq \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}x_{hr}^{v},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}x_{hr}^{f} \leq x_{it}^{f},$$
$$\sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr}z_{hr} \leq z_{it},\ \sum_{h=1}^{I}\sum_{r=1}^{t}\lambda_{hr} = 1,\ \lambda_{hr} \geq 0 \text{ for all } h \text{ and } r\}. \tag{15}$$

The value of the objective function at the optimum is an alternative estimate of $VC^t(w_{it}^v, x_{it}^f, q_{it}, z_{it})$. Dividing this estimate by observed cost yields an estimate of what we might call "super-naive" variable cost efficiency (SNVCE).

Finally, the DEA LPs described above allow production frontiers to exhibit variable returns to scale (VRS). In the efficiency literature, it is common to assume that production frontiers exhibit constant returns to scale (CRS). To impose this restriction, the right-hand sides of the constraints involving the environmental variables must be replaced with "ρz_{it}", and all instances of "$= 1$" must be replaced with "$\leq \rho$". Except in restrictive special cases (e.g., there are no environmental variables involved in the production process), the resulting LPs cannot be solved using standard DEA software packages.

7 Data

Hospital and health services in Queensland are delivered under SLAs negotiated annually between HHS managers and Queensland Health. The SLAs are signed at the beginning of each financial year and specify the output targets to be met by each HHS. These targets are informed by projections of future demand for different types of services. All targets are specified in Queensland Weighted Activity Units (QWAUs). QWAUs weight hospital activities in ways that account for their complexity and resource intensity. The QWAU is largely based on a national activity-based funding (ABF) model and a National Weighted Activity Unit (NWAU) that is used in all states and territories; it simply makes some adjustments for a number of specific features of the Queensland health system (Queensland Health, 2017).

The dataset comprises observations on input quantities, input prices, output quantities and output targets for $I = 16$ HHSs in Queensland for $T = 5$ years after the Australian National Health Reform Agreement came into effect. The Agreement came into effect on 1 July 2012, and our dataset covers the financial years from 2012/13 to 2016/17. We were able to collect data on one fixed input (x^f = beds and bed-alternatives), seven variable inputs (x_1^v = medical officers, x_2^v = nurses, x_3^v = other personal care staff, x_4^v = diagnostic and professional staff, x_5^v = administrative staff, x_6^v = domestic and other staff, x_7^v = non-labour inputs) and six outputs (q_1 = acute inpatient services, q_2 = outpatient services, q_3 = sub-acute care services, q_4 = emergency department services, q_5 = mental health services, q_6 = other interventions and procedures). Our decision to treat beds and bed-alternatives as a fixed input is consistent with literature; see, for example, Linna (1998). Quantity data on this variable were sourced from a database known as the Monthly Activity Collection (MAC) and generously supplied by Queensland Health. Quantity data on the labour inputs were sourced from a database known as the Financial and Residential Activity Collection (FRAC) and also supplied by Queensland Health. An implicit quantity index for non-labour inputs was constructed by dividing total non-salary non-capital expenditures (i.e., expenditures on drugs, medical and surgical supplies and other services) by a price index supplied by the Australian Bureau of Statistics (ABS). Price indexes for each category of staff were constructed by dividing total expenditure on salaries and wages in each category by the corresponding number of staff. On the output side, data were drawn from either the annual reports of each HHS or Service Delivery Statements provided by Queensland Health. Descriptive statistics for all variables are reported in Table 1. These statistics reveal that there is considerable variation in the inputs and outputs of different HHSs. Moreover, on average, observed outputs generally exceed the corresponding targets.

8 Results

This section reports DEA estimates of the measures of efficiency defined in Section 4 and the demand uncertainty effect defined in Section 5. The DEA LPs used to generate these results were somewhat simpler than the LPs discussed in Section 6 owing to the fact that there were no environmental variables involved in the production process. Moreover, because we only had data on $I = 16$ HHSs over $T = 5$ years, we made the assumption that there was no technical change. We used the bootstrap test of Simar and Wilson (1998) to test the null hypothesis that the (time-invariant) production frontier exhibits CRS against the alternative that it exhibits VRS. We could not reject the null hypothesis at the 5% level of significance.[3]

[3]The value of the F statistic was 0.946. We used the dea.boot function in the Benchmarking package in R to generate $B = 2000$ bootstrap samples and associated values of the test statistic. The 95th percentile of the

Table 1: Descriptive Statistics

	Variable	Mean	St. Dev.	Min	Max
Input Quantities					
x^f	Beds & alternatives	684.55	605.70	64	2354
x_1^v	Medical officers	494.28	491.55	10.87	1836.24
x_2^v	Nurses	1475.83	1334.36	81.65	4961.12
x_3^v	Other personal care staff	84.06	96.77	0.01	395.61
x_4^v	Diagnostic & professionals	359.79	395.55	0.00	1457.78
x_5^v	Administrative staff	491.07	451.92	19.37	1859.83
x_6^v	Domestic & other staff	442.93	347.81	28.96	1621.26
x_7^v	Non-labour variable inputs	1869.87	1862.03	63.48	9496.35
Input Prices					
w_1^v	Medical officers	255639.43	110478.81	166733.14	846472.96
w_2^v	Nurses	101989.13	10686.41	84165.76	138501.01
w_3^v	Other personal care staff	74128.65	37373.06	18097.83	361029.41
w_4^v	Diagnostic & professionals	103254.96	13348.06	37567.57	152142.04
w_5^v	Administrative staff	77267.43	11475.86	33475.86	128384.87
w_6^v	Domestic & other staff	66759.78	37849.16	47150.37	398962.91
w_7^v	Non-labour variable inputs	107100.00	2673.83	102800.00	110700.00
Observed Outputs					
q_1	Acute inpatients	58341.21	58371.39	1775	241887
q_2	Outpatients	14778.60	15656.30	453	74398
q_3	Sub-acute	6502.90	7286.59	172	40400
q_4	Emergency department	13479.57	10275.60	1037	40611
q_5	Mental health	9421.07	11385.27	30	56794
q_6	Interventions & prevention	7270.10	9594.35	0	41222
Output Targets					
$\hat{q}_1$	Acute inpatients	55662.06	55589.87	1797	230338
$\hat{q}_2$	Outpatients	13674.65	14031.08	500	67477
$\hat{q}_3$	Sub-acute	5875.77	5699.89	135	22011
$\hat{q}_4$	Emergency department	12811.91	9736.33	1033	39227
$\hat{q}_5$	Mental health	7845.05	8332.40	55	30068
$\hat{q}_6$	Interventions & prevention	7434.81	9971.53	0	44492

Of course, this does not mean that the null hypothesis is true. For this reason, this section reports results obtained under both the null (CRS) and alternative (VRS) hypotheses.

Selected estimates of efficiency obtained under the CRS and VRS assumptions are reported in Tables 2 and 3; results for all HHSs in all periods are reported in the Appendix. The results seem plausible. Observe from Table 2, for example, that the VCE and VITE estimates obtained under the CRS assumption are always lower than or equal to the estimates obtained under the VRS assumption. This is due to the fact that estimates of CRS frontiers always envelop estimates of VRS frontiers. Also observe from Table 3 that the VCE estimates are generally lower than the NVCE estimates, and the NVCE estimates are always lower than or equal to the SNVCE estimates. The fact that the VCE estimates are generally lower than the NVCE estimates is largely due to the fact that the VCE estimates are measures of how well the manager minimises the cost of producing an output target, while the NVCE estimates are measures of how well he/she minimises the cost of producing an observed output that is generally larger than the target. The fact that the NVCE estimates are always lower than or equal to the SNVCE estimates is due to the fact that the NVCE estimates were obtained using twice as many data points as the SNVCE estimates: the NVCE estimates were obtained by solving LP (14) while the SNVCE estimates were obtained by solving LP (15); this means that the estimated frontier that was used to compute the NVCE estimates envelops the estimated frontier that was used to compute the SNVCE estimates. Finally, observe from Table 3 that some VCE estimates are larger than the corresponding SNVCE estimates (e.g., HHSs 3 and 7 in period 1 and HHS 2 in period 3); this is because the relevant observed outputs are very much smaller than the corresponding targets.

Table 3 provides a somewhat incomplete picture of our different estimates of variable cost efficiency. A slightly more complete picture is provided by the box-and-whisker plots in Figure 4. These plots reveal that (a) the VCE estimates obtained under the CRS assumption are generally lower than the estimates obtained under the VRS assumption, (b) the VCE estimates are generally lower than the NVCE estimates, and (c) the NVCE estimates are generally lower than the SNVCE estimates. The plots also reveal a small number of outliers. The most notable outliers turn out to be observations on the Cape York Hospital and Health Service (CYHHS) and the Torres Strait-Northern Peninsula Hospital and Health Service (TSNPHHS) in periods 1 and 2. These HHSs merged to form the Torres and Cape Hospital and Health Service (TCHHS) at the end of period 2. The TCHHS is now one of Australia's

bootstrapped test statistics was 1.006.

Table 2: Selected Estimates of Efficiency

		CRS			VRS		
Period	HHS	VCE	VITE	VIAE	VCE	VITE	VIAE
1	3	0.6277	1.0000	0.6277	0.9851	1.0000	0.9851
1	7	0.8490	0.9509	0.8928	0.9219	0.9563	0.9640
1	12	0.7401	0.8113	0.9122	0.8140	0.8595	0.9470
2	10	0.9382	0.9813	0.9561	0.9394	0.9853	0.9534
2	13	0.2558	0.5493	0.4657	0.2611	0.5860	0.4455
3	2	0.8644	1.0000	0.8644	0.9956	1.0000	0.9956
5	6	0.5836	0.6683	0.8733	0.7286	0.7828	0.9308
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
min	1	0.2558	0.5492	0.4406	0.2611	0.5860	0.4017
mean		0.7471	0.8728	0.8541	0.8190	0.9194	0.8850
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 3: Selected Estimates of Variable Cost Efficiency

		CRS			VRS		
Period	HHS	VCE	NVCE	SNVCE	VCE	NVCE	SNVCE
1	3	0.6277	0.5484	0.5484	0.9851	0.7380	0.8683
1	7	0.8490	0.8087	0.8093	0.9219	0.8733	0.8774
1	12	0.7401	0.7451	0.7455	0.8140	0.8394	0.8394
2	10	0.9382	1.0000	1.0000	0.9394	1.0000	1.0000
2	13	0.2558	0.2649	0.2652	0.2611	0.2873	0.2873
3	2	0.8644	0.8230	0.8230	0.9956	0.9575	0.9749
5	6	0.5836	0.6427	0.6434	0.7286	1.0000	1.0000
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
min	1	0.2558	0.2649	0.2652	0.2611	0.2873	0.2873
mean		0.7471	0.7768	0.7949	0.8190	0.8655	0.8775
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

largest providers of health services to Australia's First Nations peoples. The data indicate that the CYHHS and TSNPHHS were using excessive quantities of non-medical staff (e.g., administrative, clerical and domestic staff). The merged HHS was able to operate with a smaller non-medical workforce.

Finally, selected estimates of CDUEs are reported in Table 4; again results for all HHSs in all periods are reported in the Appendix. The fact that the average of the CDUE estimates is less than one indicates that the use of NVCE as a measure of performance would have flattered most HHS managers. Whether a given manager in a given period would have been flattered or maligned by the use of NVCE depends on the differences between his/her (multiple) observed and target outputs. Table 4 reveals that the manager of HHS 6 would have looked relatively efficient if NVCE had been used to assess his/her performance in period 5. On the other hand, the manager of HHS 3 would have been particularly harshly (and unfairly) judged if NVCE had been used to assess his/her performance in period 1. A more complete picture of our estimates of CDUE is provided by the box-and-whisker plots in Figure 5. This figure indicates that CDUEs have generally been decreasing over time. This suggests that the use of NVCE as a measure of performance has been more flattering to HHS managers in recent periods than it has in the past. Results are quite comparable under the CRS and VRS assumptions.

Table 4: Selected Estimates of Cost-Oriented Demand Uncertainty Effects

		CRS			VRS		
Period	HHS	VCE	NVCE	CDUE	VCE	NVCE	CDUE
1	3	0.6277	0.5484	1.1447	0.9851	0.7380	1.3349
1	7	0.8490	0.8087	1.0497	0.9219	0.8733	1.0556
1	12	0.7401	0.7451	0.9933	0.8140	0.8394	0.9697
2	10	0.9382	1.0000	0.9382	0.9394	1.0000	0.9394
2	13	0.2558	0.2649	0.9658	0.2611	0.2873	0.9086
3	2	0.8644	0.8230	1.0502	0.9956	0.9575	1.0398
5	6	0.5836	0.6427	0.9081	0.7286	1.0000	0.7286
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
min	1	0.2558	0.2649	0.6586	0.2611	0.2873	0.6596
mean		0.7471	0.7768	0.9671	0.8190	0.8655	0.9493
max	16	1.0000	1.0000	1.3237	1.0000	1.0000	1.3349

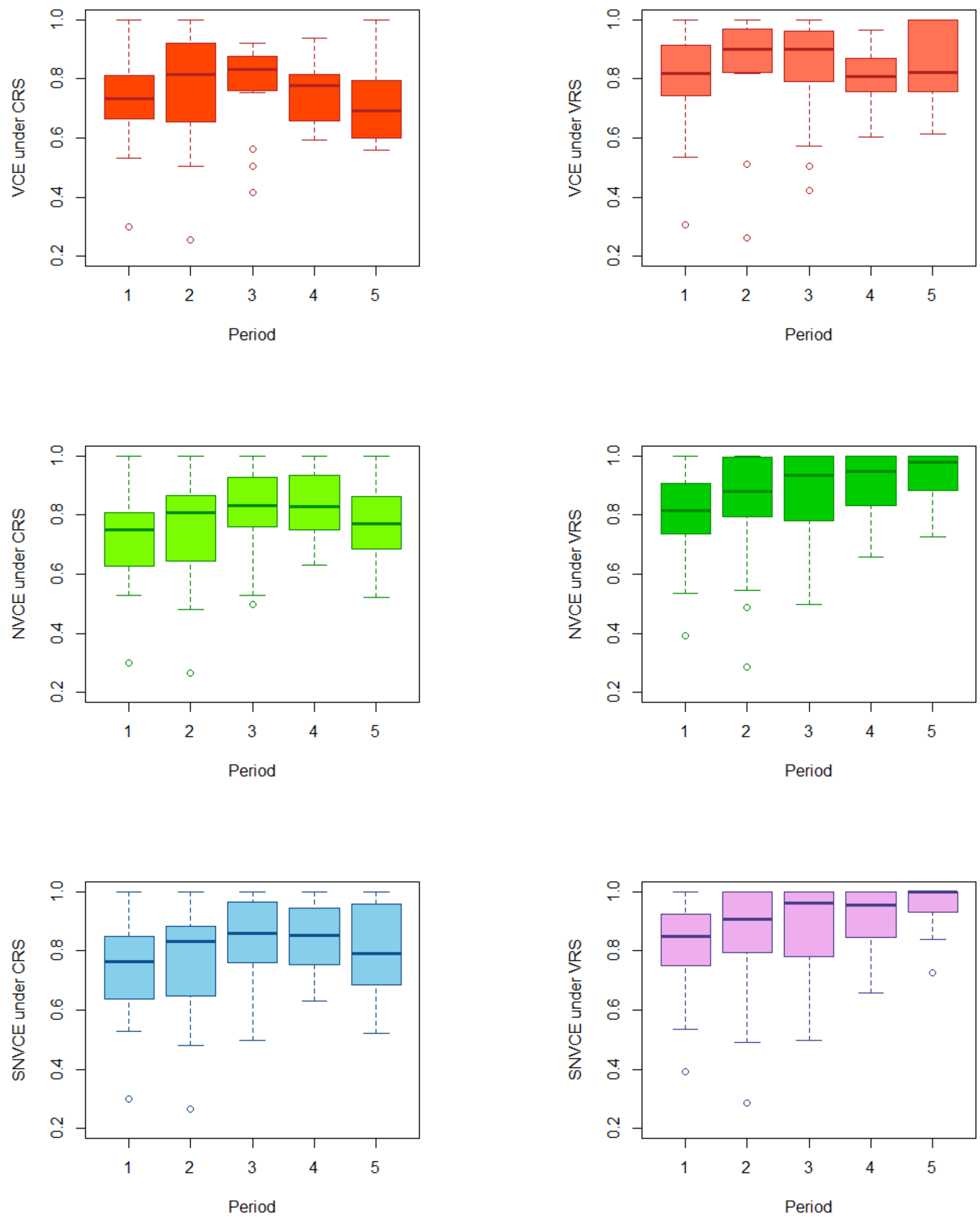

Figure 4: Estimates of Variable Cost Efficiency

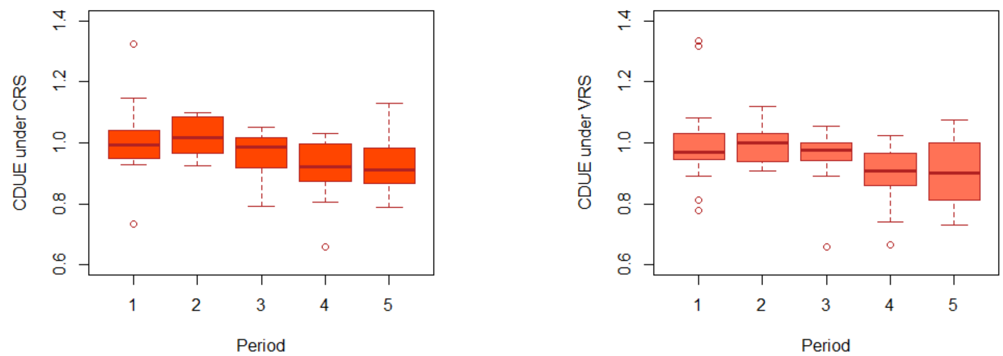

Figure 5: Estimates of Cost-Oriented Demand Uncertainty Effects

9 Conclusion

Public service managers generally have to make input choices in the face of demand uncertainty. Hospital managers, for example, must make decisions about staff, equipment and supplies in the face of uncertainty about the numbers and types of patients who will arrive at their hospital requiring treatment. In this paper, we took the view that the performance of these managers must be assessed in a way that distinguishes the effects of demand uncertainty from the effects of managerial incompetence. To do this, we began by assuming that firms are price takers in input markets, and that managers choose variable inputs in order to minimise the cost of meeting various output targets. We defined three measures of efficiency that can be used to assess the performance of these managers: cost, technical and allocative efficiency. We also identified a measure of the effect of demand uncertainty on costs. We explained how data envelopment analysis (DEA) methods could be used to estimate all these measures. In an application to Queensland hospitals and health services (HHSs), we found that estimates of cost, technical and allocative efficiency were quite different from the estimates obtained using a conventional approach that ignores demand uncertainty; we found that conventional (or naive) measures of cost and input-oriented technical efficiency tended to flatter HHS managers, and that the flattery tended to increase over time.

The methodology developed in this paper can be used to estimate the cost efficiency of any public service providers facing demand uncertainty. Arguably the main shortcoming of our work is that production frontiers and associated measures of efficiency were estimated using DEA. DEA estimators (or models) are underpinned by a number of restrictive assumptions. For example, all DEA models are underpinned by the assumption that all relevant variables are observed and measured without error, and that distance functions are locally linear. The only way to relax these assumptions is to use an estimation methodology that allows for omitted variable errors, measurement errors and functional form errors. In econometrics, these types of errors are collectively known as statistical noise. Stochastic frontier analysis (SFA) estimators explicitly allow for statistical noise. We are currently exploring ways in which SFA estimators can be used to replicate, and therefore check the robustness of, our results.

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Appendices

Table A1: Selected Estimates of Efficiency

		CRS			VRS		
Period	HHS	VCE	VITE	VIAE	VCE	VITE	VIAE
1	1	0.8352	0.9489	0.8802	0.9030	0.9911	0.9111
1	2	0.8287	0.9270	0.8939	0.9453	0.9965	0.9486
1	3	0.6277	1.0000	0.6277	0.9851	1.0000	0.9851
1	4	0.5328	0.6728	0.7920	0.5362	0.7165	0.7484
1	5	0.7481	0.9130	0.8193	0.8237	1.0000	0.8237
1	6	0.7215	0.7699	0.9372	0.7418	0.7750	0.9572
1	7	0.8490	0.9509	0.8928	0.9219	0.9563	0.9640
1	8	0.7105	0.7534	0.9430	0.7917	0.9226	0.8581
1	9	0.7938	0.8806	0.9014	0.8919	0.9754	0.9144
1	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	11	0.5760	1.0000	0.5760	0.5800	1.0000	0.5800
1	12	0.7401	0.8113	0.9122	0.8140	0.8595	0.9470
1	13	0.2980	0.6764	0.4406	0.3047	0.7586	0.4017
1	14	0.7316	0.7731	0.9463	0.8114	0.8945	0.9070
1	15	0.7315	0.9166	0.7980	0.7470	0.9180	0.8137
1	16	0.7024	0.7761	0.9050	0.8216	0.8995	0.9134
2	1	0.8269	0.8818	0.9376	0.9229	0.9908	0.9315
2	2	0.9672	1.0000	0.9672	1.0000	1.0000	1.0000
2	3	0.5529	1.0000	0.5529	1.0000	1.0000	1.0000
2	4	0.5084	0.6486	0.7839	0.5099	0.6850	0.7444
2	5	0.7589	1.0000	0.7589	0.8779	1.0000	0.8779
2	6	0.7824	0.8371	0.9347	0.8170	0.8972	0.9106
2	7	0.8400	0.9222	0.9109	0.8449	0.9224	0.9160
2	8	0.7768	0.8252	0.9415	0.8786	1.0000	0.8786
2	9	0.7997	0.8766	0.9123	0.9523	0.9893	0.9626
2	10	0.9382	0.9813	0.9561	0.9394	0.9853	0.9534
2	11	0.5037	0.6001	0.8394	0.5102	0.6346	0.8040
2	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	13	0.2558	0.5493	0.4657	0.2611	0.5860	0.4455

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Table A1 (continued).

		CRS			VRS		
Period	HHS	VCE	VITE	VIAE	VCE	VITE	VIAE
2	14	0.9581	1.0000	0.9581	0.9808	1.0000	0.9808
2	15	0.8260	0.9709	0.8507	0.8260	0.9711	0.8506
2	16	0.9037	0.9927	0.9104	0.9287	1.0000	0.9287
3	1	0.9210	0.9374	0.9826	0.9431	0.9650	0.9773
3	2	0.8644	1.0000	0.8644	0.9956	1.0000	0.9956
3	3	0.8544	1.0000	0.8544	1.0000	1.0000	1.0000
3	4	0.5621	0.7863	0.7148	0.5719	0.7934	0.7208
3	5	0.8593	1.0000	0.8593	0.9203	1.0000	0.9203
3	6	0.7672	0.8012	0.9577	0.8143	0.8924	0.9125
3	7	0.8989	1.0000	0.8989	0.9096	1.0000	0.9096
3	8	0.8417	0.9745	0.8638	0.9793	0.9876	0.9916
3	9	0.8228	0.8919	0.9226	0.9980	1.0000	0.9980
3	10	0.8895	1.0000	0.8895	0.8912	1.0000	0.8912
3	11	0.5037	0.8885	0.5669	0.5045	0.9723	0.5188
3	12	0.9182	0.9303	0.9870	0.9349	0.9565	0.9774
3	13	0.4170	0.5492	0.7594	0.4228	0.7053	0.5994
3	14	0.8201	0.8914	0.9201	0.8378	0.9435	0.8880
3	15	0.7539	0.8660	0.8706	0.7637	0.8736	0.8742
3	16	0.8067	0.8680	0.9294	0.8695	0.9124	0.9530
4	1	0.7379	0.7678	0.9610	0.7907	0.8376	0.9441
4	2	0.8637	1.0000	0.8637	0.9653	1.0000	0.9653
4	3	0.6435	1.0000	0.6435	0.7742	1.0000	0.7742
4	4	0.6124	1.0000	0.6124	0.6432	1.0000	0.6432
4	5	0.8060	0.9457	0.8523	0.8389	0.9659	0.8685
4	6	0.8006	0.8264	0.9688	0.8367	0.9153	0.9141
4	7	0.8238	0.9330	0.8830	0.8253	0.9342	0.8835
4	8	0.6588	0.8824	0.7466	0.7396	0.8827	0.8379
4	9	0.7308	0.7809	0.9358	0.8964	0.9354	0.9583
4	10	0.7950	1.0000	0.7950	0.7979	1.0000	0.7979
4	11	0.9155	1.0000	0.9155	0.9253	1.0000	0.9253
4	12	0.9369	0.9861	0.9501	0.9385	0.9865	0.9514
4	13	0.5925	0.8138	0.7281	0.6035	0.9348	0.6456
4	14	0.7887	0.8620	0.9149	0.8091	0.8887	0.9105
4	15	0.6586	0.7244	0.9093	0.6658	0.7282	0.9142
4	16	0.7623	0.8280	0.9206	0.8096	0.8547	0.9473

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Table A1 (continued).

Period	HHS	CRS VCE	CRS VITE	CRS VIAE	VRS VCE	VRS VITE	VRS VIAE
5	1	0.5651	0.6480	0.8721	0.6813	0.7594	0.8971
5	2	0.7201	0.8788	0.8194	0.8396	0.8938	0.9394
5	3	0.5603	0.6660	0.8412	1.0000	1.0000	1.0000
5	4	0.6705	1.0000	0.6705	0.8023	1.0000	0.8023
5	5	0.6861	0.7226	0.9495	0.8009	0.8979	0.8919
5	6	0.5836	0.6683	0.8733	0.7286	0.7828	0.9308
5	7	0.5739	0.7104	0.8078	0.6146	0.7155	0.8590
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	9	0.6635	0.7849	0.8454	1.0000	1.0000	1.0000
5	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	11	0.8855	0.9262	0.9561	0.8881	0.9422	0.9426
5	12	0.8175	1.0000	0.8175	0.9948	1.0000	0.9948
5	13	0.7723	0.9929	0.7779	0.8400	1.0000	0.8400
5	14	0.6944	0.7579	0.9162	0.7859	0.8206	0.9576
5	15	0.6188	0.6871	0.9006	0.6617	0.7191	0.9203
5	16	0.7057	0.7883	0.8952	0.7948	0.8272	0.9608
min	1	0.2558	0.5492	0.4406	0.2611	0.5860	0.4017
mean		0.7471	0.8728	0.8541	0.8190	0.9194	0.8850
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A2: Selected Estimates of Variable Cost Efficiency

Period	HHS	CRS VCE	CRS NVCE	CRS SNVCE	VRS VCE	VRS NVCE	VRS SNVCE
1	1	0.8352	0.8728	0.9308	0.9030	0.9371	0.9385
1	2	0.8287	0.8714	0.8928	0.9453	0.9783	1.0000
1	3	0.6277	0.5484	0.5484	0.9851	0.7380	0.8683
1	4	0.5328	0.5536	0.5539	0.5362	0.5626	0.5700
1	5	0.7481	0.8056	0.8063	0.8237	0.8507	0.8568
1	6	0.7215	0.7011	0.7227	0.7418	0.7357	0.7360
1	7	0.8490	0.8087	0.8093	0.9219	0.8733	0.8774

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Table A2 (continued).

		CRS			VRS		
Period	HHS	VCE	NVCE	SNVCE	VCE	NVCE	SNVCE
1	8	0.7105	0.7015	0.7800	0.7917	0.7877	0.8015
1	9	0.7938	0.7846	0.8875	0.8919	1.0000	1.0000
1	10	1.0000	0.7554	0.7558	1.0000	0.7598	0.7601
1	11	0.5760	0.5270	0.5270	0.5800	0.5357	0.5357
1	12	0.7401	0.7451	0.7455	0.8140	0.8394	0.8394
1	13	0.2980	0.2992	0.2992	0.3047	0.3920	0.3920
1	14	0.7316	1.0000	1.0000	0.8114	1.0000	1.0000
1	15	0.7315	0.7719	0.7725	0.7470	0.7724	0.7736
1	16	0.7024	0.7434	0.7434	0.8216	0.8791	0.9062
2	1	0.8269	0.8587	0.8647	0.9229	0.9870	0.9889
2	2	0.9672	0.8814	0.8814	1.0000	1.0000	1.0000
2	3	0.5529	0.5128	0.5128	1.0000	1.0000	1.0000
2	4	0.5084	0.5379	0.5408	0.5099	0.5444	0.5600
2	5	0.7589	0.8224	0.8224	0.8779	0.9089	0.9108
2	6	0.7824	0.7893	0.7907	0.8170	0.8827	0.8840
2	7	0.8400	0.8142	0.8169	0.8449	0.8358	0.8359
2	8	0.7768	0.7876	0.8854	0.8786	0.8703	0.9039
2	9	0.7997	0.8006	0.8392	0.9523	1.0000	1.0000
2	10	0.9382	1.0000	1.0000	0.9394	1.0000	1.0000
2	11	0.5037	0.4798	0.4811	0.5102	0.4865	0.4890
2	12	1.0000	0.9687	0.9906	1.0000	0.9875	0.9913
2	13	0.2558	0.2649	0.2652	0.2611	0.2873	0.2873
2	14	0.9581	0.8735	1.0000	0.9808	0.8776	1.0000
2	15	0.8260	0.7518	0.7527	0.8260	0.7544	0.7546
2	16	0.9037	0.8251	0.8503	0.9287	0.8729	0.8850
3	1	0.9210	1.0000	1.0000	0.9431	1.0000	1.0000
3	2	0.8644	0.8230	0.8230	0.9956	0.9575	0.9749
3	3	0.8544	0.9722	0.9722	1.0000	1.0000	1.0000
3	4	0.5621	0.5817	0.5842	0.5719	0.5919	0.5932
3	5	0.8593	0.8672	0.8674	0.9203	0.9445	0.9475
3	6	0.7672	0.7781	0.7802	0.8143	0.8667	0.8669
3	7	0.8989	0.8831	0.9548	0.9096	0.9210	1.0000
3	8	0.8417	0.8559	0.8593	0.9793	1.0000	1.0000
3	9	0.8228	0.8270	0.8704	0.9980	1.0000	1.0000
3	10	0.8895	1.0000	1.0000	0.8912	1.0000	1.0000

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Table A2 (continued).

		CRS			VRS		
Period	HHS	VCE	NVCE	SNVCE	VCE	NVCE	SNVCE
3	11	0.5037	0.4974	0.4974	0.5045	0.4983	0.4983
3	12	0.9182	1.0000	1.0000	0.9349	1.0000	1.0000
3	13	0.4170	0.5270	0.5319	0.4228	0.6410	0.6928
3	14	0.8201	0.7810	0.7910	0.8378	0.7954	0.7959
3	15	0.7539	0.7404	0.7412	0.7637	0.7620	0.7634
3	16	0.8067	0.8333	0.8579	0.8695	0.9144	0.9278
4	1	0.7379	0.8900	0.9223	0.7907	0.9778	0.9785
4	2	0.8637	0.8670	0.8671	0.9653	1.0000	1.0000
4	3	0.6435	0.6404	0.6404	0.7742	0.8822	0.8827
4	4	0.6124	0.6319	0.6319	0.6432	0.6562	0.6572
4	5	0.8060	1.0000	1.0000	0.8389	1.0000	1.0000
4	6	0.8006	0.7773	0.7798	0.8367	0.8686	0.8686
4	7	0.8238	0.8978	0.9154	0.8253	0.8982	0.9274
4	8	0.6588	0.7263	0.7335	0.7396	1.0000	1.0000
4	9	0.7308	0.7895	0.8352	0.8964	1.0000	1.0000
4	10	0.7950	0.7757	0.7757	0.7979	0.7807	0.7807
4	11	0.9155	1.0000	1.0000	0.9253	1.0000	1.0000
4	12	0.9369	0.9676	0.9678	0.9385	0.9985	0.9985
4	13	0.5925	0.6758	0.6758	0.6035	0.6874	0.6874
4	14	0.7887	0.7903	0.8165	0.8091	0.7932	0.8210
4	15	0.6586	1.0000	1.0000	0.6658	1.0000	1.0000
4	16	0.7623	0.8748	0.8936	0.8096	0.9151	0.9307
5	1	0.5651	0.6668	0.6682	0.6813	0.8327	0.8386
5	2	0.7201	0.7749	0.7751	0.8396	0.9028	0.9030
5	3	0.5603	0.5208	0.5219	1.0000	1.0000	1.0000
5	4	0.6705	0.6932	1.0000	0.8023	1.0000	1.0000
5	5	0.6861	0.8564	0.8566	0.8009	0.9694	0.9767
5	6	0.5836	0.6427	0.6434	0.7286	1.0000	1.0000
5	7	0.5739	0.7294	0.7770	0.6146	0.7882	1.0000
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	9	0.6635	0.6919	0.6919	1.0000	1.0000	1.0000
5	10	1.0000	0.8860	1.0000	1.0000	0.9305	1.0000
5	11	0.8855	1.0000	1.0000	0.8881	1.0000	1.0000
5	12	0.8175	0.8029	0.8032	0.9948	0.9555	0.9555
5	13	0.7723	0.8656	0.9174	0.8400	1.0000	1.0000

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Table A2 (continued).

		CRS			VRS		
Period	HHS	VCE	NVCE	SNVCE	VCE	NVCE	SNVCE
5	14	0.6944	0.7648	0.7659	0.7859	0.8605	0.8605
5	15	0.6188	0.6776	0.6791	0.6617	0.7256	0.7264
5	16	0.7057	0.8399	0.8401	0.7948	0.9887	0.9984
min	1	0.2558	0.2649	0.2652	0.2611	0.2873	0.2873
mean		0.7471	0.7768	0.7949	0.8190	0.8655	0.8775
max	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A3: Selected Estimates of Cost-Oriented Demand Uncertainty Effects

		CRS			VRS		
Period	HHS	VCE	NVCE	CDUE	VCE	NVCE	CDUE
1	1	0.8352	0.8728	0.9570	0.9030	0.9371	0.9636
1	2	0.8287	0.8714	0.9510	0.9453	0.9783	0.9662
1	3	0.6277	0.5484	1.1447	0.9851	0.7380	1.3349
1	4	0.5328	0.5536	0.9624	0.5362	0.5626	0.9532
1	5	0.7481	0.8056	0.9285	0.8237	0.8507	0.9682
1	6	0.7215	0.7011	1.0292	0.7418	0.7357	1.0084
1	7	0.8490	0.8087	1.0497	0.9219	0.8733	1.0556
1	8	0.7105	0.7015	1.0128	0.7917	0.7877	1.0051
1	9	0.7938	0.7846	1.0117	0.8919	1.0000	0.8919
1	10	1.0000	0.7554	1.3237	1.0000	0.7598	1.3161
1	11	0.5760	0.5270	1.0929	0.5800	0.5357	1.0828
1	12	0.7401	0.7451	0.9933	0.8140	0.8394	0.9697
1	13	0.2980	0.2992	0.9960	0.3047	0.3920	0.7774
1	14	0.7316	1.0000	0.7316	0.8114	1.0000	0.8114
1	15	0.7315	0.7719	0.9476	0.7470	0.7724	0.9671
1	16	0.7024	0.7434	0.9448	0.8216	0.8791	0.9346
2	1	0.8269	0.8587	0.9629	0.9229	0.9870	0.9350
2	2	0.9672	0.8814	1.0973	1.0000	1.0000	1.0000
2	3	0.5529	0.5128	1.0782	1.0000	1.0000	1.0000
2	4	0.5084	0.5379	0.9452	0.5099	0.5444	0.9366

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Table A3 (continued).

		CRS			VRS		
Period	HHS	VCE	NVCE	CDUE	VCE	NVCE	CDUE
2	5	0.7589	0.8224	0.9228	0.8779	0.9089	0.9659
2	6	0.7824	0.7893	0.9913	0.8170	0.8827	0.9256
2	7	0.8400	0.8142	1.0317	0.8449	0.8358	1.0109
2	8	0.7768	0.7876	0.9864	0.8786	0.8703	1.0096
2	9	0.7997	0.8006	0.9989	0.9523	1.0000	0.9523
2	10	0.9382	1.0000	0.9382	0.9394	1.0000	0.9394
2	11	0.5037	0.4798	1.0499	0.5102	0.4865	1.0487
2	12	1.0000	0.9687	1.0324	1.0000	0.9875	1.0127
2	13	0.2558	0.2649	0.9658	0.2611	0.2873	0.9086
2	14	0.9581	0.8735	1.0969	0.9808	0.8776	1.1176
2	15	0.8260	0.7518	1.0986	0.8260	0.7544	1.0950
2	16	0.9037	0.8251	1.0952	0.9287	0.8729	1.0640
3	1	0.9210	1.0000	0.9210	0.9431	1.0000	0.9431
3	2	0.8644	0.8230	1.0502	0.9956	0.9575	1.0398
3	3	0.8544	0.9722	0.8789	1.0000	1.0000	1.0000
3	4	0.5621	0.5817	0.9663	0.5719	0.5919	0.9661
3	5	0.8593	0.8672	0.9908	0.9203	0.9445	0.9744
3	6	0.7672	0.7781	0.9860	0.8143	0.8667	0.9395
3	7	0.8989	0.8831	1.0179	0.9096	0.9210	0.9876
3	8	0.8417	0.8559	0.9834	0.9793	1.0000	0.9793
3	9	0.8228	0.8270	0.9949	0.9980	1.0000	0.9980
3	10	0.8895	1.0000	0.8895	0.8912	1.0000	0.8912
3	11	0.5037	0.4974	1.0126	0.5045	0.4983	1.0125
3	12	0.9182	1.0000	0.9182	0.9349	1.0000	0.9349
3	13	0.4170	0.5270	0.7914	0.4228	0.6410	0.6596
3	14	0.8201	0.7810	1.0501	0.8378	0.7954	1.0533
3	15	0.7539	0.7404	1.0183	0.7637	0.7620	1.0021
3	16	0.8067	0.8333	0.9681	0.8695	0.9144	0.9509
4	1	0.7379	0.8900	0.8291	0.7907	0.9778	0.8087
4	2	0.8637	0.8670	0.9963	0.9653	1.0000	0.9653
4	3	0.6435	0.6404	1.0049	0.7742	0.8822	0.8776
4	4	0.6124	0.6319	0.9691	0.6432	0.6562	0.9801
4	5	0.8060	1.0000	0.8060	0.8389	1.0000	0.8389
4	6	0.8006	0.7773	1.0300	0.8367	0.8686	0.9632
4	7	0.8238	0.8978	0.9176	0.8253	0.8982	0.9188

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Table A3 (continued).

Period	HHS	CRS VCE	CRS NVCE	CRS CDUE	VRS VCE	VRS NVCE	VRS CDUE
4	8	0.6588	0.7263	0.9071	0.7396	1.0000	0.7396
4	9	0.7308	0.7895	0.9257	0.8964	1.0000	0.8964
4	10	0.7950	0.7757	1.0249	0.7979	0.7807	1.0221
4	11	0.9155	1.0000	0.9155	0.9253	1.0000	0.9253
4	12	0.9369	0.9676	0.9683	0.9385	0.9985	0.9399
4	13	0.5925	0.6758	0.8767	0.6035	0.6874	0.8780
4	14	0.7887	0.7903	0.9979	0.8091	0.7932	1.0201
4	15	0.6586	1.0000	0.6586	0.6658	1.0000	0.6658
4	16	0.7623	0.8748	0.8714	0.8096	0.9151	0.8847
5	1	0.5651	0.6668	0.8474	0.6813	0.8327	0.8182
5	2	0.7201	0.7749	0.9293	0.8396	0.9028	0.9300
5	3	0.5603	0.5208	1.0759	1.0000	1.0000	1.0000
5	4	0.6705	0.6932	0.9672	0.8023	1.0000	0.8023
5	5	0.6861	0.8564	0.8012	0.8009	0.9694	0.8261
5	6	0.5836	0.6427	0.9081	0.7286	1.0000	0.7286
5	7	0.5739	0.7294	0.7868	0.6146	0.7882	0.7798
5	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	9	0.6635	0.6919	0.9590	1.0000	1.0000	1.0000
5	10	1.0000	0.8860	1.1287	1.0000	0.9305	1.0747
5	11	0.8855	1.0000	0.8855	0.8881	1.0000	0.8881
5	12	0.8175	0.8029	1.0183	0.9948	0.9555	1.0411
5	13	0.7723	0.8656	0.8923	0.8400	1.0000	0.8400
5	14	0.6944	0.7648	0.9078	0.7859	0.8605	0.9133
5	15	0.6188	0.6776	0.9131	0.6617	0.7256	0.9119
5	16	0.7057	0.8399	0.8402	0.7948	0.9887	0.8038
min	1	0.2558	0.2649	0.6586	0.2611	0.2873	0.6596
mean		0.7471	0.7768	0.9671	0.8190	0.8655	0.9493
max	16	1.0000	1.0000	1.3237	1.0000	1.0000	1.3349